The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization
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2052-36

Summer College on Plasma Physics

10 - 28 August 2009

Introduction to drift waves

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Introduction to drift waves

the experimentalist‘s view of things

I. The drift wave mechanism - overview

II. Linear drift wave dynamics

III. Non-linear drift wave dynamics

IV. Summary

Summer College on Plasma Physics

Drift-wave related talks during the summer college:

~~Cowley~~	~~transport in Tokamaks~~
~~Porkolab~~	~~turbulence in Tokamaks~~
Jenko	gyrokinetic simulation
Hahm	gyrokinetic theory
Hubbard	edge transport barriers
Tynan	turbulence in magnetically confined plasmas
Weiland	turbulent transport

This is a very basic introduction to drift waves.

Drift waves – intro comments

„drift“ refers to the diamagnetic drift in magnetized plasmas with $\nabla n \neq 0$

$$v_{de} = \frac{T_e}{eB_0}\left[\frac{1}{n}\cdot\nabla n\right] = \frac{T_e}{eB_0}\nabla \ln n = \frac{T_e}{eB_0}\cdot 1/L_n$$

density gradient length

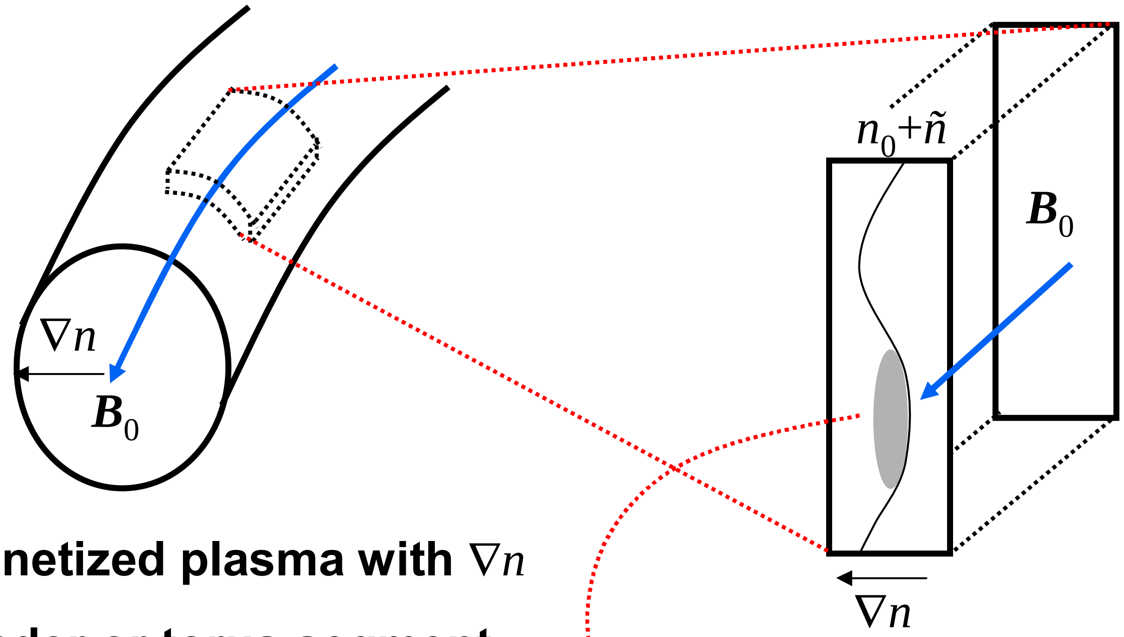
drift waves …

- **are „universal“ instabilites of magnetized plasmas,**
- **are electrostatic in low β plasmas,**
- **lead to fluctuations in n, φ and T,**
- **have a relatively long wavelength $\lambda_\perp > \omega_{ci}$**
- **propagate at v_{de} with frequencies $< \omega_{ci}$ (low frequency)**
- **are candidates for explaning anomalous diffusion $\perp \boldsymbol{B}_o$ with $D \sim \lambda^2\omega$**
- **are the fundamental instability for edge turbulence in fusion devices**

Drift waves – brief history

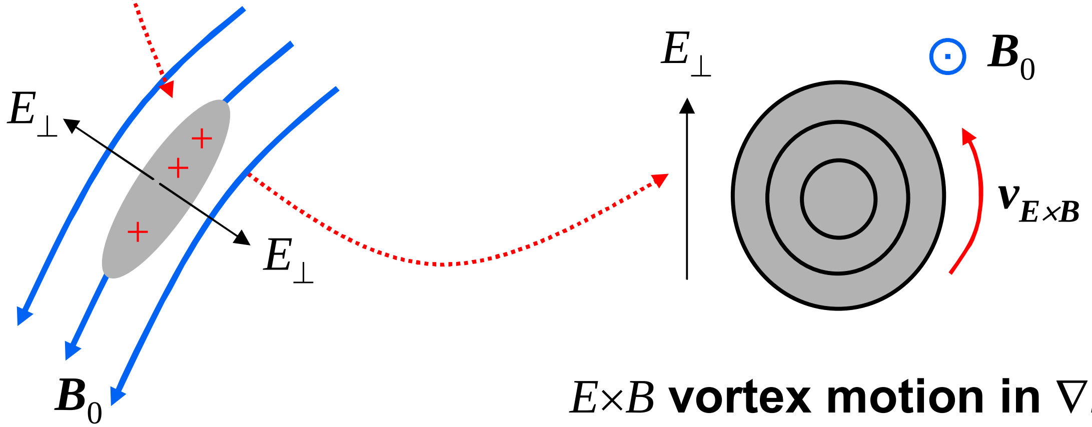
- **Bohm: anomalous diffusion could be due to $E \times B$ fluctuations**
  (1949)
- **low-frequency waves propagating $\perp B_0$ observed in laboratory**
  (1961)
- **first linear theories for low-frequency drift waves**
  (1959-1965)
- **systematic experiment/theory comparison in laboratory**
  (1965-present)
- **linear theory for cylindrical and toroidal geometry**
  (1970-1989)
- **non-linear drift wave theory and drift wave turbulence research**
  (1978-present)
- **importance of edge turbulence for H-mode in tokamaks realized**
  (1982)
- **advanced non-linear drift wave models and computer simulation**
  (1990-present)
- **detailed turbulence measurements and comparison with theory**
  (present)

Drift wave concetual elements



- local slab geometry
- ∇n region
- density perturbation

- magnetized plasma with ∇n
- cylinder or torus segment



- electron motion $\parallel B_0$
- perp. electric field

$E\times B$ vortex motion in ∇n

Drift waves - a pictoral approach

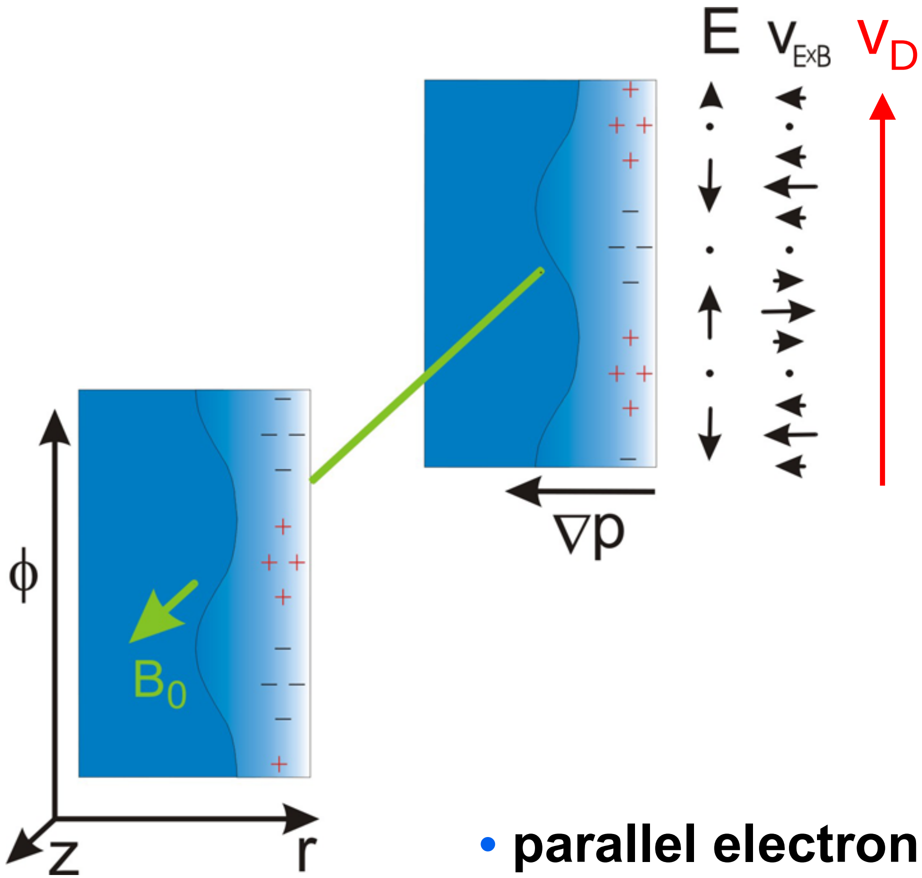
- ∇n **perperdicular to** $\boldsymbol{B}_0$
- **density perturbation** $\tilde{n}$ **in region of strong** ∇n
- **tight coupling between dynamics** $\perp$ **and** $\parallel$ **to** $\boldsymbol{B}_0$
- **positive perturbation** $n_0 + \tilde{n} > n_0 \Rightarrow$ **electrons move away** $\Rightarrow \tilde{\phi} > 0$
  positive perturbation $n_0 + \tilde{n} < n_0 \Rightarrow$ **electrons move towards** $\Rightarrow \tilde{\phi} < 0$
- $\tilde{\boldsymbol{E}}_\perp = -\nabla_\perp \tilde{\phi}$ **leads to ion polarisation current** $\boldsymbol{j}_p = \frac{\rho}{B^2} \frac{d\tilde{\boldsymbol{E}}_\perp}{dt}$
- $\tilde{\boldsymbol{E}}_\perp \times \boldsymbol{B}_0$ **drift causes advection** $\perp \boldsymbol{B}_o$ **in direction of perturbation**
- **linear stable for adiabatic electron response – drift wave**

$$\angle(\tilde{n}, \tilde{\phi}) = 0 \qquad \tilde{n} = n_0 \exp\left(\frac{e\tilde{\phi}}{k_b Te}\right)$$

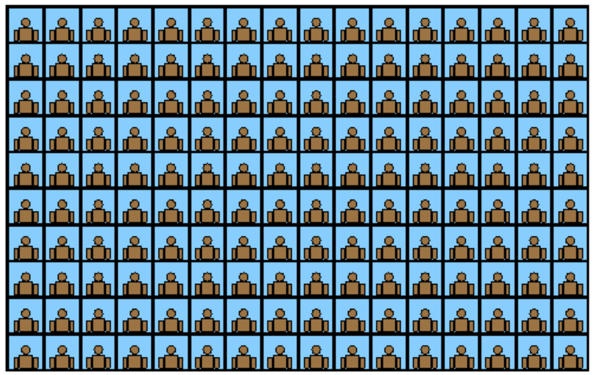
- **linear unstable for non-adiabatic electrons – drift instability**
  $\angle(\tilde{n}, \tilde{\phi}) > 0$ **advection amplifies perturbation**
- **drift waves are not associated with single particle motion**

Drift wave principles

- radial density gradient ∇p and $E \times B$ motion …



- propagation $\perp B_0$
- spatial scale $k_\perp \rho_s \sim 1$



Mexican wave* like

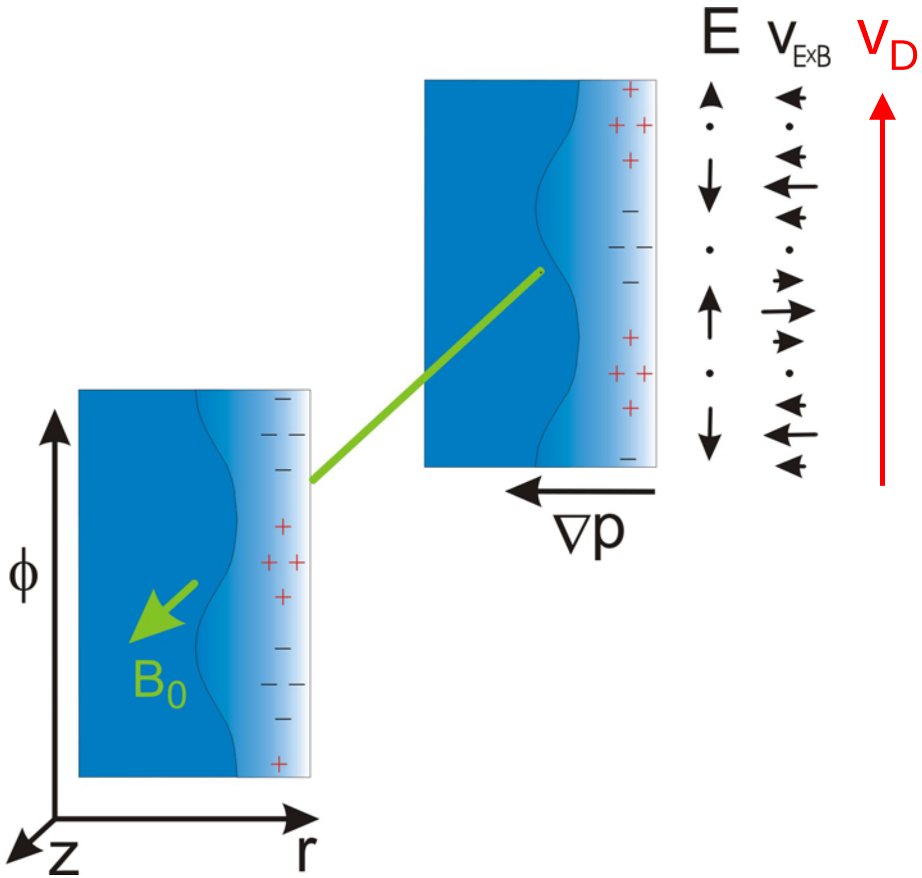
(not single particle motion)

- parallel electron response ⇒ local electric field E_ϕ
- local $E \times B$ drift ⇒ propagation of perturbation

*Farkas et al. Nature **419** 131-132 (2002)*

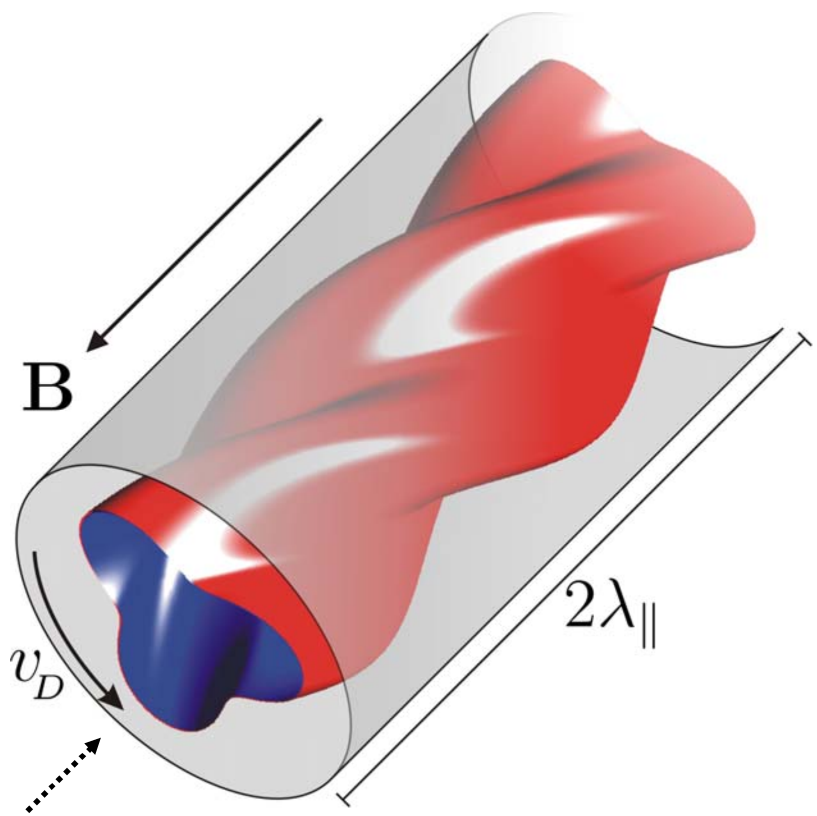
Drift mode formation

- **mode structure owing to periodic boundary conditions**



in cylindrical geometry:

azimuthal eigenmode structure

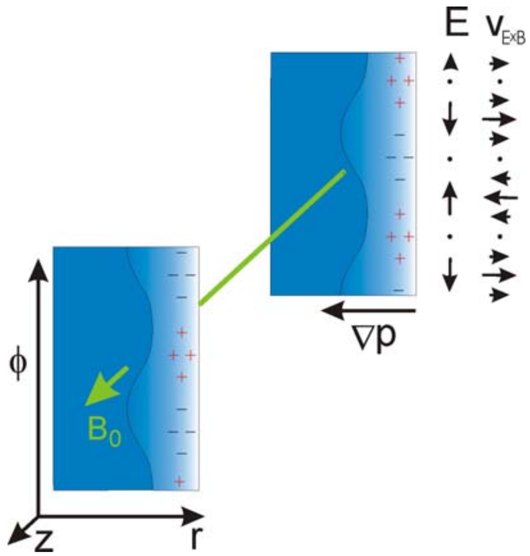


m=3 mode

note $k_{\|} \neq 0$

$k_{\|}/k_{\perp} << 1$

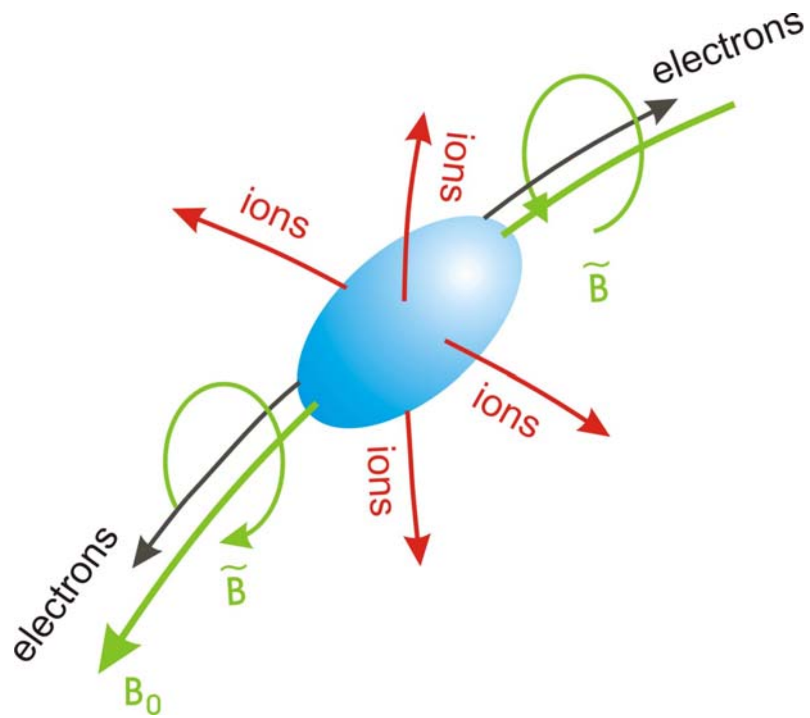
Drift wave complex currents



- electron response current balanced by ion polarization drift current

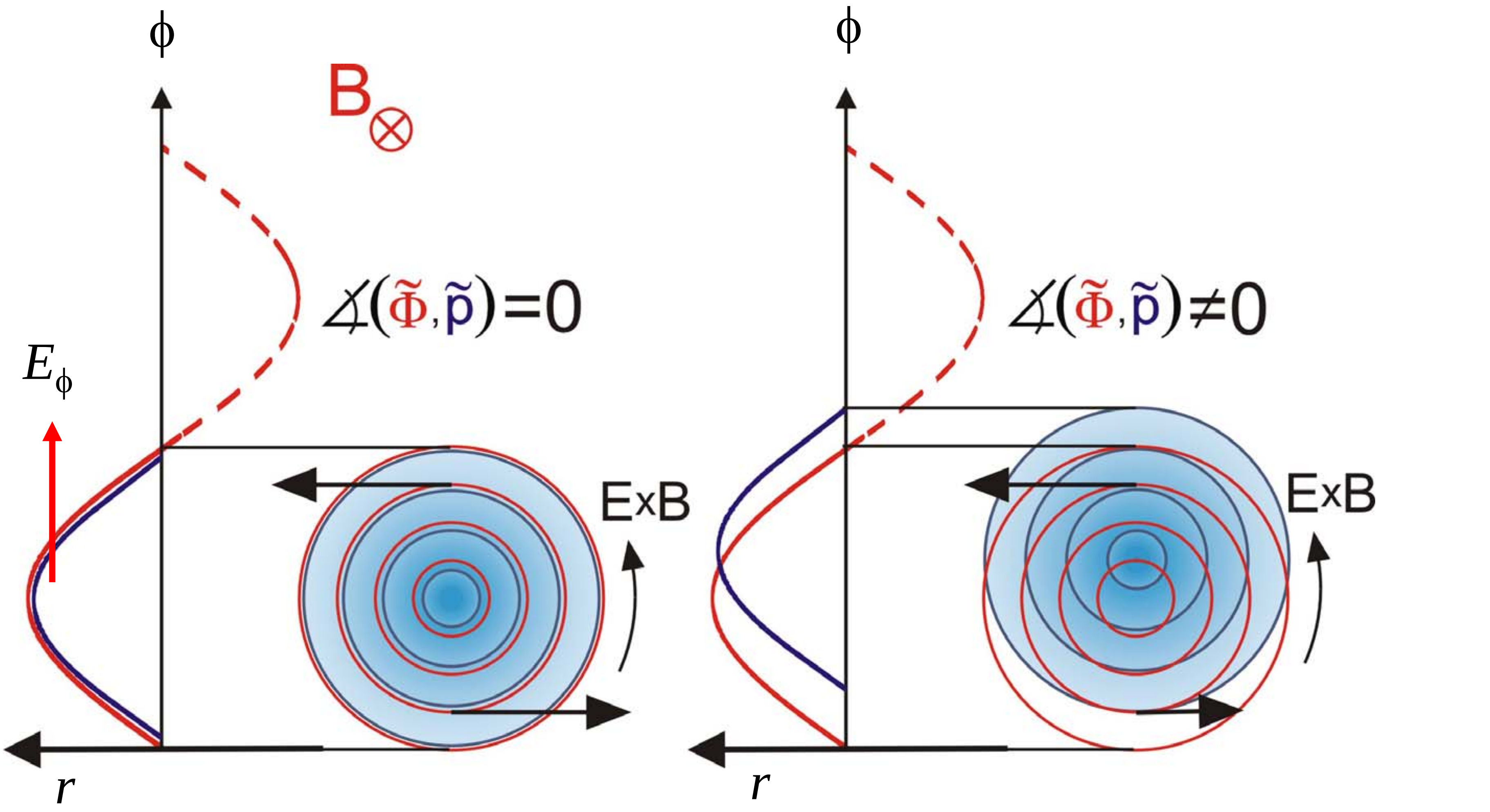
$$\nabla \cdot J = \nabla \cdot J_{\perp} + \nabla \cdot J_{\parallel} = 0$$

