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Drift turbulence & structure generation

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***Drift* turbulence**

&

structure *generation*

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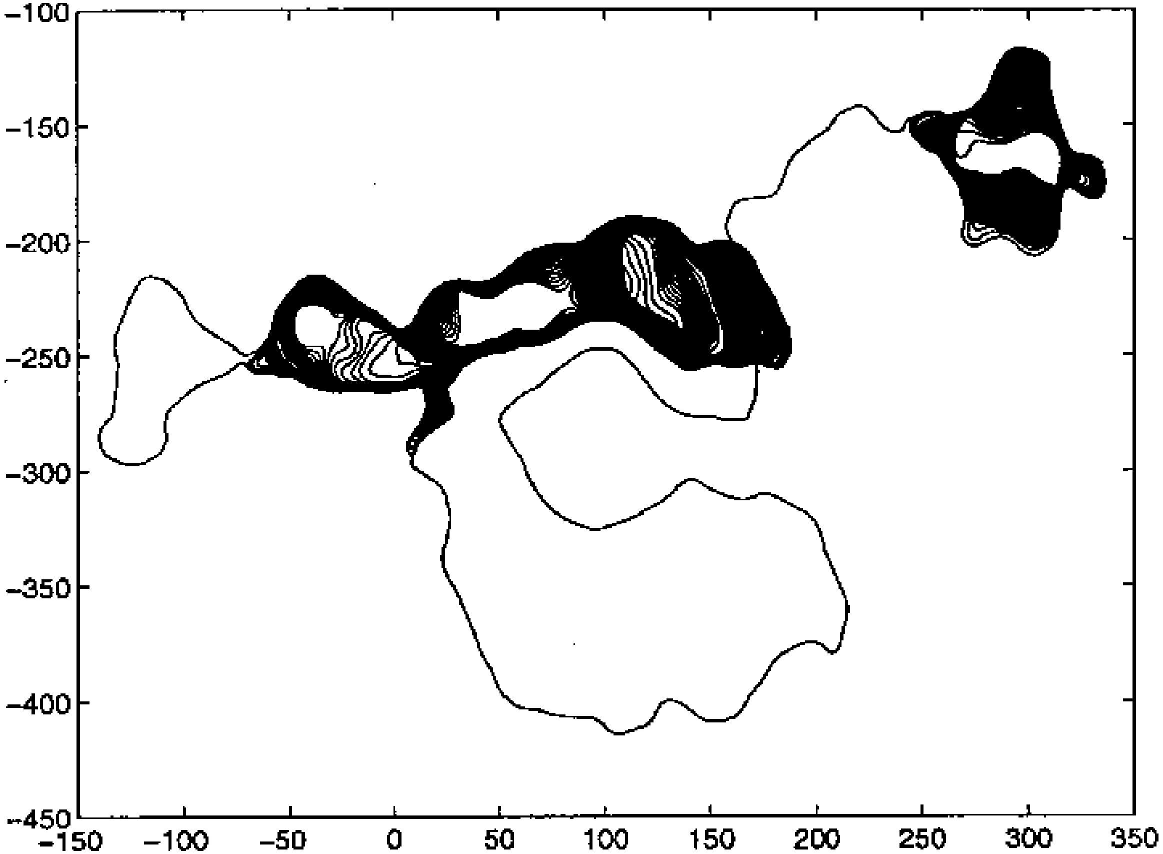
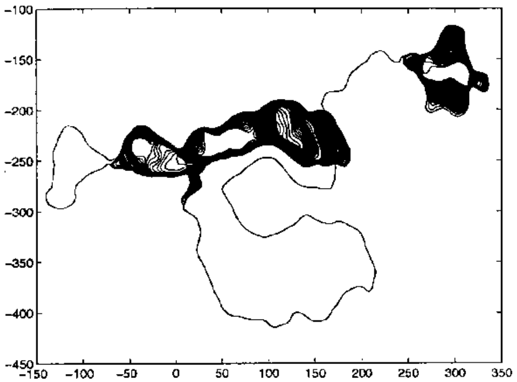
Typical trajectory in the plane perpendicular to the magnetic field generated by the ExB stochastic drift in turbulent plasmas



Random sequence of <u>trapping</u> events and long jumps

(completely different of the trajectories in Gaussian stochastic processes)



We have developed in the last decade semi-analytical methods, which are able to determine the statistics of such trajectories

Trapping *completely changes the statistics of trajectories:*

• memory effects

(long tailes of the correlation of the Lagrangian velocity)

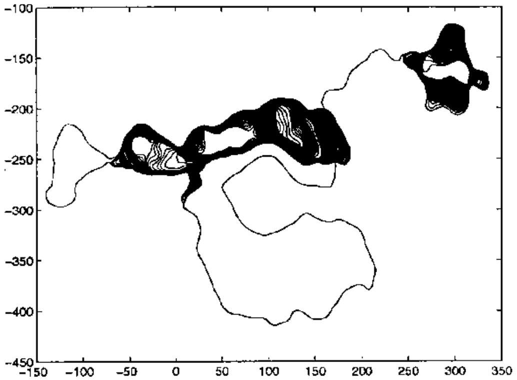
• anomalous transport regimes

(diffusion coefficients with different dependence on the parameters, even opposite)

• non-Gaussian distribution of displacements

• high degree of coherence

• trajectory structures



? *What is the influence of these trajectories on the evolution of the turbulence ?*

We show here that trapping has very strong effects on turbulence and that a *complex evolution appears beyond the quasi-linear limit*.

