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Electric Probes in Magnetized and Flowing Plasmas

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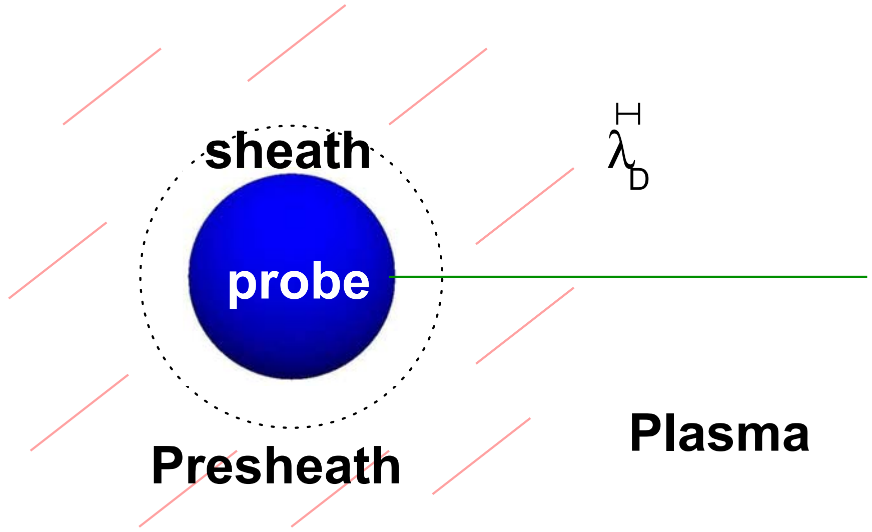
Electric (Langmuir) Probes

Probe in Plasma

Plasma strongly perturbed locally

Sheath: Quasineutrality is violated over a distance of about $4\lambda_D$.

Presheath: Density is perturbed over a bigger, quasineutral region: a few probe radii.



[Debye Length: $\lambda_D^2 = \epsilon_0 T / n_e e^2$.]

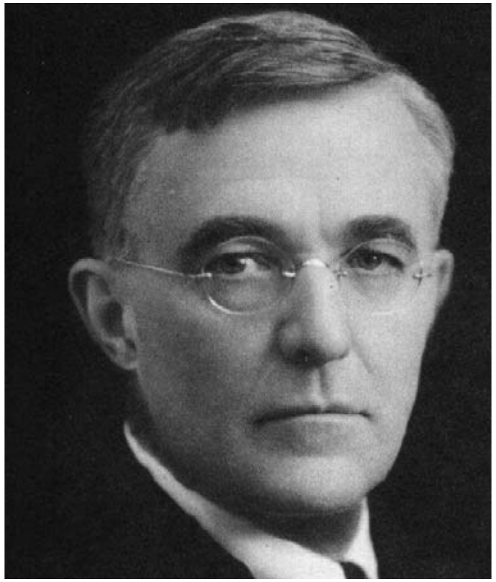
Varying voltage is applied to the probe through lead.

Current is a function of voltage applied.

Measurements are fairly easy/cheap.

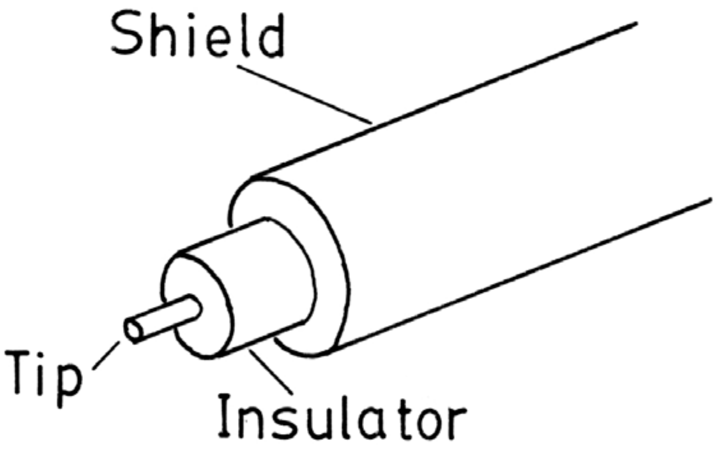
Interpretation/theory is quite difficult.

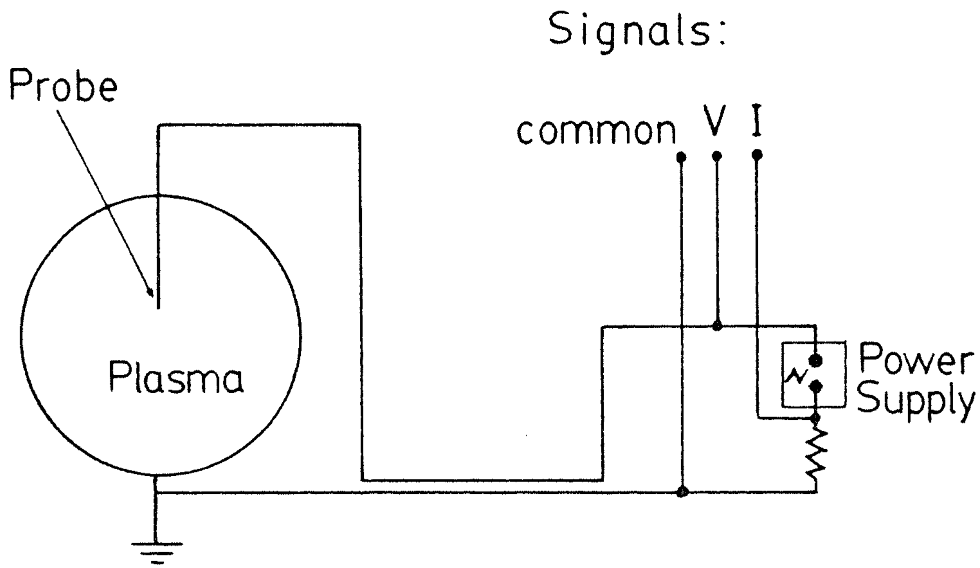
Irving Langmuir, who coined the name "Plasma":



http://www.ee.nmt.edu/~langmuir/pics/yunglang.jpg

Probe Implementation





Issues:

- Avoiding melting the probe!
- Defining cleanly the effective current collector size.
- Avoiding excessive perturbation/pollution of plasma.
- Unknown potential difference between plasma and wall/ground.
- RF rectification in RF plasmas.
- Practical construction of vacuum-compatible apparatus.
- Electronics and data acquisition/analysis [comparatively easy].

Currents to the probe

Total electric current *from* probe (made up of particles flowing *to* probe):

$$\underset{\text{current}}{\mathsf{I}} = \underset{\text{charge}}{\mathsf{e}}\ \underset{\text{area}}{\mathsf{A}}\ (\underset{\text{e flux density}}{\Gamma_e} - \underset{\text{i flux density}}{\Gamma_i})$$

Electrons are governed by a Boltzmann factor, provided they are mostly repelled. So

$$\boxed{\Gamma_e = \frac{1}{4}\bar{v}_e n_e = \sqrt{T_e/2\pi m_e}\, n_\infty \exp(e\phi_p/T_e)}$$

where ϕ_p is probe potential relative to unperturbed plasma in the vicinity of the probe, which is called "∞" (from probe's perspective).

Ion flux Γ_i, is much smaller than $\frac{1}{4}\bar{v}_e n_e$ because ions move slower by factor $\sqrt{m_e/m_i}$. Consequently, if we are drawing a current near zero, ϕ_p must be substantially negative, repelling electrons.

Also Γ_i is rather independent of ϕ_p (because attracted),

provided that $\lambda_D \ll$ probe size.

Call the ion current I_{si}. Ion saturation current.

Current/Voltage Characteristic

When repelling electrons:

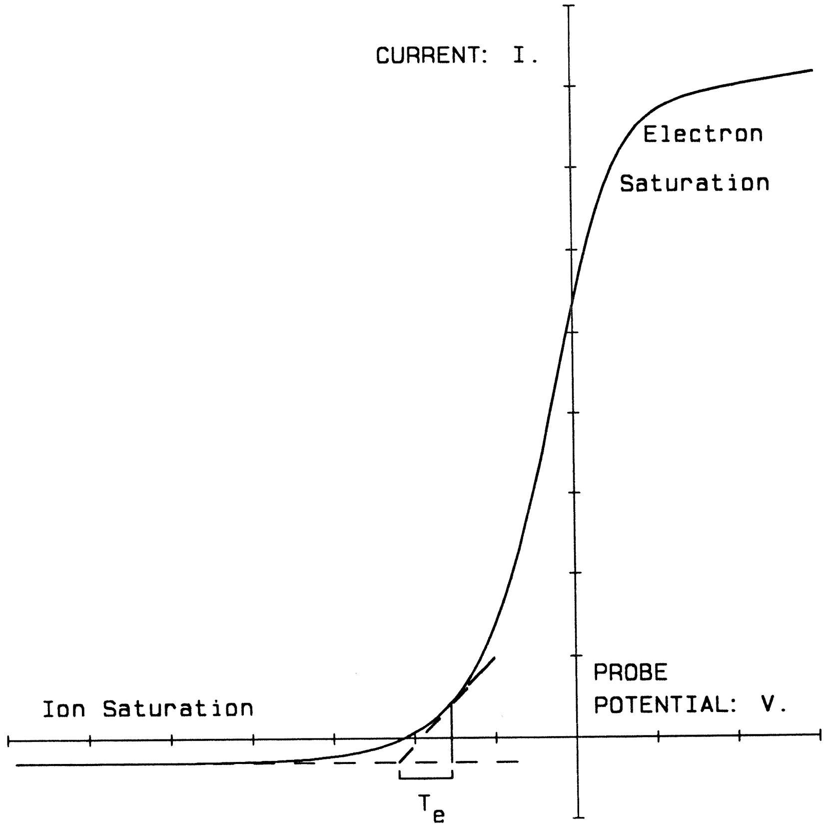
$(\mathsf{I}-\mathsf{I}_{si}) = eA\Gamma_e \propto \exp(e\phi_p/T_e)$.

Obtain electron temperature from slope of plot of

$$\boxed{\ln(\mathsf{I}-\mathsf{I}_{si}) = \underbrace{(e/T_e)}_{\text{slope}}\phi_p + \text{const.}}$$

or simple geometric construction.

(In region away from electron saturation).



T_e measurement relatively reliable because does not need absolute current.

But assumes electrons Maxwellian.

In some plasmas electrons have a distorting non-thermal tail on distribution function.

We get plasma **density** from absolute current magnitude (best) I_{si}.

This requires much tougher non-linear theory of sheath/presheath.

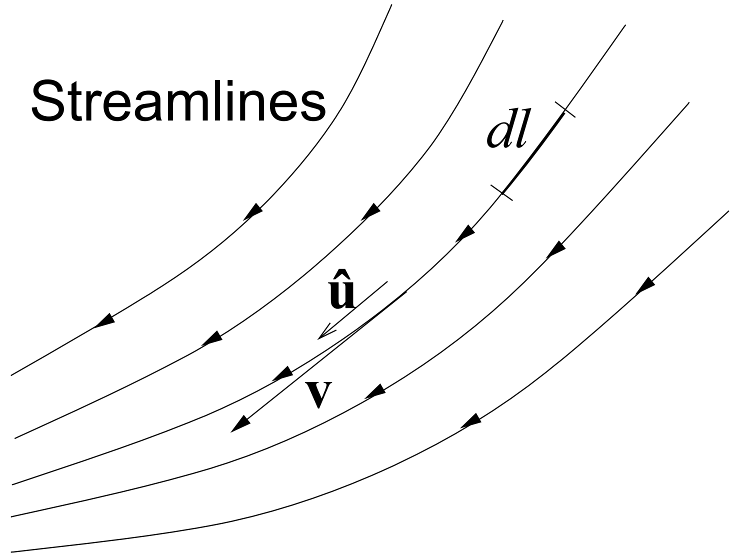
Fluid Theory of Presheath

Simplest equations of the ion fluid:

Continuity: $\nabla.(n\mathbf{v}) = S$ (particle source — may be zero)

Momentum: $\nabla.(nm_i\mathbf{v}\mathbf{v}) + qn\nabla\phi = m_i S_m$ (momentum source — ditto)

[Ignore ion pressure (and viscosity). Justified for $T_i \ll T_e$, if orbits don't cross.]



