Summer College on Plasma Physics

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Periodic nonlinear surface waves in plasmas: nonlinear electrostatic oscillations in a sharp plasma interface

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Periodic nonlinear surface waves in plasmas:

nonlinear electrostatic oscillations in a sharp plasma interface

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Plan of the talk

- Linear plasma surface waves
- Gradov-Stenflo equation
- New periodic solutions to the Gradov-Stenflo equation
Linear electrostatic surface plasma waves

\[ z < 0, \quad \varepsilon = 1, \quad \phi = \phi_0 \exp(ikx + kz) \quad \text{vacuum} \]

\[ z > 0, \quad \varepsilon = 1 - \frac{\omega_p^2}{\omega^2}, \quad \phi = \phi_0 \exp(ikx - kz) \quad \text{plasma} \]
Properties

1) Propagation parallel to the surface $z = 0$
2) Restricted basically to that surface (since $k > 0$)
3) Satisfy $\nabla \cdot (\varepsilon \nabla \phi) = 0$
4) From the later: continuity of $\varepsilon \frac{\partial \phi}{\partial z}$ at $z=0$
Linear dispersion relation

Therefore: \[ -k = (1 - \frac{\omega_p^2}{\omega^2}) \ k \]

or \[ \omega = \frac{\omega_p}{\sqrt{2}} \]

Mismatch factor: \[ \Delta = 2 - \frac{\omega_p^2}{\omega^2} \]
• Reasonable agreement with experiments
• Physics of low-temperature *bounded* plasmas: not as well developed as that of high-temperature fusion plasmas
• Many industrial applications (e.g. TV screens)
Nonlinear plasma surface waves

- A veritable zoo of special solutions
- Some sort of expansion in powers of the amplitude of the electrostatic potential

$\Rightarrow$

nonlinear corrections to the dielectric function
The Gradov-Stenflo equation

\[
\frac{8ik^2}{\omega} \frac{\partial \phi_0}{\partial t} + \frac{\partial^2 \phi_0}{\partial x^2} - \frac{1}{\phi_0} \left( \frac{\partial \phi_0}{\partial x} \right)^2 + \\
+ 2 \beta \gamma k |\phi_0| \phi_0 + \beta^2 |\phi_0|^2 \phi_0 + 2k^2 \Delta \phi_0 = 0
\]
Derivation

Sharp plane plasma boundary at $z = 0$, or: a plasma with a fixed homogeneous ionic background for $z > 0$ and vacuum for $z < 0$.

Scalar potential: $\phi(x, z, t) \exp[i(kx - \omega t)]$
For $z > 0$:

$$\phi = \phi_0(x,t) \exp[-kz + \int_0^z dz' k_z(x,z')]$$

$$k_z \ll k$$

$\phi_0$: a slowly varying envelope.

Plus the analytic continuation for $z < 0$. 
• From the cold plasma fluid equations and a perturbative treatment:

\[
\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} + 2i \frac{\omega_p^2}{\omega^3} \frac{\partial}{\partial t} \ln \phi - \frac{\beta^2}{2k^2} |\phi|^2
\]

• Assumptions: \( \frac{\partial}{\partial t} \ln \phi \ll \omega, \quad \frac{\partial}{\partial x} \ln \phi \ll k \)
Previously known exact solutions: *stationary localized structures* (without the $\sim \gamma$ and $\sim \Delta$ terms).


Similar to the recently derived *oscillon* solutions (in an external periodic flow oscillating at twice the natural surface wave frequency), but that’s another history!
A peculiar property

• Remark:

\[ \phi_0 \neq 0 \]

• Remember Kibble‘s objection in the context of dissipative quantum mechanics

• The scalar potential is small, but nonzero!
A conservation law

It can be shown that

\[
\frac{d}{dt} \int |\ln \phi_0|^2 \, dx = 0,
\]

\[
\int |\ln \phi_0(x, t)|^2 \, dx = \int |\ln \phi_0(x, 0)|^2 \, dx
\]
Exact periodic solution

• Let $\psi = \ln \phi_0$

• A modified nonlinear Schrödinger equation:

$$\frac{8i k^2}{\omega} \frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + 2 \gamma \beta k |\exp \psi| + \beta^2 |\exp(2\psi)| + 2k^2 \Delta = 0$$
Madelung transformation

\[ \psi = A \exp(iS), \]

\[ A = A(X), \quad S = S(X), \]

\[ X = x - ut, \]

\[ \phi = \ln \left( \frac{A}{A_0} \right), \quad A_0 > 0 \]
Sagdeev potential

\[
\frac{d^2 \varphi}{dX^2} = - \frac{dV}{d\varphi},
\]

\[
V = V(\varphi) = 32 \left( \frac{k^2 u}{\omega} \right)^2 \varphi^2 + 2 \gamma \beta k A_0 (e^\varphi - \varphi - 1) + \frac{\beta^2 A_0^2}{2} (e^{2\varphi} - 2\varphi - 1)
\]
Stable or bistable oscillations
Phase space trajectories in the bistable case
Near the bottom trajectories:
Near separatrix trajectories:
To conclude

Madelung transformation for the Gradov-Stenflo equation

⇒

exact translating oscillatory solution

Possible improvements: dissipative or transverse effects; physical modeling of parameters; moving boundaries...