- Ampère's law $\mu_0 J_{\parallel} = -\nabla_{\perp}^2 \Psi$



- tight coupling of ⊥ and || dynamics
- electron current || magnetic field
- ion current ⊥ magnetic field
- electron current ⇒ magnetic flutter

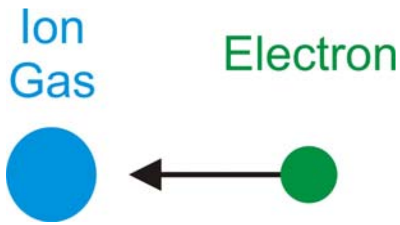
Drift wave E × B vortex



adiabatic electron response ⇒ potential and pressure in phase

non-adiabatic electron response ⇒ potential and pressure out of phase

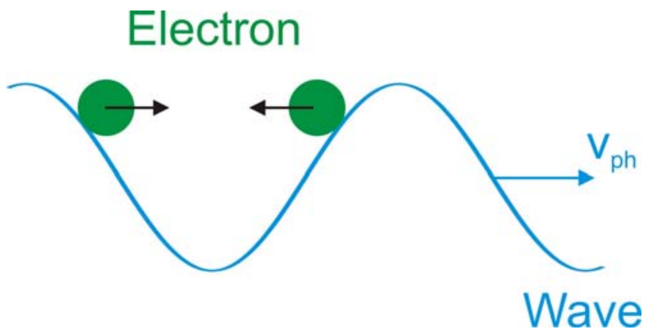
Non-adiabatic electron response



friction — **collisional drift waves**

Horton, Rev. Mod. Phys. 71, 1999

$\Rightarrow \quad \nu_{ei}\ ,\ \nu_{en}$



kinetic effects — **Landau damping**

Jenko et al., Phys. Plasmas 6, 1999

$\Rightarrow \quad v_{ph} \sim v_{e,th}$

induction — **drift Alfvén waves**

Scott, PPCF 39,1997

$\Rightarrow \quad v_A < v_{e,th} \quad \Leftrightarrow \quad \beta > m_e/M_i$

Nonlinear model – Hasegawa-Mima

adiabatic parallel electron response $\frac{\tilde{n}}{n} = \frac{e\tilde{\Phi}}{T_e} \Rightarrow$

two-dimensional one-field model = Hasegawa-Mima equation

Hasegawa & Mima, Phys. Fluids 21(1), 1978

linearized

$$\left\{(1 - \rho_s^2 \nabla_\perp^2)\frac{\partial}{\partial t} + v_D \frac{\partial}{\partial y}\right\} \frac{e\tilde{\Phi}}{T_e} = 0$$

dispersion relation

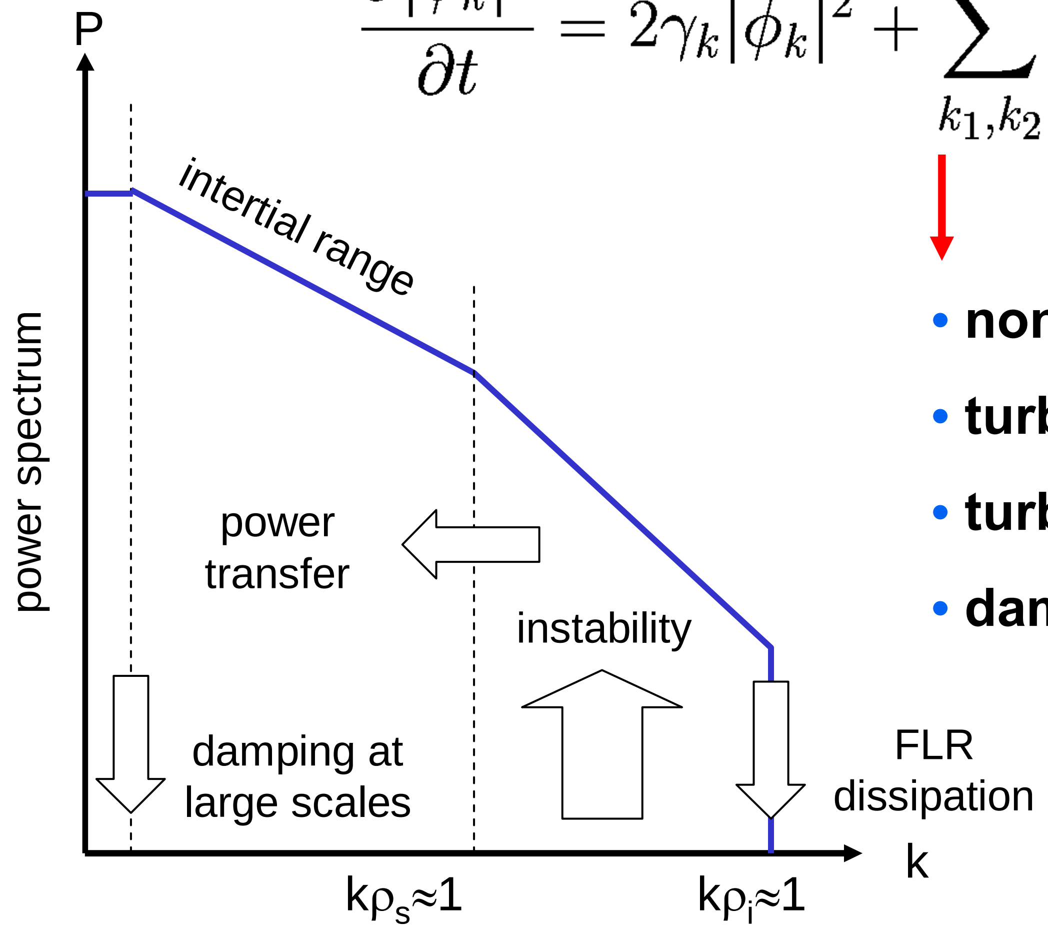
$$\omega = \frac{v_D k_y}{1 + \rho_s^2 k_\perp^2}$$

- **analogue to Euler equation for neutral fluids**
- **linearly stable ~ adiabatic electrons (no collisions, Landau damping)**
- **non-linearly unstable**
- **simplest 2d turbulence model for drift waves**

HM in Fourier space – 3WI

The three wave interaction scheme in k-space

$$\frac{\partial |\phi_k|^2}{\partial t} = 2\gamma_k |\phi_k|^2 + \sum_{k_1,k_2} T_k(k_1,k_2)\,\langle \phi_k^* \phi_{k_1} \phi_{k_2} \rangle$$



- **nonlinear drift instability**
- **turbulent k-spectrum**
- **turbulent cascade**
- **damping and dissipation**

Hasegawa-Wakatani model

non-adiabatic electron response ~ iδ-model n and φ out of phase

two-dimensional two-field model: Hasegawa-Wakatani model

Hasegawa & Wakatani, Phys. Rev. Lett. 50, 1983

plasma potential
$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \vec{V}_{E\times B}\cdot\nabla\nabla_\perp^2\phi = \tilde{\sigma}\left(\phi - n\right) + \mu_w\nabla_\perp^4\phi$$

plasma density
$$\frac{\partial}{\partial t}n + \vec{V}_{E\times B}\cdot\nabla\left(N_0 + n\right) = \tilde{\sigma}\left(\phi - n\right) + \mu_n\nabla_\perp^2 n$$

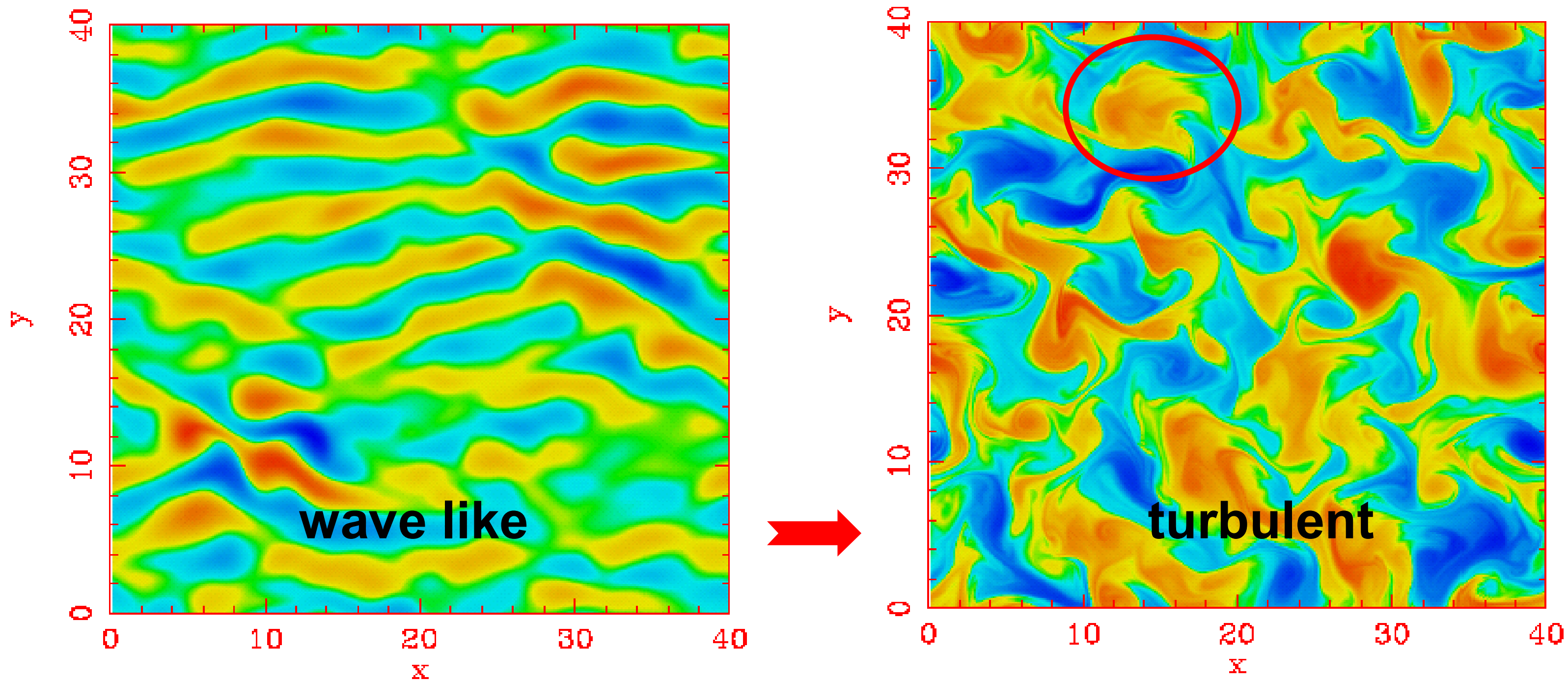
resistive coupling

- **collisional drift wave model**
- **coupling between plasma density and potential by collisions**
- **linearly unstable**
- **advanced drift wave turbulence model ~ transport**

Hasegawa-Wakatani model cont.

plasma potential
$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \vec{V}_{E\times B}\cdot\nabla\nabla_\perp^2\phi = \tilde{\sigma}\left(\phi - n\right) + \mu_w\nabla_\perp^4\phi$$

plasma density
$$\frac{\partial}{\partial t}n + \vec{V}_{E\times B}\cdot\nabla\left(N_0 + n\right) = \tilde{\sigma}\left(\phi - n\right) + \mu_n\nabla_\perp^2 n$$



Ref: V. Naulin, Risoe, Denmark

Summary

- Drift waves are important - edge turbulence and transport
- A distinct two-fluid phenomenon
- Perpendicular and parallel dynamics tightly coupled
- $E \times B$ vortex in region with strong ∇n perp B_0
- Linear stable for adiabatic electrons
- Linear unstable for non-adiabatic electrons
- Hasegawa-Mima one field model - adiabatic e‘s
- Hasegawa-Wakatani two field model – non-adiabatic e‘s
- More in the next lecture …

Thank you for your kind attention.

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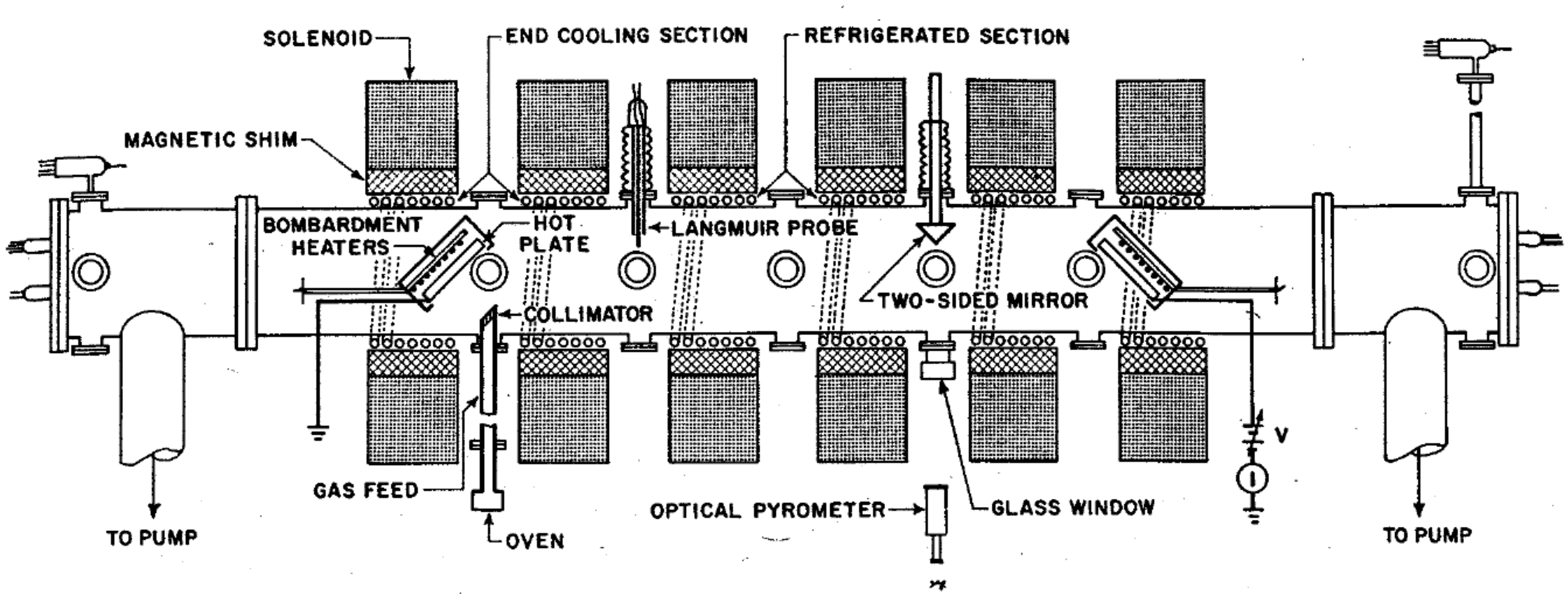
Experiments on drift waves

A selection of basic work

Linear device: VINETA

Earlier work



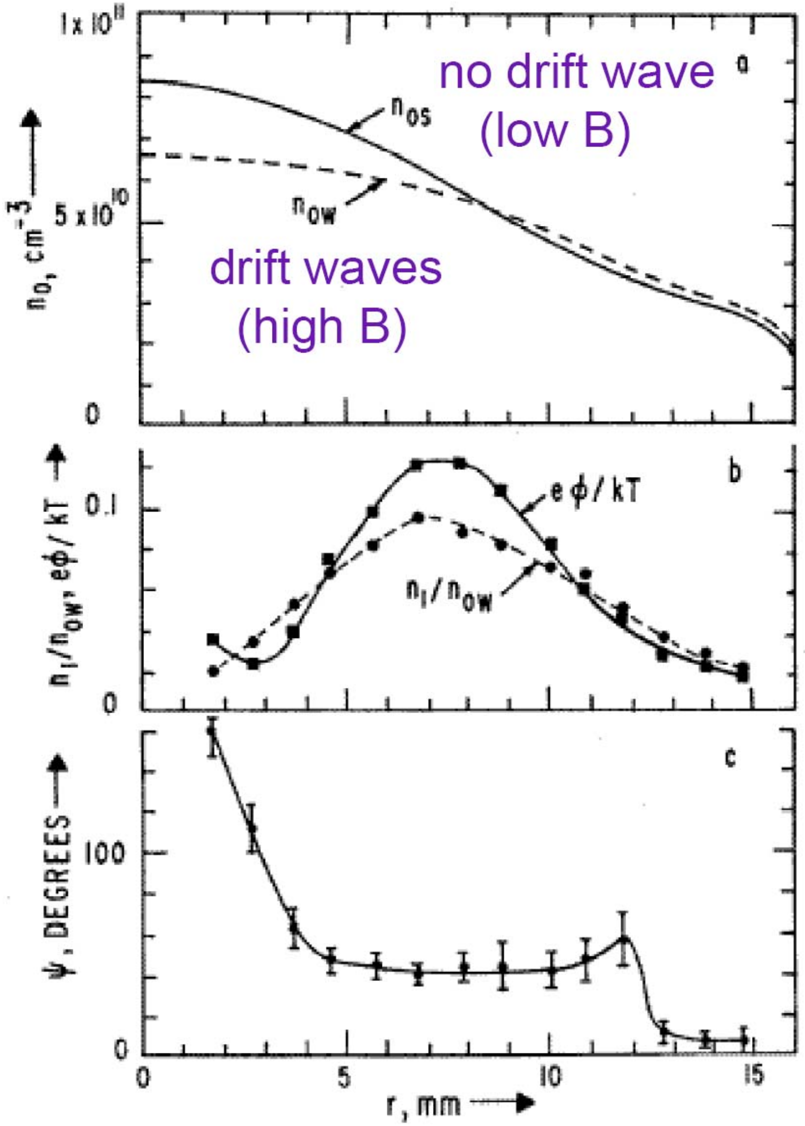
- **Q-machine with K or Cs plasma**
- **relatively low density n ~ $10^4 \ldots 10^7$ m^{-3}**
- **isothermal** $T_i = T_e = 0.25\,\text{eV}$

