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International Centre for Theoretical Physics**



**2053-43**

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Volcanic and Seismic Hazards in East Africa**

*17 - 28 August 2009*

**Volcano Deformation - models and examples**

Tim Wright  
*University of Leeds*  
*U.K.*

# Volcano Deformation – Models and Examples

Tim Wright, University of Leeds, UK

Juliet Biggs, University of Oxford, UK

[t.j.wright@leeds.ac.uk](mailto:t.j.wright@leeds.ac.uk)

[Juliet.biggs@earth.ox.ac.uk](mailto:Juliet.biggs@earth.ox.ac.uk)



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OXFORD



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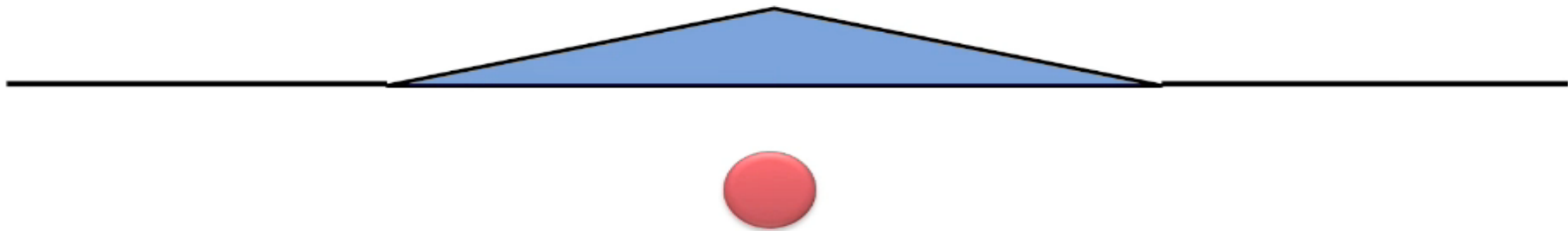
# A simple volcanic system

## 1. Charging the system



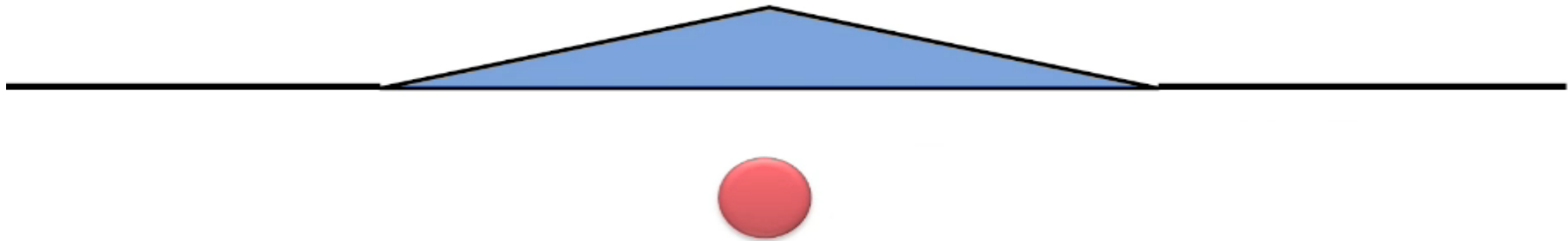
# A simple volcanic system

## 1. Discharging the system (a. eruptions)



# A simple volcanic system

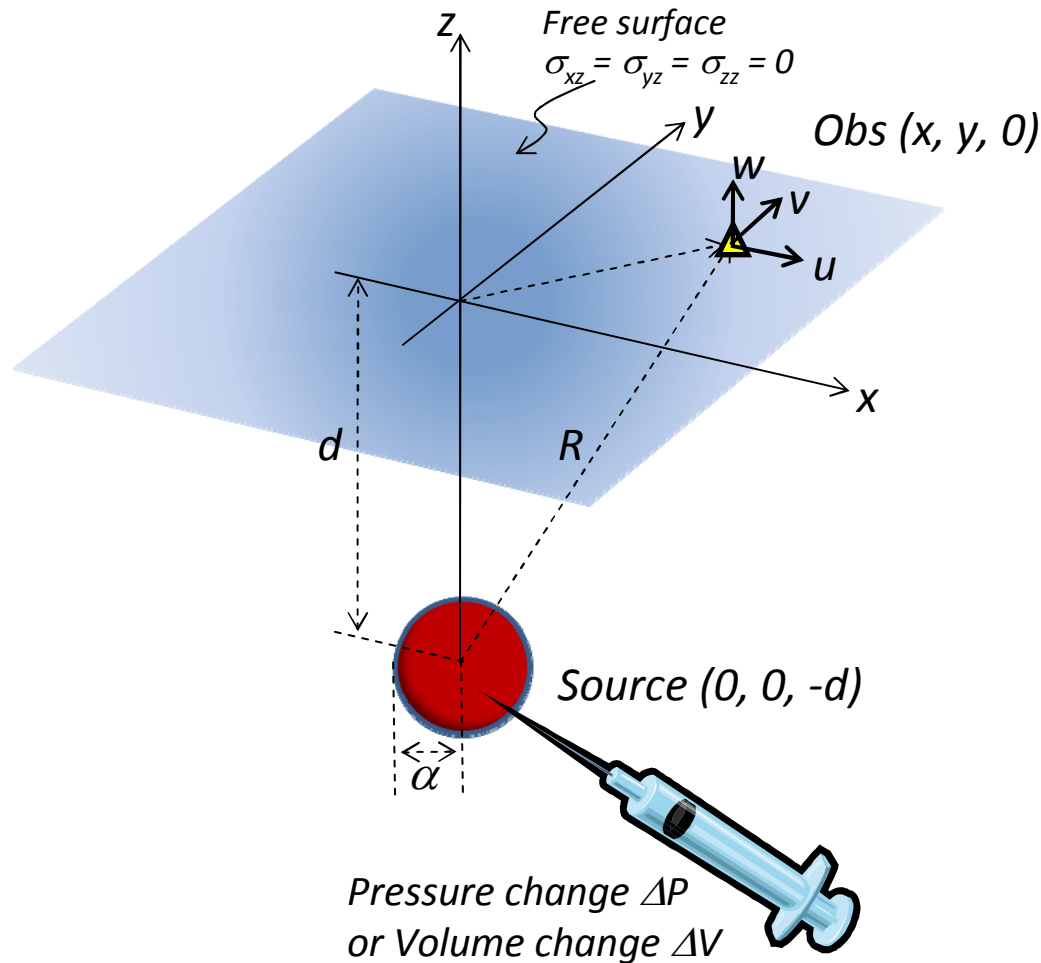
## 1. Discharging the system



# Outline

- Simple volcanic deformation sources
  - Point Pressure ('Mogi' Model for magma chambers)
  - Elastic Dislocations ('Okada' model for Dykes and Sills)
  - Penny-shaped crack (for sills)
  - Other models
- Limitations of Volcanic Geodesy
- Comparison of InSAR and GPS
- Alaskan Case Study

# Simple Sources: 1. The Mogi Model



## Assumptions

1. Isotropic elastic half space (Poisson's ratio  $\nu$ ; Shear modulus  $\mu$ )
2.  $\alpha \ll d$  (i.e. spherical point source)
3. Incompressible magma

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \alpha^3 \Delta P \frac{(1-\nu)}{\mu} \begin{pmatrix} x/R^3 \\ y/R^3 \\ z/R^3 \end{pmatrix}$$

But  $\Delta V \approx \frac{\Delta P}{\mu} \pi \alpha^3$  so

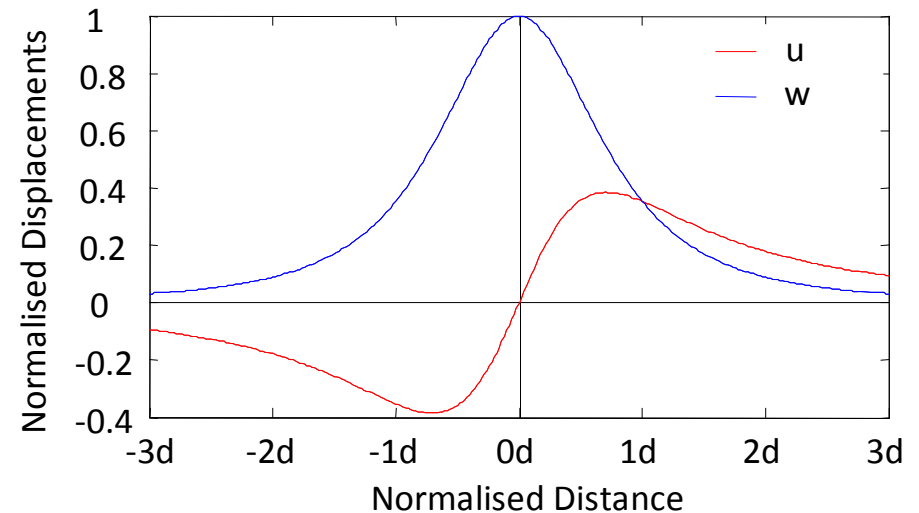
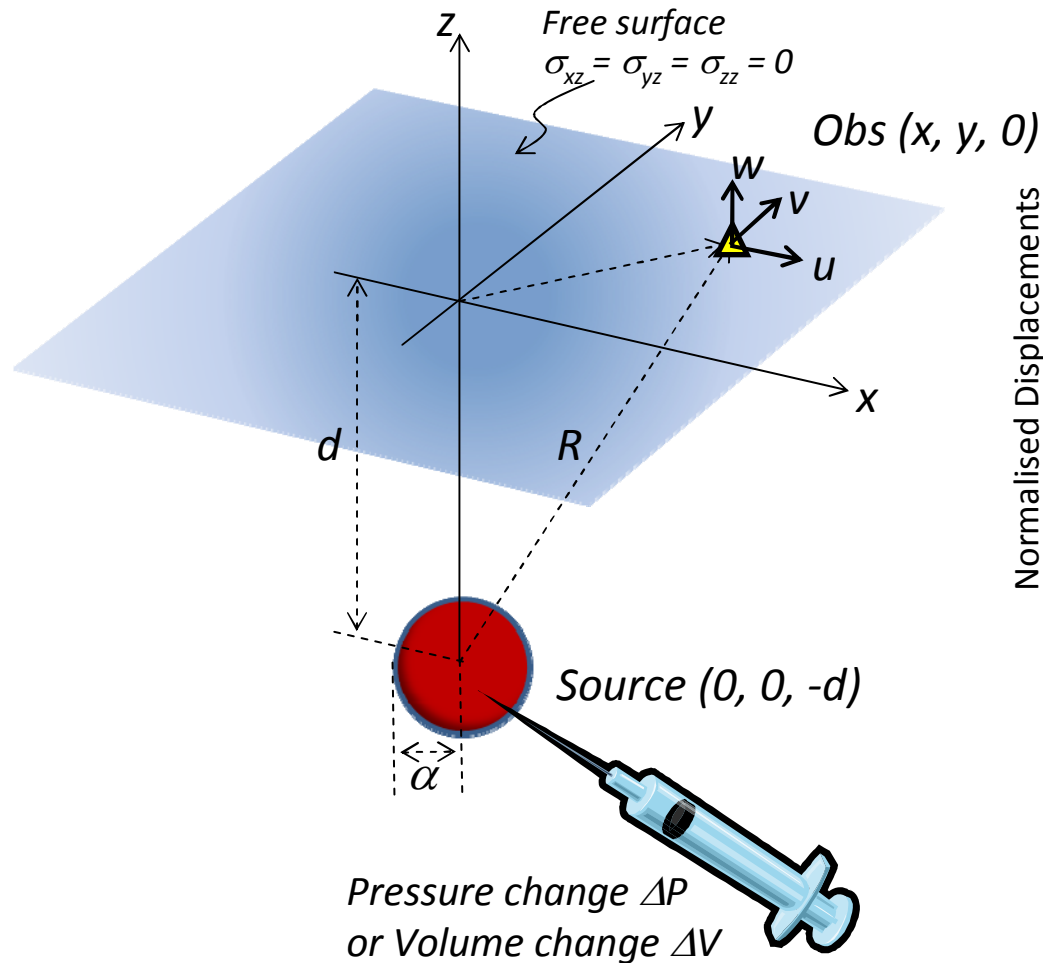
$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \Delta V \frac{(1-\nu)}{\pi} \begin{pmatrix} x/R^3 \\ y/R^3 \\ z/R^3 \end{pmatrix}$$

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Mogi, K. (1958), *Bull. Earthq. Inst. U. Tokyo*, 36, 99-134

Delaney, P., McTigue, D. (1994) *Bull. Volcanology*, 56 417-424

# Simple Sources: 1. The Mogi Model

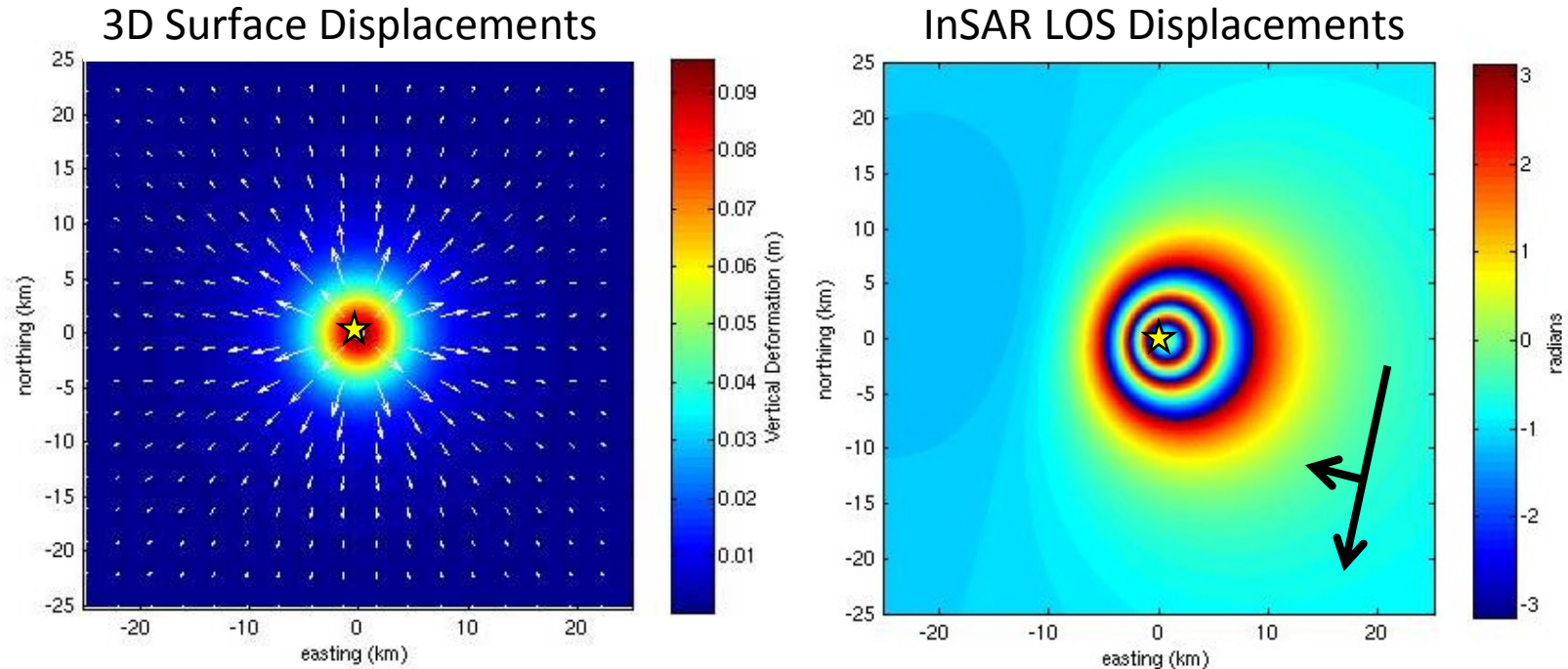


- Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
 Mogi, K. (1958), *Bull. Earthq. Inst. U. Tokyo*, 36, 99-134  
 Delaney, P., McTigue, D. (1994) *Bull. Volcanology*, 56 417-424