Write $\mathbf{v} = v\hat{\mathbf{u}}$,

so $\hat{\mathbf{u}}$ is the unit-vector for streamline.

Expand $\hat{\mathbf{u}}$.(Momentum equation) eliminating $\nabla.(n\mathbf{v})$ from Continuity.

$$n\ \hat{\mathbf{u}}.\nabla\left(\frac{v^2}{2} + \frac{q}{m_i}\phi\right) \quad + v^2\underbrace{\hat{\mathbf{u}}.[(\hat{\mathbf{u}}.\nabla)\hat{\mathbf{u}}]}_{\equiv 0} \quad = S_m + S\mathbf{v}$$

When sources S, S_m are zero, simply energy conservation $\frac{v^2}{2} + \frac{q\phi}{m_i} = \text{const.}$

Structure of the differential equations

Define the Mach Number $\mathbf{M} \equiv \mathbf{v}/c_s$, where $c_s^2 = qT_e/em_i$

and use Boltzmann electrons and quasineutrality: $n/n_\infty = \exp(e\phi/T_e)$, so that $\nabla\phi = (T_e/e)\nabla \ln n$. And write $\hat{\mathbf{u}}.\nabla \equiv d/d\ell$ as the derivative along streamline. Then equations become:

$$[\nabla.(n\mathbf{M}) =] \qquad n\frac{dM}{d\ell} + M\frac{dn}{d\ell} = nM\nabla.\hat{\mathbf{u}} + S/c_s$$

$$[n\hat{\mathbf{u}}.\nabla(M^2/2 + \ln n) =] \qquad nM\frac{dM}{d\ell} + \frac{dn}{d\ell} = (S_m + Sv)/c_s^2$$

Think of this as a matrix equation:

$$\underbrace{\begin{pmatrix} n & M \\ nM & 1 \end{pmatrix}}_{\text{matrix}} \begin{pmatrix} dM/d\ell \\ dn/d\ell \end{pmatrix} = \begin{pmatrix} nM\nabla.\hat{\mathbf{u}} + S/c_s \\ (S_m + Sv)/c_s^2 \end{pmatrix}$$

at the place where the determinant of the coefficient matrix is zero, $dM/d\ell$, $dn/d\ell$ become infinite. Quasi-neutral equations break down where $1 - M^2 = 0$ ($M = \pm 1$). This point is the sheath edge. $M = \pm 1$ is called the **Bohm Condition**:

- Ions enter the sheath at the sound speed.

Deduction of Sheath-Edge Values

Substitute Bohm velocity,

$$v = c_s$$

into energy equation[†],

$$\frac{v^2}{2} + \frac{q\phi}{m_i} = 0.$$

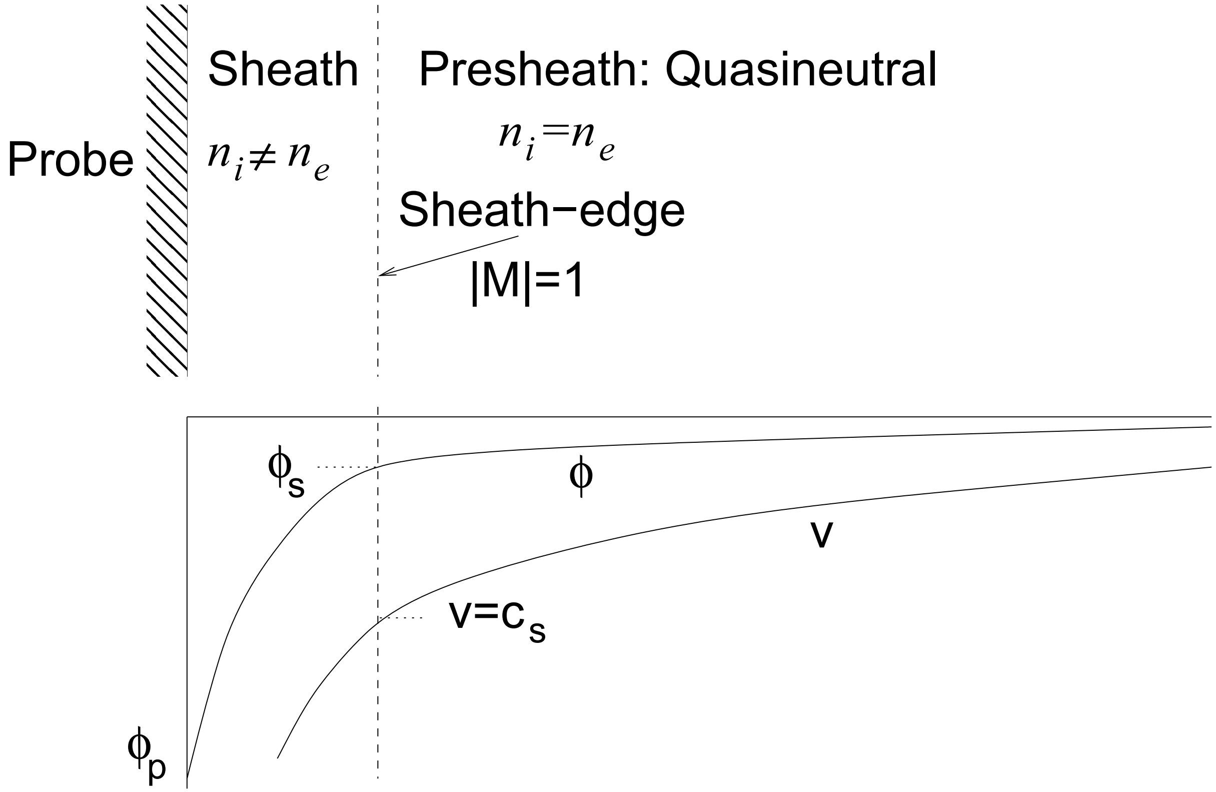
Hence derive

$$\phi_s = -T_e/2e$$

$$n_s = n_\infty \exp(-1/2)$$

Hence ion flux-density crossing sheath edge (and approx at probe):

Zoomed-in view of an assumed thin sheath



$$\boxed{\Gamma_i = n_s v_s = n_s M_s c_s = \exp(-\tfrac{1}{2}) n_\infty \sqrt{\frac{qT_e}{em_i}} \quad \text{Key standard result.}}$$

Deduce density from I_{si}: $\quad n_\infty = I_{si}/[qA \exp(-\frac{1}{2})\sqrt{\frac{qT_e}{em_i}}]$ $\quad$ (A is sheath/probe area*).

[T_i does not enter, even when finite, for unmagnetized probes.]

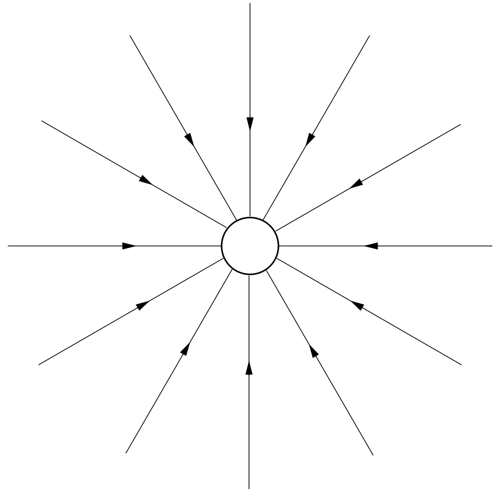
[† zero-source, hence collisionless. * perpendicular to v]

Need for sources or flow convergence

$$\begin{pmatrix} n & M \\ nM & 1 \end{pmatrix} \begin{pmatrix} dM/d\ell \\ dn/d\ell \end{pmatrix} = \underbrace{\begin{pmatrix} nM\nabla.\hat{\mathbf{u}} + S/c_s \\ (S_m + Sv)/c_s^2 \end{pmatrix}}_{\text{source vector}}$$

To have a non-uniform solution, there must be a non-zero source vector (RHS).

Therefore if S and S_m are zero, we require non-zero $\nabla.\hat{\mathbf{u}}$.
For spherical or cylindrical geometry, no problem.
A simple solution exists.



Planar (slab) geometry has no convergence. Hence no meaningful quasineutral 1-d planar source-free soln.

There is no such thing as a collisionless planar electric probe.
[Finite plane size always matters.]

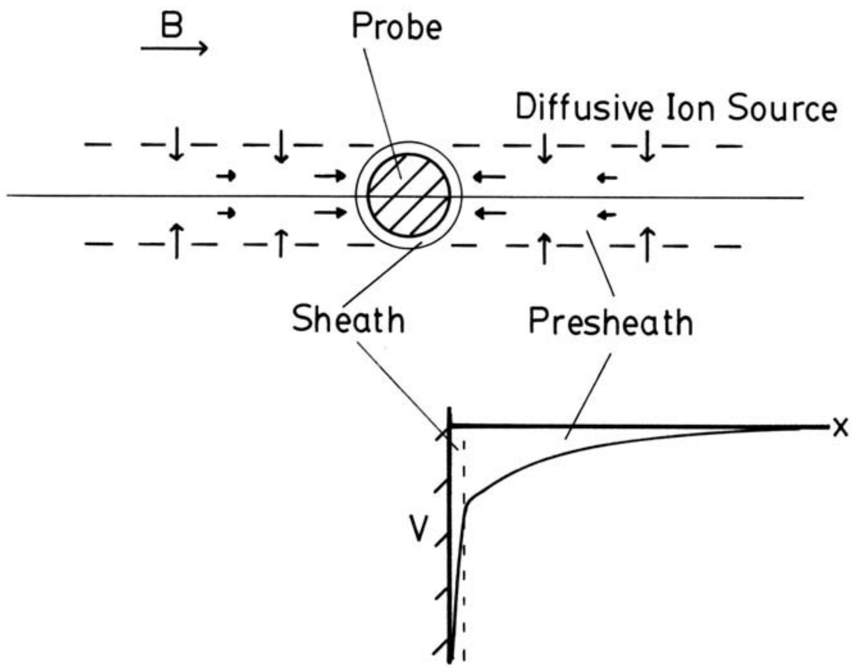
Magnetized plasmas have practically one-dimensional ion dynamics, along the field.
This paradox determines the physics of magnetized probes.

Magnetized Probes

When Ion Larmor Radius (ρ_i) $\ll$ Probe Radius (a),
ions move predominantly along field, not across it. They are almost 1-D.

Result: Presheath highly elongated along the field, until

parallel flow to probe is made up by small perpendicular cross-field divergence.



Parallel dynamics governed by same fluid equations. Cross-field gives sources.