Hendel et al. Phys. Rev. Lett. 18, 439 (1967) and Hendel et al. Phys. Fluids 11, 2426 (1968)

Earlier work

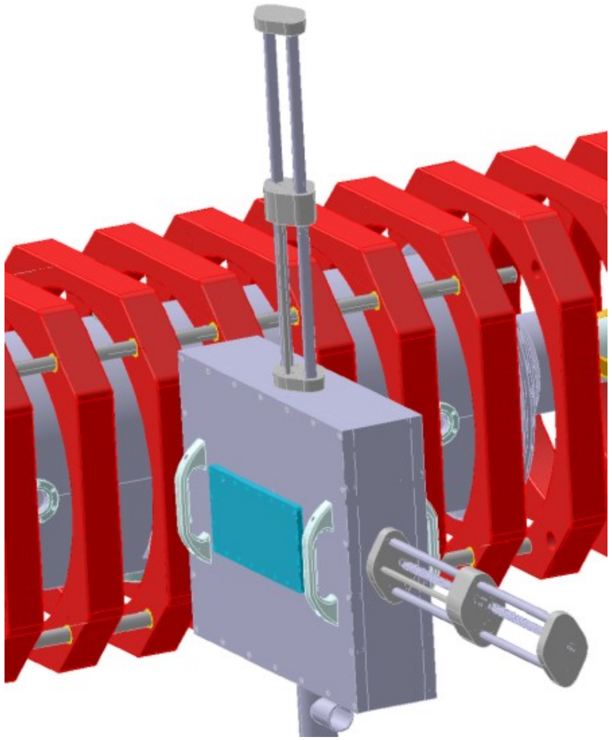
A few comments:

- **nearly coherent drift mode**
- **localized in high ∇n region**
- $e\delta\varphi/k_bT \approx \delta n/n$ **Boltzmann satisfied**
- δn **leads** $\delta\varphi$
- **expected from linear theory**
- **collisional drift wave**
- **destabilized by electron resistivity**
- **stabilized by ion viscosity $\perp$B**
- **unstable when** $k\cdot\rho_i$~0.5
- **saturated instability**



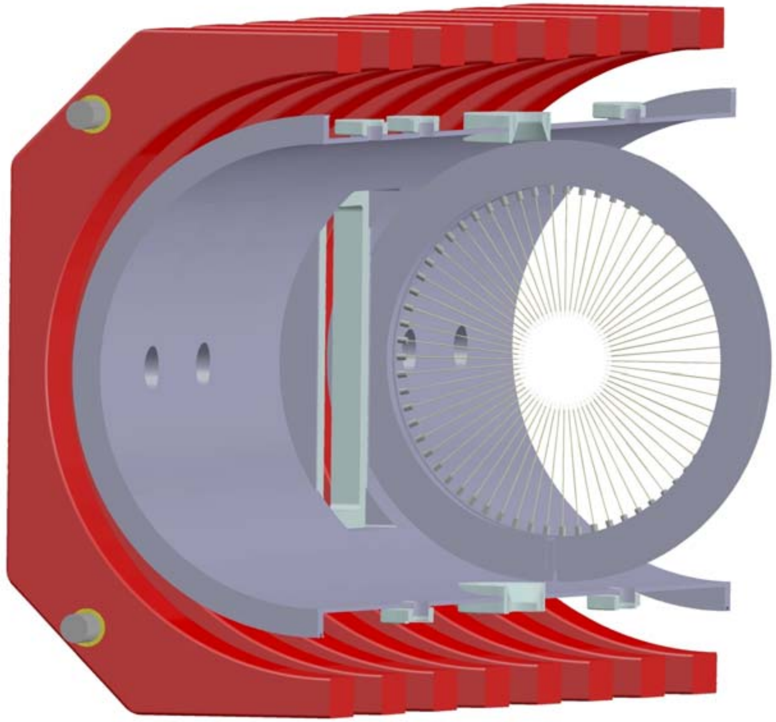
Linear device: VINETA

azimuthal single probe positioning system



- **2D profiles ✳n⅄ and n(t)**
- **2D correlation functions**

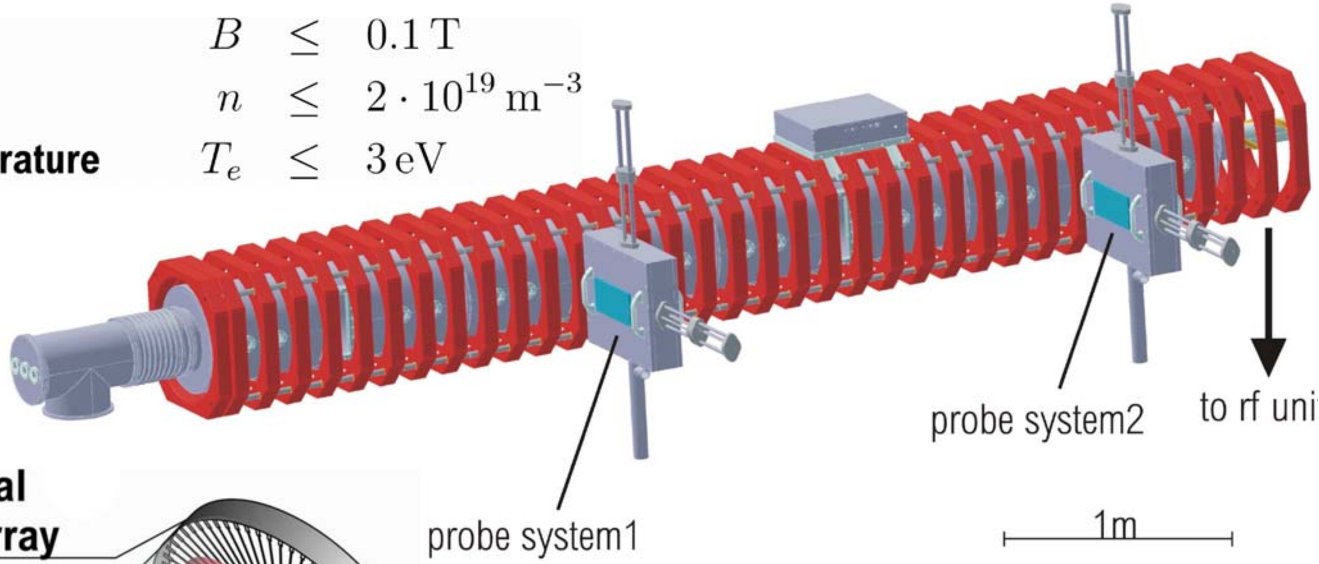
azimuthal 64 probe array

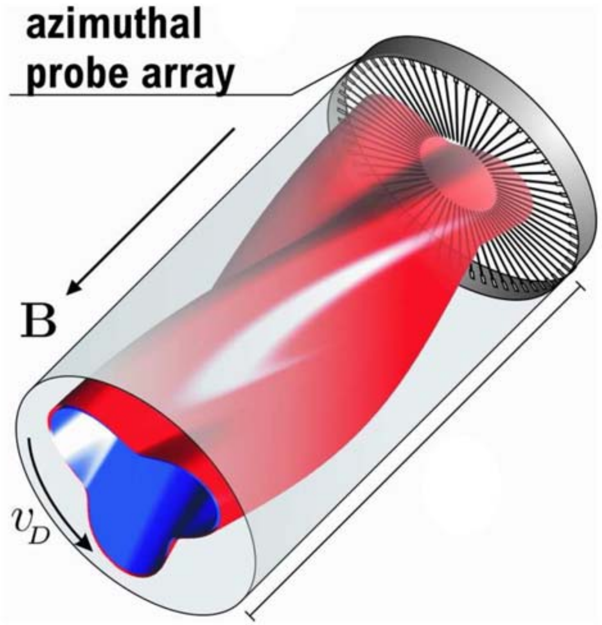


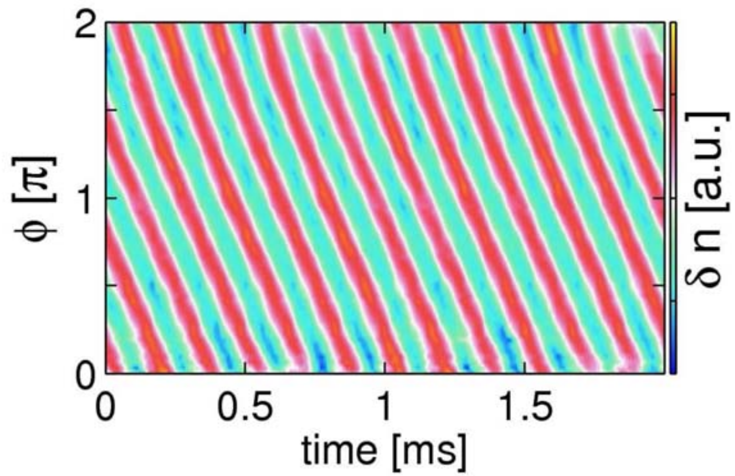
- **density fluctuations on azimuthal circumference**

Space-time data

magnetic field	B	$\leq$	$0.1\,\mathrm{T}$
density	n	$\leq$	$2 \cdot 10^{19}\,\mathrm{m}^{-3}$
electron temperature	T_e	$\leq$	$3\,\mathrm{eV}$







Space-time data

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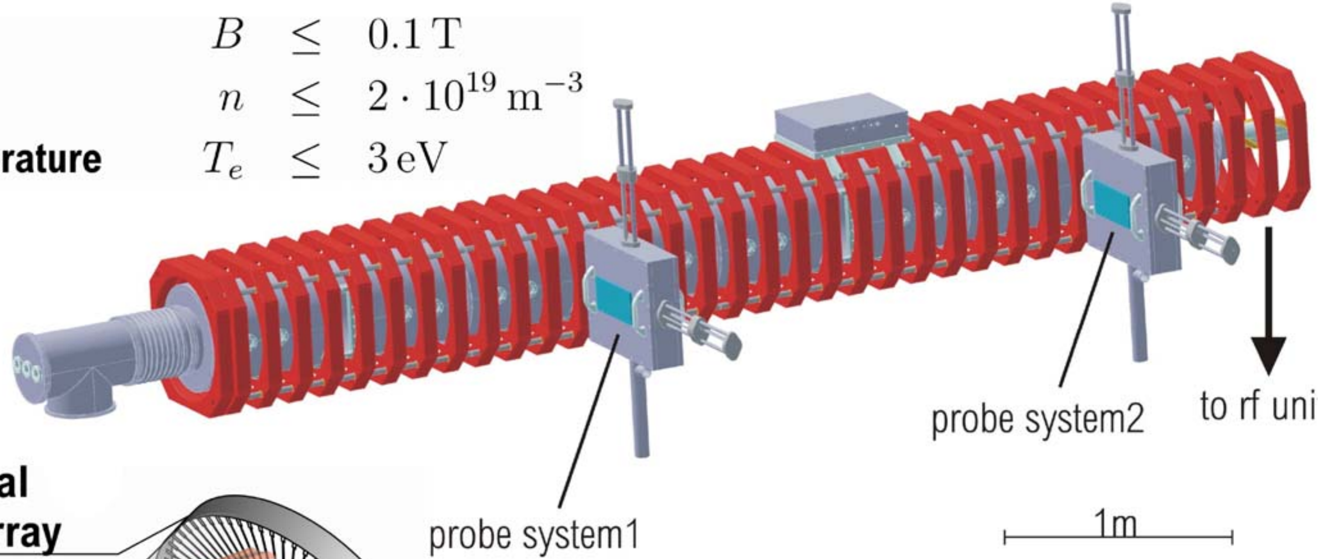
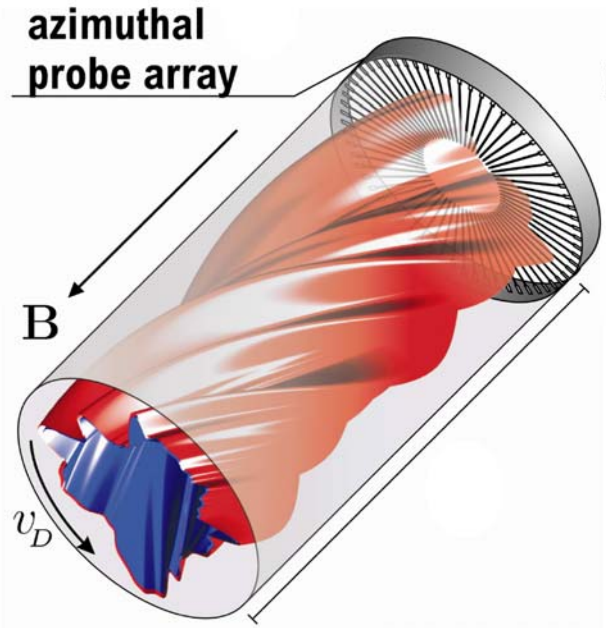
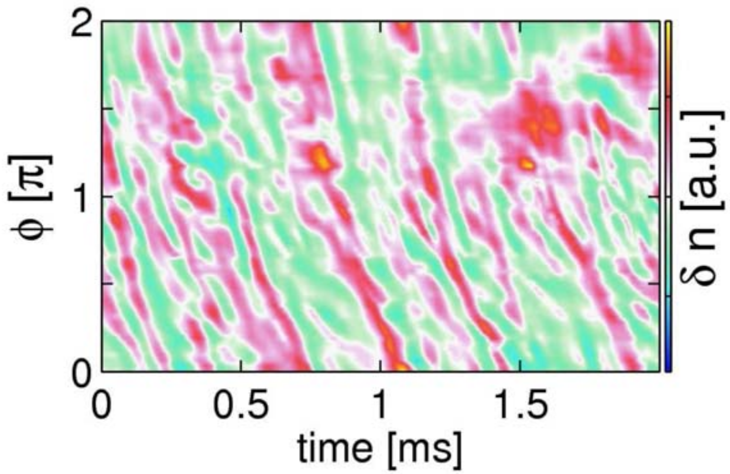
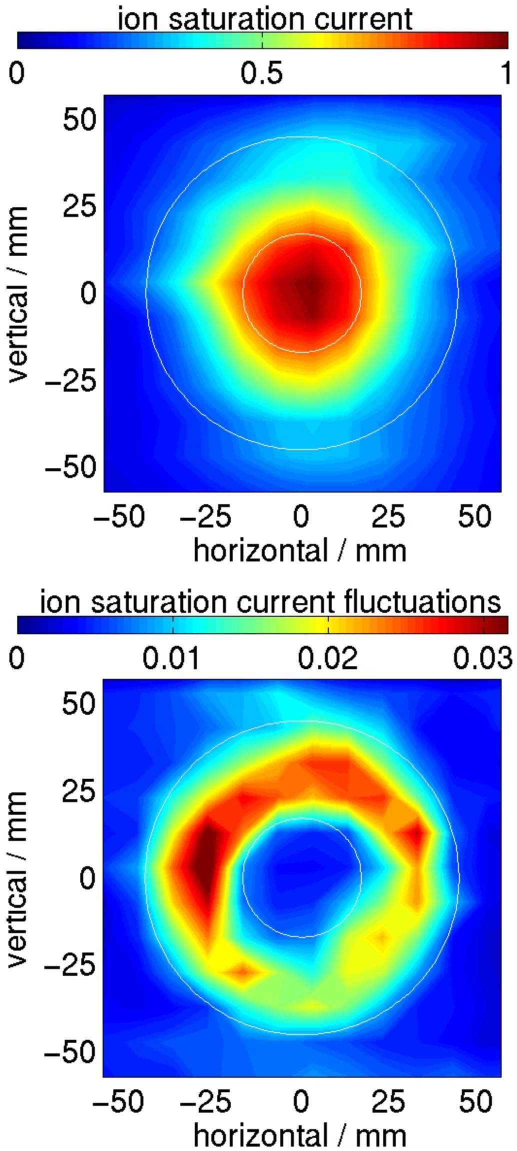
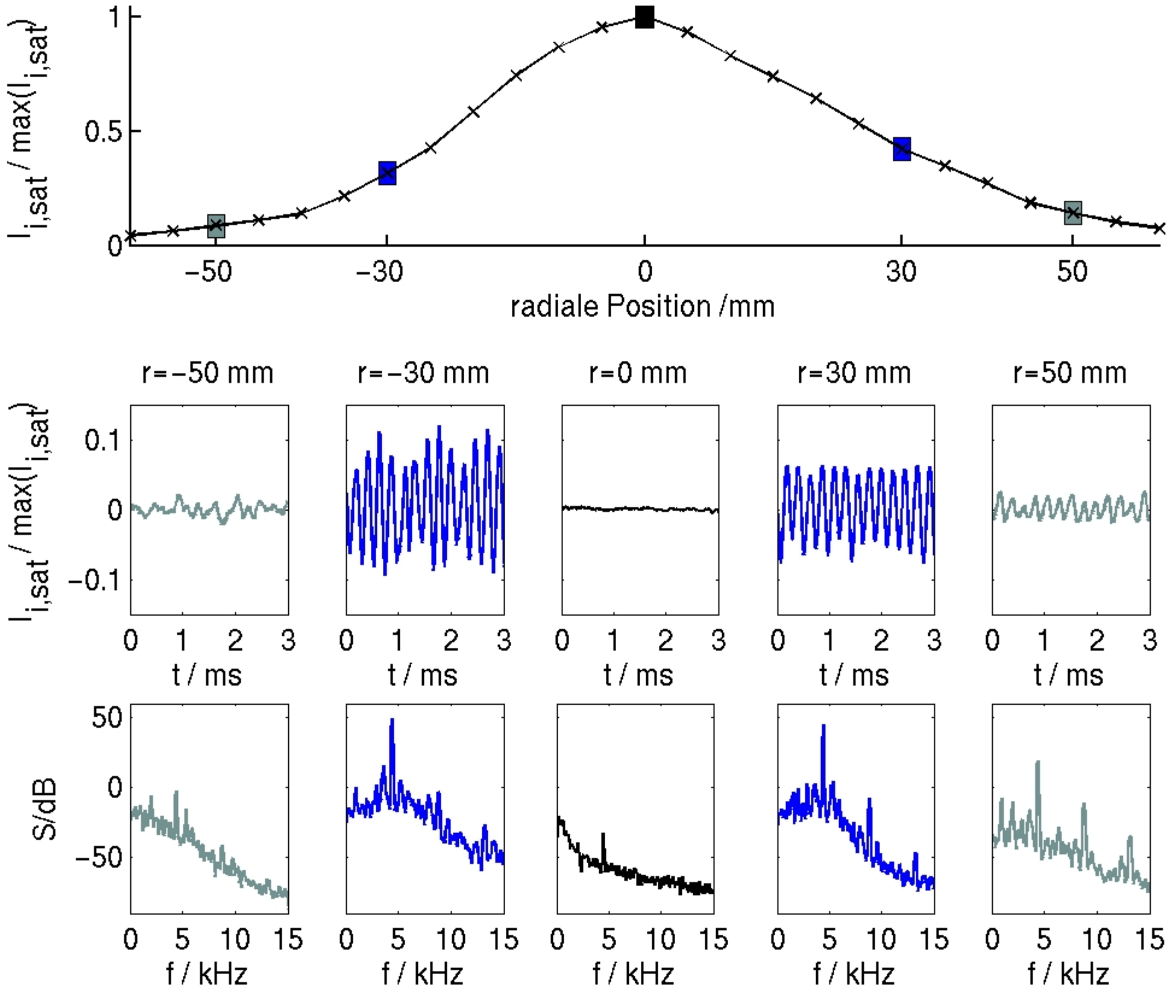