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \Delta V \frac{(1-\nu)}{\pi} \begin{pmatrix} x / R^3 \\ y / R^3 \\ z / R^3 \end{pmatrix}$$



# Simple Sources: 1. The Mogi Model



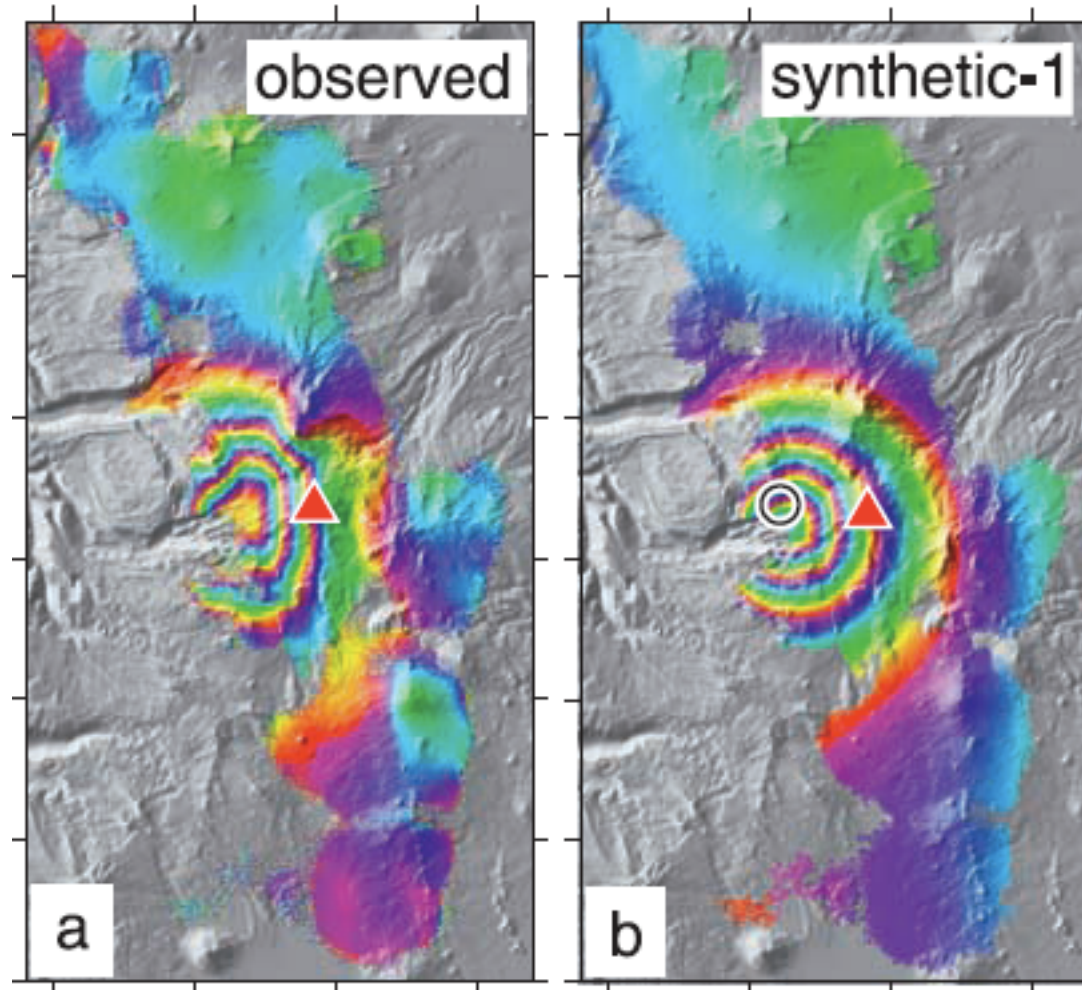
Source Depth = 5 km; Volume =  $10^7 \text{ m}^3$   
Maximum uplift  $\approx 95 \text{ mm}$

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Mogi, K. (1958), *Bull. Earthq. Inst. U. Tokyo*, 36, 99-134

Delaney, P., McTigue, D. (1994) *Bull. Volcanology*, 56 417-424

# Simple Sources: 1. The Mogi Model



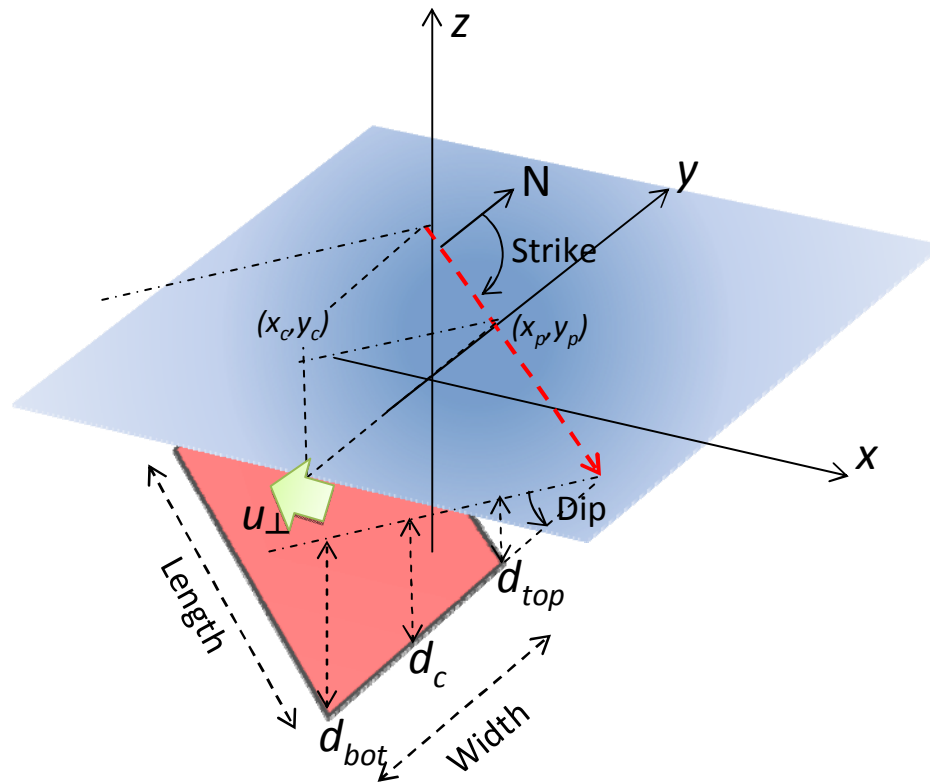
Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Mogi, K. (1958), *Bull. Earthq. Inst. U. Tokyo*, 36, 99-134

Delaney, P., McTigue, D.(1994) *Bull. Volcanology*, 56 417-424

Wicks, C., Dzurisin, D. et al (2002), *GRL*, 27.

# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)



## Assumptions

1. Isotropic elastic half space
2. Uniform opening (for dykes/sills) on rectangular dislocation

## Parameters

8 parameters needed to define dislocation:

<i>Strike, Dip</i>	[2]
Location $(x_c, y_c)$ or $(x_p, y_p)$	[2]
$d_c$ and <i>Width</i> (or $d_{top}, d_{bot}$ )	[2]
<i>Length</i>	[1]
<i>Opening</i> ( $u_{\perp}$ )	[1]

Can simplify

e.g. *Length* = *Width* for sills;

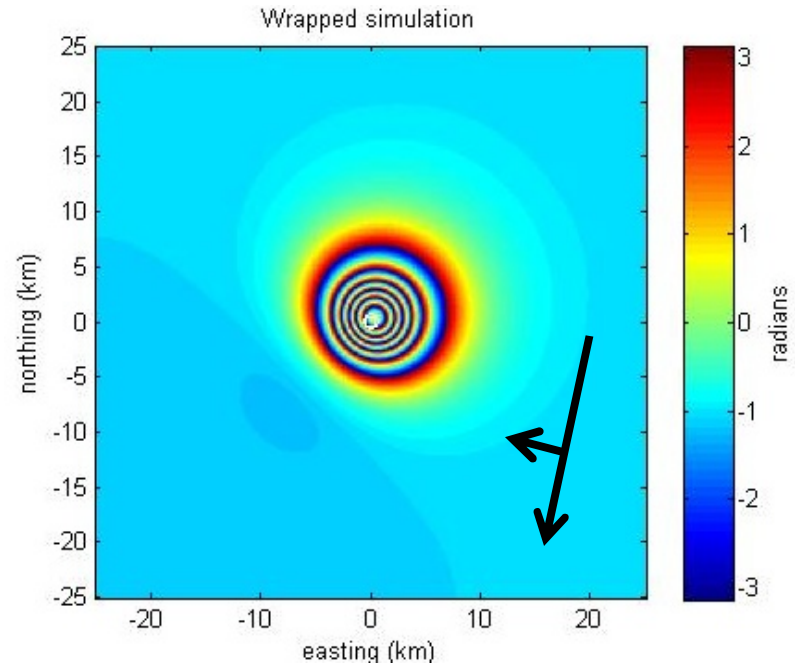
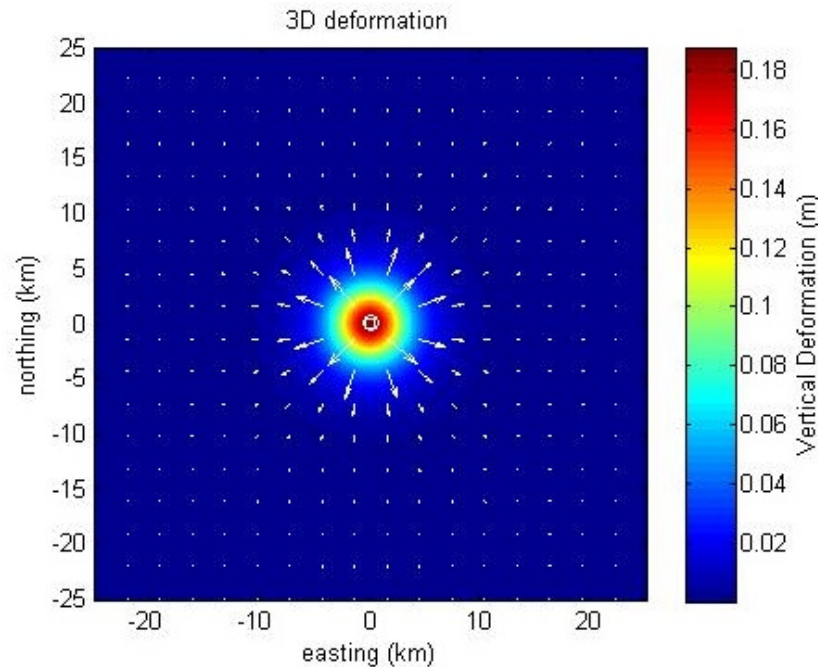
*Dip* = 0 for sills, 90 for dykes

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Okada, Y. (1985). *Bull. Seis. Society of America* 75 (4), 1135-1154 [surface]

Okada, Y. (1992). *Bull. Seis. Society of America* 82 (2), 1018-1040 [internal]

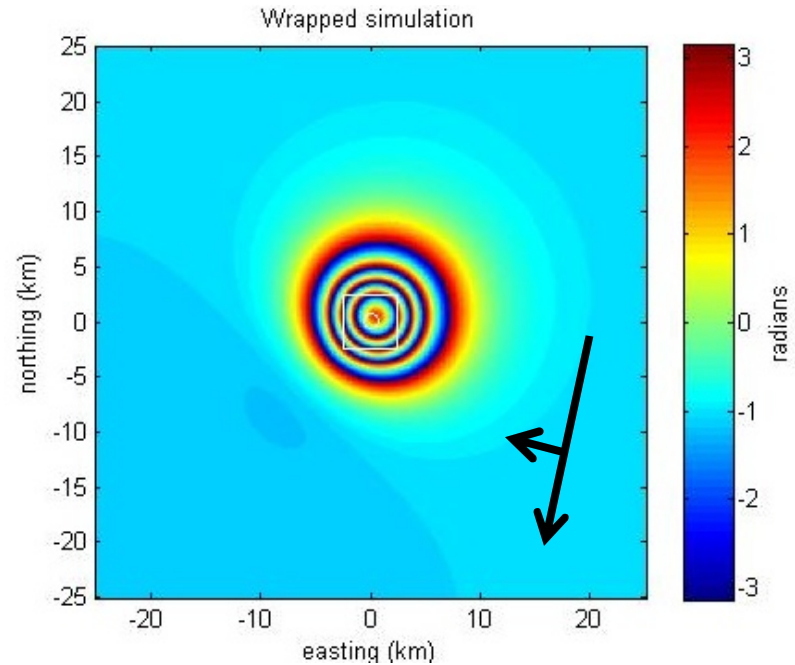
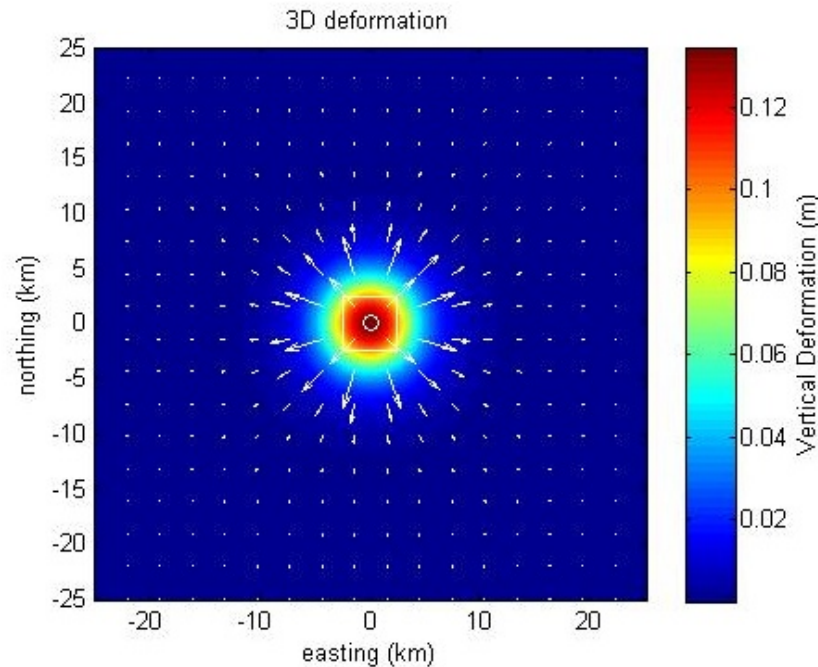
# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – **sills** or dykes)