These results determine a new perspective on two important paradigms on the nonlinear evolution of drift turbulence:

- **inverse cascade;**
- **zonal flows.**

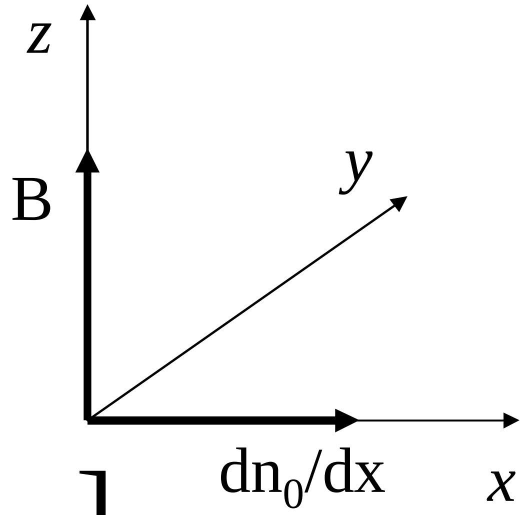
Content

Test modes in turbulent plasma

[The frequency and the growth rates of the test modes are determined as function of the characteristics of the background turbulence]

1. The problem
2. Why the ExB drift leads to non-standard statistics of trajectories
3. Effects of the background turbulence through the mode propagator
   - quasi-linear (no trapping)
   - weak trapping
   - strong trapping
5. Effects of the background turbulence through the fluctuations of the diamagnetic velocity
6. Conclusions

1) The problem



• ***Drift instability*** in cartezian slab geometry, with constant magnetic field and density gradient in collisionless plasmas.

-The limit of zero Larmor radius of the ions:

- stable drift waves $\omega = k_y V_*, \quad \gamma = 0$
- potentials that move with the diamagnetic velocity

$$\varphi(\vec{x}, z, t) = \varphi_0(\vec{x} - V_* t, z), \quad \left[\ \varphi_0(\vec{x}, z) \text{ is the initial condition} \right]$$

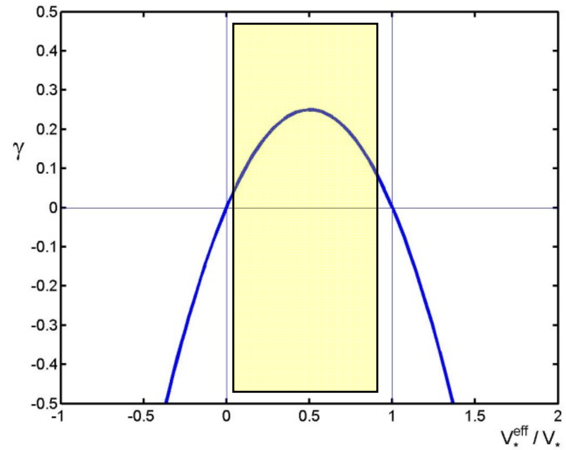
- The instability is produced by the combined effect of the non-adiabatic response of the electrons and of the finite Larmor radius (FLR) of the ions

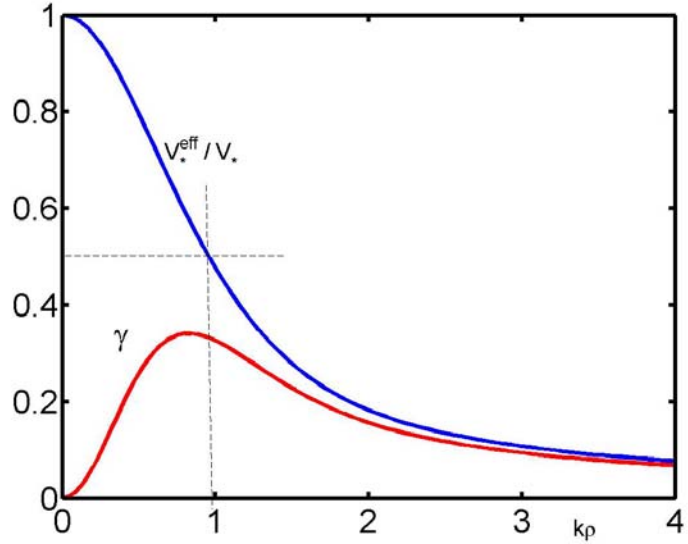
$$\omega = k_y V_*^{eff}$$

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^{\,2}(V_* - V_*^{eff})V_*^{eff}}{2 - \Gamma_0}$$

$$V_*^{eff} = V_* \frac{\Gamma_0}{2 - \Gamma_0}$$

$$\Gamma_0 = \Gamma_0\left(\frac{k_\perp^{\,2} \rho_{Li}^{\,2}}{2}\right), \qquad V_* = \frac{T}{e n_0 B} \frac{d n_0}{dx}$$



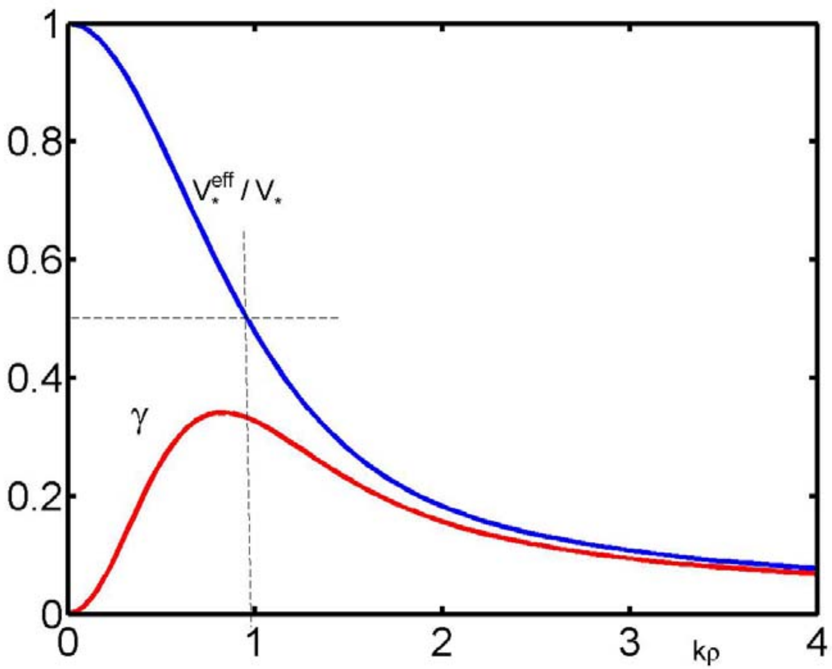


$$\gamma_{\max} = \frac{\sqrt{\pi} k_y^{\,2}}{|k_z| v_{Te} (2 - \Gamma_0)} \frac{1}{4} \qquad \left[V_*^{eff} = V_*/2, \quad k\rho \cong 1 \right]$$