$S = -\nabla_\perp.(n\mathbf{v}_\perp)$

$S_m = -\nabla_\perp.(n\mathbf{v}_\perp v_\|) + \overbrace{\nabla_\perp.(\eta\nabla_\perp v_\|)}^{\text{shear viscosity}} / m_i$

Assume heuristic diffusion: $n\mathbf{v}_\perp = -D\nabla_\perp n$, and approx $\nabla_\perp \to 1/a$, then

$$\begin{pmatrix} n & M \\ nM & 1 \end{pmatrix} \begin{pmatrix} dM/d\ell \\ dn/d\ell \end{pmatrix} = \frac{n_\infty D}{a^2 c_s} \begin{pmatrix} 1 - n/n_\infty \\ (M_\infty - M)(1 - n/n_\infty + \alpha) \end{pmatrix}$$

where $\alpha = \eta/(n_\infty m_i D)$ is ratio of diffusivity of particles/momentum (~ 1).
Parameter $a^2 c_s/D$ determines presheath scale length, but not flux.

Flux is Along the Field

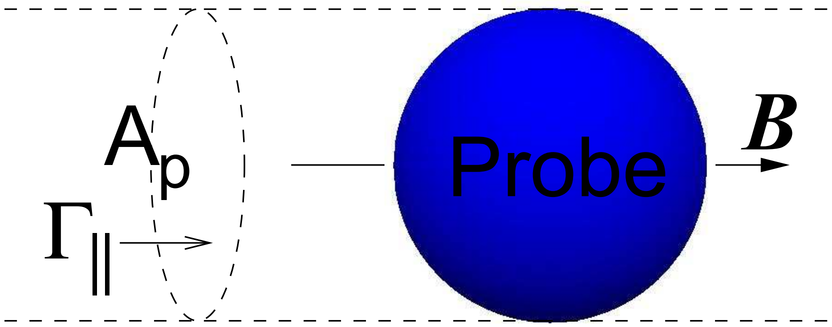
Coefficient matrix is unchanged, and so is the Bohm condition.

But it is the **parallel** velocity that reaches $c_s \equiv \sqrt{(ZT_e + T_i)/m_i}$ at sheath edge.

The **parallel** ion flux density is then $\quad \Gamma_{\|} = n_s c_s = f\ n_\infty c_s$.

And the ion current to each probe face is

$$\boxed{I_{si} = q \underbrace{A_p}_{\text{projected area}} f\ n_\infty c_s\ .}$$



But because there are sources, energy conservation does not apply, and the coefficient f is no longer $\exp(-1/2)$. Greater potential drop is required.

It has to be found by solving (numerically) the differential equations.

It depends on the parallel velocity in the external plasma: M_∞.

A good fit to the numerical results (with $\alpha = 1$) is $\quad \boxed{f = \exp(-1-1.1M_\infty)}$

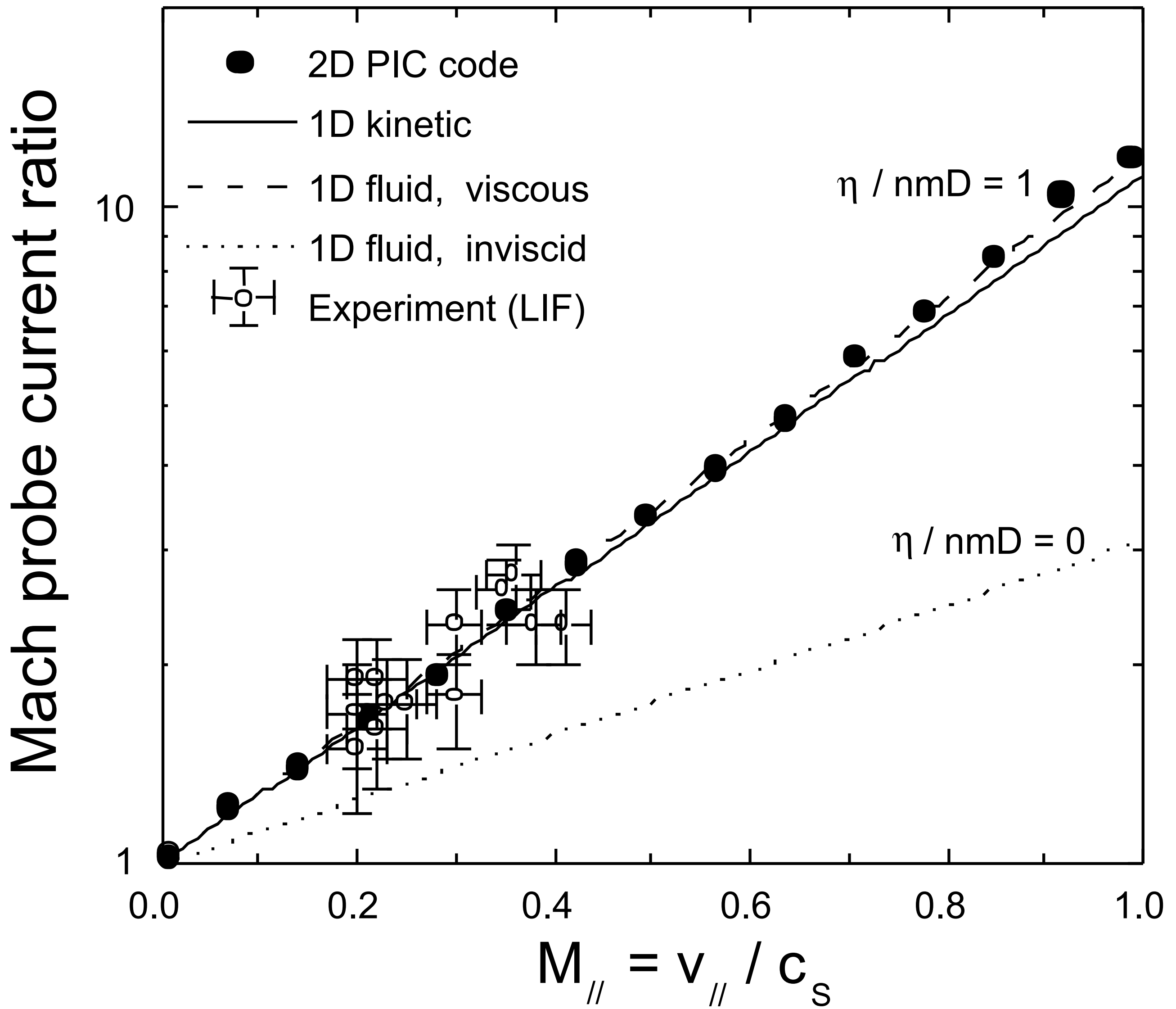
There is more flux to the upstream side than to the downstream side.

If we measure the ratio, that tells us M_∞: $\quad \boxed{\Gamma_{\|up}/\Gamma_{\|down} = R = \exp(M_\infty/M_c).}$

Where $M_c = 0.45$ is the calibration factor.

Independent Velocity Measurements

using laser induced fluorescence confirm the calibration



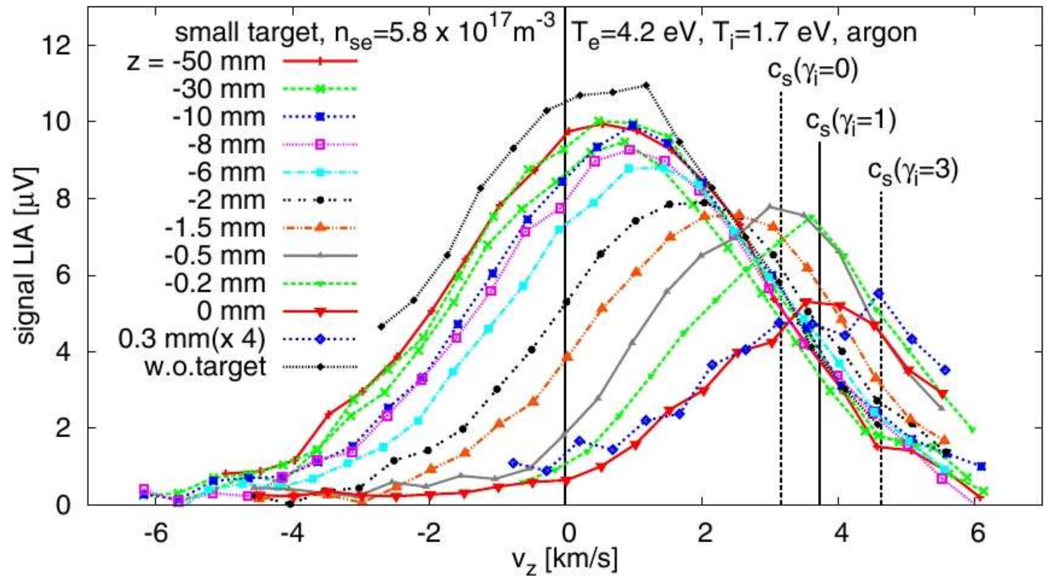
Show that using $\alpha = 1$ gives decent agreement with experiment.

Also, kinetic calculations give very similar calibration, showing that the fluid approximations are accurate.

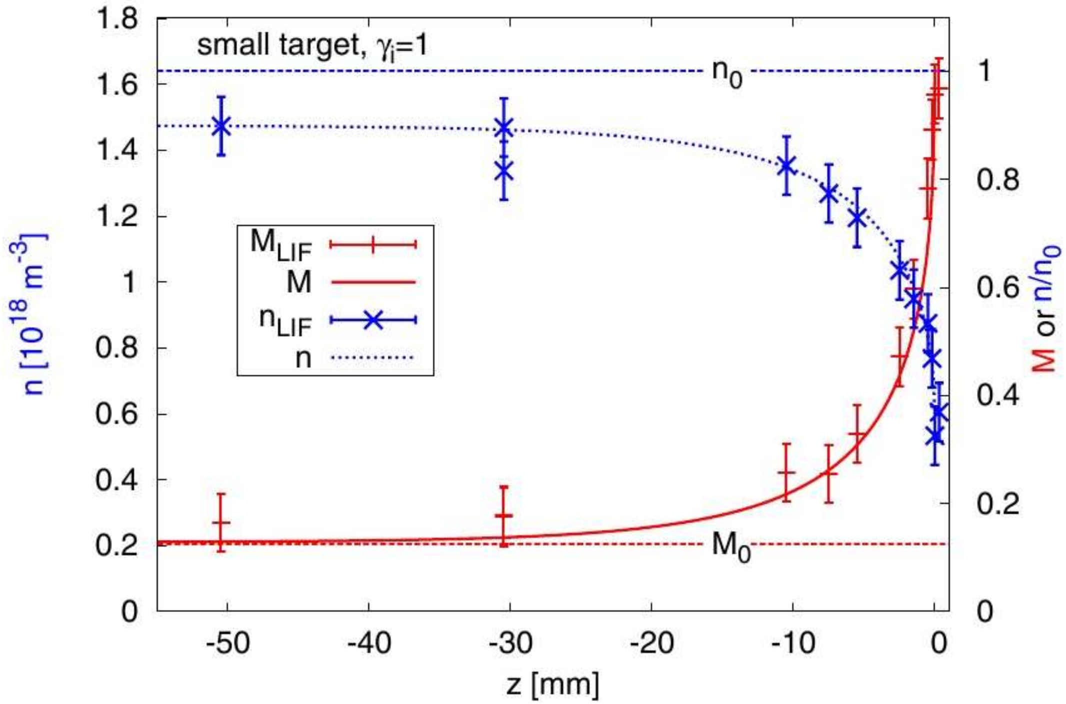
Gunn et al, Phys Plasmas, 8, 1995 (2001)

Spatial Profile of Presheath Confirmed

LIF measurements of the ion parallel distribution function at various parallel distances from a probe.



The spatial variation of the density and velocity, as a function of distance from the probe, agrees with the theory, giving confidence in the physics. But the diffusivity is a free fitting parameter.