Sill 1: Dip = 0, Strike = 0,  $x=y=0$ ,  $d_c = 5$  km  
Length=Width=1 km  
Opening = 10 m [Volume =  $10^7$  m<sup>3</sup>]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Okada, Y. (1985). Bull. Seis. Society of America 75 (4), 1135-1154 [surface]  
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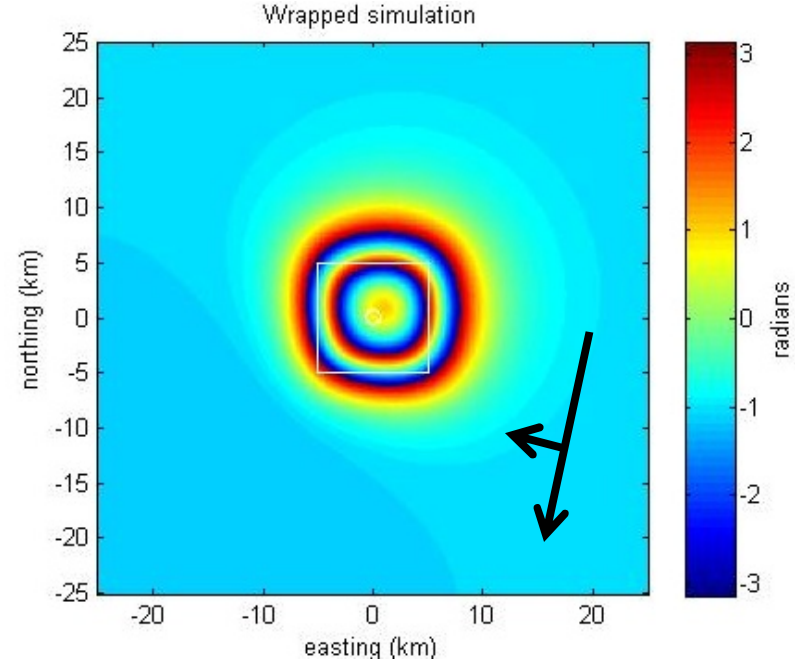
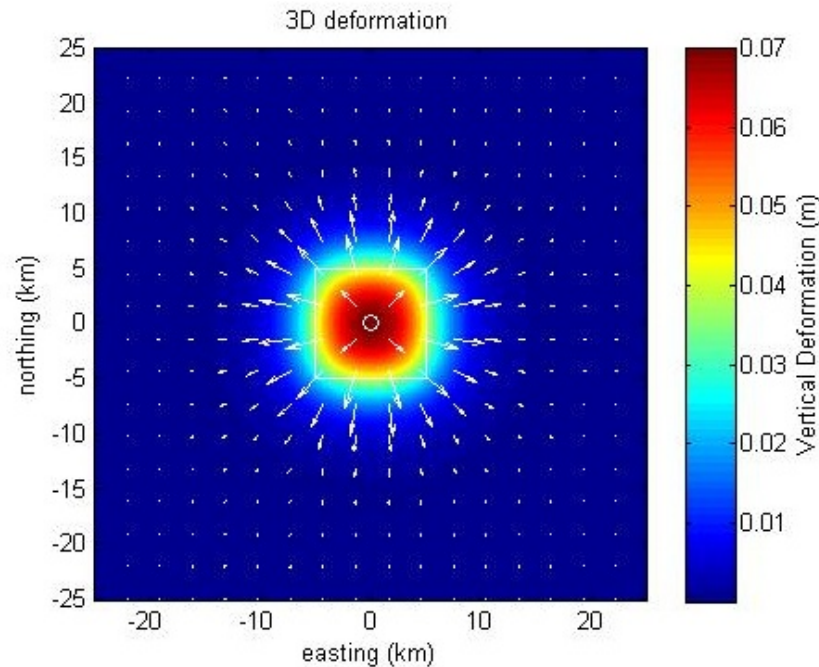
# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – **sills** or dykes)



Sill 2: Dip = 0, Strike = 0,  $x=y=0$ ,  $d_c = 5$  km  
Length=Width=5 km  
Opening = 0.4m [Volume =  $10^7$  m<sup>3</sup>]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Okada, Y. (1985). Bull. Seis. Society of America 75 (4), 1135-1154 [surface]  
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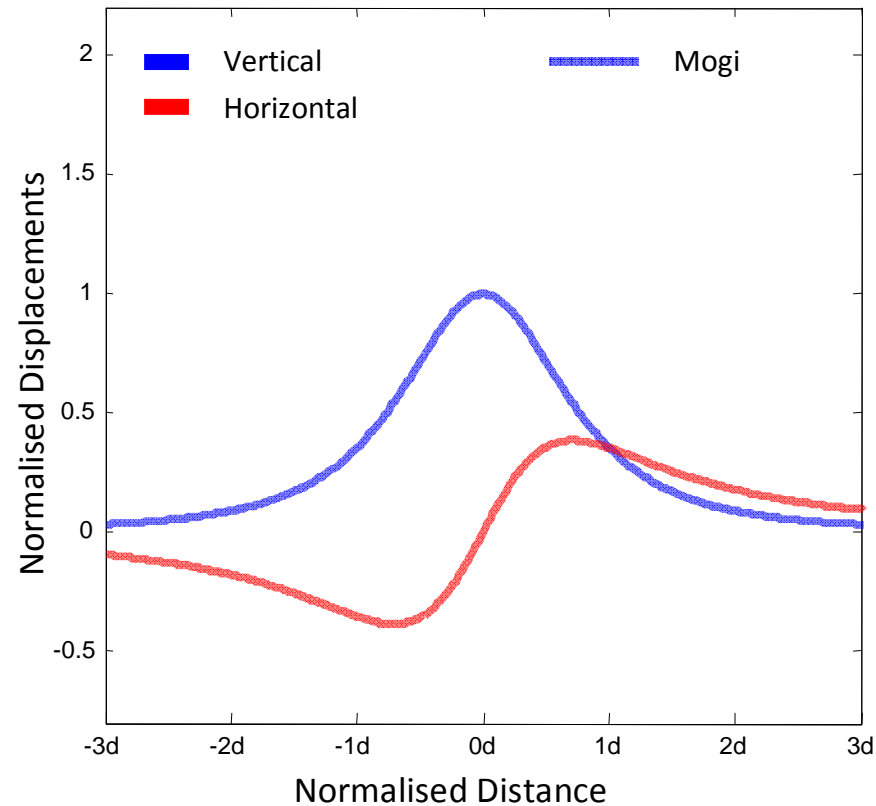
# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – **sills** or dykes)



Sill 3: Dip = 0, Strike = 0,  $x=y=0$ ,  $d_c = 5$  km  
Length=Width=10 km  
Opening = 0.1 m [Volume =  $10^7$  m<sup>3</sup>]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Okada, Y. (1985). Bull. Seis. Society of America 75 (4), 1135-1154 [surface]  
Okada, Y. (1992). Bull. Seis. Society of America 82 (2), 1018-1040 [internal]

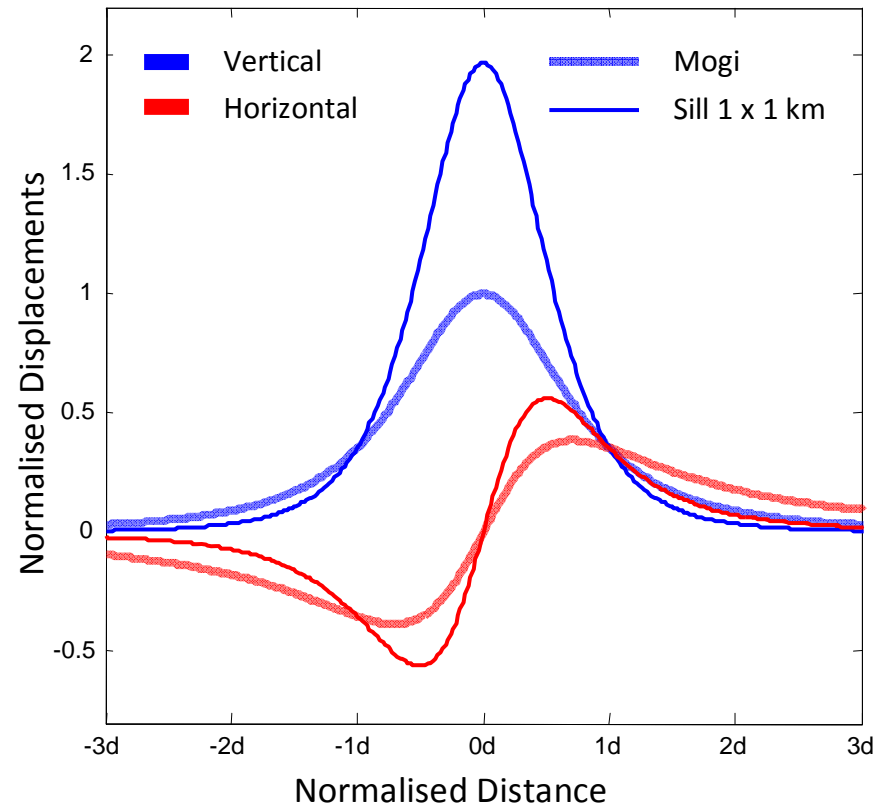
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Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Okada, Y. (1985). *Bull. Seis. Society of America* 75 (4), 1135-1154 [surface]  
Okada, Y. (1992). *Bull. Seis. Society of America* 82 (2), 1018-1040 [internal]

- Vertical deformation normalised by Mogi uplift
- Distance normalised by depth of source

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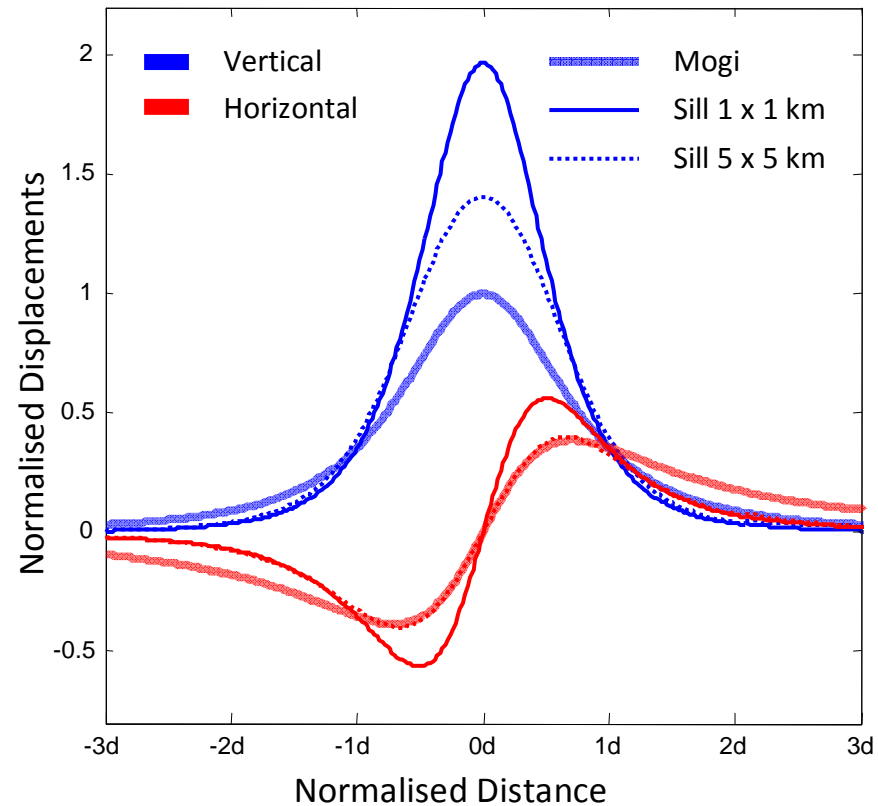


Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Okada, Y. (1985). *Bull. Seis. Society of America* 75 (4), 1135-1154 [surface]  
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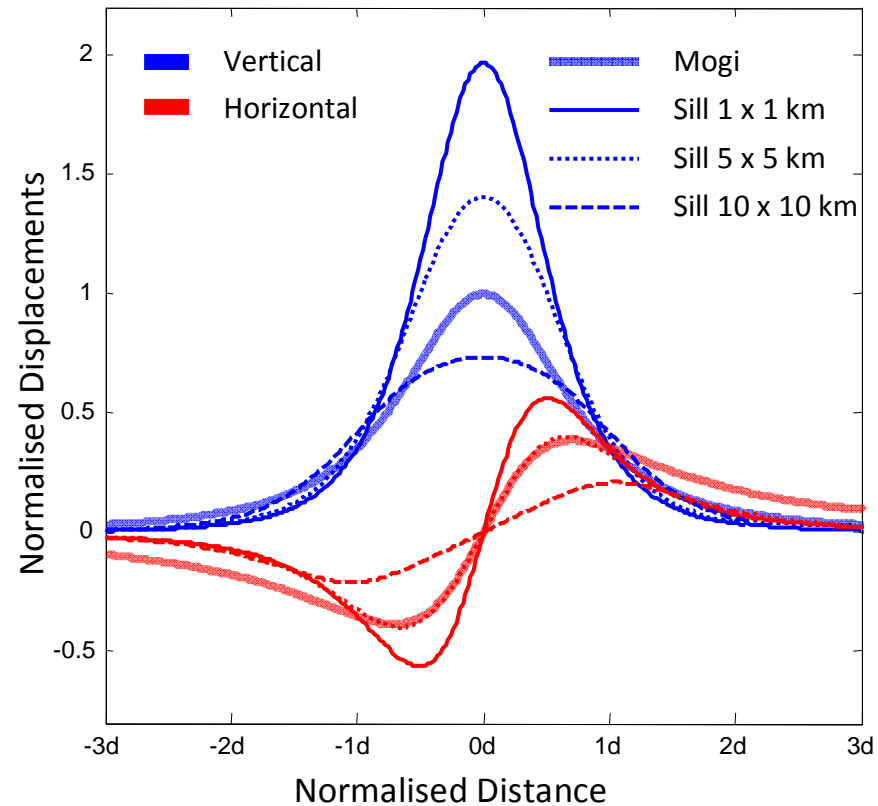
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- Vertical deformation normalised by Mogi uplift
- Distance normalised by depth of source

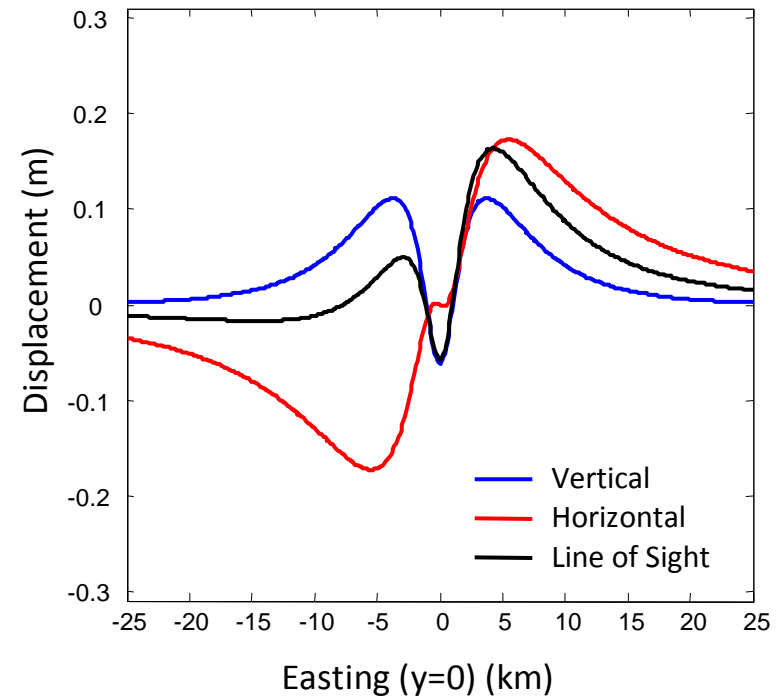
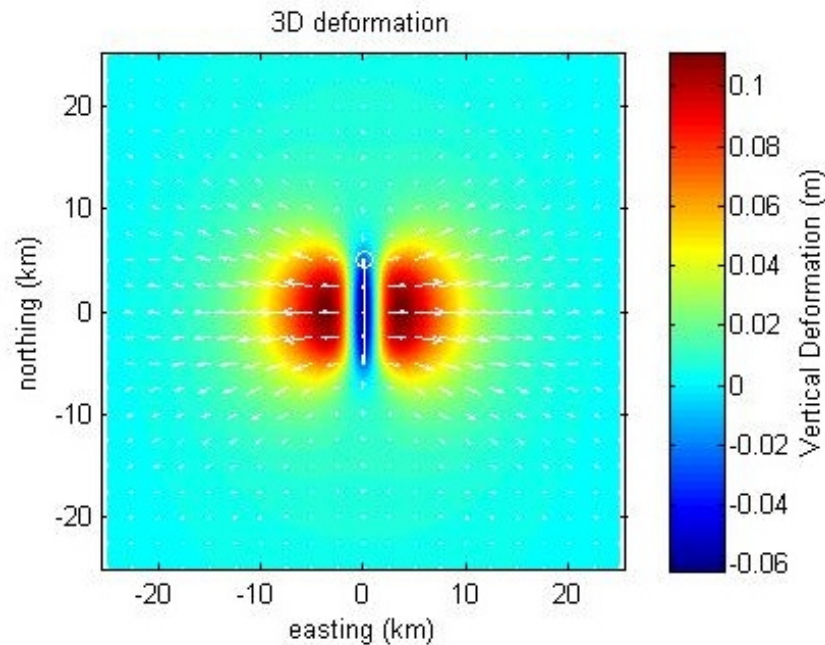
# Source volume vs uplift volume

Delaney and McTigue (1994) showed:

1. For sills,  $\text{source volume change} = \text{surface uplift volume}$
2. For Mogi,  $\text{source volume change} = \text{surface uplift volume} / 2(1-\nu)$   
e.g. For  $\nu = 0.25$ ,  $\text{source volume} = (2/3) * \text{surface uplift volume}$

These relationships are only true if the Magma is incompressible. See Johnson et al (2000) for details [also summarised in Dzurisin (2007)]

# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)



Dyke 1: Dip = 90, Strike = 0,  $x=y=0$

$d_{\text{top}} = 2 \text{ km}$ ,  $d_{\text{bot}} = 8 \text{ km}$

Length = 10 km

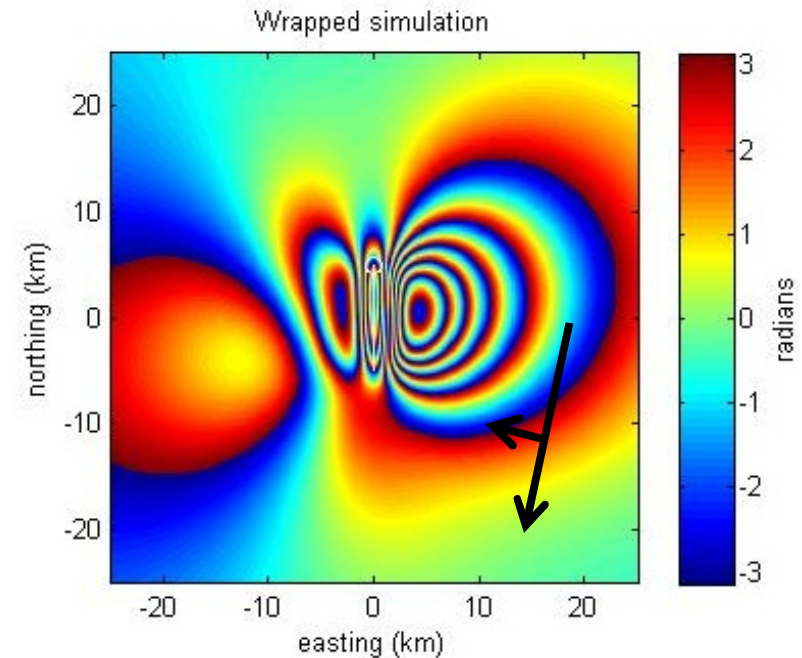
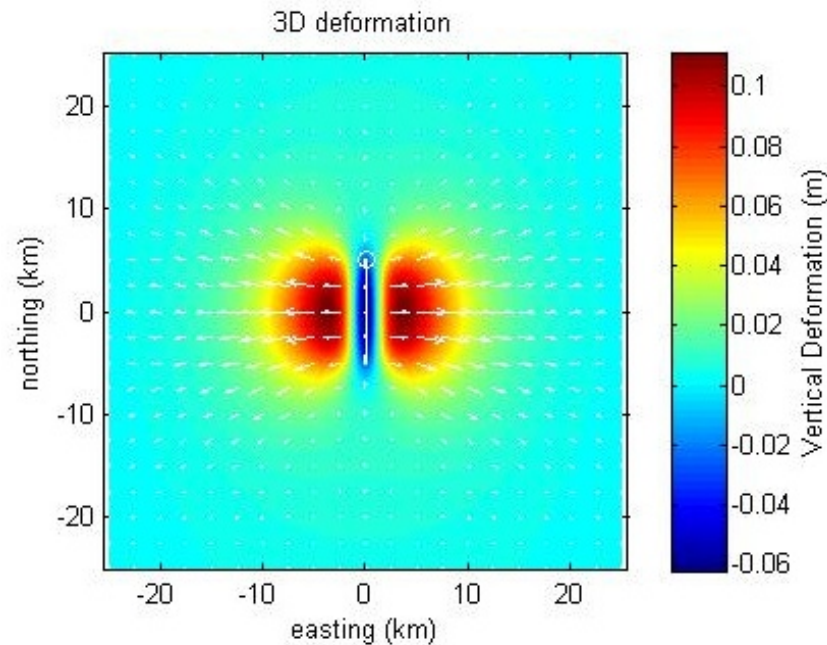
Opening = 1 m [Volume =  $6 \times 10^7 \text{ m}^3$ ]

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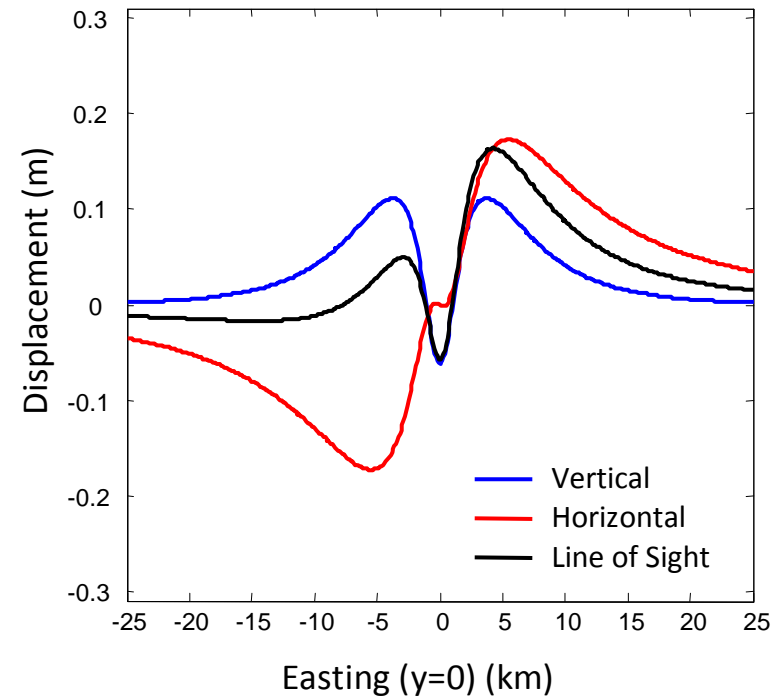
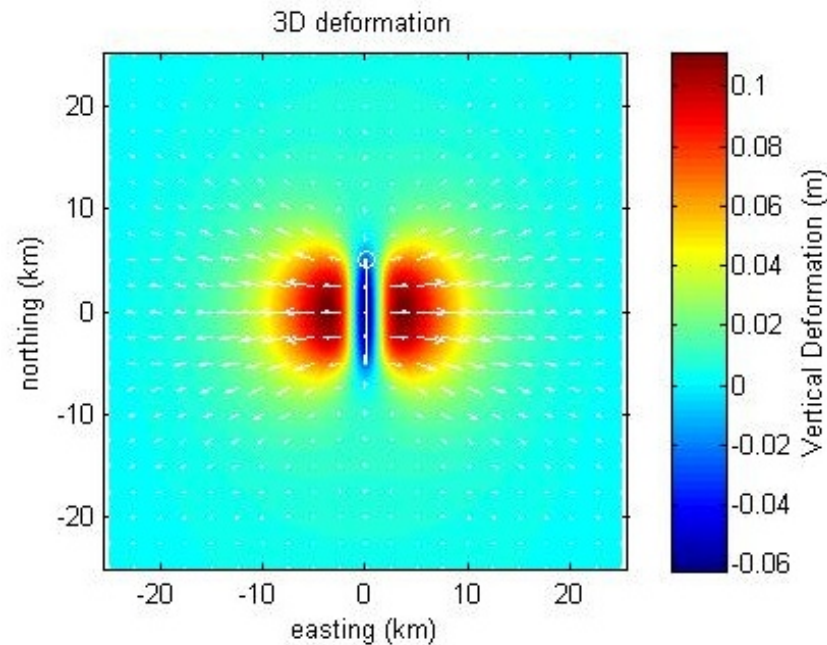
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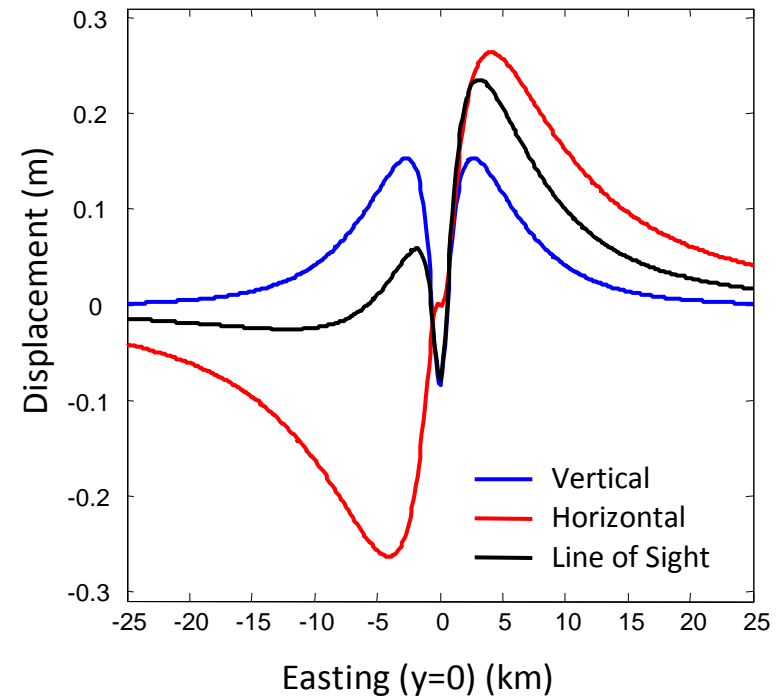
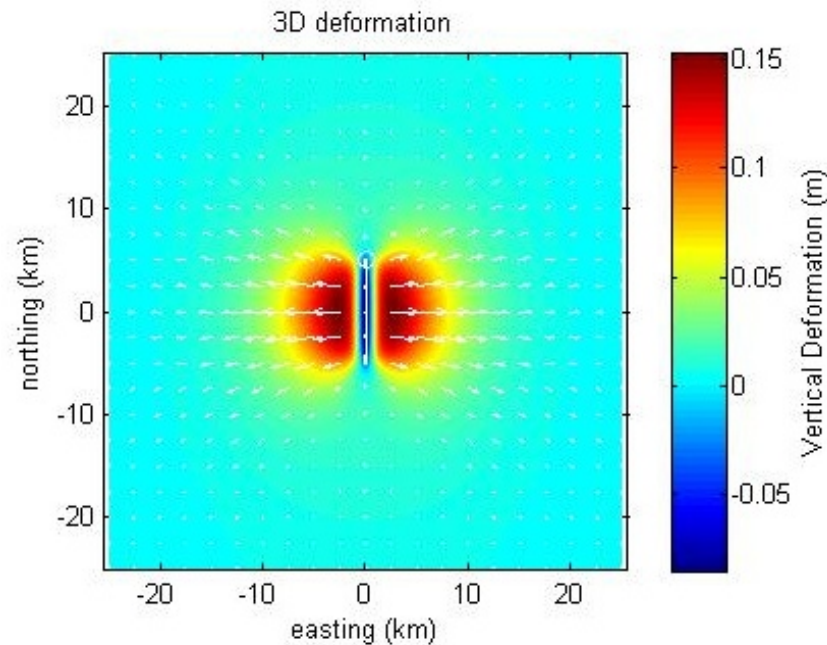
Opening = 1 m [Volume =  $6 \times 10^7 \text{ m}^3$ ]