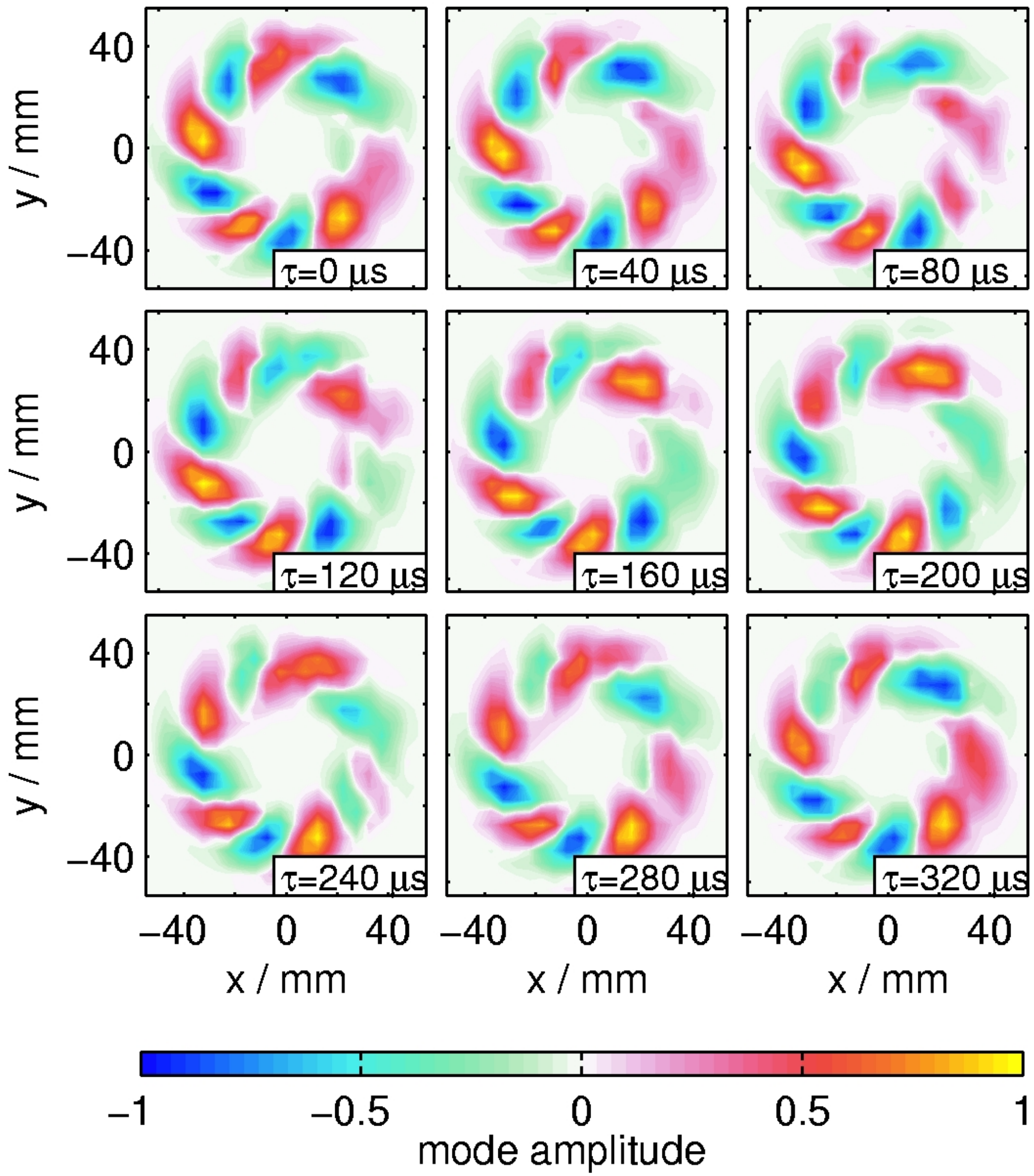


Basic fluctuation characteristics

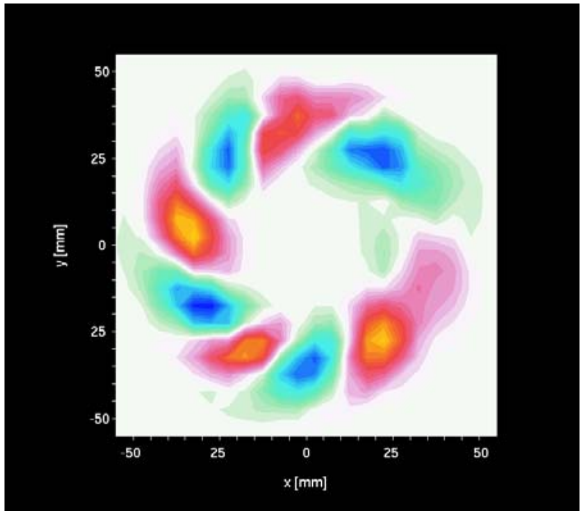




Azimuthal mode structure



- **propagation in** v_{ed}
- **fluctuation** $\tilde{n}/n \sim 10\%$
- **mode structure**
- **azimuthally sheared**



Linear global model

eigenvalue equation

Ellis et al., Plasma Physics ***22****, 1980*

$$\partial_{rr}\phi + \left(\frac{1}{r} - \kappa(r) + RD(r)\right)\partial_r\phi + \left(Q(r) - \frac{m^2}{r^2}\right)\phi = 0$$

with

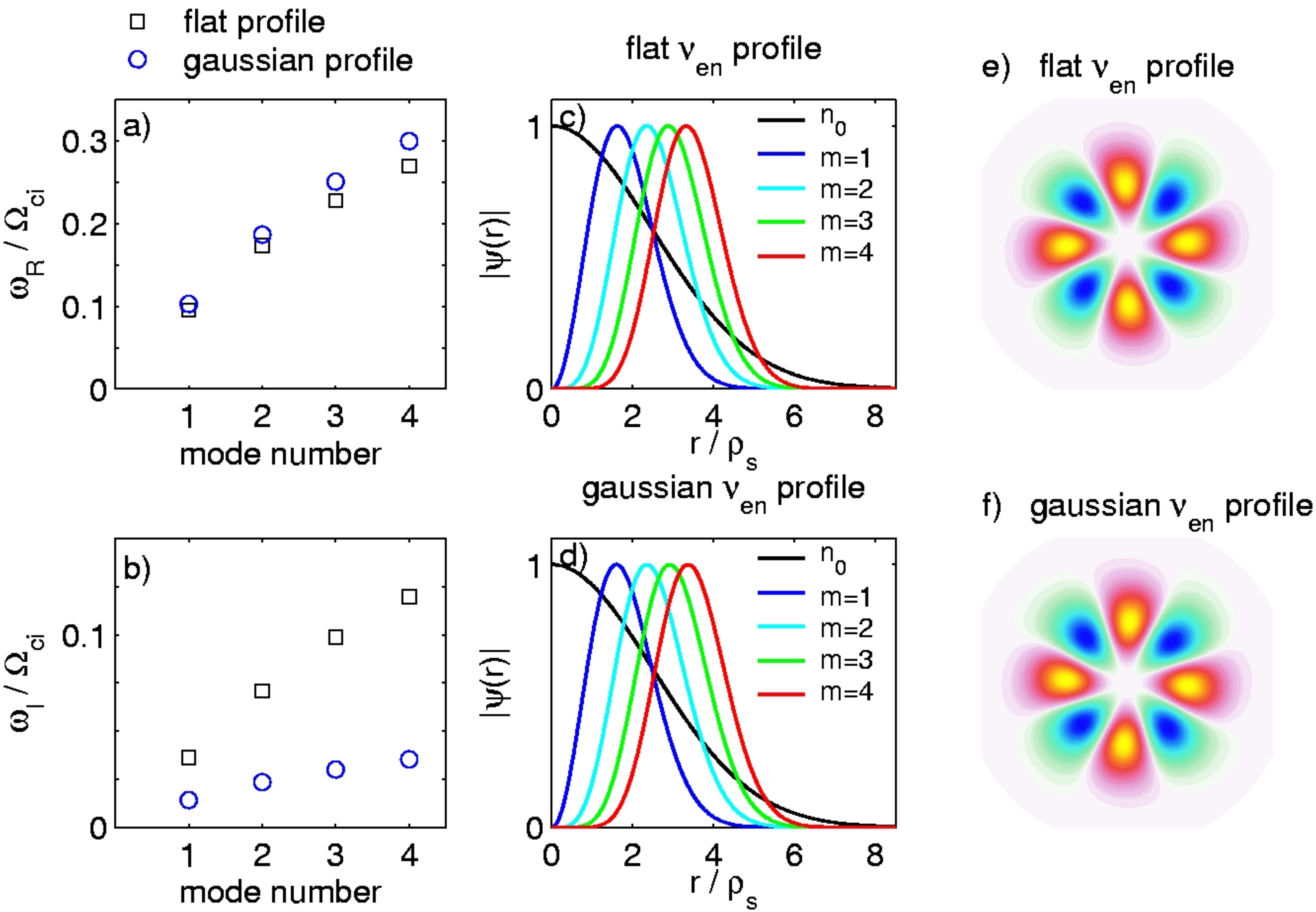
$$RD(r) := i\,\frac{1}{\tilde{\omega} + i\nu_{in}}\left(\frac{\omega^* + iP}{\tilde{\omega} - \omega_1 + iP}\right)\nu_{in}\, r\, V_p$$

$$Q(r) := \frac{1}{\tilde{\omega} + i\nu_{in}}\left[\omega^* + \frac{m}{r}S_p - \tilde{\omega}\,\frac{\omega^* + iP}{\tilde{\omega} - \omega_1 + iP}\right];\ P = P(\nu_e)$$

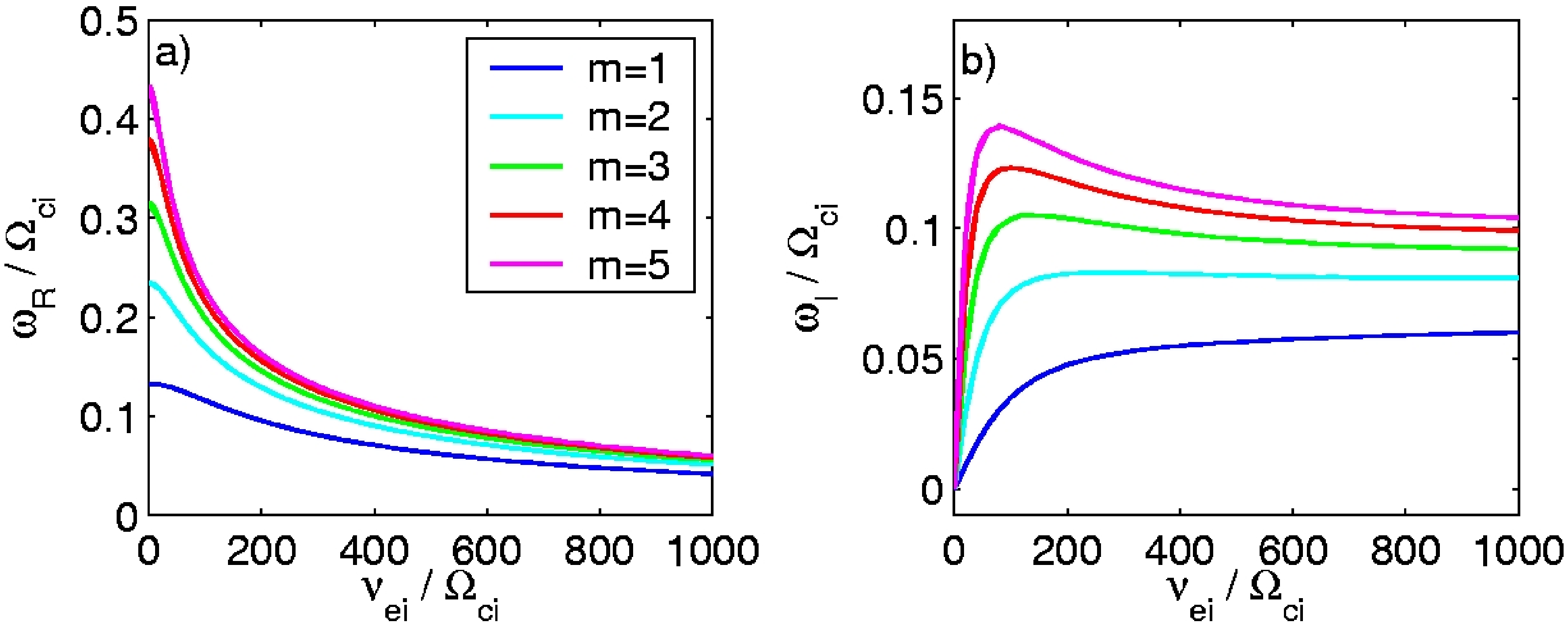
- **important:** $\nu = \nu(r)$
- **solve for eigenfrequencies & eigenmodes**

$$\omega = \omega_R + i\omega_I$$
$$\psi(r) = \psi_R(r) + i\psi_I(r)$$

Eigenvalue solutions



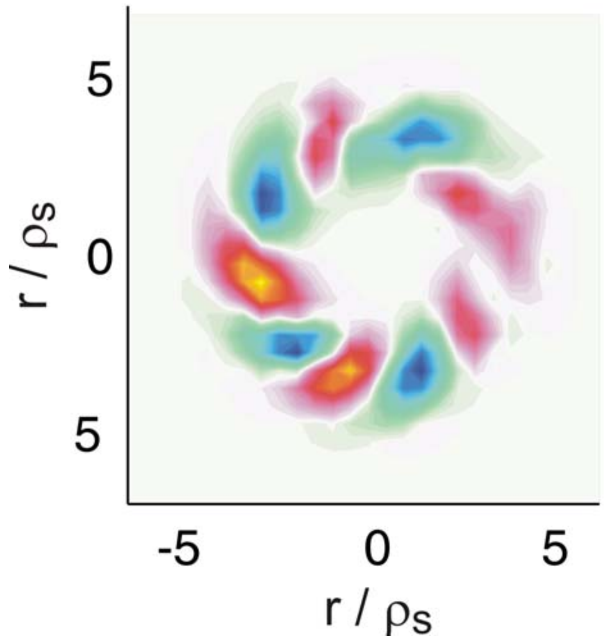
Role of collisionality

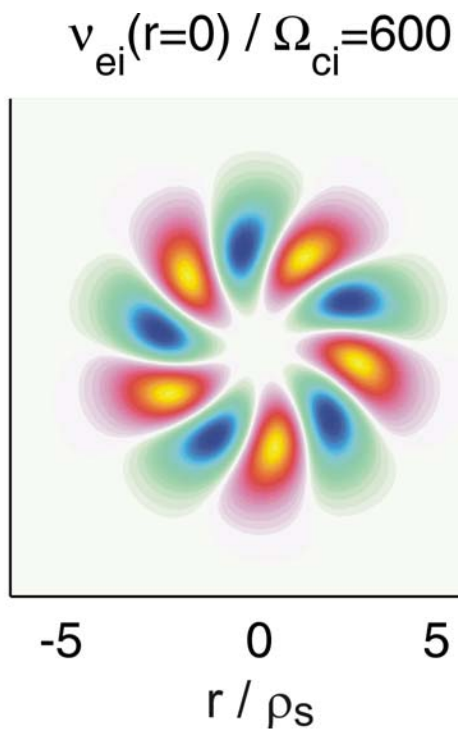


- **frequency of drift mode decreases considerably**
- **growth rate for m=1 mode much smaller**
- **generally not observed in VINETA as single coherent mode**

Detailed mode structure

measurement ↔ **model**

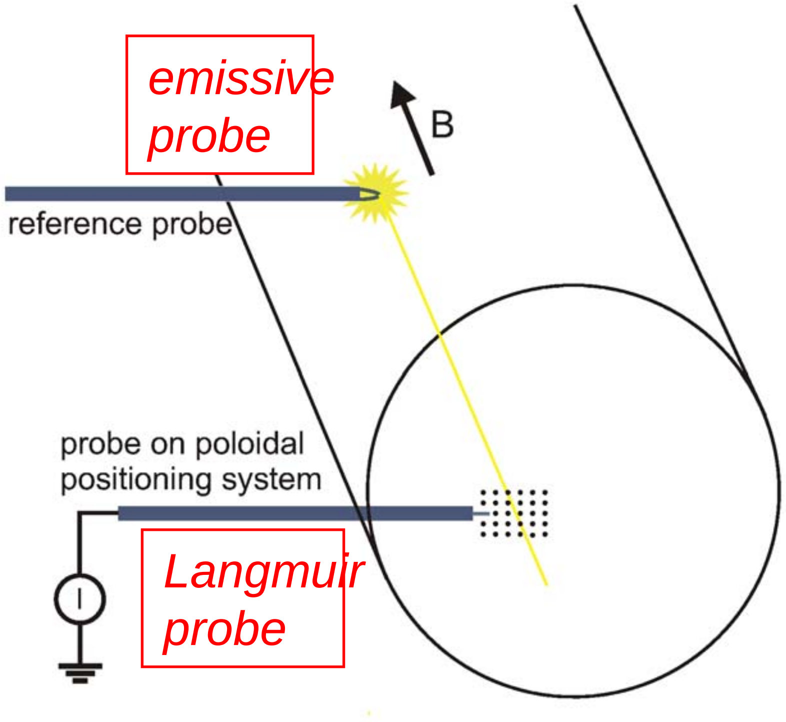




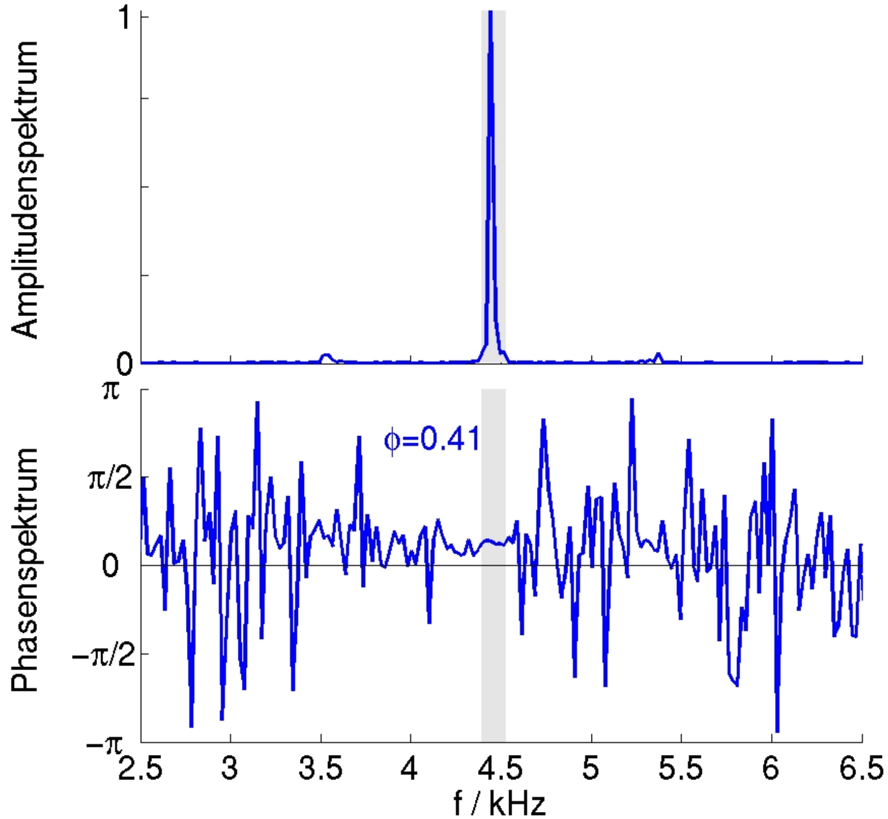
sheared mode structure owing to radial collisionality profile