• We consider a turbulent plasma with given statistical characteristics of the stochastic potential and study linear test modes

• ***AIM: to find the effect of trajectory trapping on test modes***

The growth rates and frequencies of the test modes are determined as function of the statistical characteristics of the background turbulence, which are estimated from the growth rates



Modes on quiescent plasma:

• k-dependent effective drift velocity, but with an average value for the most un-stable modes $\longrightarrow$ potential that drifts with $V_d \approx V_*^{eff} / 2$ and changes due to k-dependence and $\gamma, \quad V_*^{eff}$

The existence of trapping is determined by the amplitude of the ExB drift: it appears when V is larger than the velocity of the potential V_d

The <u>trapping parameter</u> for drift type turbulence is $\overline{V}_d = V_d / V$

The ordering of the characteristic times is

QL (no trapping): $\tau_{II}^{e} << \tau_{*} < \tau_{fl} < \tau_{c} < \tau_{II}^{i}$

NL (trapping) : $\tau_{II}^{e} << \tau_{fl} < \tau_{*} < \tau_{c} << \tau_{II}^{i}$

$$\tau_{*} = \lambda_c / V_d \cong 4\pi / k_2 V_*, \qquad \tau_{II}^{e} = 2\pi / k_z v_e, \quad \tau_{fl} = 2\pi / k_2 V$$

Thus

- ***nonlinear effects appear in the ion trajectories if*** $V_d < V$
- ***electron trajectories have quasiliniar statistics***

Test modes on ***turbulent state of the plasma*** with a potential that has known EC $\phi(\vec{x}, z, t)$

• The system is perturbed with a small potential $\delta\phi(\vec{x}, z, t) = \delta\phi_{k\omega} \exp\left(i\vec{k}\cdot\vec{x} + ik_z z - i\omega t\right) << \phi,$

which is small enough such that the trajectories are not much perturbed.

• the Vlasov (drift kinetic) equation for the response at the perturbation is linearized

$$\partial_t g^\alpha - \frac{\nabla\phi\times\vec{e}_z}{B_0}\cdot\nabla g^\alpha + v_z\partial_z g^\alpha = \frac{\nabla\delta\phi\times\vec{e}_z}{B_0}\cdot\nabla\left(n_0 F_M^\alpha + h^\alpha\right) + \frac{e_\alpha n_0 F_M^\alpha}{T_\alpha}\partial_t\delta\phi$$

• The solution is obtained by the characteristic method as integral along the trajectories determined by the unperturbed potential

$$g^\alpha(\vec{x}, z, \vec{v}, t) = \frac{e_\alpha n_0 F_M^\alpha}{T_\alpha}\int_{-\infty}^{t} d\tau\left[\partial_t - \left(V_* + \tilde{V}_*\right)\cdot\nabla\right]\delta\phi\left(\vec{x}^\alpha(\tau), z^\alpha(\tau), \tau\right)$$

Fluctuating of the diamagnetic velocity
determined by the background turbulence

$$\tilde{\vec{V}}_* = \frac{T_\alpha}{e_\alpha n_0 B_0}\vec{e}_z \times \nabla\overline{h}^\alpha$$

$$\frac{d\vec{x}^{\alpha}}{d\tau} = -\frac{\nabla\phi(\vec{x}-\vec{V}_d t)\times\vec{e}_z}{B_0}, \quad \frac{dz^{\alpha}}{d\tau} = v_z^{\alpha}$$

$$\vec{x}^{\alpha}\left(t;\vec{x},z\right) = \vec{x}, \quad z^{\alpha}\left(t;\vec{x},z\right) = z$$

The background turbulence produces the ***stochastic ExB drift in the trajectories*** *(characteristics)*

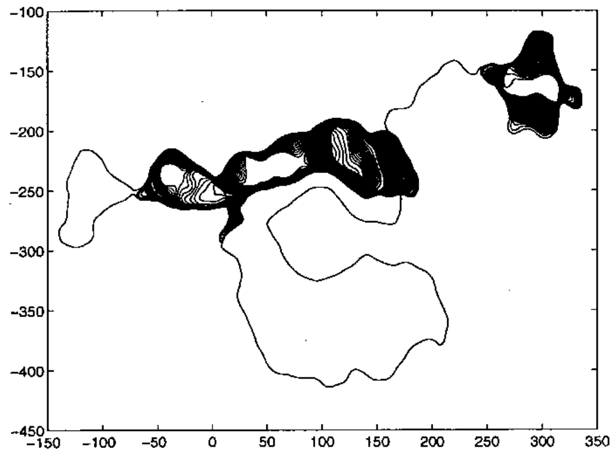
- the characteristic times ordering is used for simplifications: the parallel motion of the ions and the ExB drift for electrons are neglected

The average propagator and the average effect of the diamagnetic velocity fluctuations are determines using trajectory statistics

$$\Pi = \int_{-\infty}^{t} d\tau \underbrace{\left\langle \exp\left(i\vec{k}\cdot\left(\vec{x}(\tau)-\vec{x}\right)\right)\right\rangle}_{\text{Average over the trajectories}} \exp(i\omega(t-\tau))$$

2) *Why the ExB drift leads to non-standard statistics of trajectories?*

There are two strong constraints for the statistical methods:



(A) The invariance of the potential for the static case

(Hamiltonian equation of motion)

$$\frac{d\phi(\vec{x}(t),t)}{dt} = \cancel{v_i(\vec{x}(t),t)\frac{\partial\phi(\vec{x}(t),t)}{\partial x_i}} + \frac{\partial\phi(\vec{x}(t),t)}{\partial t} = \frac{\partial\phi(\vec{x}(t),t)}{\partial t}$$

- ***static potential***: invariance of the potential and ***permanent trapping*** on the contour lines;
- ***weakly perturbed potential*** : approx. invariance of the potential and ***temporary trapping*** (time variation, collisions, average velocity, parallel motion, etc.)
- *strong perturbations*: no trapping.