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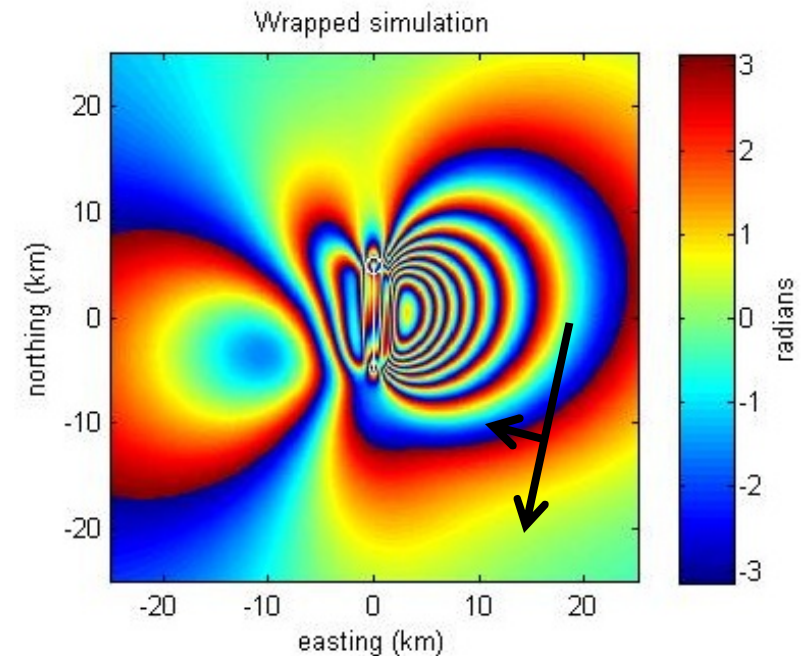
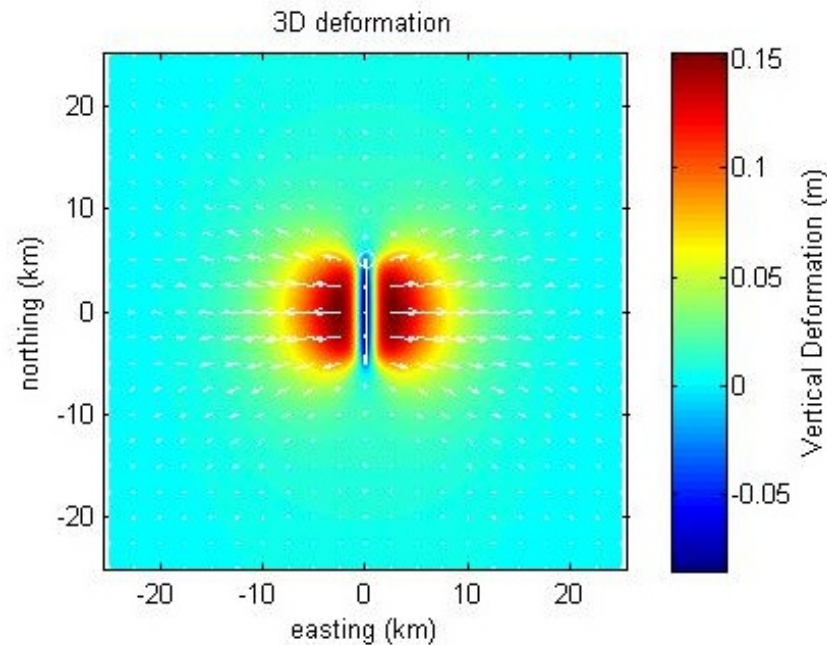
# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or **dykes**)



Dyke 2: Dip = 90, Strike = 0,  $x=y=0$   
 $d_{\text{top}} = 1 \text{ km}$ ,  $d_{\text{bot}} = 8 \text{ km}$   
Length = 10 km  
Opening = 1 m [Volume =  $7 \times 10^7 \text{ m}^3$ ]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Okada, Y. (1985). Bull. Seis. Society of America 75 (4), 1135-1154 [surface]  
Okada, Y. (1992). Bull. Seis. Society of America 82 (2), 1018-1040 [internal]

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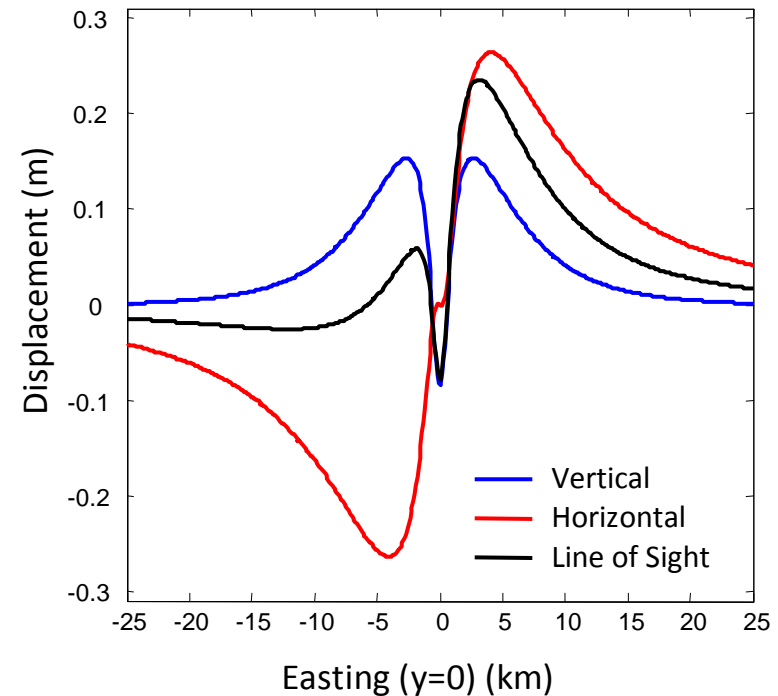
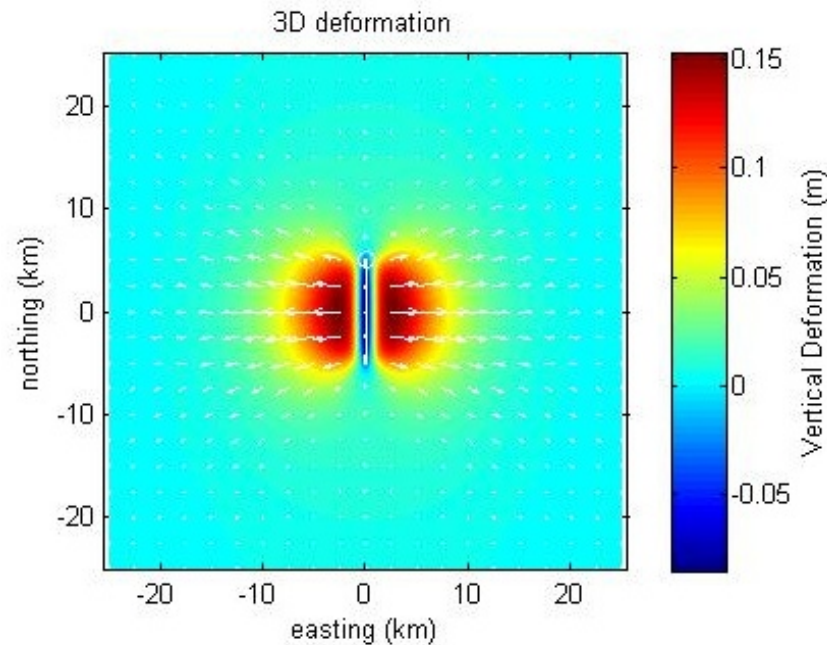
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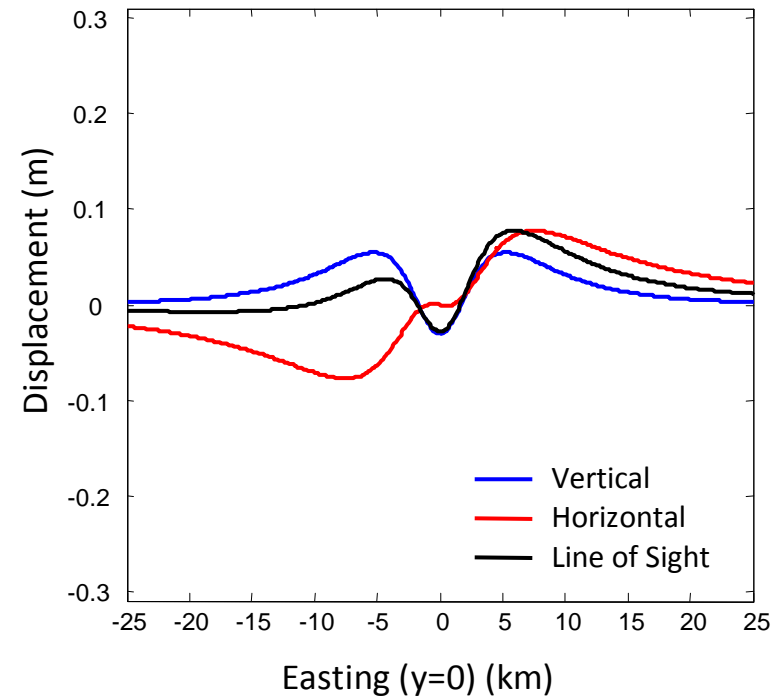
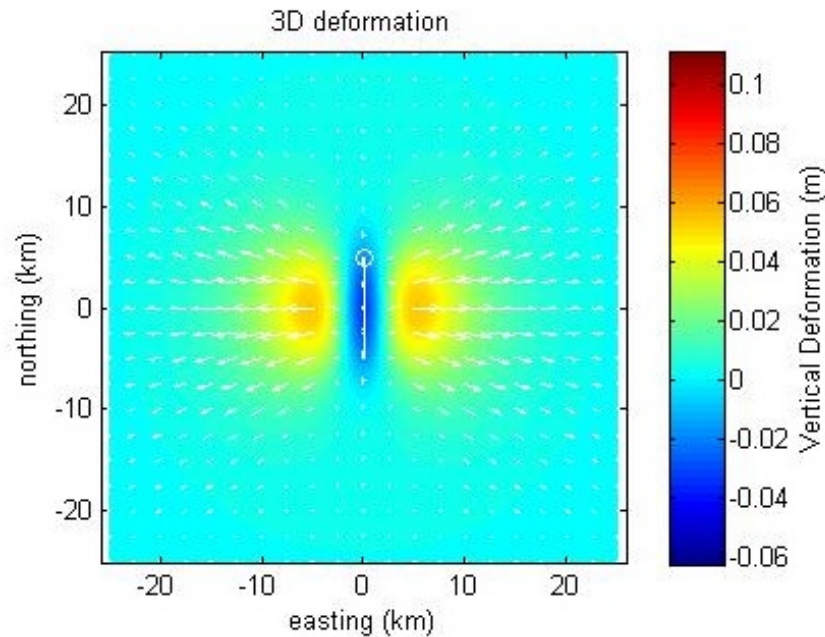
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Length = 10 km  
Opening = 1 m [Volume =  $7 \times 10^7 \text{ m}^3$ ]

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Okada, Y. (1985). Bull. Seis. Society of America 75 (4), 1135-1154 [surface]  
Okada, Y. (1992). Bull. Seis. Society of America 82 (2), 1018-1040 [internal]

# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)