Lunt, Fussmann & Waldeman, PRL, 100 (2008)

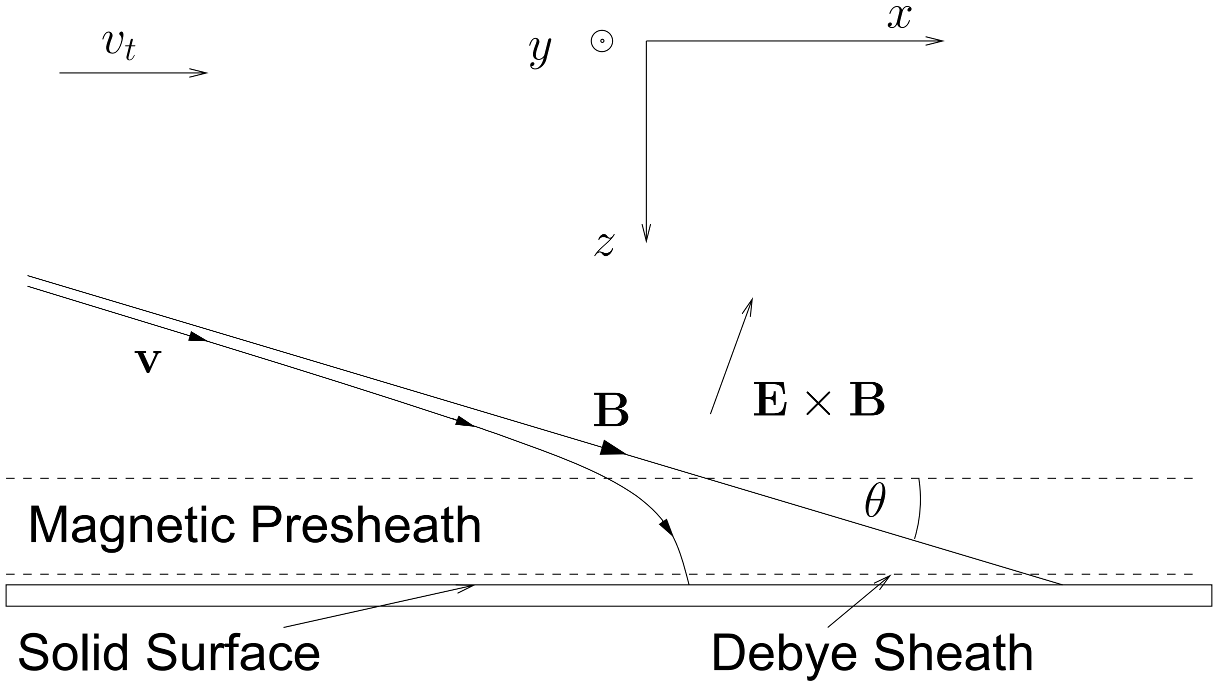
Oblique Impact Angle

Causes formation of additional Magnetic Presheath

With oblique impact, a **magnetic presheath** arises. Ions enter it with $v_{||} = c_s$.

In this region, about ρ_i thick, ions accelerate across the magnetic field.

Their velocity perpendicular to surface reaches c_s at the Debye sheath edge.



One can show by Galilean transformation that if there is a background perpendicular $\mathbf{E} \wedge \mathbf{B}$ drift, $M_\perp$, it causes the ion flux collection coefficient to become

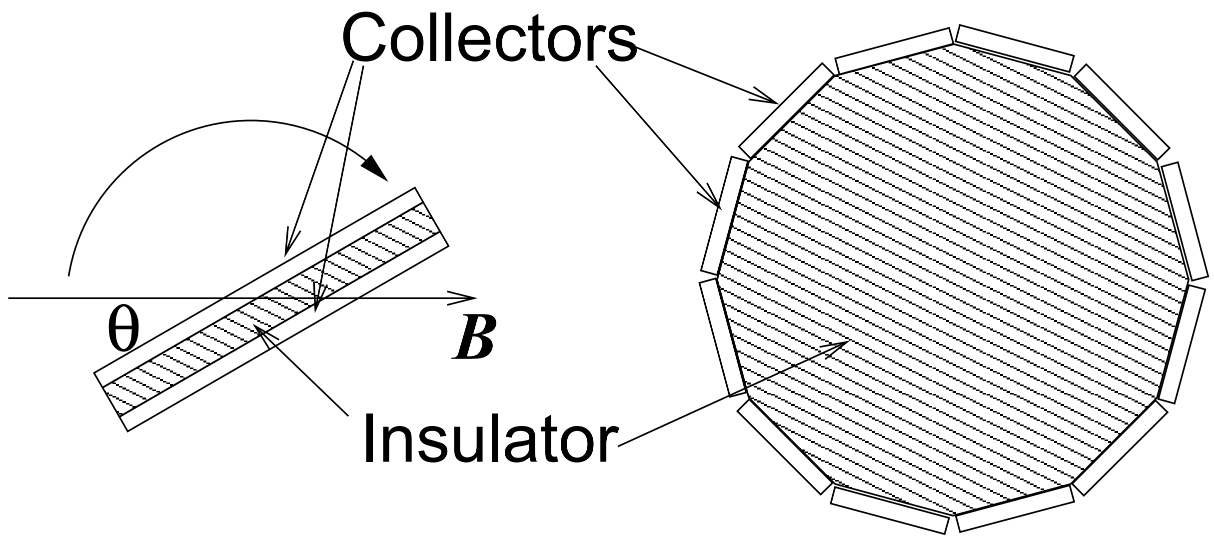
$$\boxed{f_d(M_{||\infty}, M_\perp) = f(M_{||\infty} - M_\perp \cot\theta) = \exp[-1 - 1.1(M_{||\infty} - M_\perp \cot\theta)]}$$

Thus in principle, by measuring at different θ, we may deduce $M_\perp$ as well as $M_{||}$.

Mach Probes Measure flow

by measuring separately the upstream and downstream fluxes to deduce M_∞.

Electrodes facing in different directions collect different ion flux densities, depending on the external plasma flow.



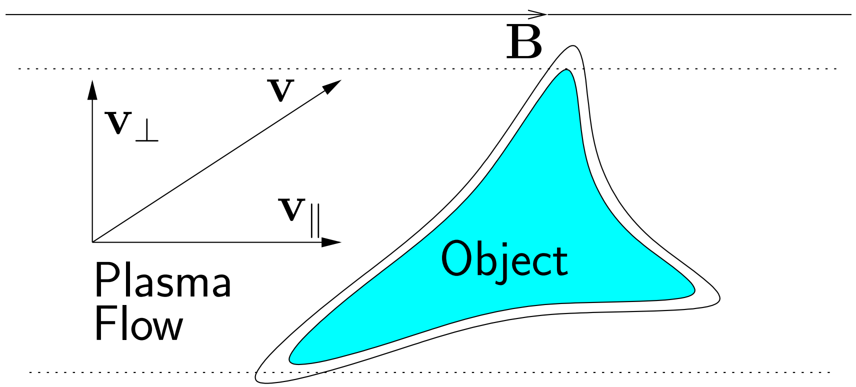
External pre-existing plasma flow or object motion

Flow past the object is some combination of parallel and perpendicular ion drifts.

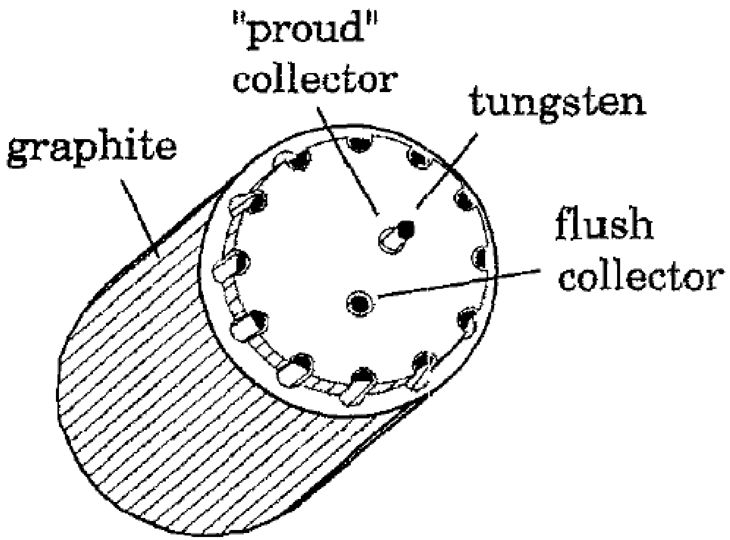
Large flows in tokamak scrape-offs.