(B) The *statistical invariance* of the Lagrangian velocity due to $\nabla\cdot\vec{v}(\vec{x},t)=0$

$$P\left[\vec{v}(\vec{x}(t),t)\right] = P\left[\vec{v}(\vec{x}(0),0)\right] = P\left[\vec{v}(\vec{x},t)\right]$$

The existing analytical methods
(Corrsin Approximation, DIA, functional integration) are not compatible with the conditions (A) and (B). They do not describe trapping and lead to diffusive transport in in the static potential

❑ ***The decorrelation trajectory method (DTM)*** *1998*

(M.Vlad, F. Spineanu, J. H. Misguich, R. Balescu, “Diffusion with intrinsic trapping in 2-d incompressible stochastic velocity fields”, **Physical Review E 58** (1998) 7359)

- DTM is based on a set of simple (deterministic) trajectories determined from the Eulerian correlation EC of the stochastic potential.
- main consequences of condition (A) are fulfilled, condition (B) is not;
- ***Main physical result***: ***trapping produces memory effects*** ***(long-time correlation of the Lagrangian velocity)*** and subdiffusive transport for static potential.

❑ *The nested subensemble method* ***(NSM)*** *2004*

(M. Vlad, F. Spineanu, “Trajectory structures and transport”, **Physical Review E 70** (2004) 056304(14))

- NSM is a systematic expansion based on dividing the space of realizations of the stochastic potential in nested subensembles. NSM yields detailed statistical information.
- all consequences of (A) are fulfilled, condition (B) is improved (but not enough in order 2)
- ***Main physical result***: ***trapping determines coherence*** in the stochastic motion and ***quasi-coherent trajectory structures***.

❑*The velocity on structure method* ***(VS)*** *2009*

- ***Both conditions (A) and (B) are respected***;
- Trajectory structures are confirmed and better described
- ***Main physical result***: ***the distribution of displacements until decorrelation is determined*** (the 1-step pdf).

It is strongly non-Gaussian in the trapping regime **(K>>1)**.

3) Effects of the background turbulence through the propagator

Turbulent plasma with small amplitude (quasilinear turbulence $V < V_d$ **)**

- the trajectories are Gaussian
- the propagator is

$$\Pi \cong i \frac{n_f}{\omega + i k_i^2 D_i}$$

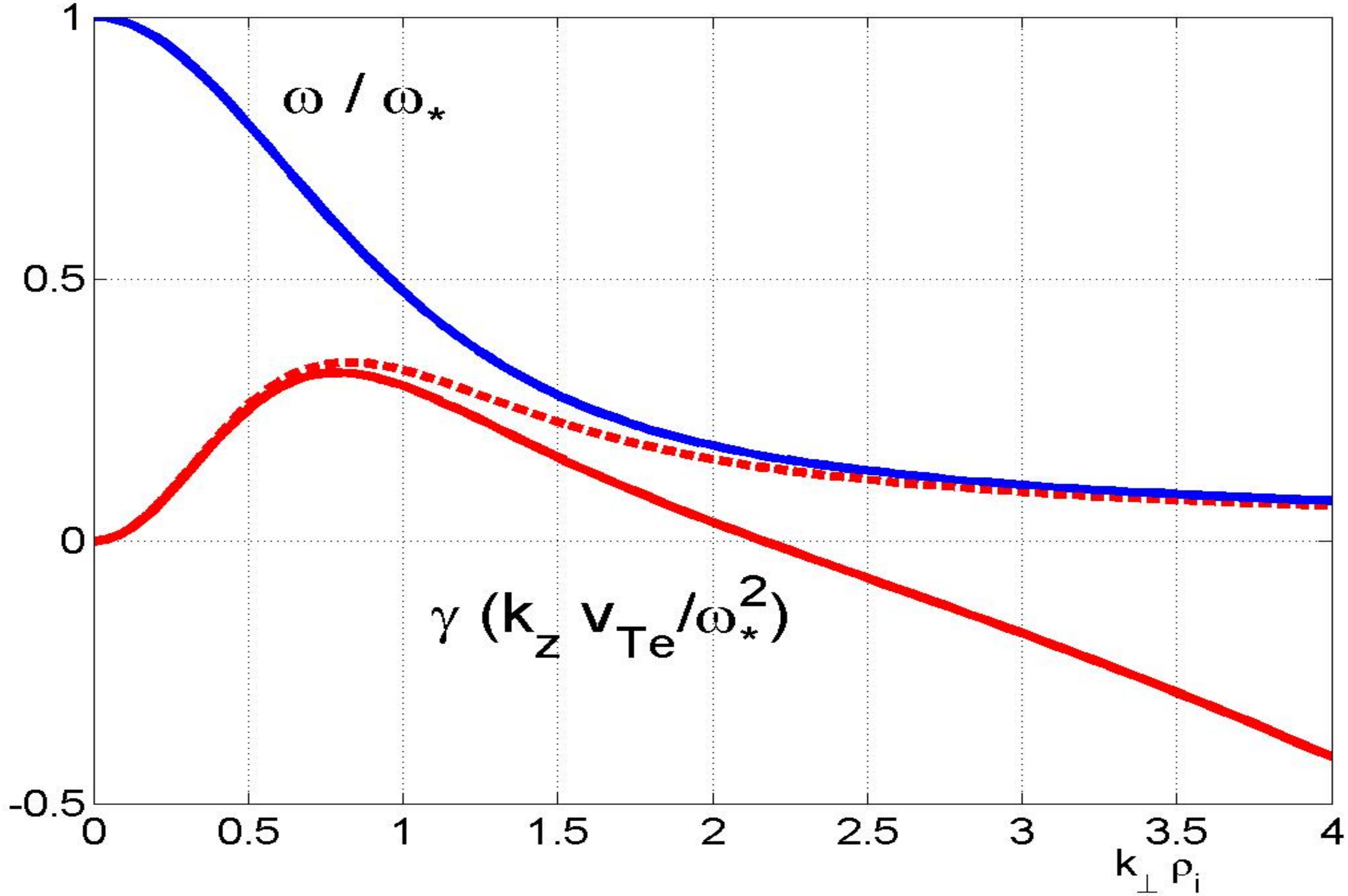
$$\omega = k_y V_*^{eff}$$

$$V_*^{eff} = V_* \frac{\Gamma_0}{2 - \Gamma_0}$$

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^2 (V_* - V_*^{eff}) V_*^{eff}}{2 - \Gamma_0} - \frac{2 k_i^2 D_i}{2 - \Gamma_0}$$