Dyke 3: Dip = 90, Strike = 0,  $x=y=0$

$d_{\text{top}} = 4$  km,  $d_{\text{bot}} = 8$  km

Length = 10 km

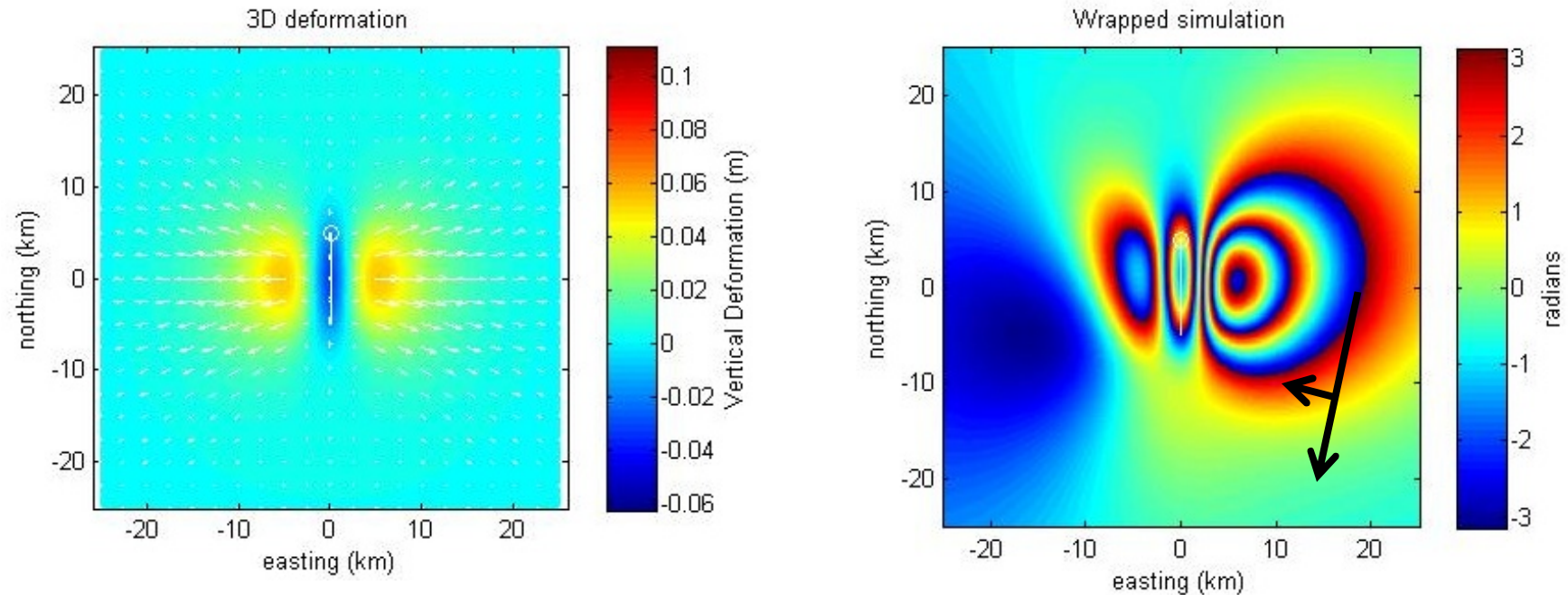
Opening = 1 m [Volume =  $4 \times 10^7$  m<sup>3</sup>]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Okada, Y. (1985). *Bull. Seis. Society of America* 75 (4), 1135-1154 [surface]

Okada, Y. (1992). *Bull. Seis. Society of America* 82 (2), 1018-1040 [internal]

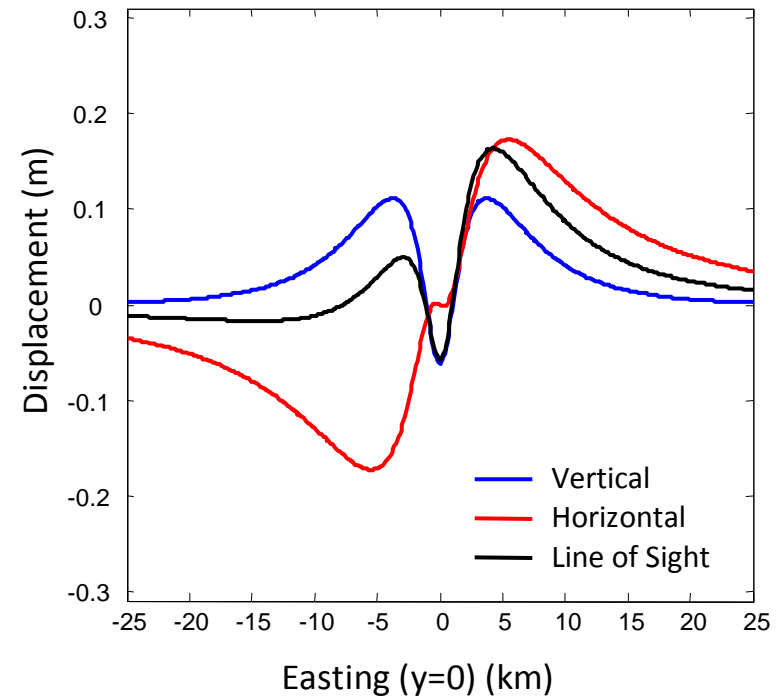
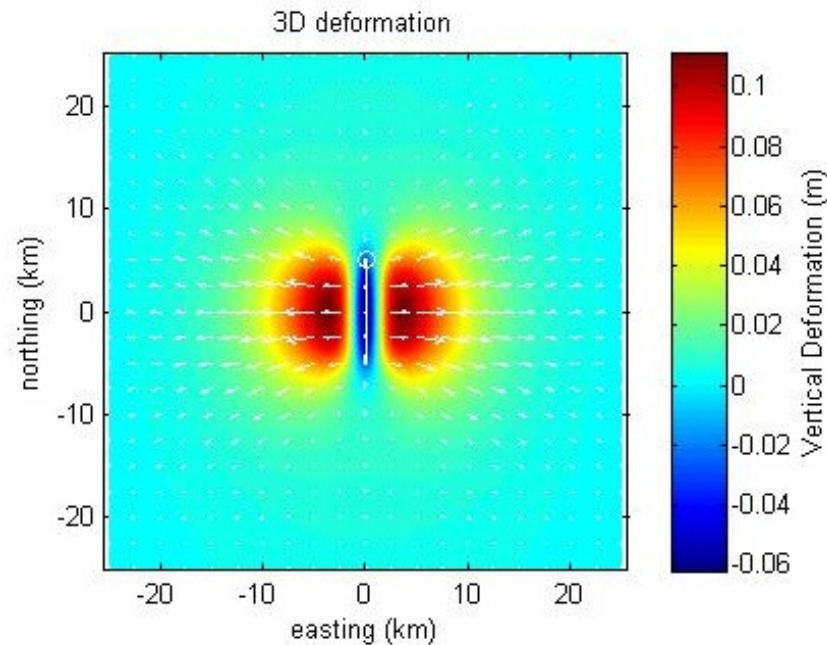
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Dyke 3: Dip = 90, Strike = 0,  $x=y=0$   
 $d_{\text{top}} = 4$  km,  $d_{\text{bot}} = 8$  km  
Length = 10 km  
Opening = 1 m [Volume =  $4 \times 10^7$  m<sup>3</sup>]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Okada, Y. (1985). Bull. Seis. Society of America 75 (4), 1135-1154 [surface]  
Okada, Y. (1992). Bull. Seis. Society of America 82 (2), 1018-1040 [internal]

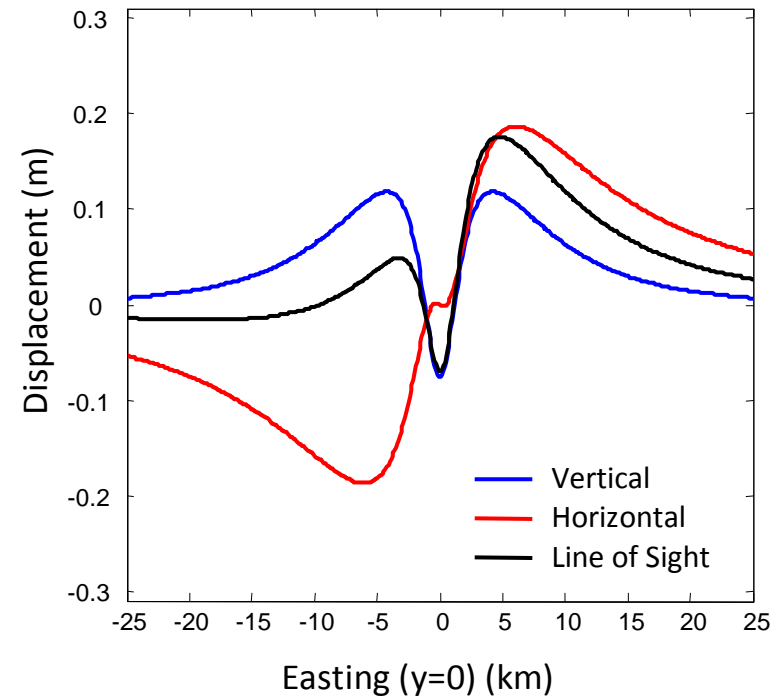
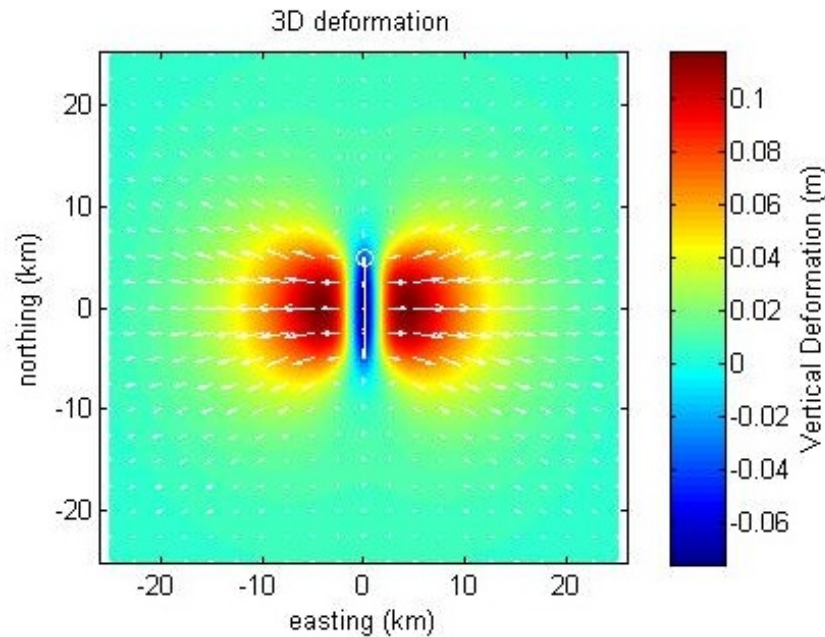
# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)



Dyke 1: Dip = 90, Strike = 0,  $x=y=0$   
 $d_{\text{top}} = 2 \text{ km}$ ,  $d_{\text{bot}} = 8 \text{ km}$   
Length = 10 km  
Opening = 1 m [Volume =  $6 \times 10^7 \text{ m}^3$ ]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Okada, Y. (1985). Bull. Seis. Society of America 75 (4), 1135-1154 [surface]  
Okada, Y. (1992). Bull. Seis. Society of America 82 (2), 1018-1040 [internal]

# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)



Dyke 4: Dip = 90, Strike = 0,  $x=y=0$

$d_{\text{top}} = 2 \text{ km}$ ,  $d_{\text{bot}} = 12 \text{ km}$

Length = 10 km

Opening = 1 m [Volume =  $10 \times 10^7$

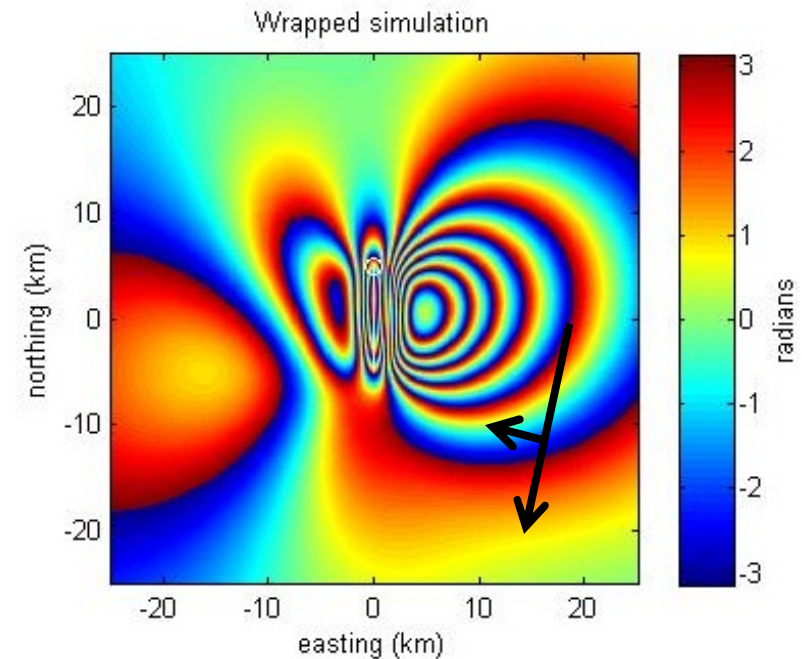
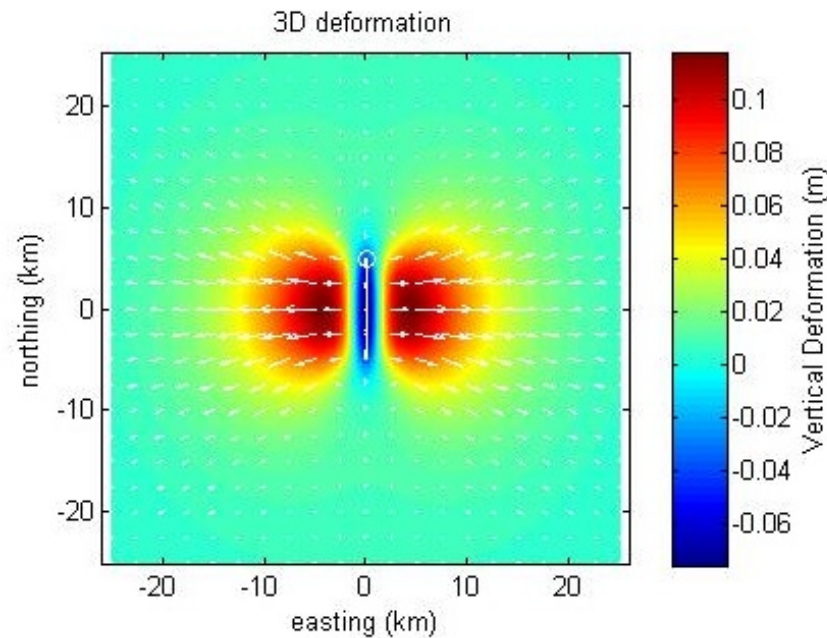
$\text{m}^3$ ]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

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Dyke 4: Dip = 90, Strike = 0,  $x=y=0$