Parallel wavelength

alignment of probes along magnetic field

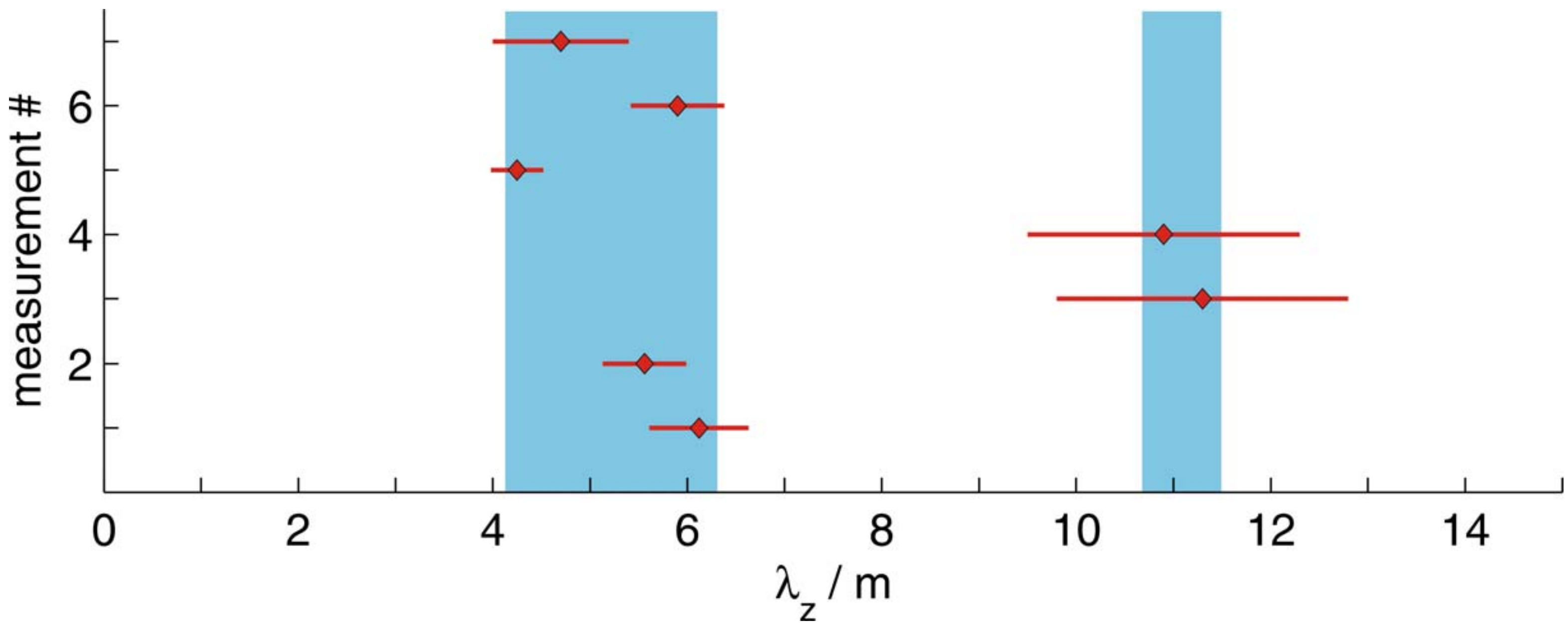


phase shift along magnetic field



- **alignment accuracy ≤ 1mm**

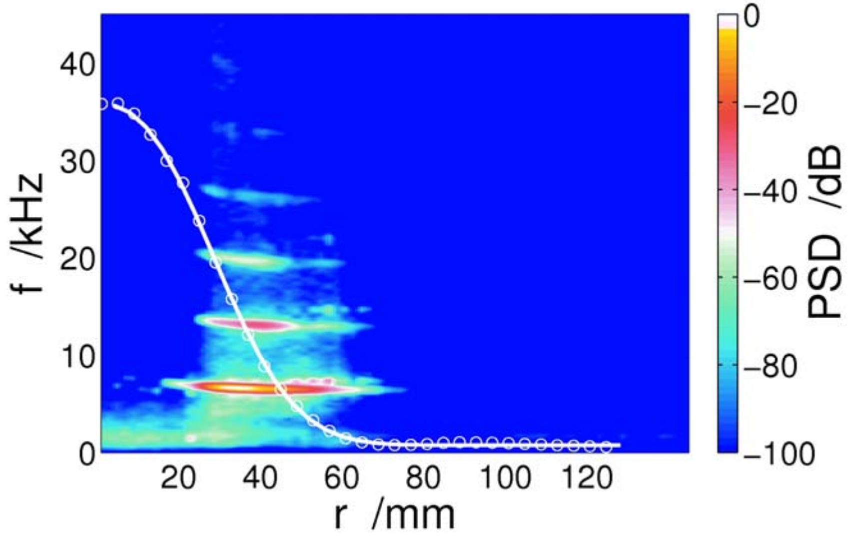
Parallel wavelength



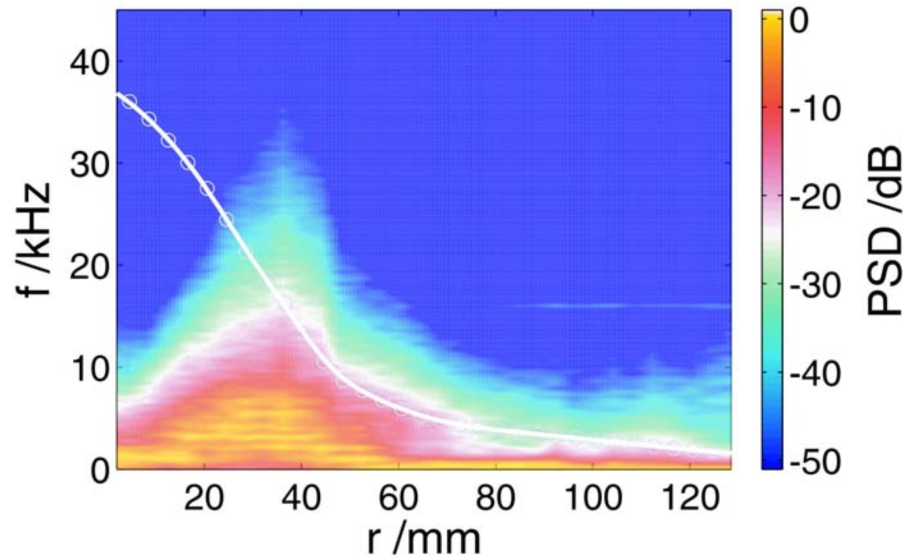
- $k_{||} \neq 0$
- **phase shift & axial separation provides parallel wavelength** λ_z
- **wavelengths group at** $L_{||}$ **and** $2L_{||}$
- **important proof to observe *really* drift waves**

Drift wave turbulence

drift wave mode



drift wave turbulence



radially resolved power

spectra

- coherent fluctuations
- fluctuations well localized
- spectrum is peaked
- higher harmonics

- incoherent fluctuations
- fluctuations spread
- spectrum is broad
- power-law decrease