Plasma flow $\equiv$ Object motion

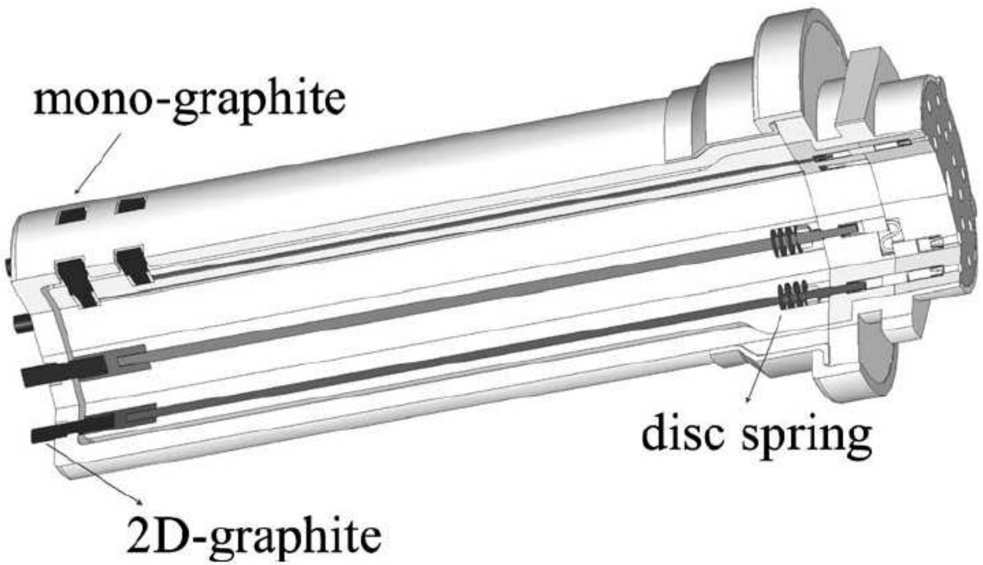
Space shuttle surface-charging



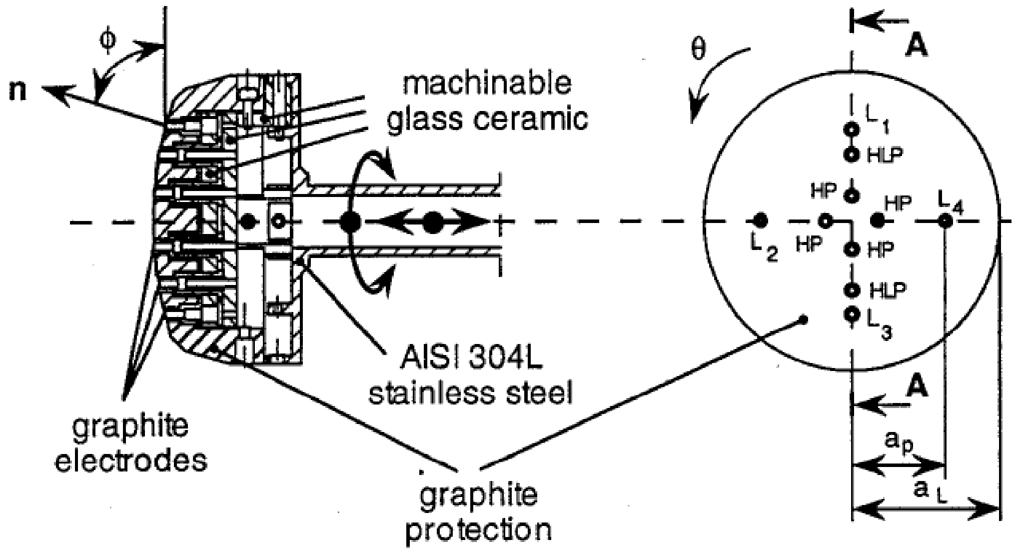
Practical Mach probe shapes complicated



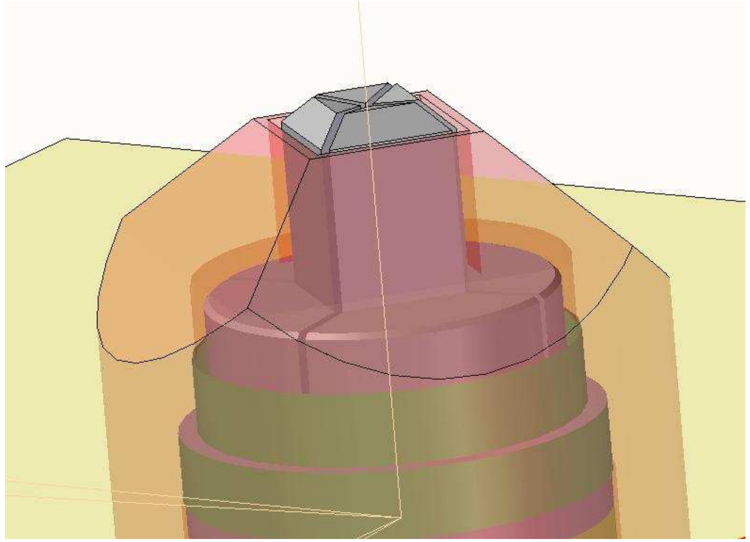
MacLatchy et al 1992 Gundestrup



Peleman et al 2006 (Tokamak)



Antoni et al 1996 (RFP)



Smick and LaBombard 2008

Turbulence scales larger than probe

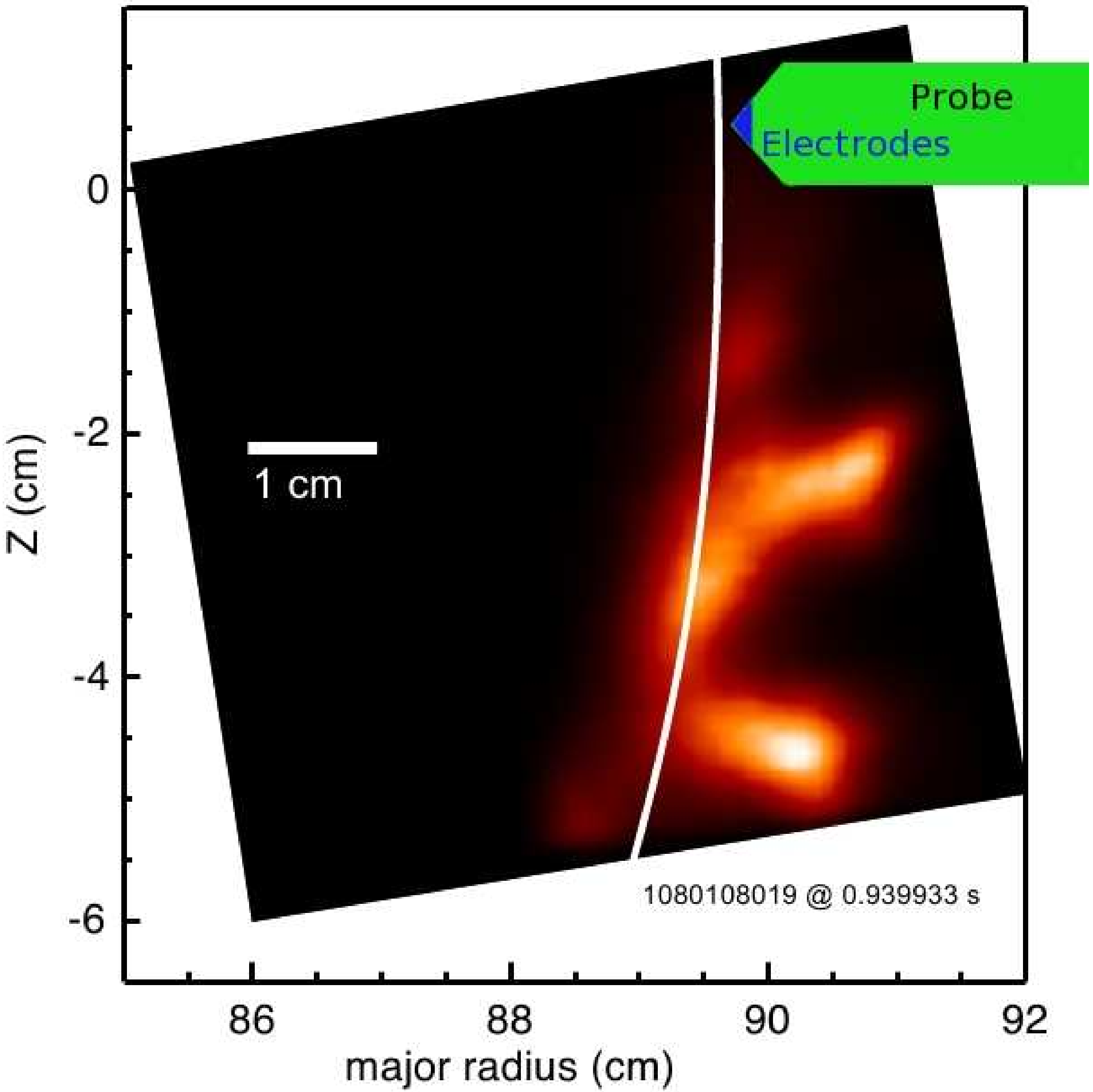
C-Mod scanning probe head (schematic)

compared in scale with

Example of imaged turbulence eddies in SOL.

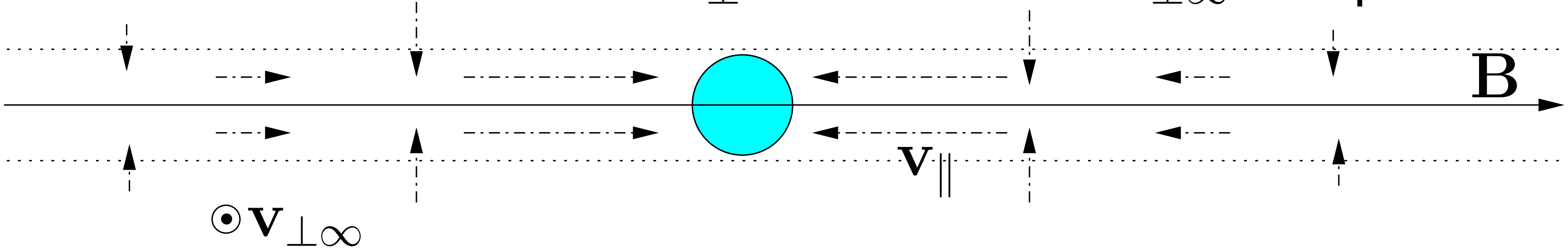
Probe body is comparable in size.

Probe electrodes are smaller.



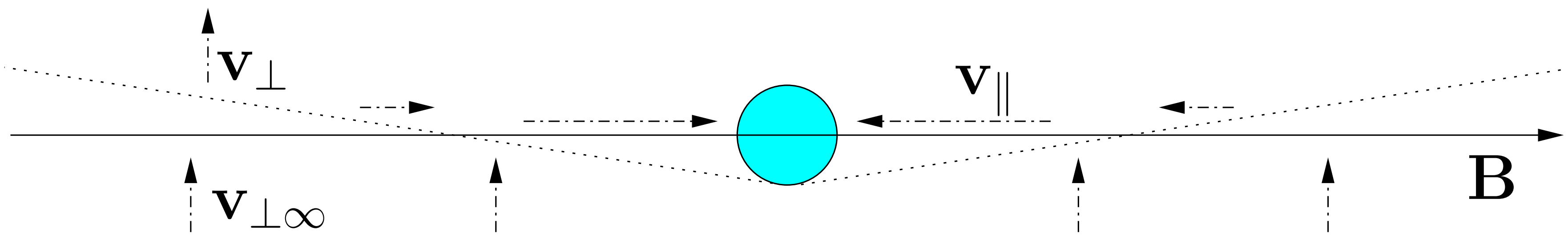
Rigorous Convective Treatment

Old: Heuristic Diffusion $\mathbf{v}_\perp = D\nabla \ln n + \mathbf{v}_{\perp\infty}$. Elliptic.



External $n = n_\infty$, $M_\parallel = M_{\parallel\infty}$

New: Rigorous Convection: $\mathbf{v}_\perp(\mathbf{x})$ solved. Hyperbolic.



Instead of invoking heuristic cross-field diffusion, **ignore diffusion**.

Treat pure imposed transverse-drift, solve self-consistently, no heuristics.

More satisfactory physics. Rigorous mathematics. Analytic solution!

Result consistent. Dependence on diamagnetic drifts discovered.

Hyperbolic system. Analyze by "Characteristics"

Define $c_s^2 = (ZT_e + T_i)/m$ and $\mathbf{M} = \mathbf{v}/c_s$: Mach Number. Eliminate ϕ. Then 3-D, anisotropic ion fluid equations become

$$\mathbf{M}.\nabla \ln n + \nabla_{\|} M_{\|} = 0$$

$$\mathbf{M}.\nabla M_{\|} + \nabla_{\|} \ln n = 0$$

which can be rearranged to display explicitly the characteristics.

$$(\mathbf{M}.\nabla + \nabla_{\|})(\ln n + M_{\|}) = 0$$

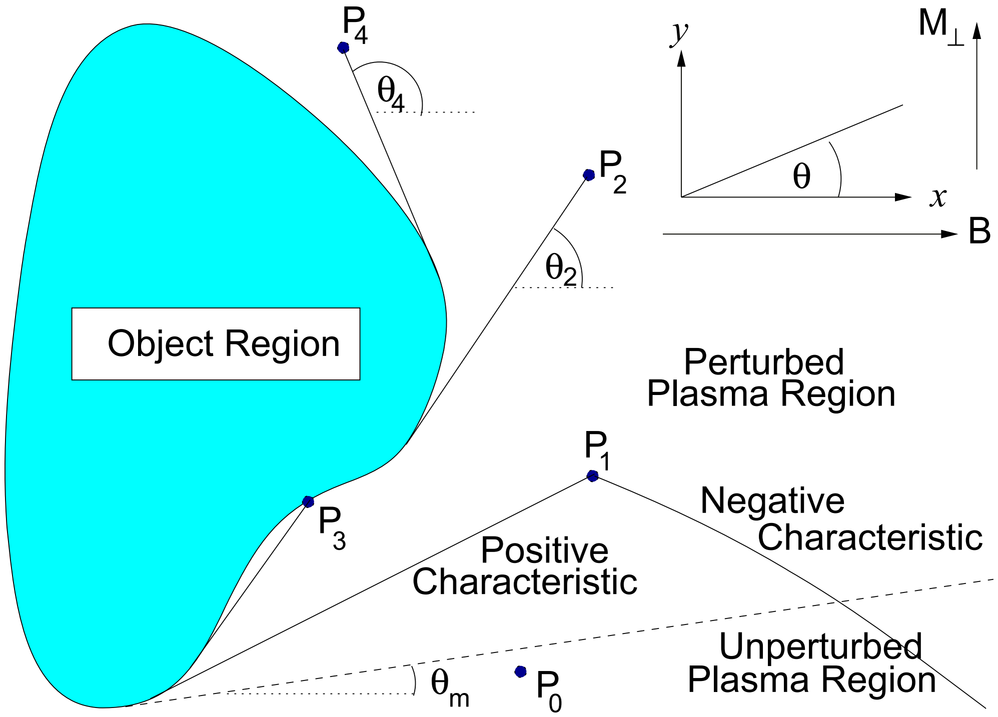
$$(\mathbf{M}.\nabla - \nabla_{\|})(\ln n - M_{\|}) = 0.$$

Thus the quantities $(\ln n \pm M_{\|})$
are constant along their corresponding characteristics: $d\mathbf{x} = (\mathbf{M} \pm \mathbf{B}/B)ds$.