$d_{\text{top}} = 2 \text{ km}$ ,  $d_{\text{bot}} = 12 \text{ km}$

Length = 10 km

Opening = 1 m [Volume =  $10 \times 10^7$

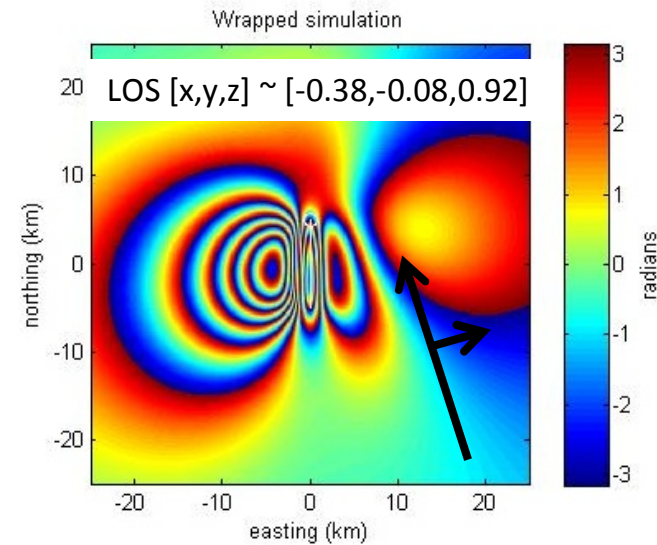
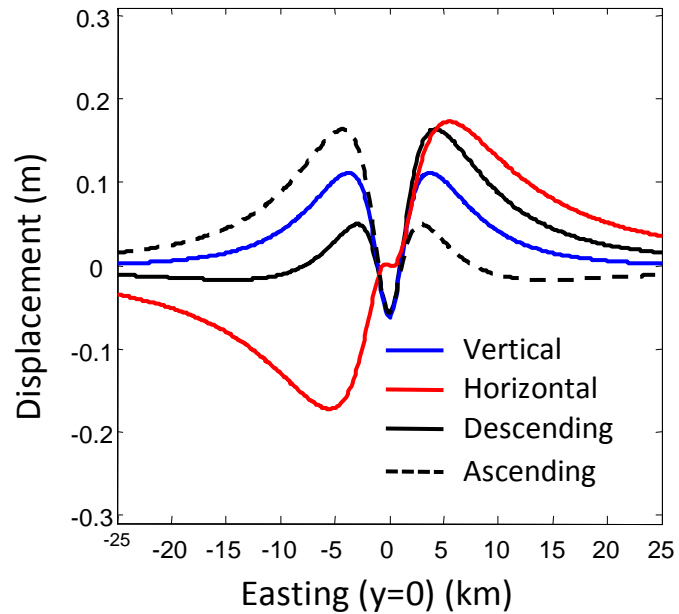
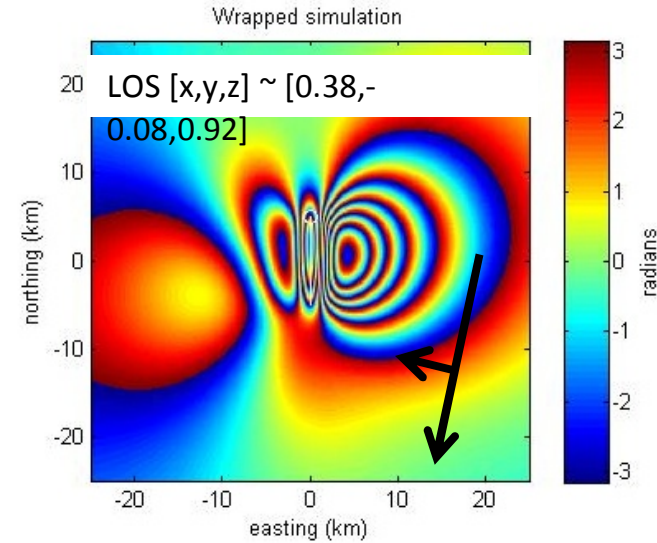
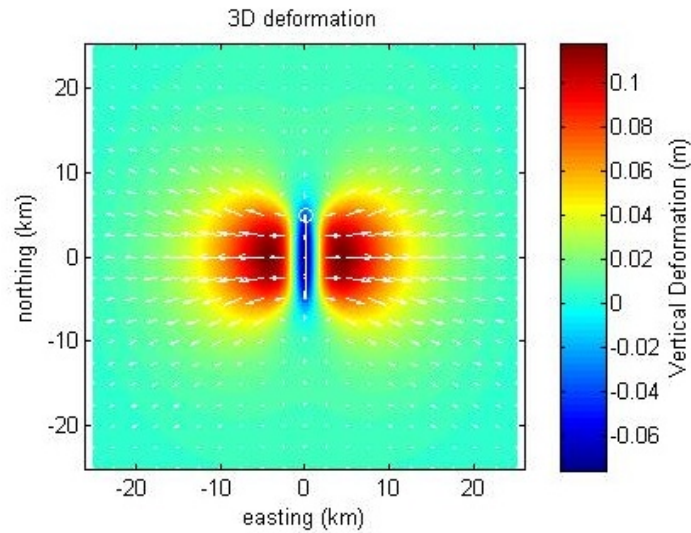
$\text{m}^3$ ]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

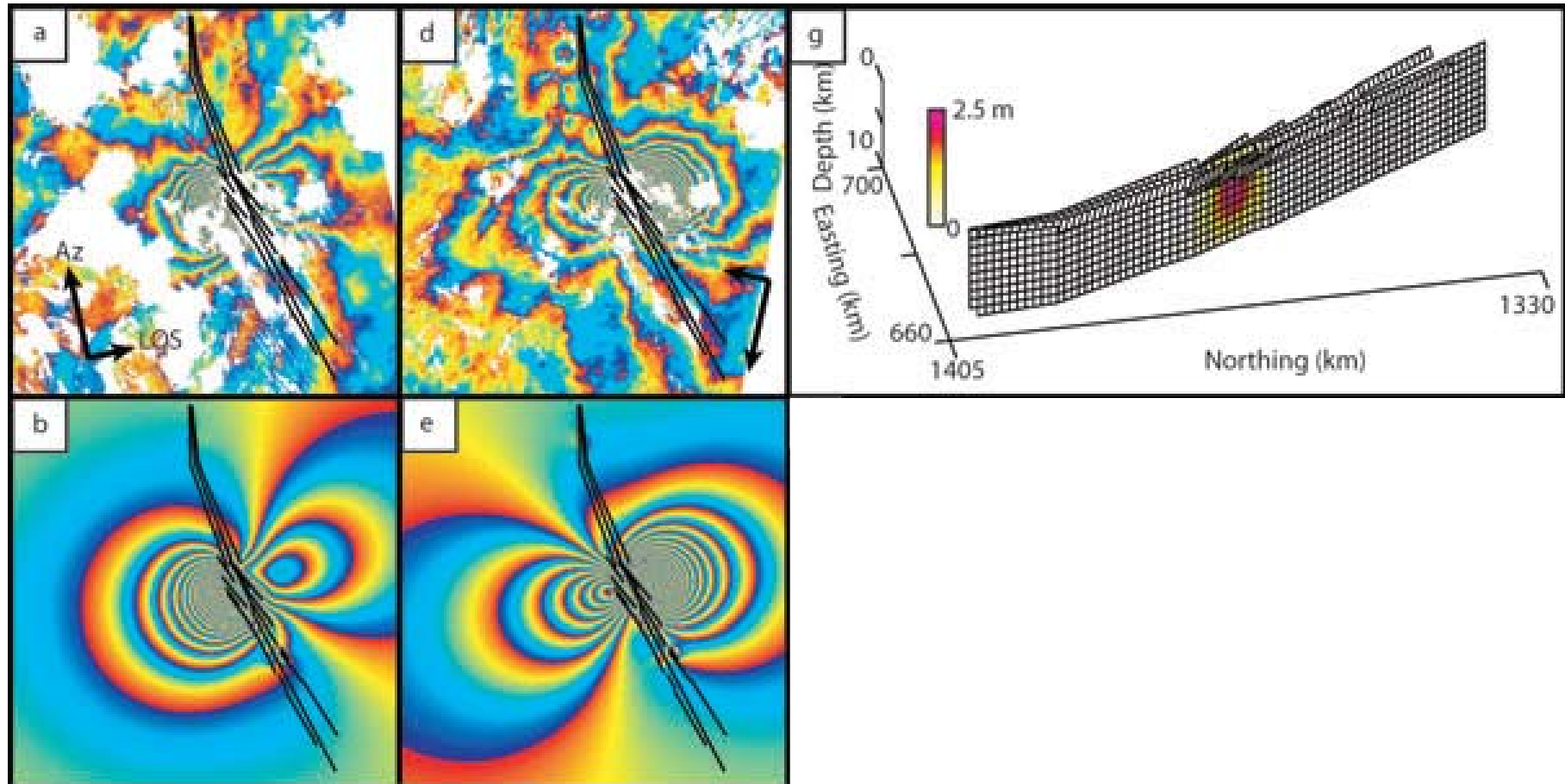
Okada, Y. (1985). *Bull. Seis. Society of America* 75 (4), 1135-1154 [surface]

Okada, Y. (1992). *Bull. Seis. Society of America* 82 (2), 1018-1040 [internal]

# Ascending vs Descending



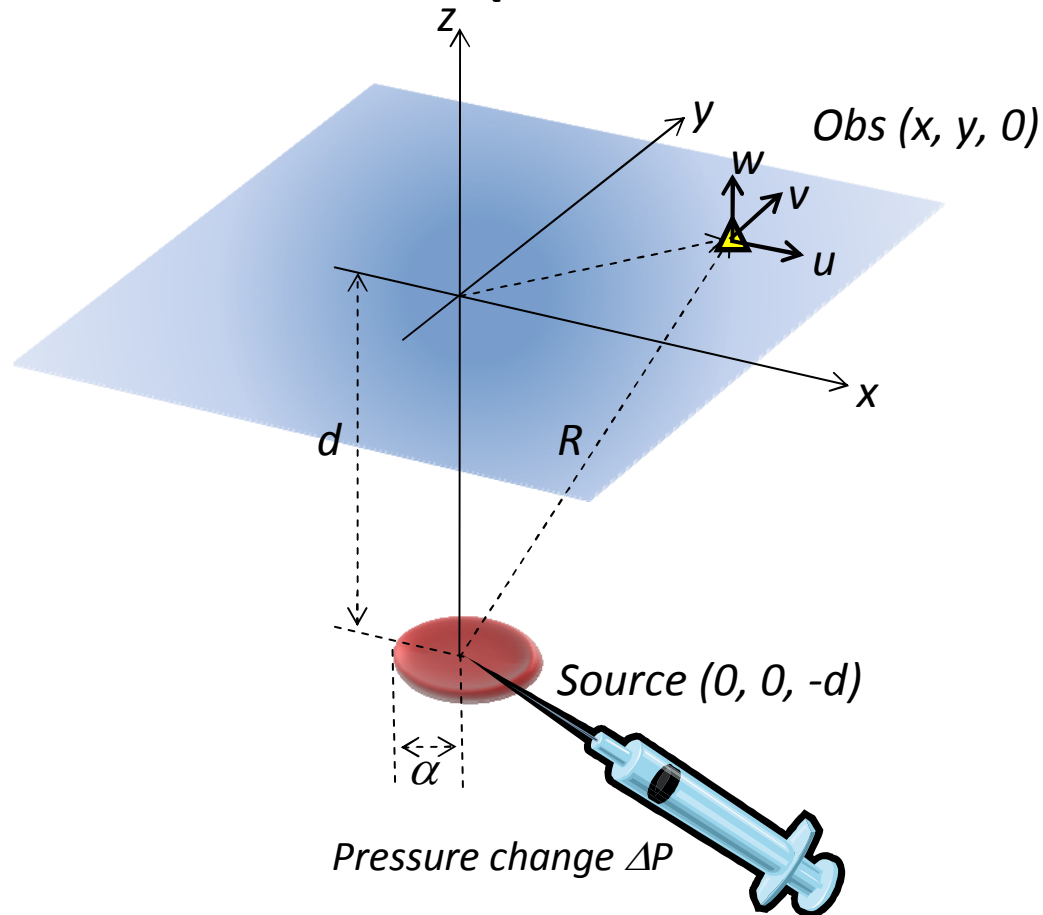
# Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or **dykes**)



June 2006 Dyke, Afar, Ethiopia. Hamling et al. (2009), Geophysical J. Int.



# Simple Sources: 3. Penny-shaped crack (‘Fialko’ model for sills)



## Assumptions

1. Isotropic elastic half space (Poisson's ratio  $\nu$ ; Shear modulus  $\mu$ )
2. Uniformly pressurized horizontal crack

## Parameters

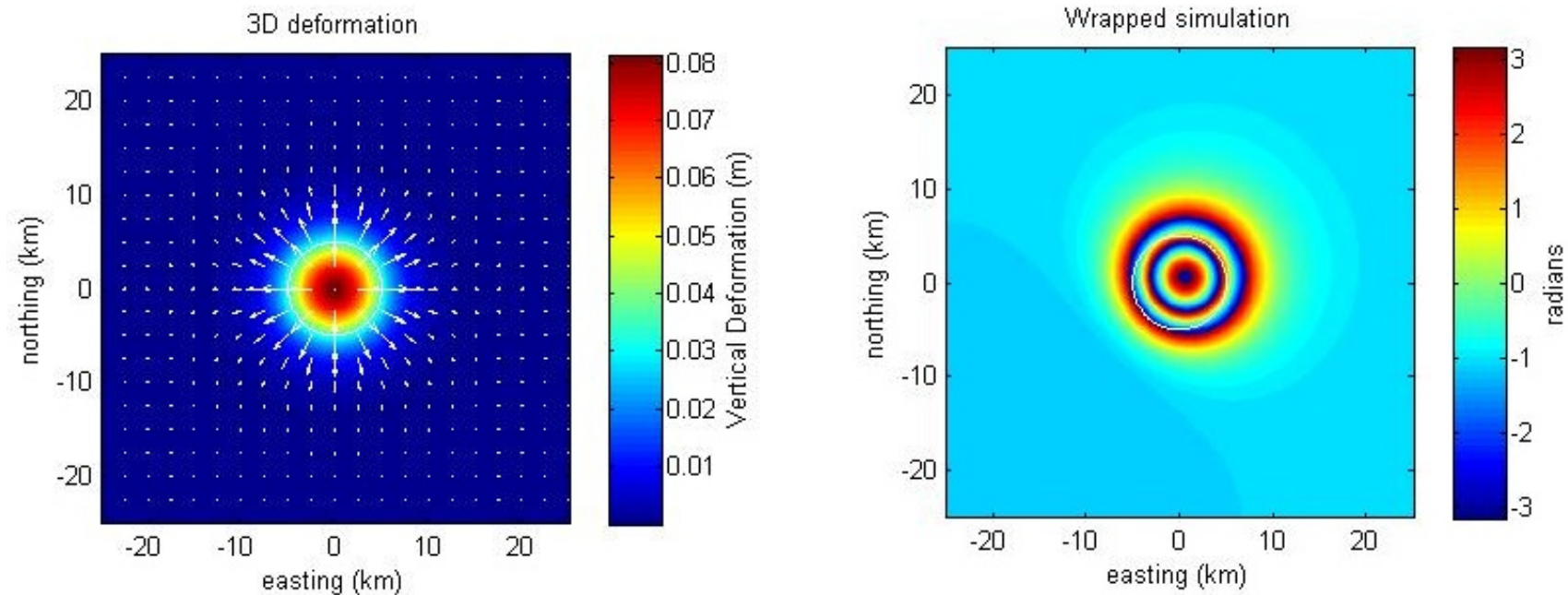
1. Depth
2. Radius of source
3. Pressure

For full equations see Fialko et al., 2001.