Damping due to ion diffusion

Damping of the test modes at large k (resonance broadening - Dupree)

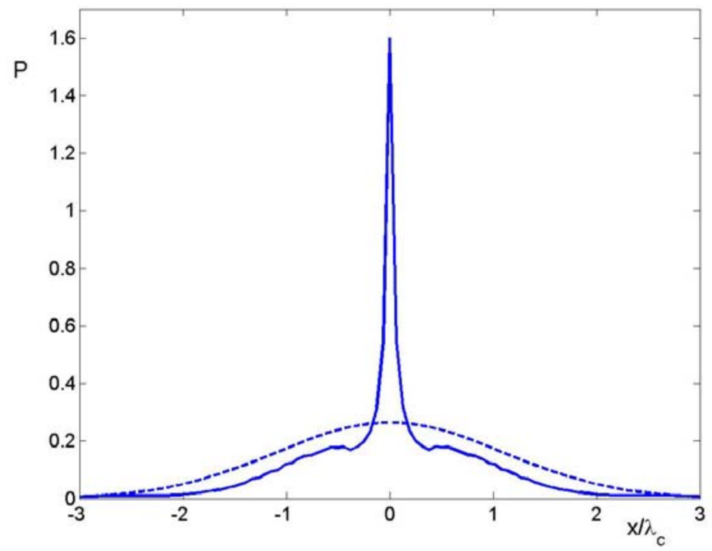


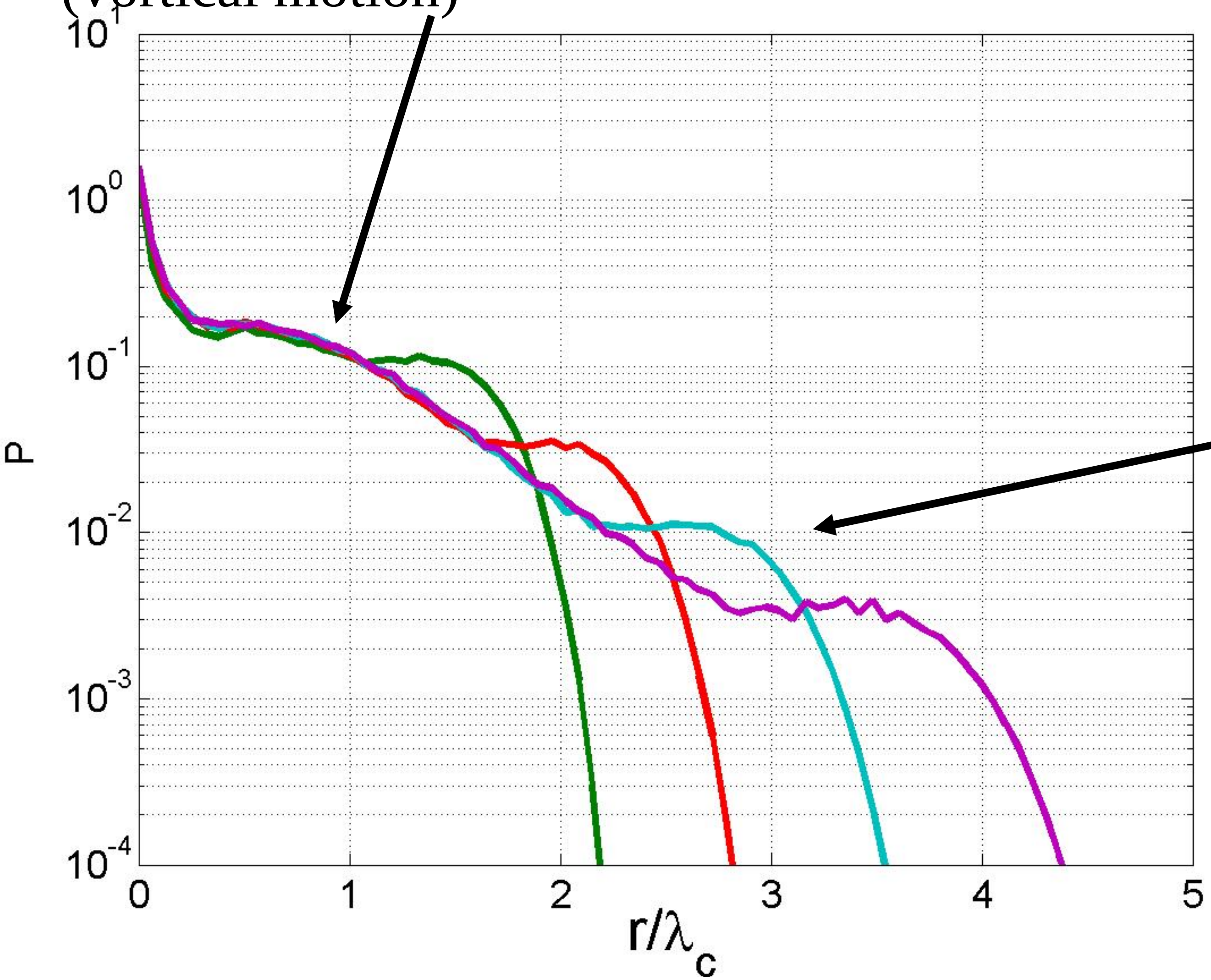
The amplitude of the potential continues to grow and the correlation length is of the order of the Larmor radius

Weak trapping $\left[V \geq V_d, \quad n_{tr} << n_f\right]$

The probability of displacements for static potential

Time invariant part formed by trapped trajectories (vortical motion)





Moving maximum of trajectories that are not in a structure at that time (radial motion). Decreasing number of free particles.