Integration along characteristics is the secret to solving hyperbolic systems.

Solution by Characteristics

Assume $\mathbf{M}_\perp$ simply uniform (in y-direction). Take $\mathbf{B}$ along x-axis. 2-D problem. Analyse the higher-x side of the object. Two characteristics pass through any point.



Can show rigorously* that

$$\mathrm{M}_{\|} = \text{const}, \ \ln \mathrm{n} = \text{const}.$$

along straight +ve characteristics, that are tangent to object.
And that solution depends only on the slope of that characteristic

$$\mathrm{M}_{\|} = \mathrm{M}_\perp \cot\theta - 1$$
$$\ln \mathrm{n} = \ln \mathrm{n}_\infty - \mathrm{M}_{\|} + \mathrm{M}_{\|\infty}$$

The ion flux to the object at the plasma boundary, per unit perpendicular area is

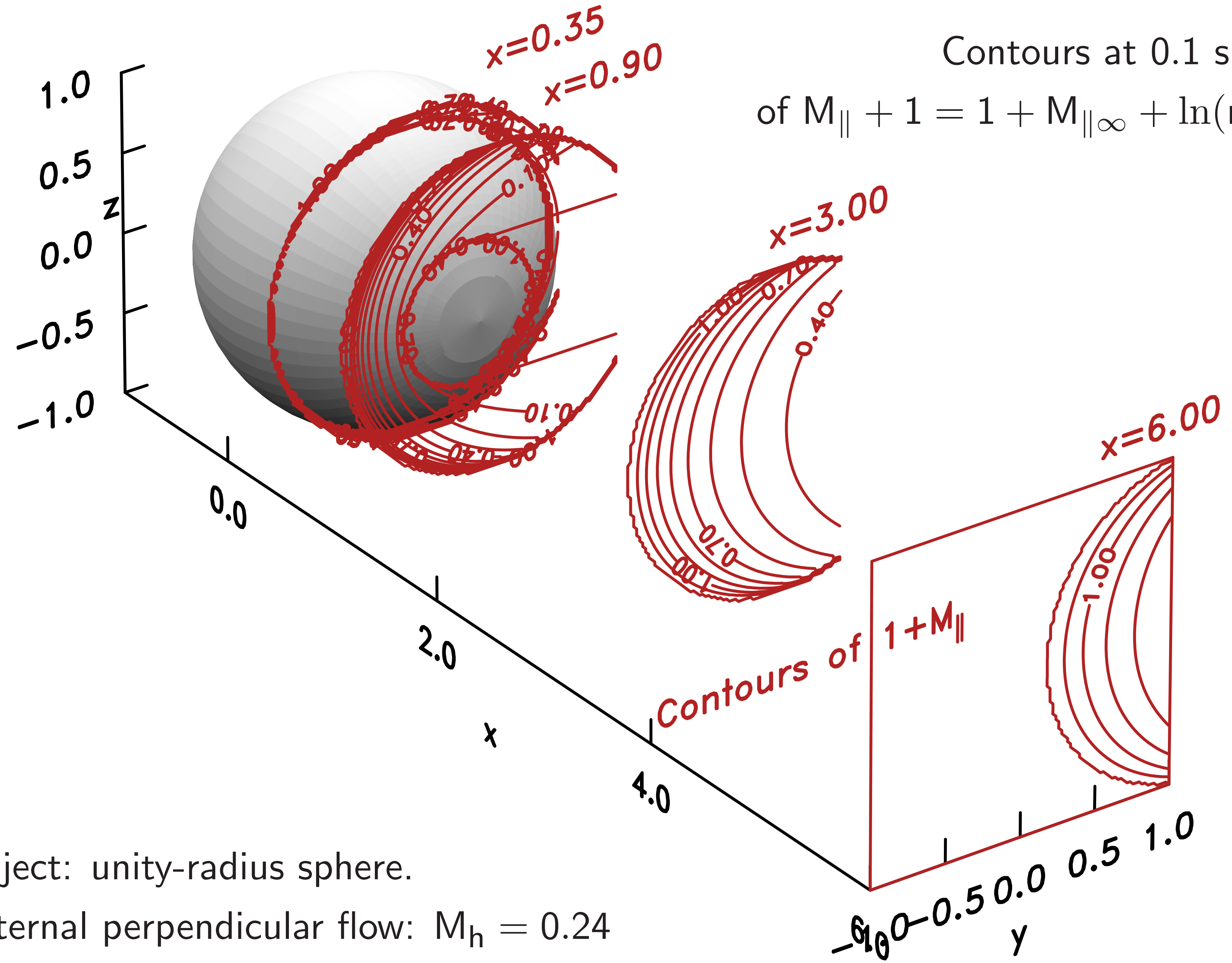
$$\Gamma_{\|} = \mathrm{n}c_s = \mathrm{n}_\infty c_s \exp(-1 - \mathrm{M}_{\|} + \mathrm{M}_\perp \cot\theta)$$

with θ the local surface tangent (in convex regions, not at P_3).

*I H Hutchinson Phys Rev Lett 101, 035004 (2008)

Example 3-D Solution: Sphere

Contours at 0.1 spacing
of $M_{\|} + 1 = 1 + M_{\|\infty} + \ln(n/n_{\infty})$



Object: unity-radius sphere.

External perpendicular flow: $M_h = 0.24$

External parallel flow: $M_{\|\infty} = 0.2$

Diamagnetic drift along contour lines

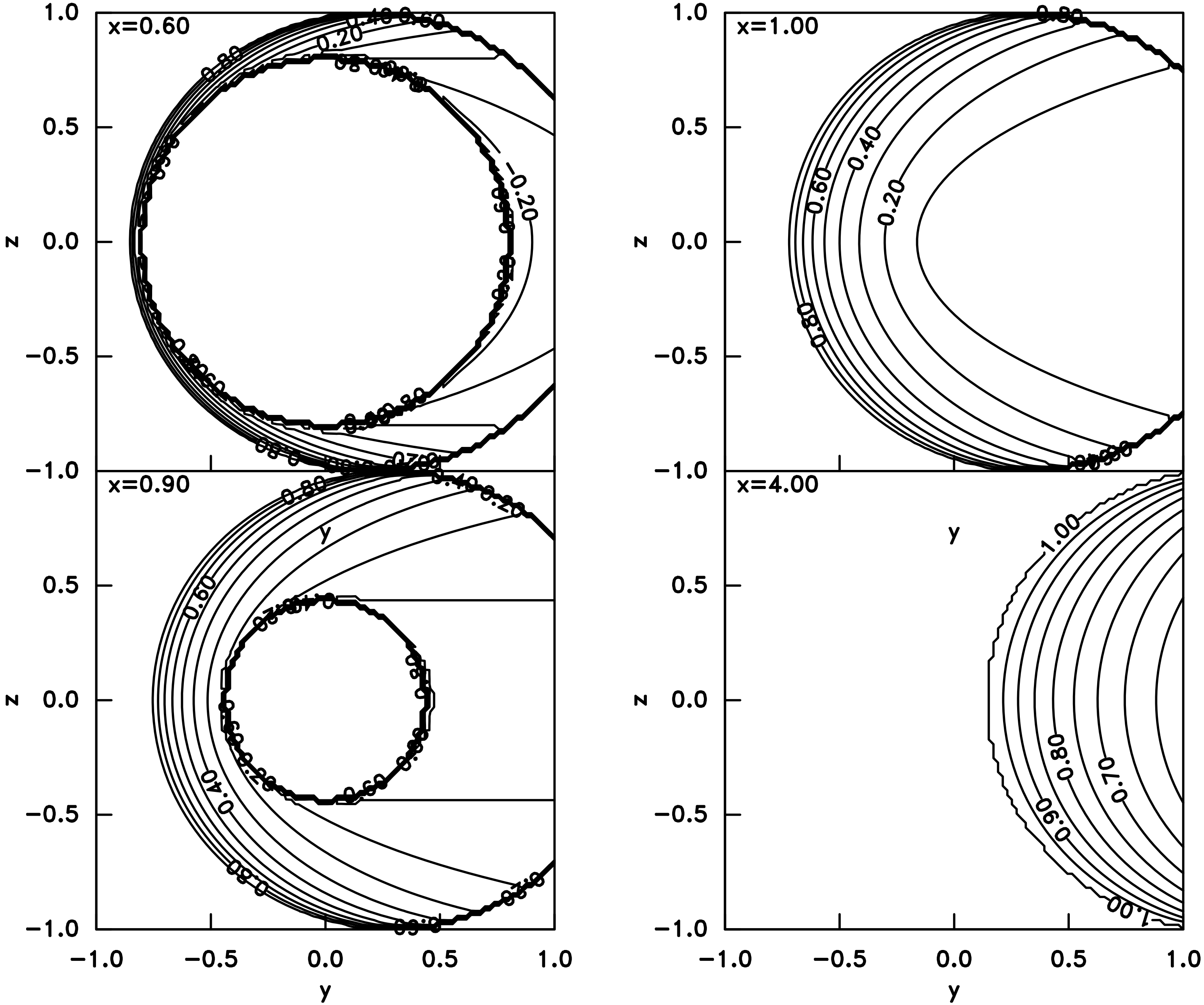
Sphere.