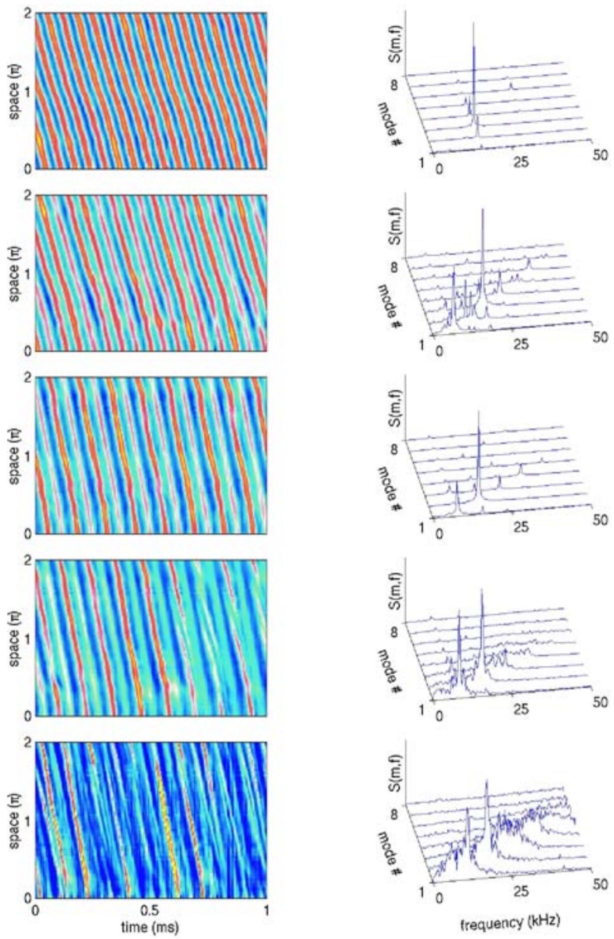
A transition to turbulence

control parameter

$$\epsilon = \frac{U_g - U_{gc}}{U_g}$$

onset drift wave

separation grid bias



⑧-*t* - diagram **spectrum**

increas of grid bias

increase of plasma current

A transition to turbulence

$\varepsilon = 0.13$

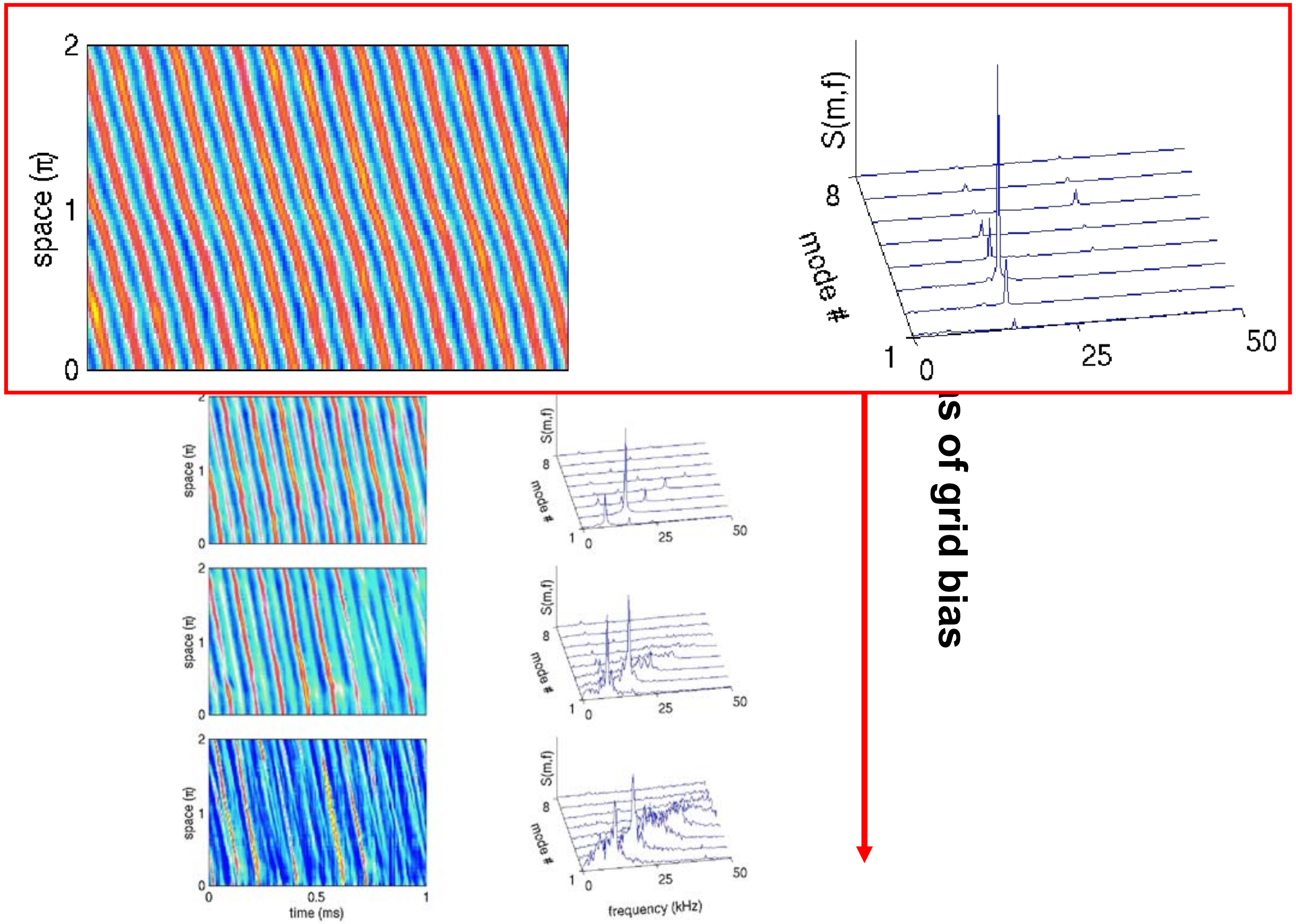
m=3 mode

$\varepsilon = 0.49$

$\varepsilon = 0.62$

$\varepsilon = 0.75$

$\varepsilon = 1.01$



⑧-*t* - **diagram** **spectrum**

A transition to turbulence

$\varepsilon = 0.13$

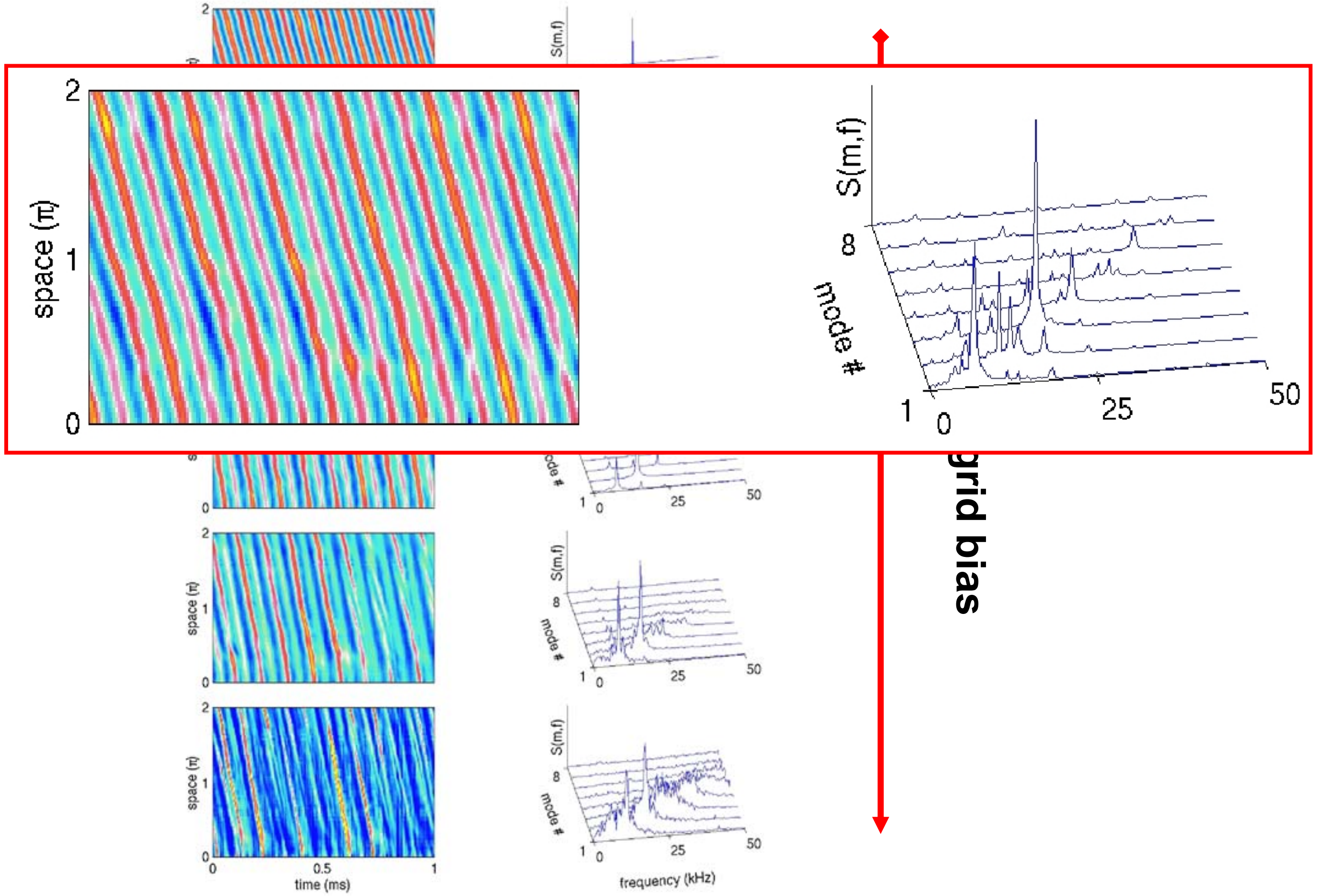
$\varepsilon = 0.49$

2 modes

$\varepsilon = 0.62$

$\varepsilon = 0.75$

$\varepsilon = 1.01$



ϑ-t - diagram spectrum

A transition to turbulence

$\varepsilon = 0.13$

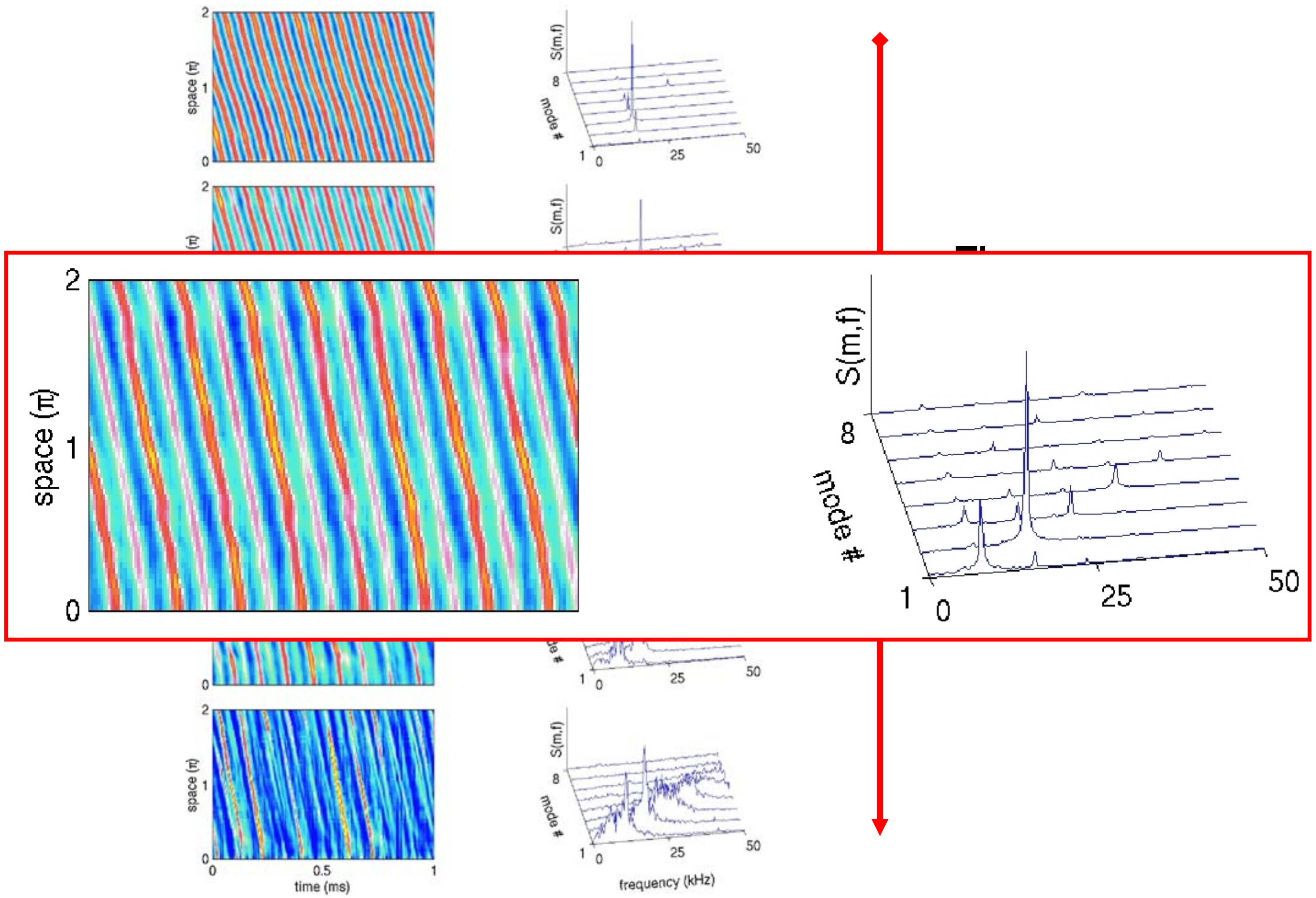
$\varepsilon = 0.49$

$\varepsilon = 0.62$

mode-lock

$\varepsilon = 0.75$

$\varepsilon = 1.01$



⑧-*t* - **diagram** **spectrum**

A transition to turbulence

$\varepsilon = 0.13$

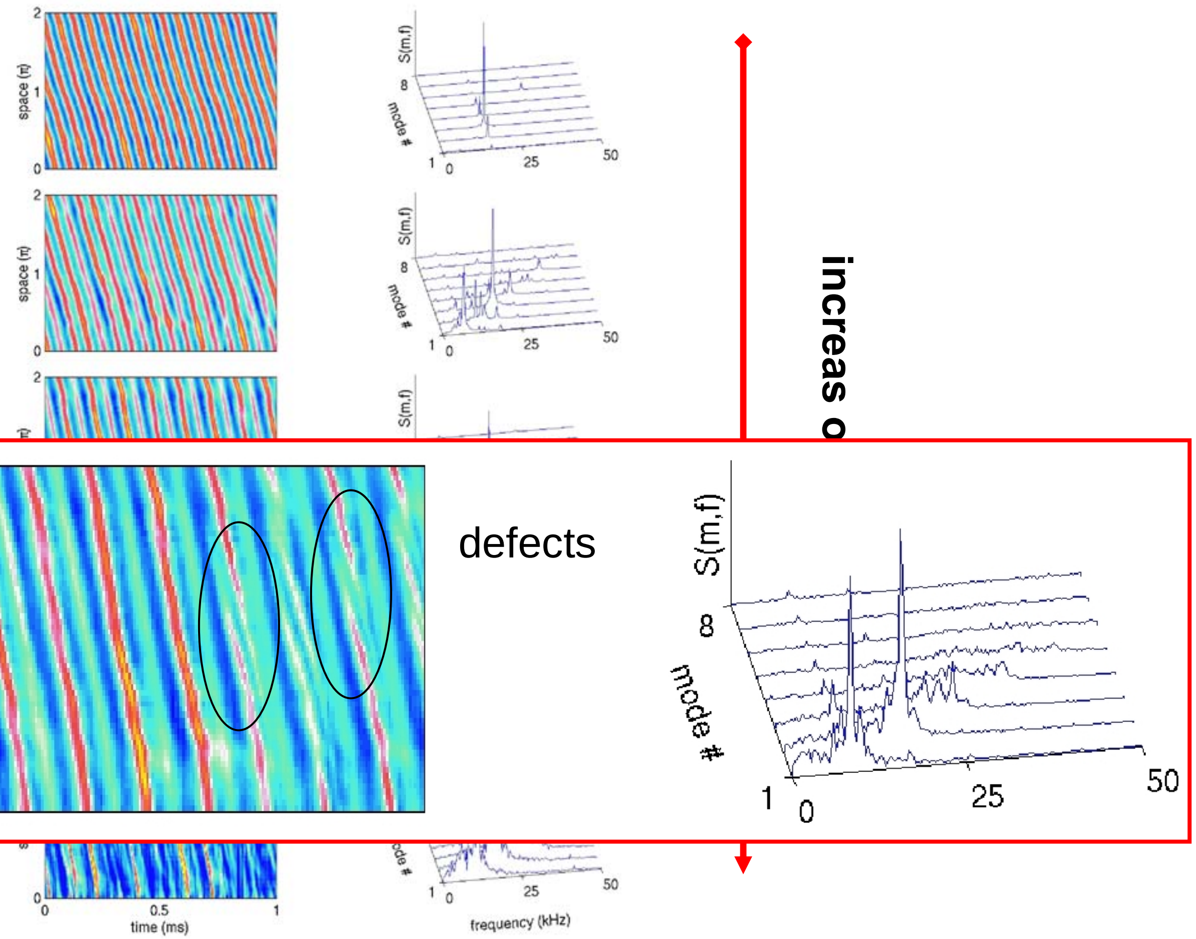
$\varepsilon = 0.49$

$\varepsilon = 0.62$

$\varepsilon = 0.75$

chaos

$\varepsilon = 1.01$



⑧-*t* **- diagram** **spectrum**

A transition to turbulence

$\varepsilon = 0.13$

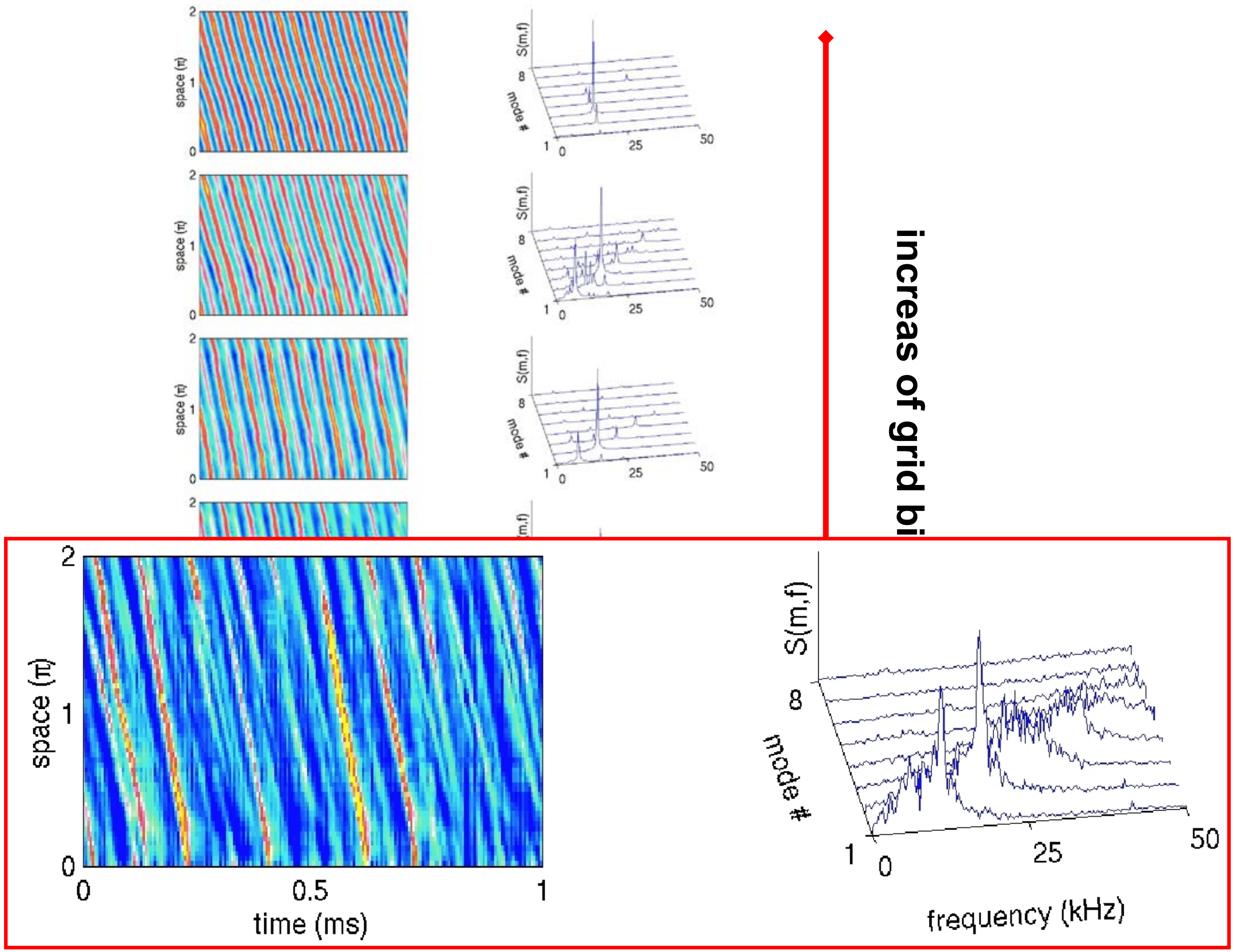
$\varepsilon = 0.49$

$\varepsilon = 0.62$

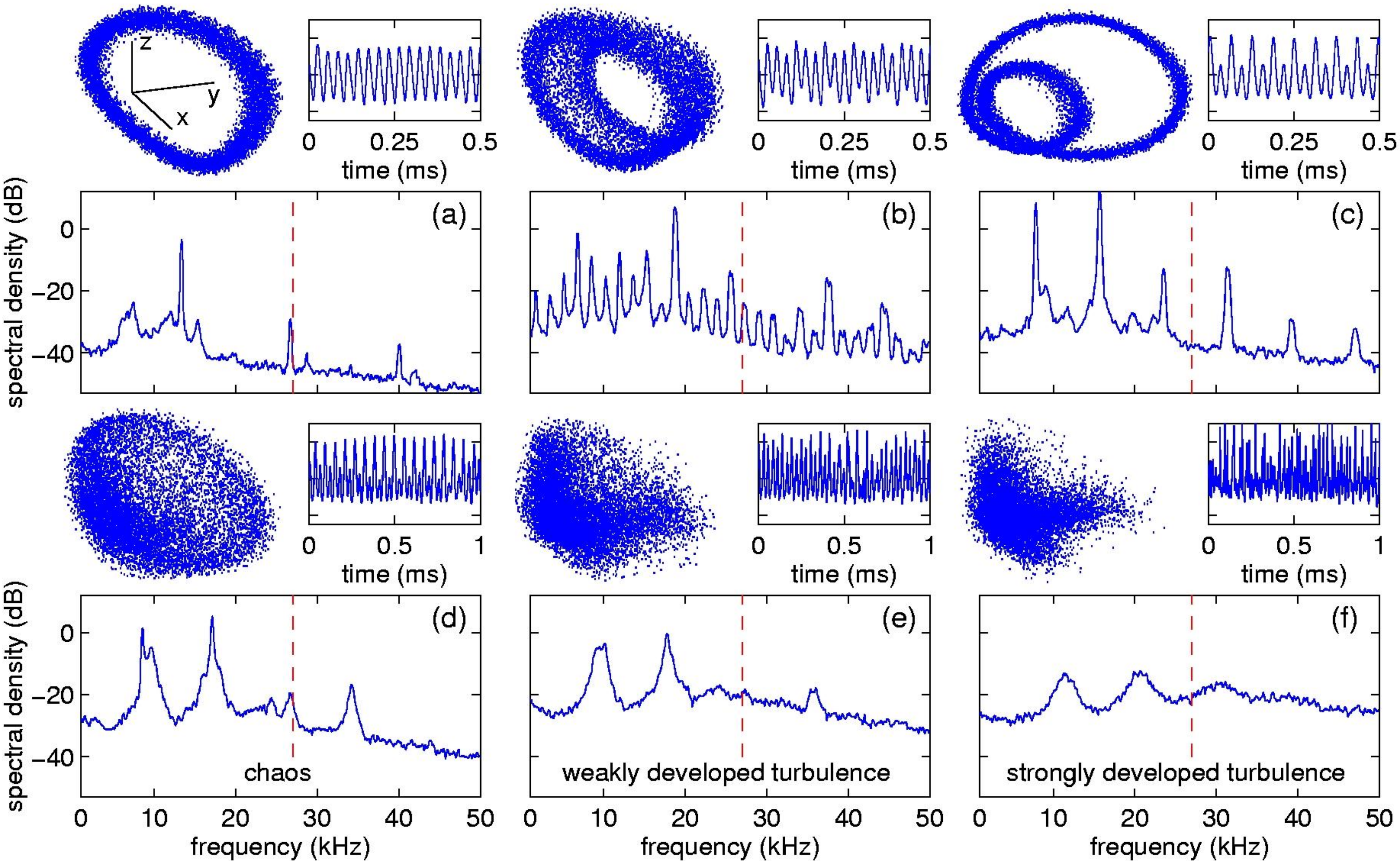
$\varepsilon = 0.75$

$\varepsilon = 1.01$

turbulence

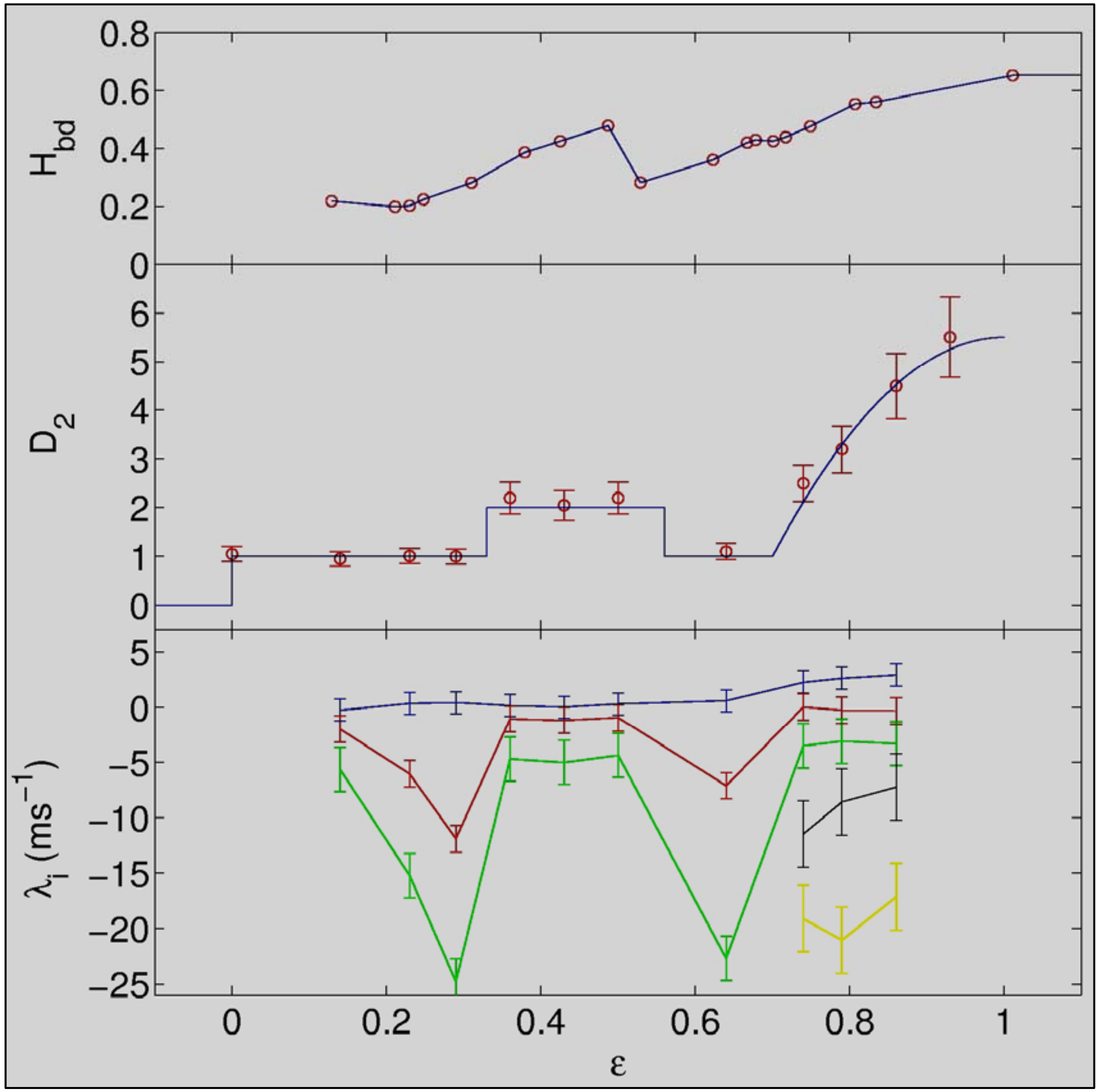


Phase space



z
y
x
0 0.25 0.5
time (ms)
spectral density (dB)
0
−20
−40
(a)
(b)
(c)
0 0.5 1
time (ms)
(d)
(e)
(f)
chaos
weakly developed turbulence
strongly developed turbulence
0 10 20 30 40 50
frequency (kHz)

Phase space analysis



SVD entropy

+

correlation dimension

+

Lyapunov exponents

„weak“ drift wave turbulence

- **Phase space analysis ◆ dimension, stability ✱ scenario**
  T.K. et al., PRL 79, 3913 (1997), Plasma Phys. Controlled Fusion 39, B145 (1997)

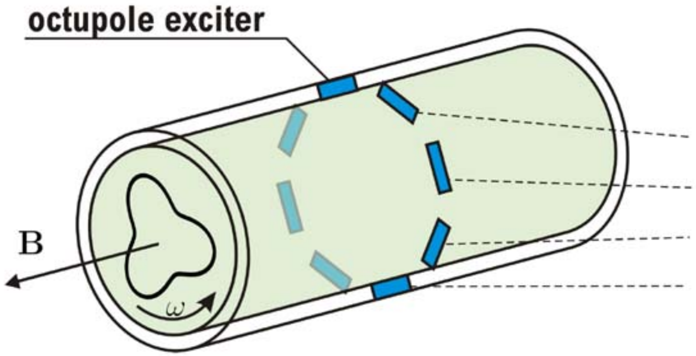
- **Ruelle-Takens-Newhouse (RTN) transition scenario**
  Newhouse, Ruelle, Takens, Commun. Math. Phys. 64, 35 (1978)

- **RTN was already found in earlier drift wave models**
  Wersinger, Finn, Ott, Phys. Fluids 23, 1142 (1980)
  Biskamp, He, Phys. Fluids 28, 2172 (1985)

- **Drift wave chaos exists in transition regime only**
  - turbulence is high-dimensional D~100
  - phase space analysis impossible

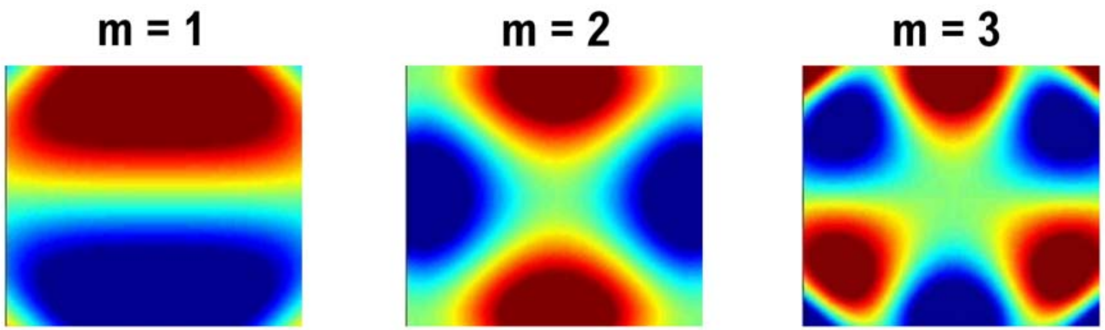
- **Quick transition to weakly developed turbulence**
  Manneville, Dissipative Structures and Weak Turbulence, Academic Press 1990

Control of drift wave turbulence



$$U_1 = A\sin(\omega t)$$
$$U_2 = A\sin(\omega t + \delta)$$
$$U_3 = A\sin(\omega t + 2\delta)$$
$$U_4 = A\sin(\omega t + 3\delta)$$
$$\ldots$$

- **mode control by phase shift:** $\delta = \pm\frac{2\pi \cdot m}{8}$
- **Nyquist limit:** $m_{ex} < 4$



T.K., Schröder, Block et al., *Phys. Plasmas* 8, 1961 (2001)

Schröder, T.K., Block, Piel, Bonhomme, Naulin, *Phys. Rev. Lett.* 86, 5711 (2001)

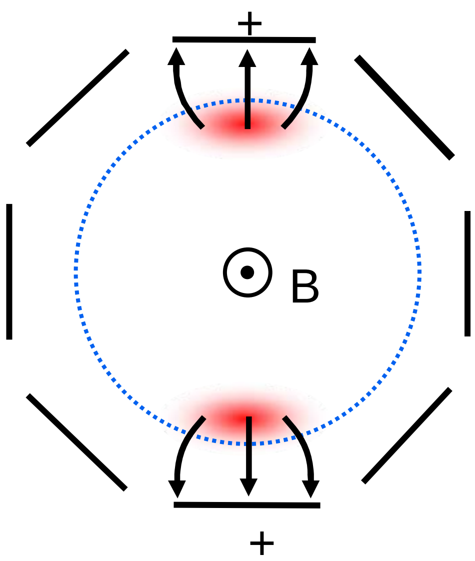
Model: rotating current profile

extended HW-model (2d)

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \vec{V}_{E\times B}\cdot\nabla\nabla_\perp^2\phi = \tilde{\sigma}\left(\phi - n\right) + S + \mu_w\nabla_\perp^4\phi$$

$$\frac{\partial}{\partial t}n + \vec{V}_{E\times B}\cdot\nabla\left(N_0 + n\right) = \tilde{\sigma}\left(\phi - n\right) + S + \mu_n\nabla_\perp^2 n$$

$$S = A\sin(\pi r/r_0)\sin(m_d\Theta - \omega_d t)$$



- **rotating electron current profile** $|| B$
- **azimuthal mode structure (m=2)**
- **radial localisation**

Model: rotating current profile

extended HW-model (2d)

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \vec{V}_{E\times B}\cdot\nabla\nabla_\perp^2\phi = \tilde{\sigma}\left(\phi - n\right) + S + \mu_w\nabla_\perp^4\phi$$