- The distribution of displacements is approximated by two Gaussian functions

$$P(x,t) \cong G_{tr}(x) + G_f(x,t), \quad \left\langle x_i^{\,2}(t) \right\rangle = S_i^{\,2} + 2D_i t$$

(one for the trapped particles (with fixed width) and one of diffusive type)

S_i is the size of the structures of trapped trajectories

- the propagator is

$$\Pi \cong i \exp(-k_i^{\,2} S_i) \frac{1}{\omega + i k_i^{\,2} D_i}$$

$$\omega = k_y V_*^{eff}, \quad \gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^{\,2}(V_* - V_*^{eff}) V_*^{eff}}{2 - \Gamma_0 \mathfrak{I}} - k_i^{\,2} D_i \frac{2}{2 - \Gamma_0 \mathfrak{I}}$$

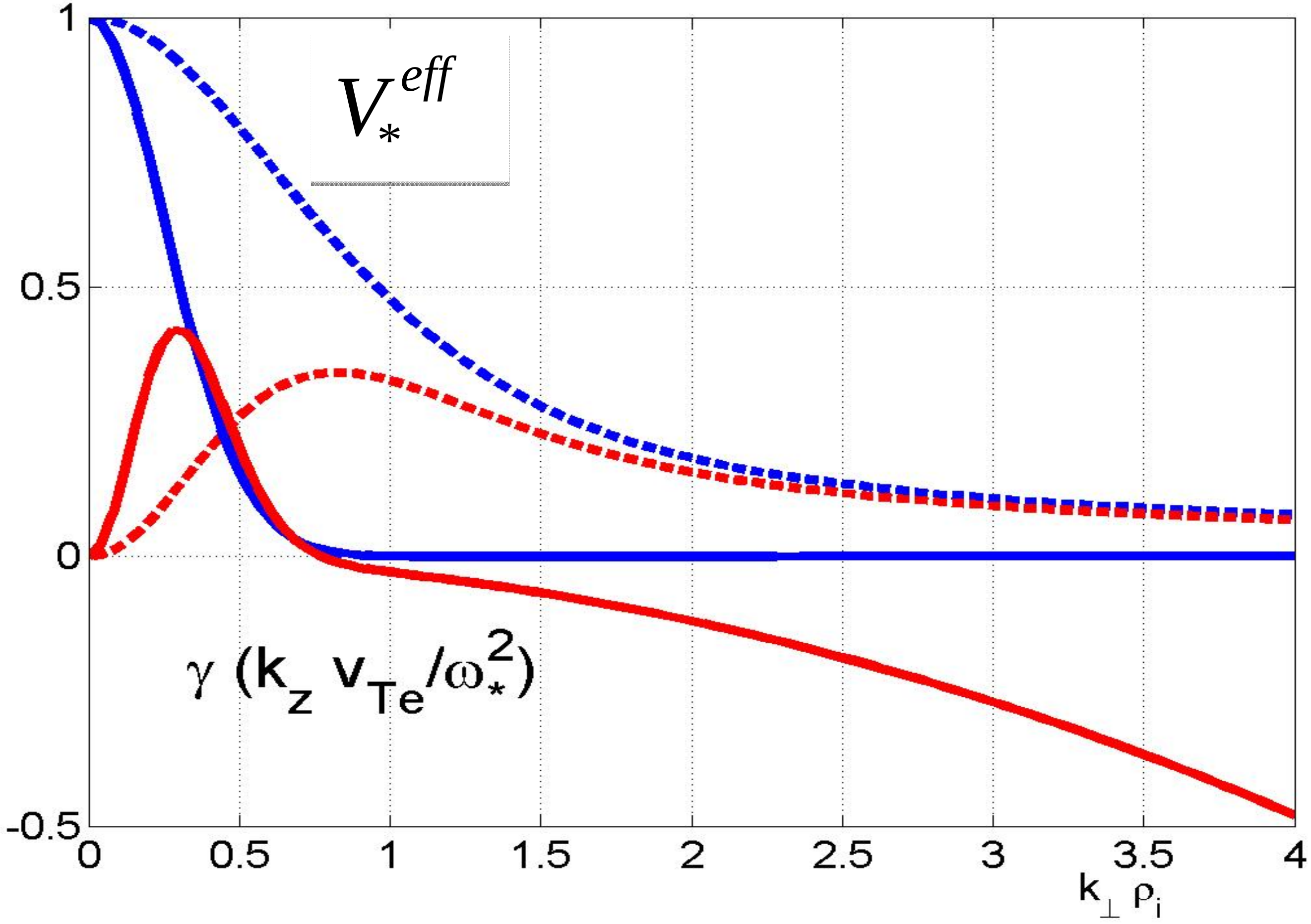
$$V_*^{eff} = V_* \frac{\Gamma_0 \mathfrak{I}}{2 - \Gamma_0 \mathfrak{I}}$$

Decrease of the effective diamagnetic velocity

$$\mathfrak{I} = \exp\left(-\frac{1}{2} k_i^{\,2} S_i^{\,2} (\overline{V_*}) \right)$$

Effect of ion trajectory structures

Displacement of the unstable range toward small k due to trajectory structures
while the maximum growth rate is not changed