$M_{\parallel\infty} = 0$

$M_h = 0.3$

$M_\parallel + 1$ contours

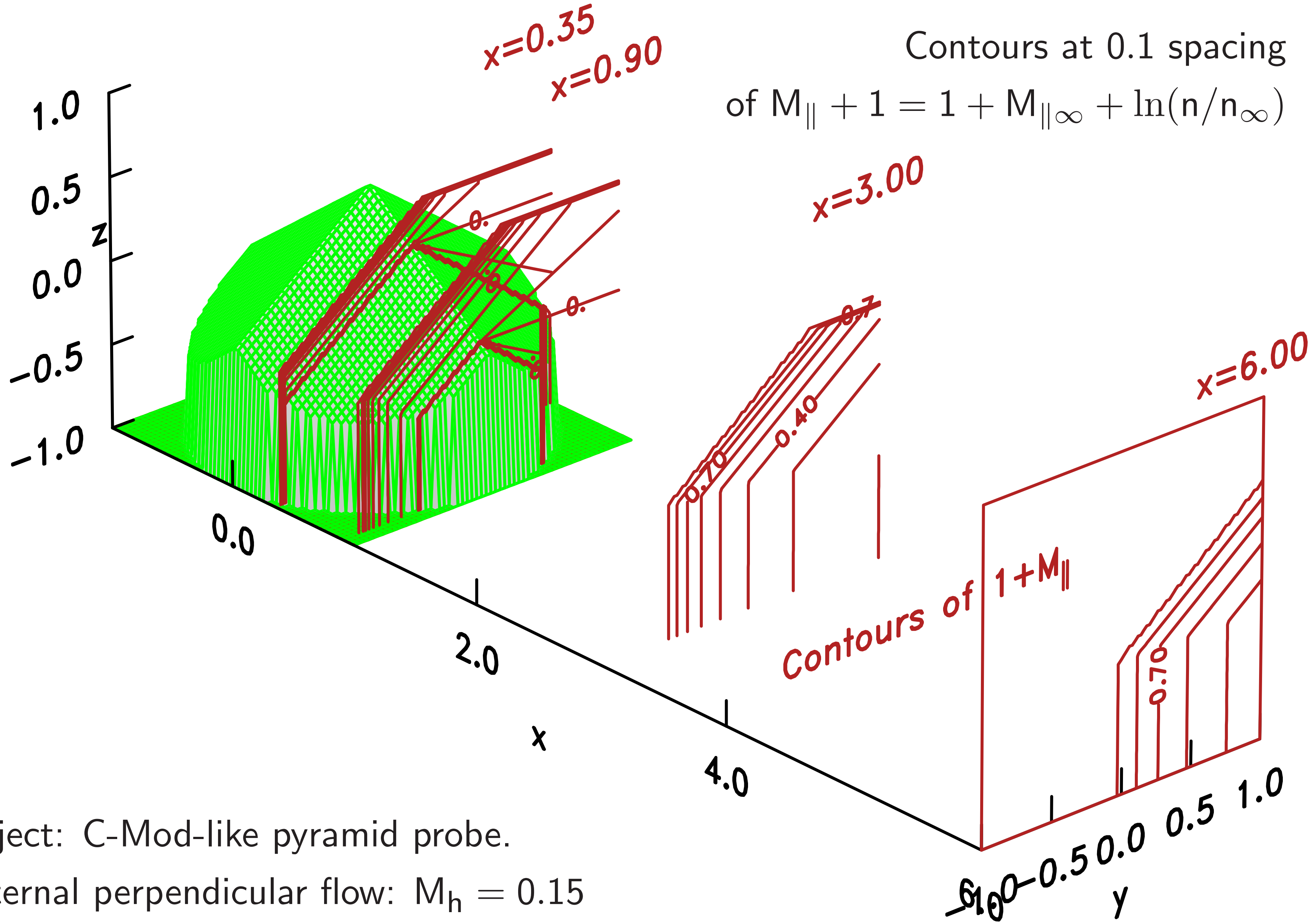
Coincide with $\ln n$ contours.



Example 3-D Solution: Pyramid Probe

Contours at 0.1 spacing

of $M_{\|} + 1 = 1 + M_{\|\infty} + \ln(n/n_{\infty})$



Object: C-Mod-like pyramid probe.

External perpendicular flow: $M_h = 0.15$

External parallel flow: $M_{\|\infty} = -0.1$

∇nT External Diamagnetic Drifts

Introduce heavy complications:

- Drifts in direction of ∇nT introduce flux changes
- Important effects arise from displacements in the magnetic presheath
- Temperature gradients imply gradients in c_s.
- Electron temperature gradients induce extra E-fields and drifts.

With major effort one can integrate pages of equations like this
$\frac{d}{dM_\parallel}(\delta z/2L_n) \equiv r$; so that $\left.\frac{d}{c_s dt}\right|_\pm \frac{\delta z}{2L_n} = \left.\frac{d}{c_s dt}\right|_\pm M_\parallel \cdot \frac{d}{dM_\parallel}\frac{\delta z}{2L_n} = r \left.\frac{d}{c_s dt}\right|_\pm M_\parallel \ldots$
but I'm sparing you all of that* and skipping to the bottom line.

"It can be shown that" the presheath displacements have asymmetries small enough to be ignored when discussing up-stream to down-stream differences ($\pm\cos\theta$).

As a result one can derive analytically a simple but essentially complete formula for the flux to the object.

* I H Hutchinson, Physics of Plasmas, 15, 123503, 2008

Final Full Flux Density

$$\ln\left\{\frac{\Gamma_{\parallel p}}{n c_s}\right\} = \underbrace{-1 - M_{\parallel\infty}}_{\text{parallel flow}} + \left[\underbrace{(1 + M_{\parallel\infty})M_{Te}}_{T_e-\text{gradient effect}} + M_{Di} + \underbrace{M_E}_{E\times B} - \underbrace{\left(\frac{1-\sin\alpha}{1+\sin\alpha}\right) M_D}_{\text{MPS displacement}}\right] \cot\theta$$

with

$M_E = \mathbf{E}\wedge\mathbf{B}/c_s B^2.\hat{\mathbf{y}}$ .... Electric field drift, agreeing with intuition.

$M_{Di} = -\nabla p \wedge \mathbf{B}/(c_s n Z e B^2).\hat{\mathbf{y}}$ .... Total ion diamagnetic drift. Intuitive.

$M_{Te} = \nabla T_e \wedge \mathbf{B}/(c_s e B^2).\hat{\mathbf{y}}$ .... Electron diamagnetic due to T_e gradient.

$M_D = M_{Di} - M_{De}$ .... Difference between ion and electron diamag drifts.

α .... Angle between $\mathbf{B}$ and surface.

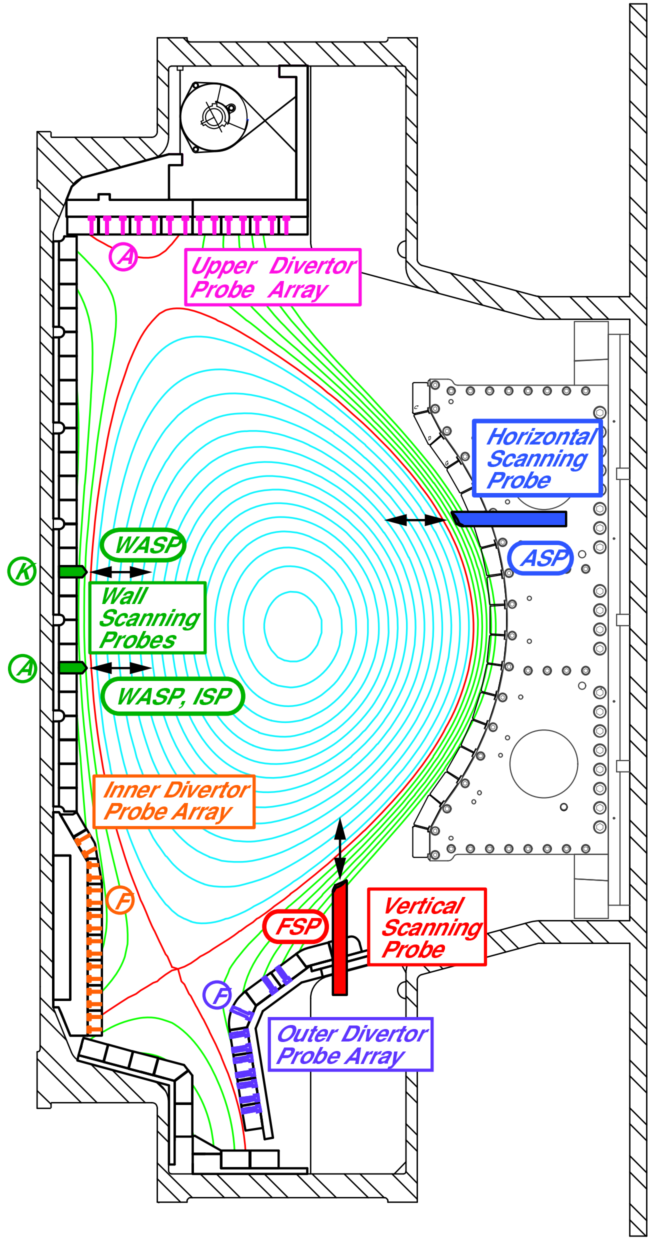
θ .... Angle in x-y-plane between $\mathbf{B}$ and surface.

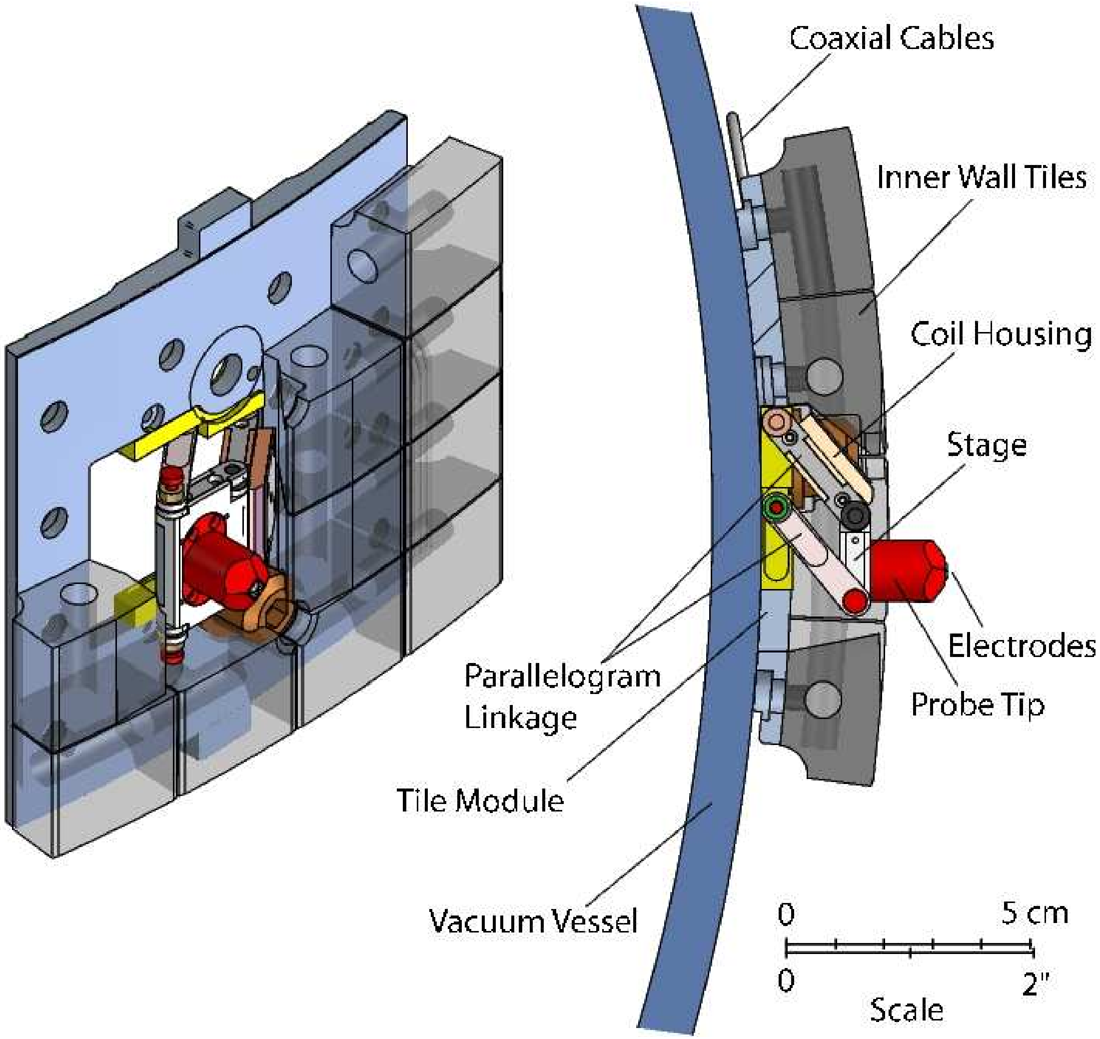
The quantity in [ ] is what a transverse Mach probe measures.