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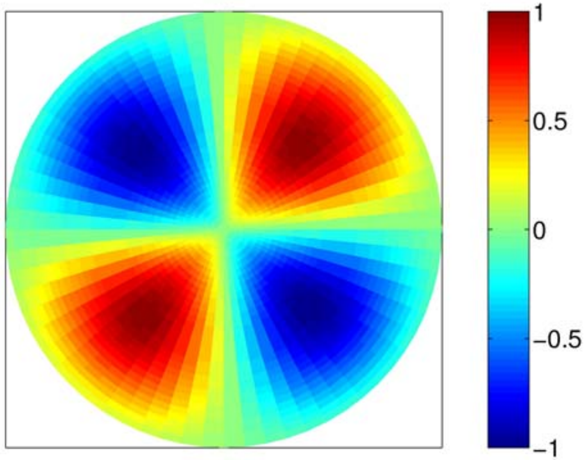
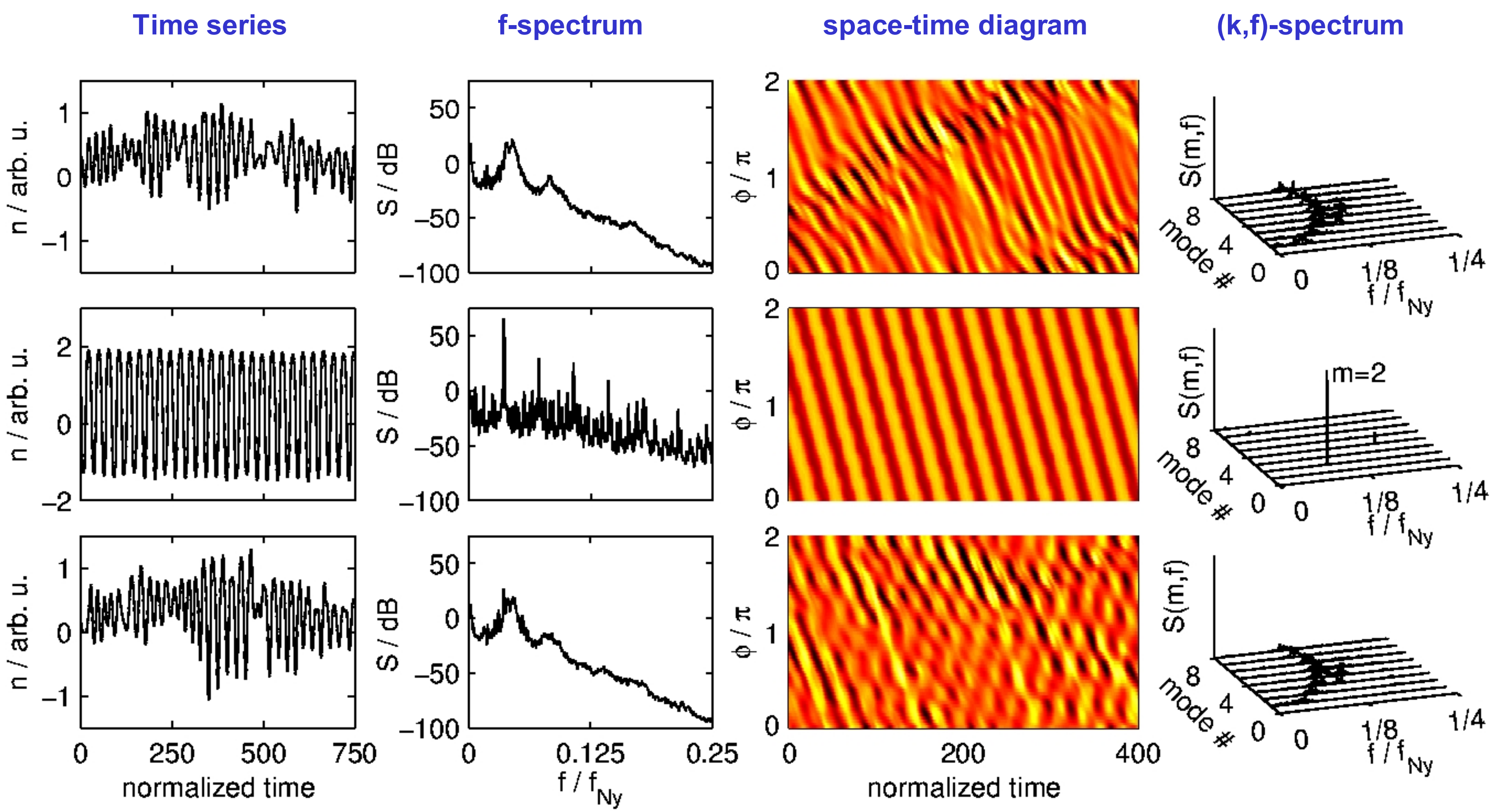


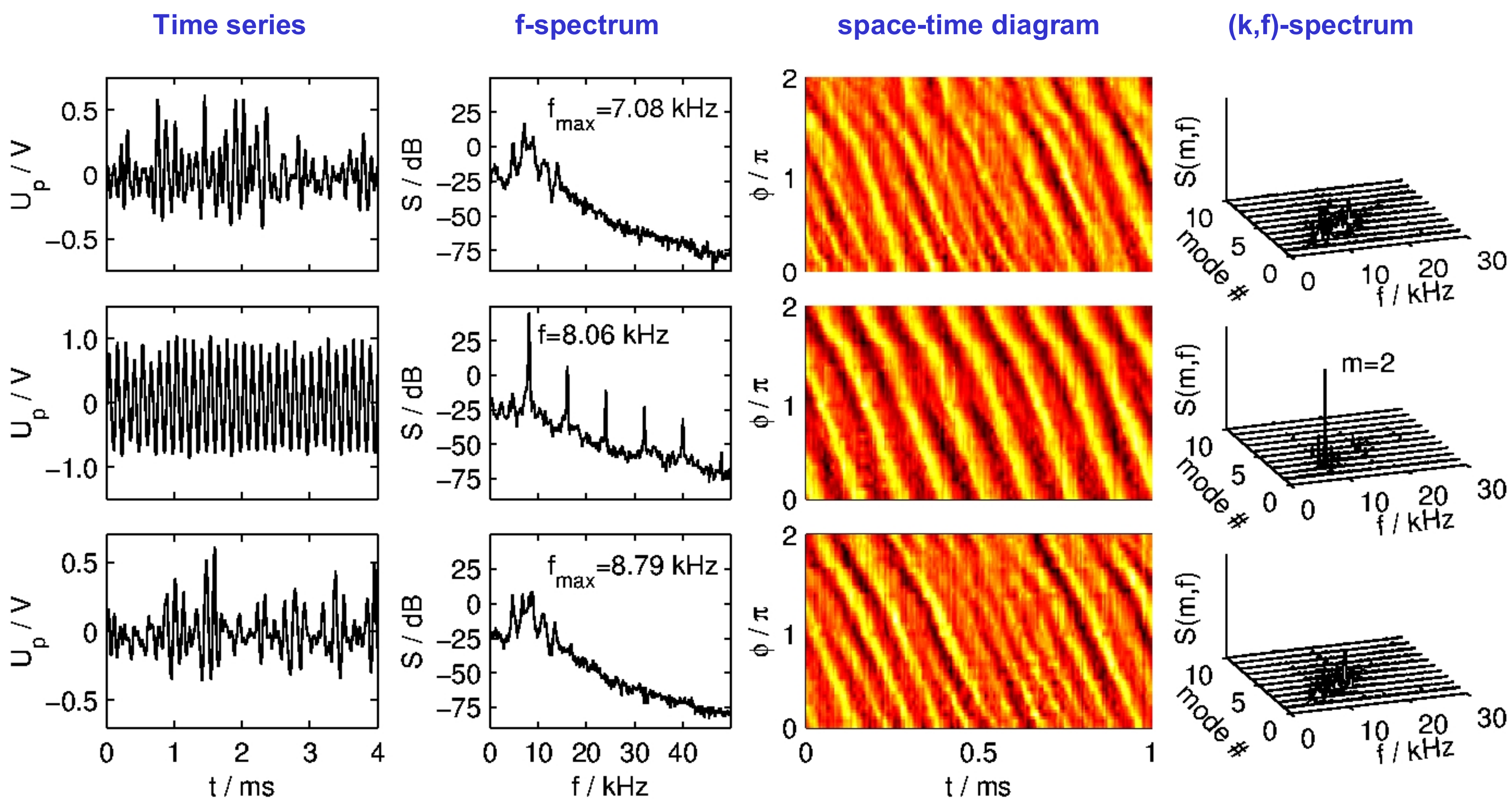
- **rotating electron current profile $||\, B$**
- **azimuthal mode structure (m=2)**
- **radial localisation**

Drift wave sync‘ - model



- no external field
- co-rotating field
- counter-rotating field

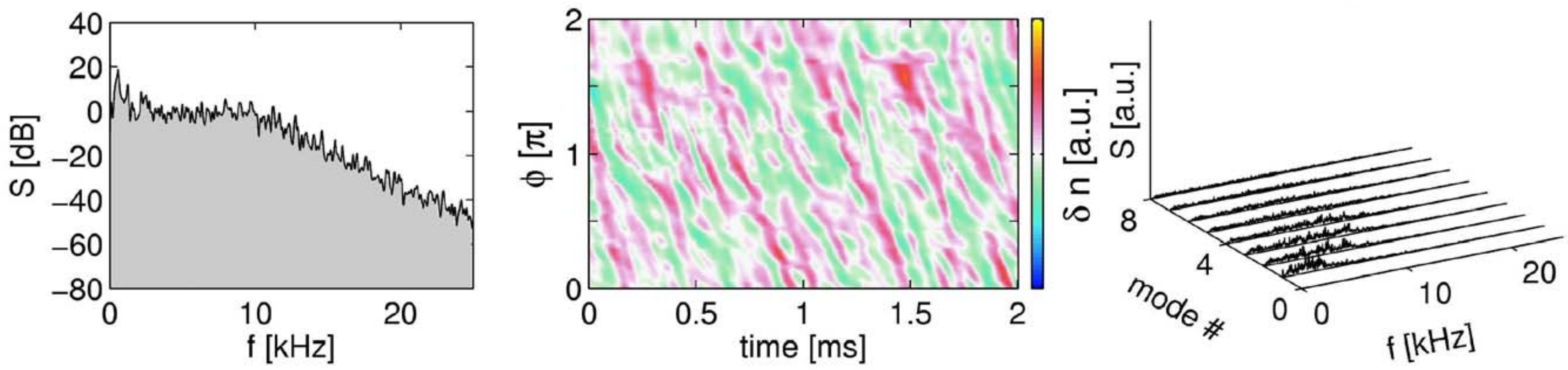
Drift wave sync‘ - experiment



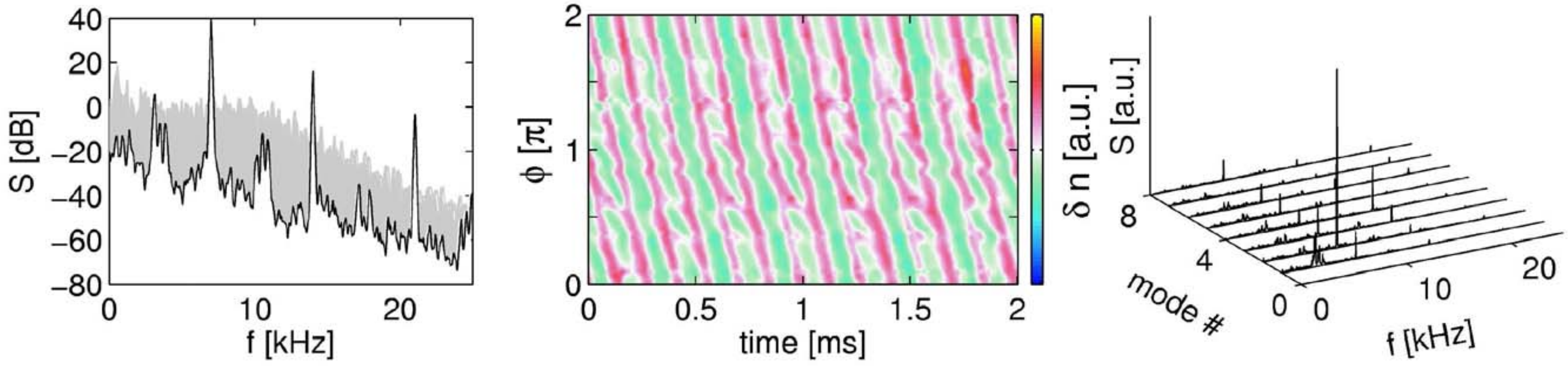
- no external field
- co-rotating field
- counter-rotating field

Synchronising turbulence

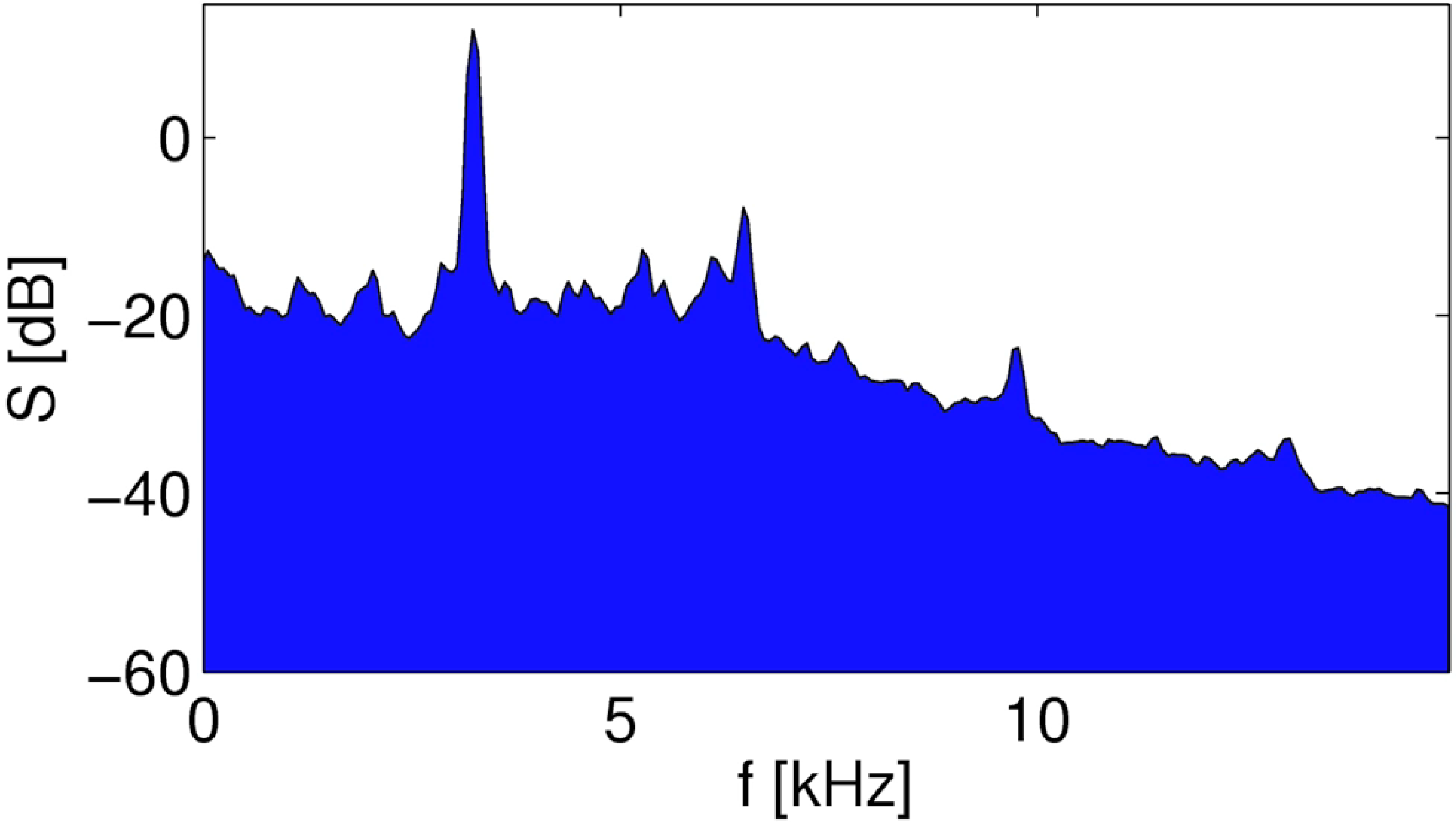
f-spectrum **spatiotemporal plot** **mode-f-spectrum**



$V_{ex} = 20$ V $f_{ex} = 8.4$ kHz $m_{ex} = 2$



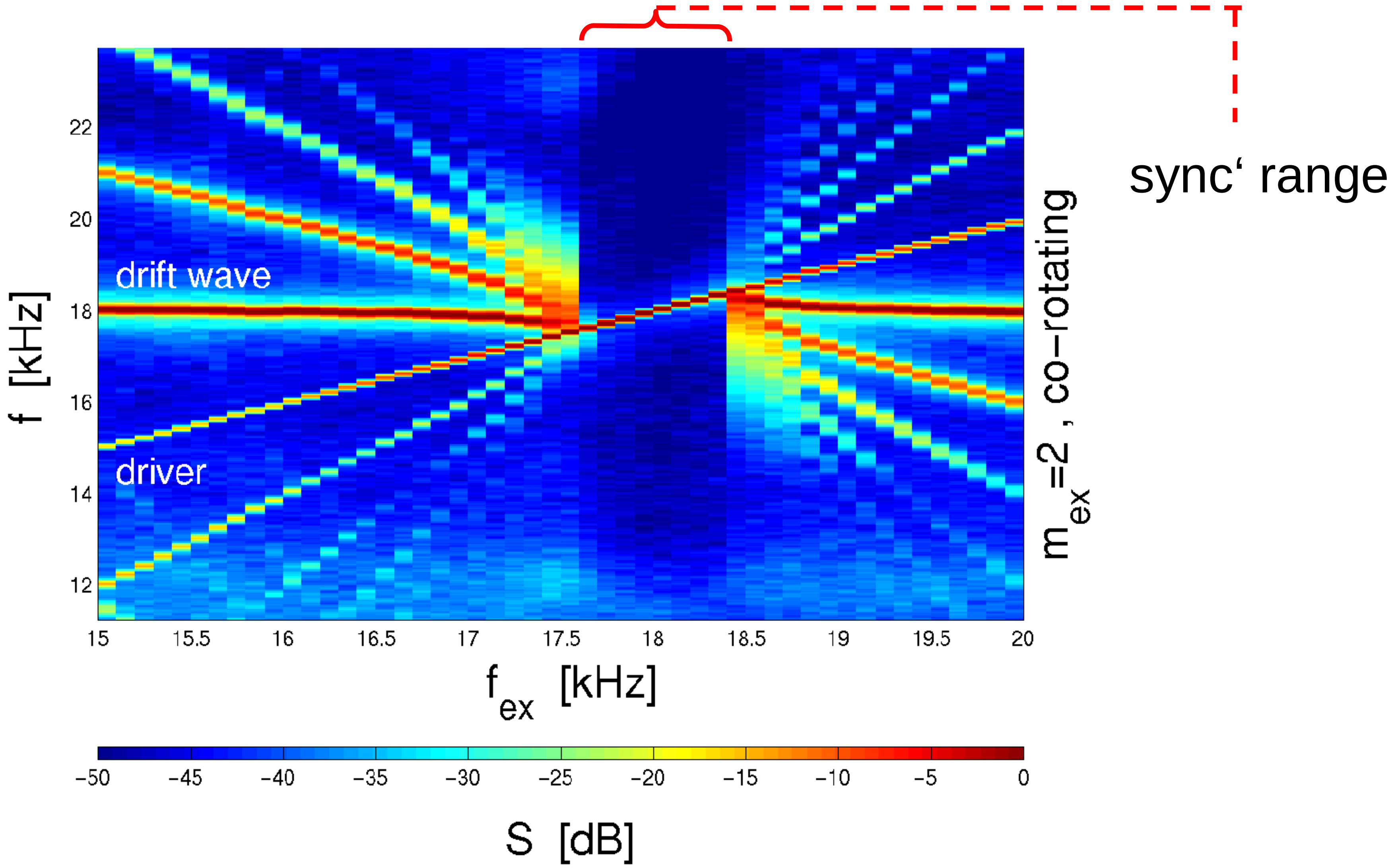
Single mode synchronisation



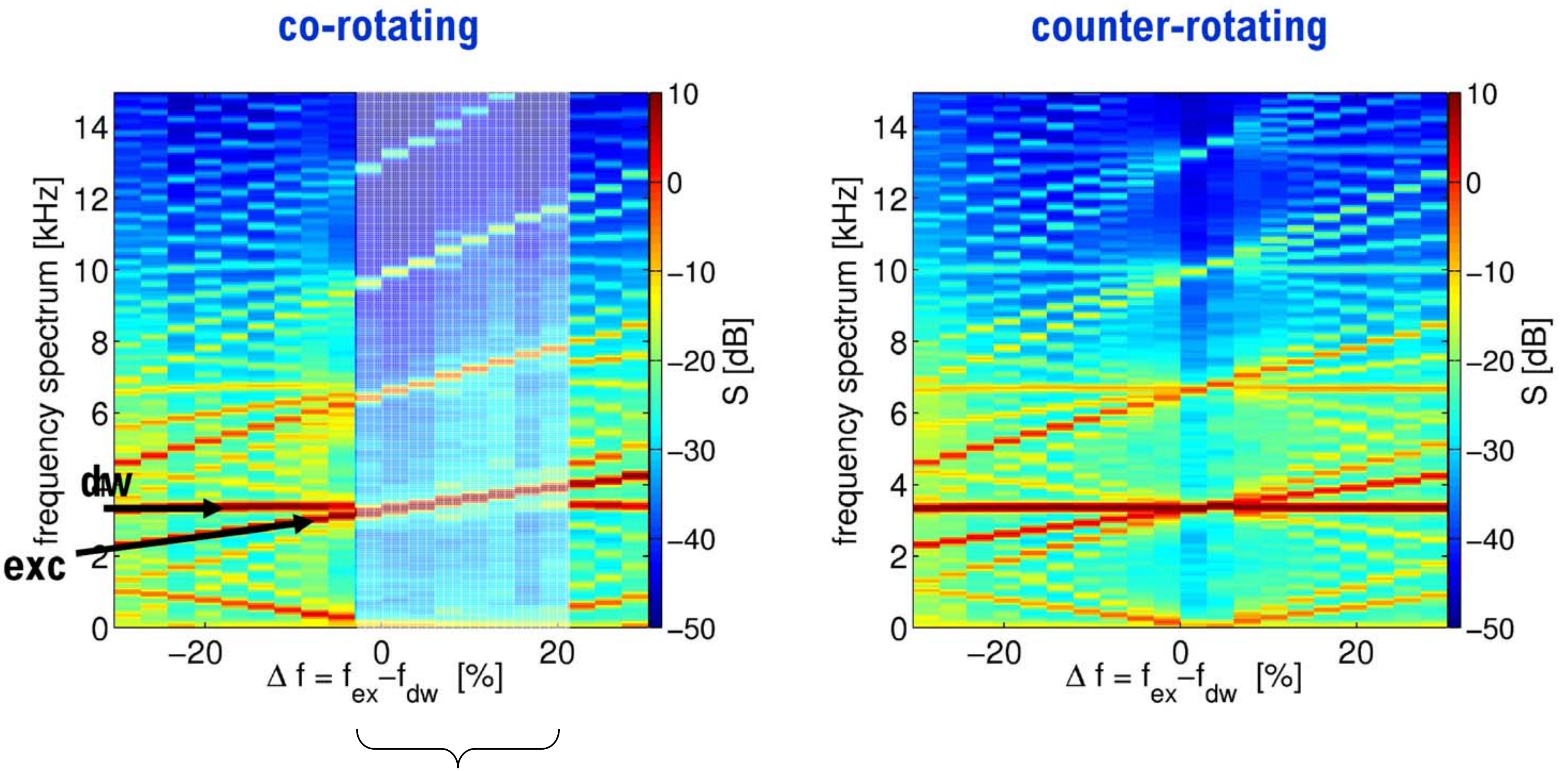
without external drive with external drive