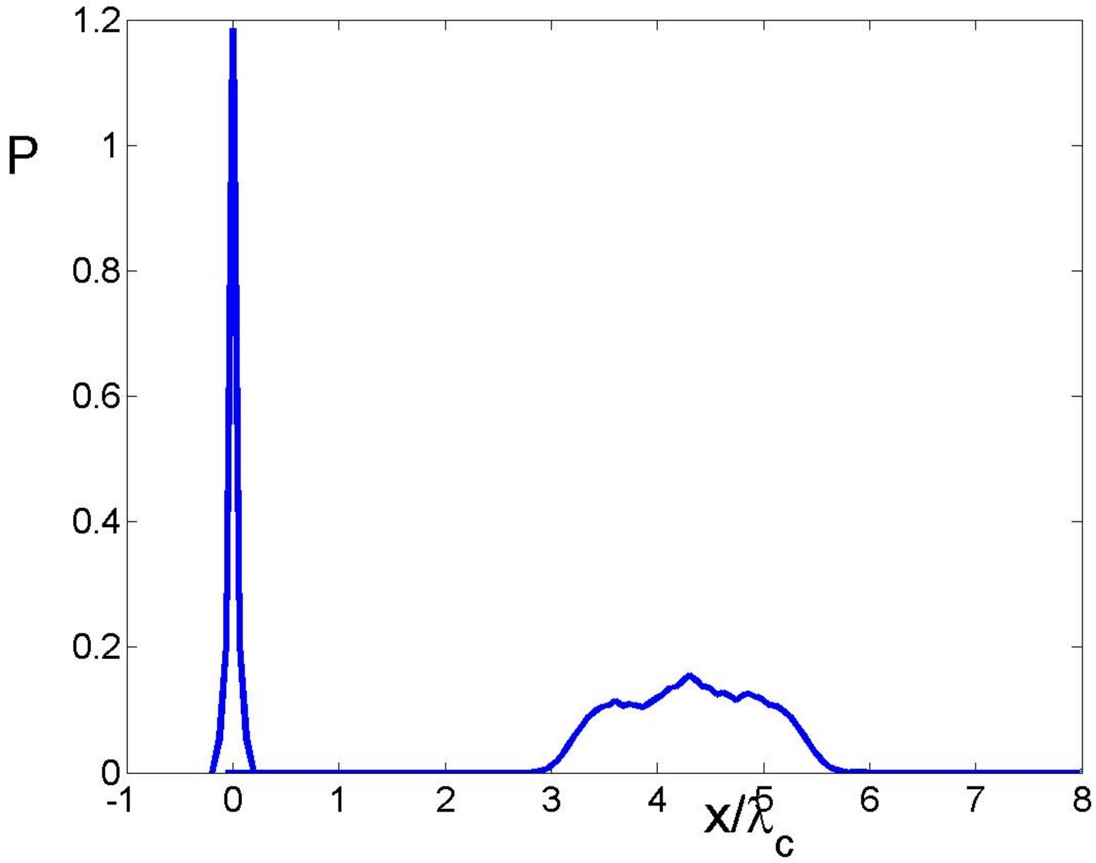


Both the amplitude of the turbulence and correlation length increase
The inverse cascade appears as the shift of the unstable wave numbers

Strong trapping $\left[V > V_d, \quad n_{tr} \approx n_f\right]$

The moving stochastic potential determines ion flows

Trapped particles move with V_d and free particles move in opposite direction



$$\left\langle x(t)\right\rangle = 0$$

$$\left\langle x(t)\right\rangle = n_{tr}V_* t + n_f V' t = 0$$

$$V' = -V_* \frac{n_{tr}}{n_f}$$

$\left\langle x^2(t)\right\rangle$ is large and has ballistical time-dependence (because of this separation)

0

$\leftarrow V_d$ $\quad\quad$ $V' \rightarrow$

- The distribution of displacements is approximated by two Gaussian functions

$$P(x,t) \cong n_{tr} G(x, y - V_d t) + n_f G_f(x, y - V' t),$$

- the propagator is $$\Pi \cong i \exp(-k_i^2 S_i)\left[\frac{n_{tr}}{\omega + k_y V_*} + \frac{n_f}{\omega + k_y V' + i k_i^2 D_i}\right]$$

$$\omega = k_y V_*^{eff}$$

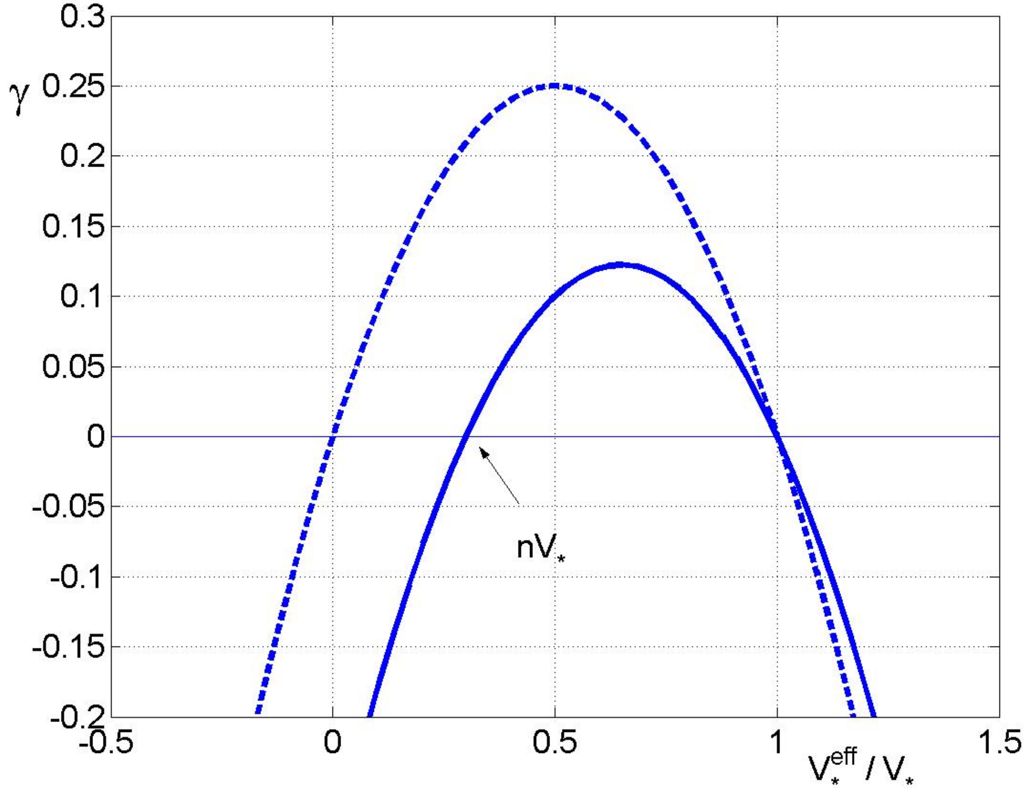
$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^2 (V_* - V_*^{eff})(V_*^{eff} \boxed{- nV_*})}{2 - \Gamma_0 \Im} - k_i^2 D_i \frac{2 - \Gamma_0 \Im n_{tr}}{2 - \Gamma_0 \Im}$$