C.f. prior diffusive treatment* $\quad \ln\left\{\frac{\Gamma_{\parallel p}}{n c_s}\right\} = -1 - 1.1(M_{\parallel\infty} - M_\perp \cot\theta)$

* I H Hutchinson Phys Rev A, 37 (1988) 4358; IHH "Principles of Plasma Diagnostics" (2002).

Probe Deployment on Tokamaks



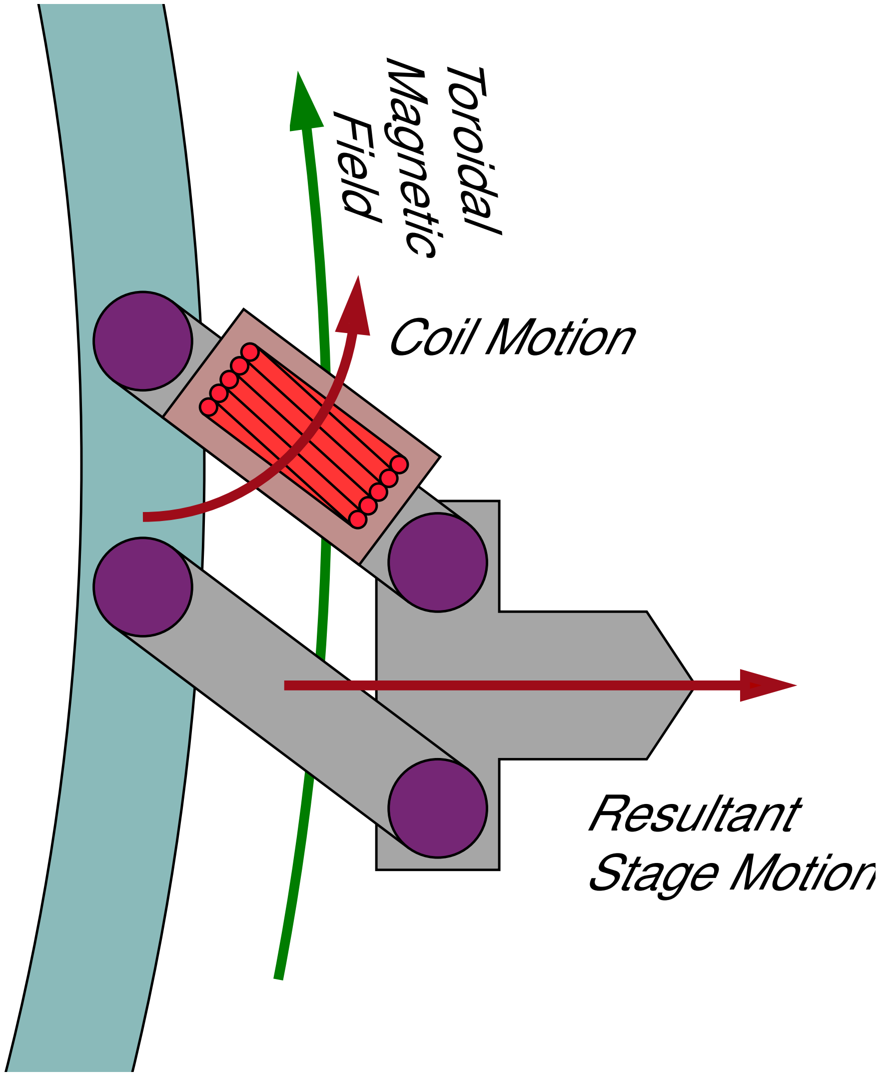


Use of rapidly reciprocating probes to minimize the heat load.

Courtesy LaBombard and Smick

Reciprocation Mechanisms

WASP Parallelogram Linkage



Electromagnetic drive using the ambient field is very effective for a compact reciprocation drive.

More traditional drives are pneumatically driver rod with spring stops to give rapid velocity reversal.
Reciprocation in $\sim$ 10ms.

Characteristic Interpretation

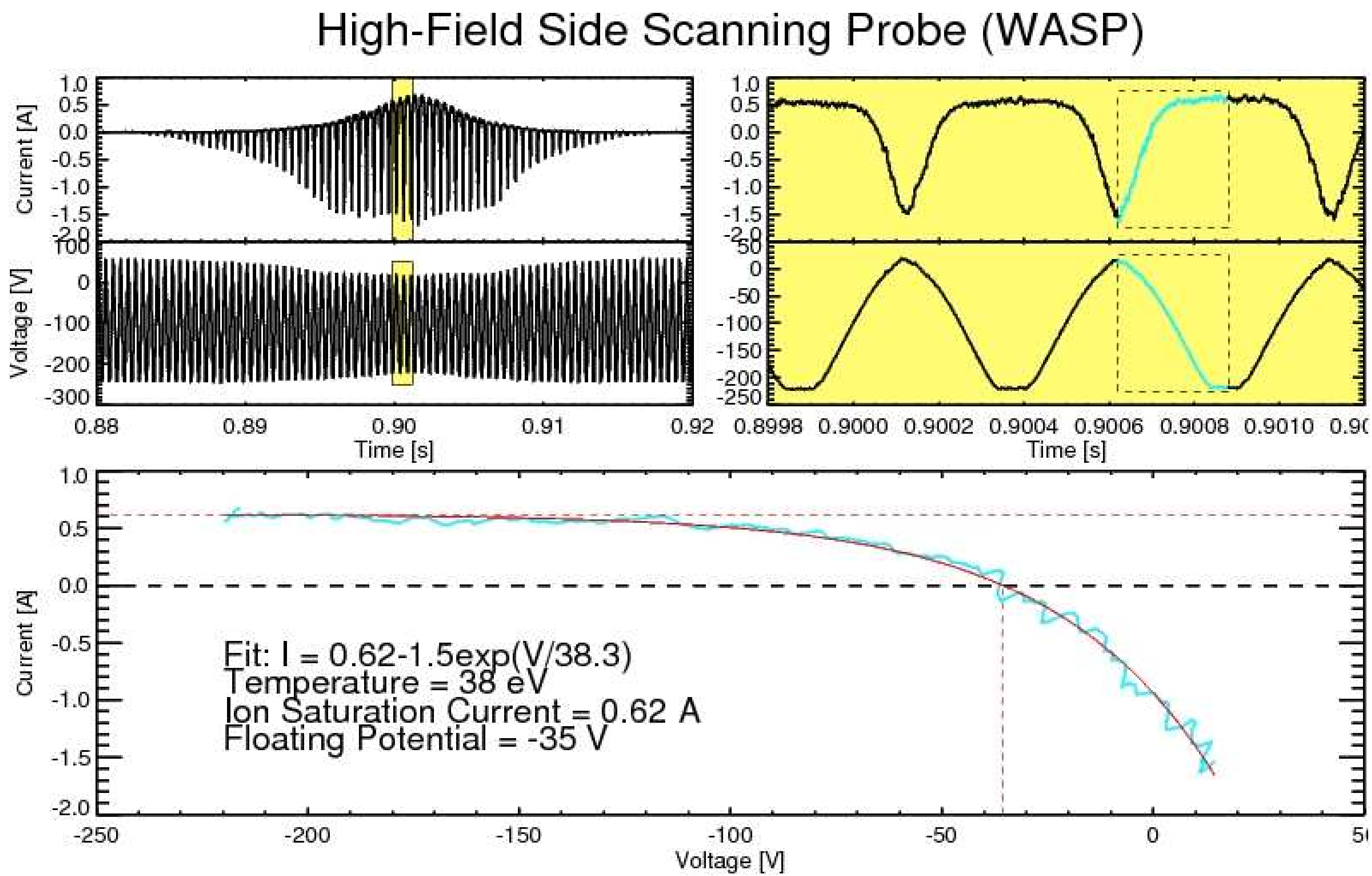
Insertion

$\sim$ 10ms.

Voltage sweep

$\sim$ 0.2ms.

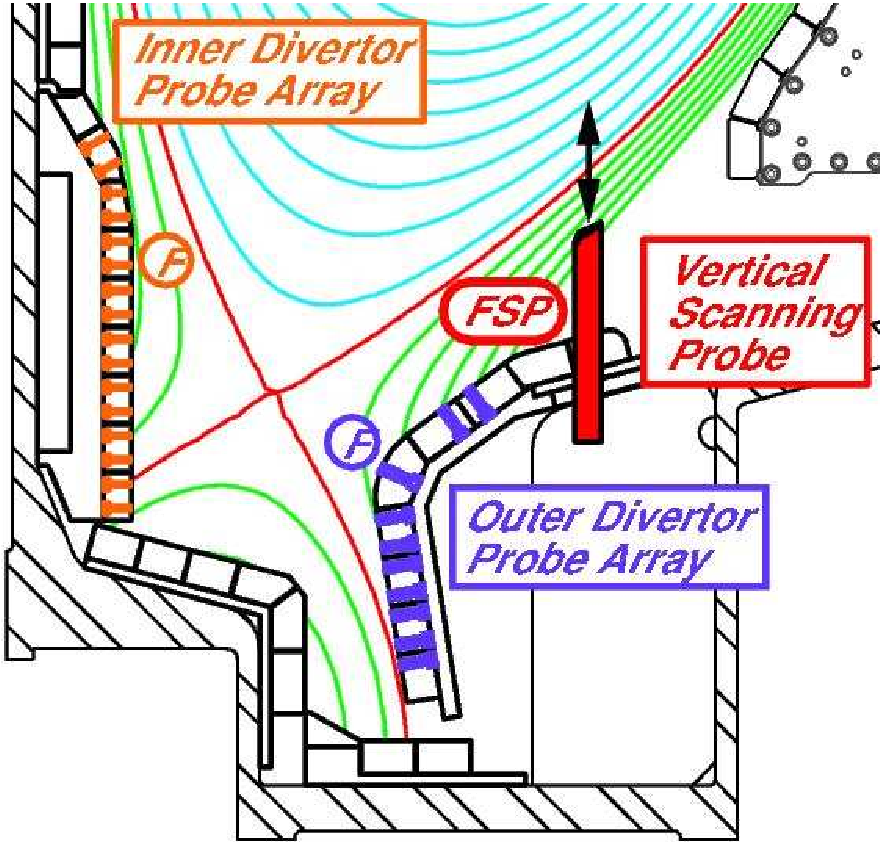
Currents inverted (from probe) here.



Electron currents limited to not significantly more than I_{si} to minimize heat load.

Still quite a significant power supply is required (300W), depending on probe size.

Embedded Probes



Probes embedded into divertor surfaces with their tips flush or nearly flush with the surface are protected from strongest heat flux by the divertor itself.

Unfortunately such probes encounter serious difficulties of interpretation if they are exactly flush. So usually designed slightly 'proud'.

Special Probes

"Plug probes" (Katsumata). Try to exclude electron collection.

Retarding Field analysers. More complicated resonant analysers.

Double probes, Triple probes, and other combinations.

Summary of Probe Diagnostics

One of the first and most simply-implemented plasma measurements.
Still one of the most complicated to interpret.

Magnetized probe theory now is almost as well-developed as unmagnetized.

Spatial resolution and simultaneous T, n [and v] measurements.

Literature is massive and unhelpful. A critical approach is essential.

Because the costs of probe experiments are small, many people try to publish expt and interpretation algorithms.

It is very hard to do anything really innovative in probe expt and electronics.
You have to know the literature quite well.