movie

Single mode synchronisation

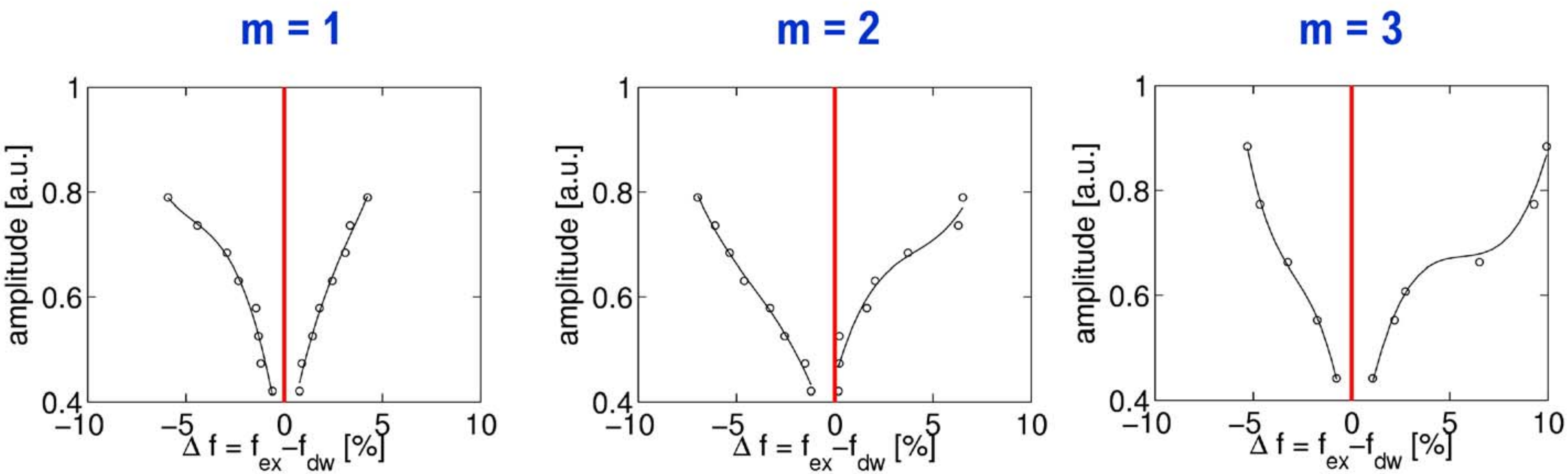


Single mode synchronisation



synchronisation range

Arnold‘d tongues



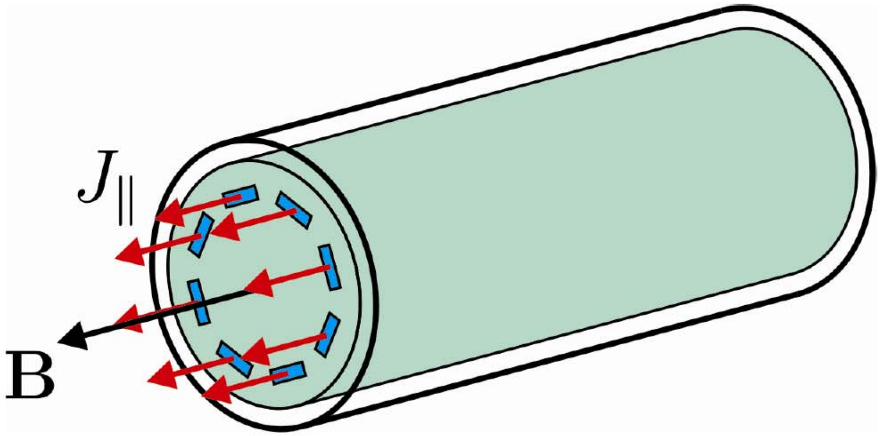
Summary of findings:

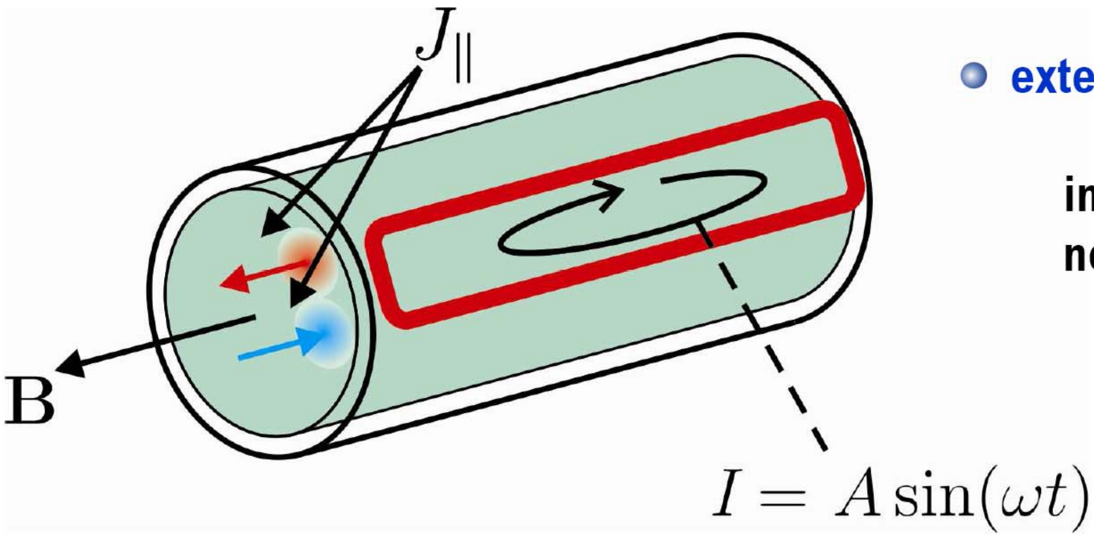
- **drift modes can be synchronised**
- **features very much like driven non-linear oscillator**
- **space-time modulation required**
- **mechanism: rotating || B current profile – at rest in wave frame**

Exciter schemes

- **electric exciter:**

  electrodes draw current
  direct contact with plasma





- **external magnetic field:**

  induction of parallel currents
  no contact with plasma

Conclusion

- **Drift waves are universal instabilities in magnetized plasmas**
- **Magnetic field geometry plays a significant role (not discussed)**
- **Linear space-time dynamics is well understood**
- **Non-linear models usually prodict fully developed turbulence**
- **Spatio-temporal chaos plays a role in the transition to turbulence**
- **Taming turbulence:**
    - **rotating electric (magnetic?) fields**
    - **synchronised drift mode on expense of turbulence**
    - **space-time oscillator behavior**

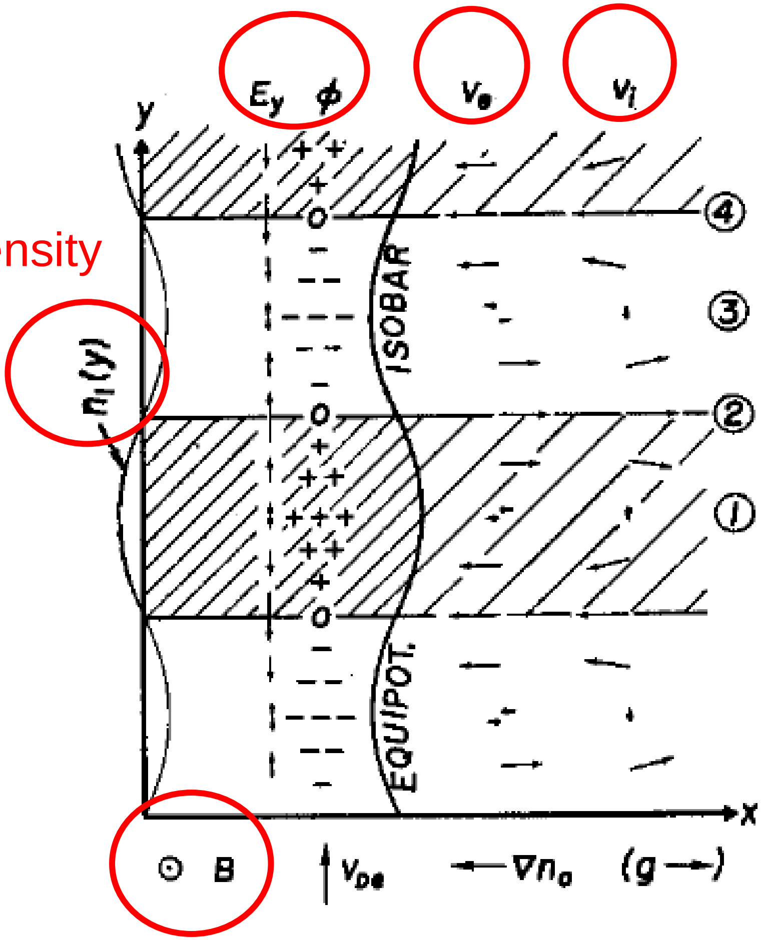
Credits to: O. Grulke, C. Schröder (MPI Greifswald); D. Block, A. Piel (U Kiel); G. Bonhomme (U Nancy); V. Naulin (Risoe); T. Dudok de Wit (U Orleans)

Drift wave basic elements

electric field
plasma potential

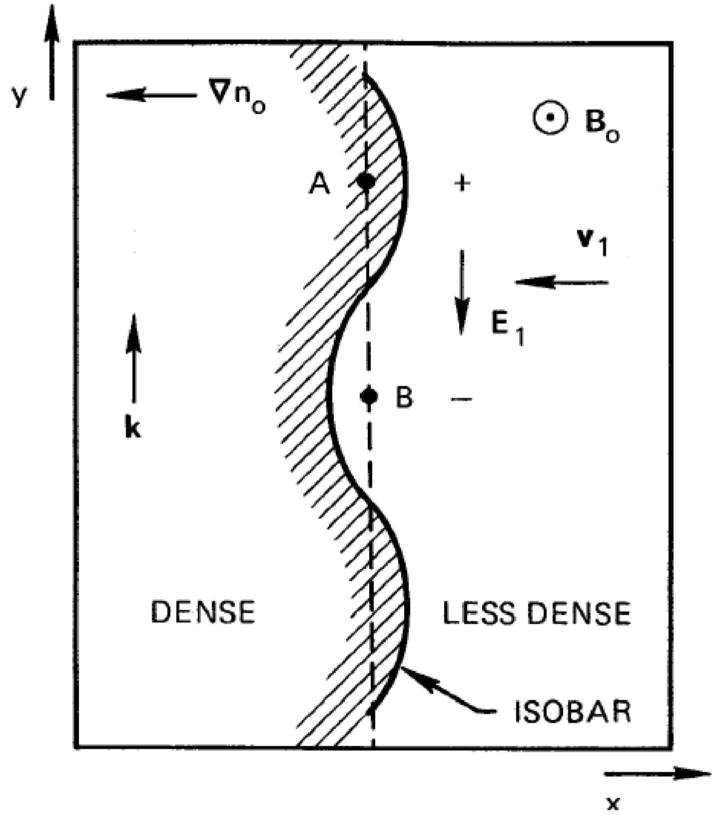
electron drift
ion drift

density



ambient magnetic field

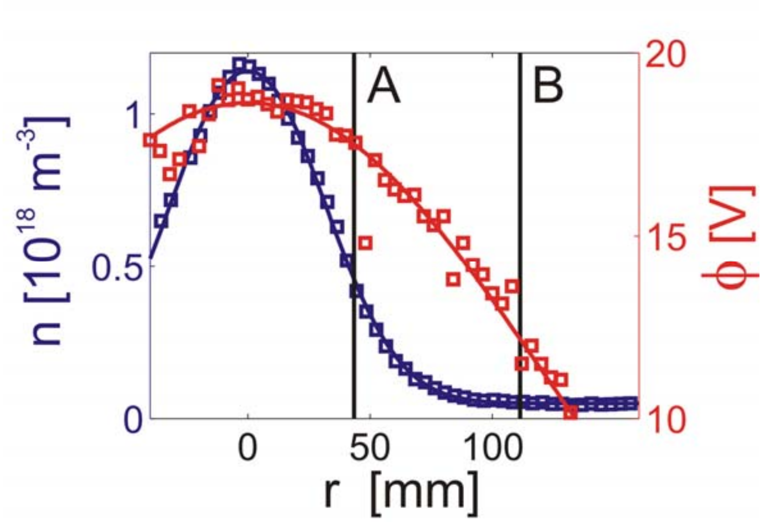
simplified diagram



Chen, Phys. Fluids 8(5), 912 (1965)

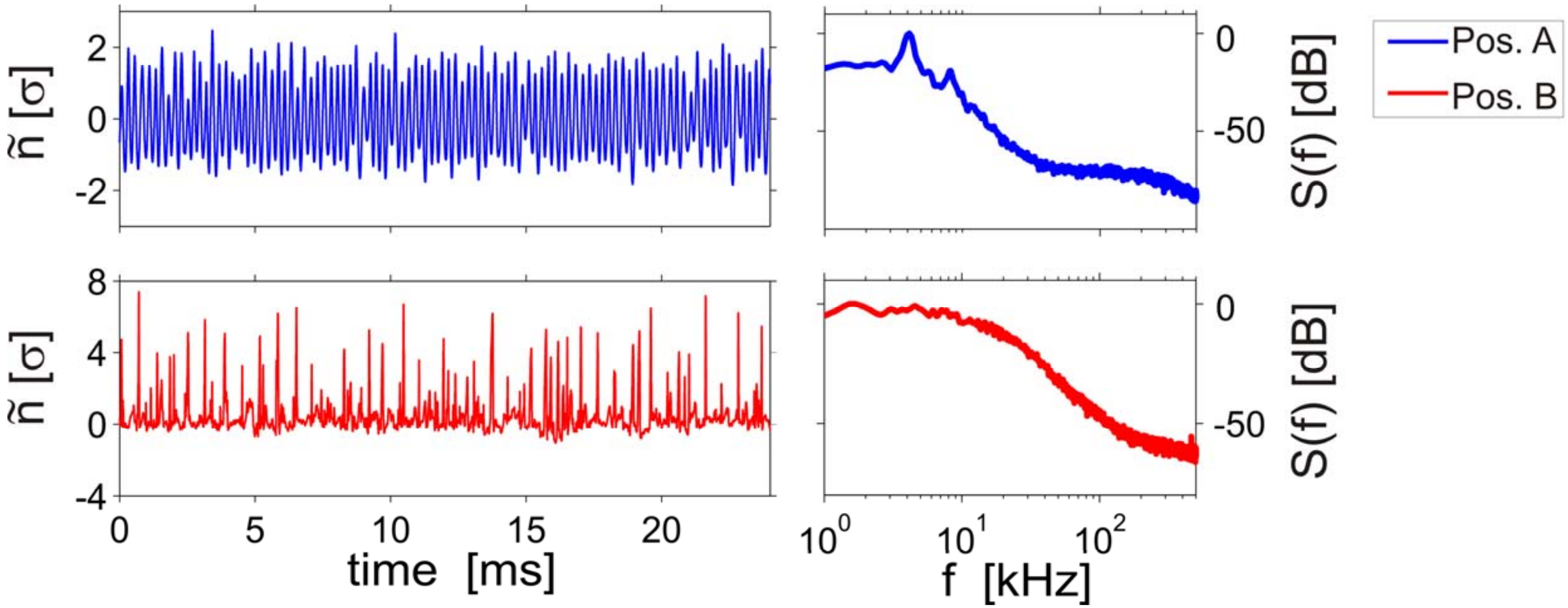
Chen, Introduction to Plasma Physics

Intermittency

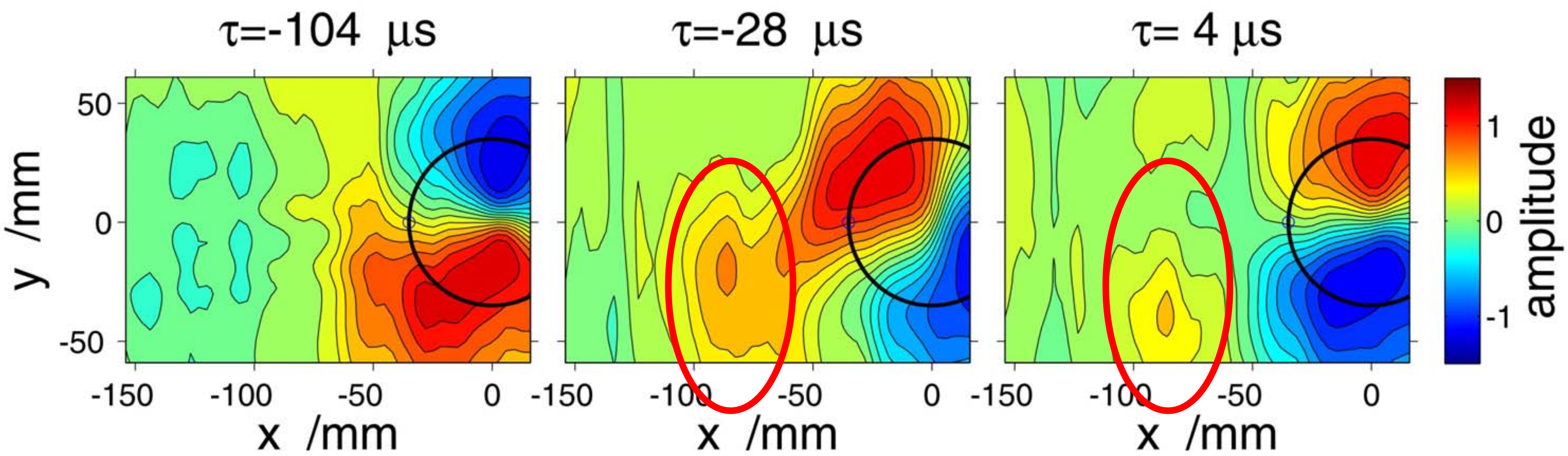


Observation:

- **quasi-coherent fluctuations in the gradient region**
- **strongly intermittent fluctuations in the far plasma edge**



Intermittency



- conditional correlation analysis
  used to reconstruct spatiotemporal dynamics
- quasi-coherent m=1 mode pattern dominates
- mode-coupling analysis (bicoherence) suggests
  inverse energy transfer
- plasma peels-off and is transported into edge region