The ion flows modify both the growth rate and the effective diamagnetic velocity

$$V_*^{eff} = V_* \frac{\Gamma_0 \Im \boxed{(1-n) + 2n}}{2 - \Gamma_0 \Im}$$

$$n = n_{tr} / n_f$$

The ion flows induced by the moving potential determines the increase of the effective diamagnetic velocity and the decrease of the growth rate



$$V_*^{eff}\left(\Gamma_0\Im, n\right) = V_*^{eff}\left(\Gamma_0\Im, 0\right) + nV_*$$

For n $\longrightarrow$ 1
the growth rate is negative

The ion (zonal) flows stabilize the drift modes

5) Effects of the background turbulence through the fluctuations of the diamagnetic velocity

• they appear due to the coupling of the non-adiabatic response of the ions to the background potential with the mode through trajectories

• the fluctuations of the diamagnetic velocity determine a new term in the growth rate of the modes:

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y{}^2 (V_* - V_*^{eff})(V_*^{eff} - nV_*)}{2-\Gamma_0 \Im} - k_i{}^2 D_i \frac{2-\Gamma_0 \Im n_{tr}}{2-\Gamma_0 \Im} + \boxed{k_i k_j R_{ij} \frac{2V_*}{[2+n(2-\Gamma_0\Im)](2-\Gamma_0\Im)}}$$

$$R_{ij} = \int_\tau^t d\theta' \int_{-\infty}^{\tau-\theta'} d\theta M_{ij}(|\theta|), \quad M_{ij}(|\theta|) = \left\langle v_i(0,0) \frac{\partial}{\partial y} v_j(\vec{x}(\theta), \theta) \right\rangle$$

• it is much more complicated and it was not possible to obtain a simple evaluation;

•The component R_{11} corresponds to modes with $k_2 = 0$ (zonal flow modes).

• quasilinear turbulence: all the components of the tensor are zero

• for weak trapping $\left\lfloor V \geq V_d, \quad n_{tr} << n_f \right\rfloor$ the non-diagonal component appear but only for non-isotropic turbulence; $R_{11} = 0$

• for strong trapping $\left\lfloor V > V_d, \quad n_{tr} \approx n_f \right\rfloor$ the anizotropy induced by particle flows with the moving potential leads to $R_{11} > 0$

Thus, zonal flow modes (with $k_2 = 0, \quad \omega = 0$) ***are generated due to ion flows. As n increases to 1, the asymmetry induced by the ion flows disappears and*** $R_{11} \rightarrow 0$

The process does not appear as a competition between drift modes and zonal flow modes, but as the effect of the ion flows induced by the moving potential.

Note

- These are simplified estimations that do not take into account the strongly non-isotropic diffusion coefficients and the non-isotropy of the turbulence.

Conclusion

- ***Trapping determines a strong and complex influence on test modes in turbulent plasmas producing large scale potential cells and zonal flows.***

The nonlinear growth rate of the test modes on drift turbulence is

$$\omega = k_y V_*^{eff}$$

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y{}^2 (V_* - V_*^{eff})(V_*^{eff} - nV_*)}{2 - \Gamma_0 \Im} - k_i{}^2 D_i \frac{2 - \Gamma_0 \Im n_{tr}}{2 - \Gamma_0 \Im} + k_i k_j R_{ij} V_*^{eff}$$

$$V_*^{eff} = V_* \frac{\Gamma_0 \Im (1-n) + 2n}{2 - \Gamma_0 \Im}$$

$$\Im = \exp\left(-\frac{1}{2} k_i{}^2 S_i{}^2 (V) \right)$$

$$R_{ij} = \int_\tau^t d\theta' \int_{-\infty}^{\tau-\theta'} d\theta \, M_{ij}(|\theta|), \quad M_{ij}(|\theta|) = \left\langle v_i(0,0) \frac{\partial}{\partial y} v_j(\vec{x}(\theta), \theta) \right\rangle$$

Trajectory trapping determines several types of effects; some of them acts simultaneously, others at different stages of the evolution.

(1) Dupree turbulent damping (V>0)

$$\omega = k_y V_*^{eff}$$

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^2 (V_* - V_*^{eff})(V_*^{eff} - n V_*)}{2 - \Gamma_0 \Im} - k_i^2 D_i \frac{2 - \Gamma_0 \Im n_{tr}}{2 - \Gamma_0 \Im} + k_i k_j R_{ij} V_*^{eff}$$

(3) Fluctuations of V_*

$$V_*^{eff} = V_* \frac{\Gamma_0 \Im (1 - n) + 2n}{2 - \Gamma_0 \Im}$$

(3) Ion flows $V >> V_*,\ n_{tr} \approx n_f$

$$\Im = \exp\left(-\frac{1}{2} k_i^2 S_i^2 (V) \right)$$

(2) Trajectory structures

$V > V_*,\ n_{tr} << n_f; \quad \Gamma_0 \to \Gamma_0 \Im$

$$R_{ij} = \int d\theta \int_{-\infty}^{-\theta} d\theta M_{ij}(|\theta|) \quad M_{ij}(|\theta|) = \left\langle v_i(0,0) \frac{\partial}{\partial y} v_j\left(\vec{x}(\theta), \theta\right) \right\rangle$$

❑ ***Trapping determines strong changes in the turbulence that are eliminates later in the evolution. Drift turbulence does not saturate but has a complex oscillatory behaviuor.***