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Fialko, Y. et al. (2001), *Geophys. J. Int.*, 146, 181-190

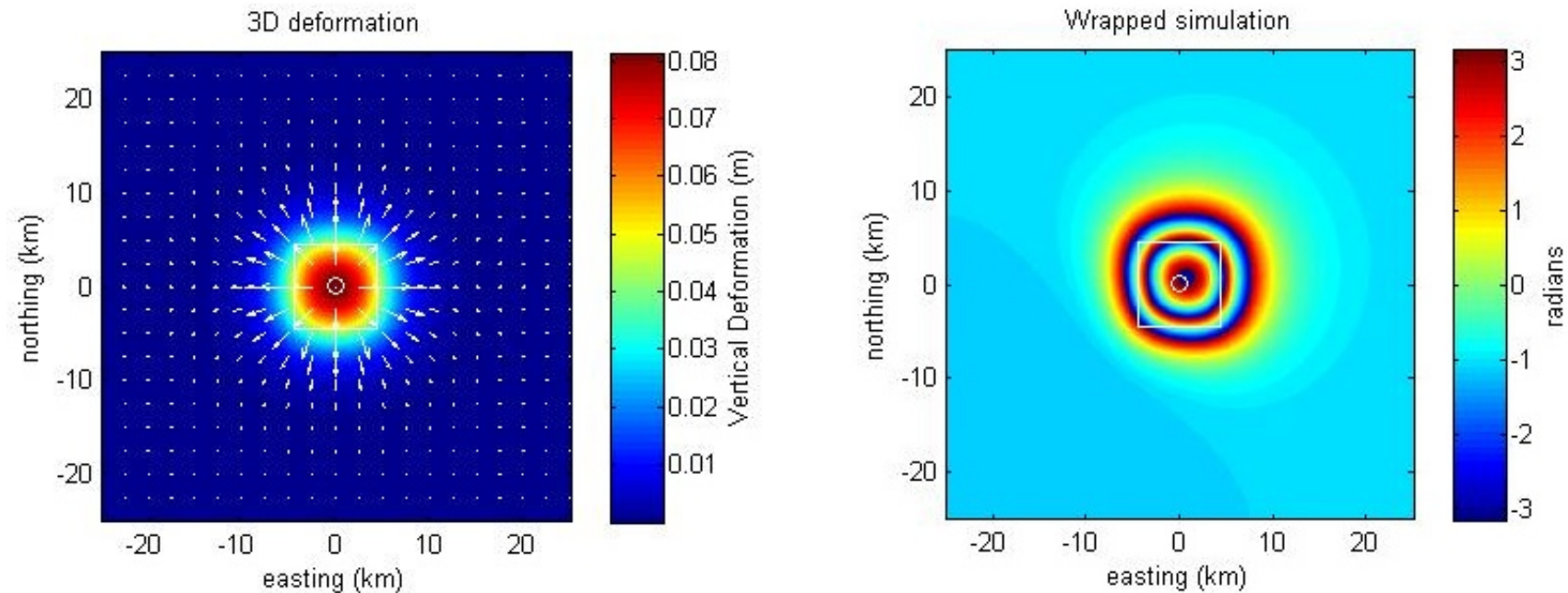
# Simple Sources: 3. Penny-shaped crack (‘Fialko’ model for sills)



Penny Sill: Source Depth = 5 km; Radius = 5 km;  
Pressure = 0.77 MPa

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.  
Fialko, Y. et al. (2001), *Geophys. J. Int.*, 146, 181-190

# Simple Sources: 3. Penny-shaped crack (‘Fialko’ model for sills)



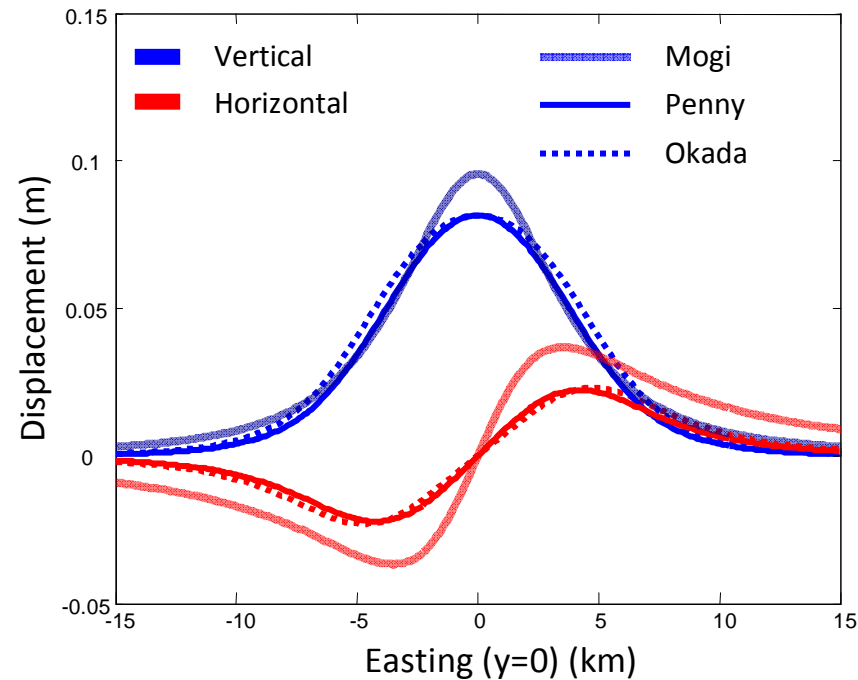
Penny Sill: Source Depth = 5 km; Radius = 5 km;  
Pressure = 0.77 MPa

Okada Sill: Source Depth = 5 km; Length = Width = 8.86 km ( $= \sqrt{\pi 5^2}$ )  
Opening = 0.13 m ( $= 10/(\pi 5^2)$ ) [Volume =  $10^7$  m<sup>3</sup>]

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Fialko, Y. et al. (2001), *Geophys. J. Int.*, 146, 181-190

# Simple Sources: 3. Penny-shaped crack (‘Fialko’ model for sills)

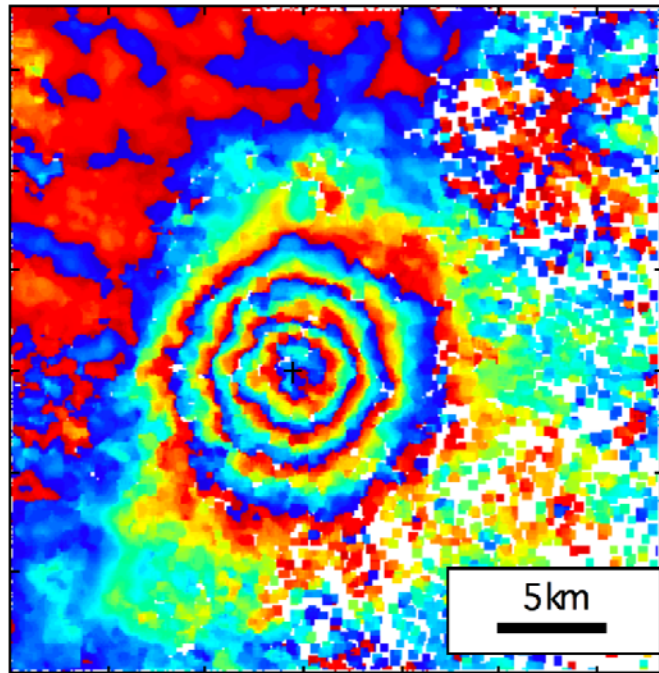


Penny Sill: Source Depth = 5 km; Radius = 5 km;  
Pressure = 0.77 MPa

Okada Sill: Source Depth = 5 km; Length = Width = 8.86 km ( $= \sqrt{\pi 5^2}$ )  
Opening = 0.13 m ( $= 10/(\pi 5^2)$ ) [Volume =  $10^7$  m<sup>3</sup>]

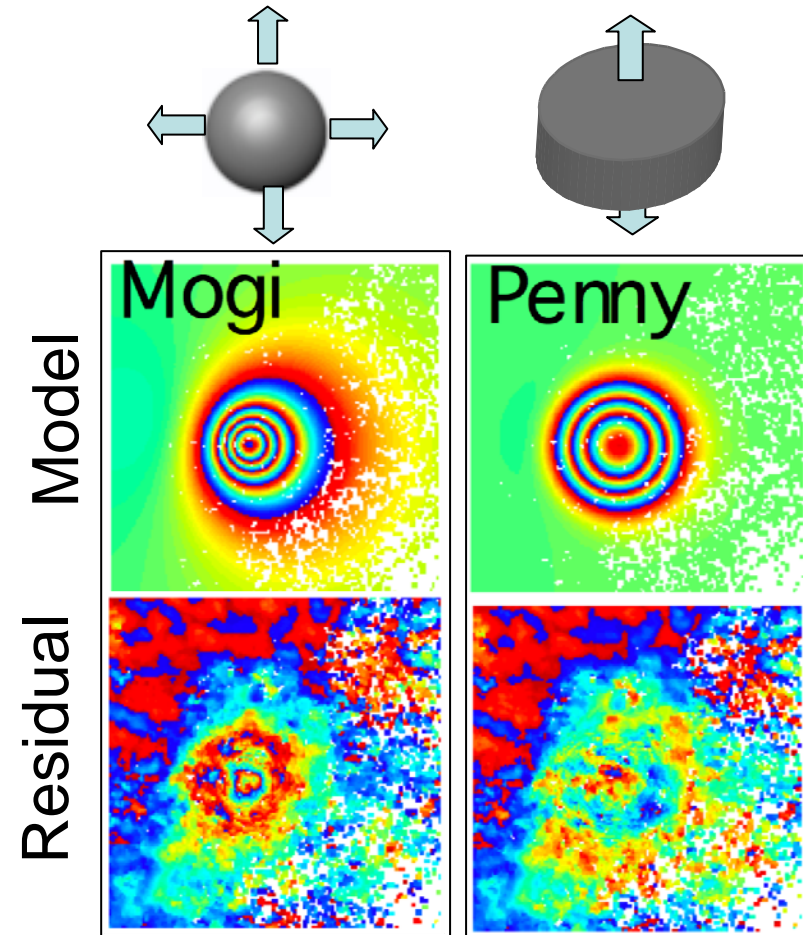
Mogi Point: Source Depth = 5 km; Volume Change =  $10^7$  m<sup>3</sup>

# Simple Sources: 3. Penny-shaped crack ('Fialko' model for sills)



Stack of 5 images

Biggs et al. (2009), *Geology*, In Press.



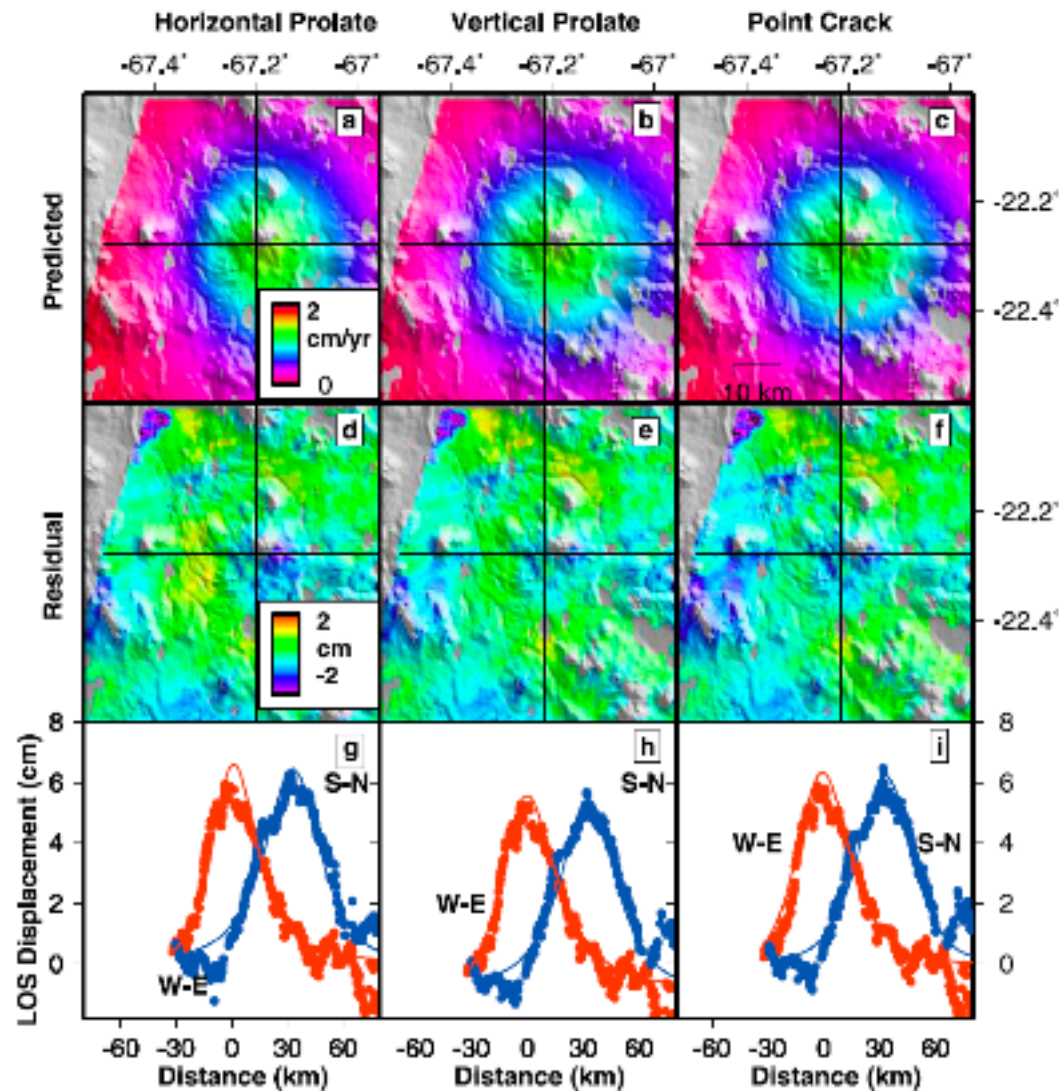
Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Fialko, Y. et al. (2001), *Geophys. J. Int.*, 146, 181-190

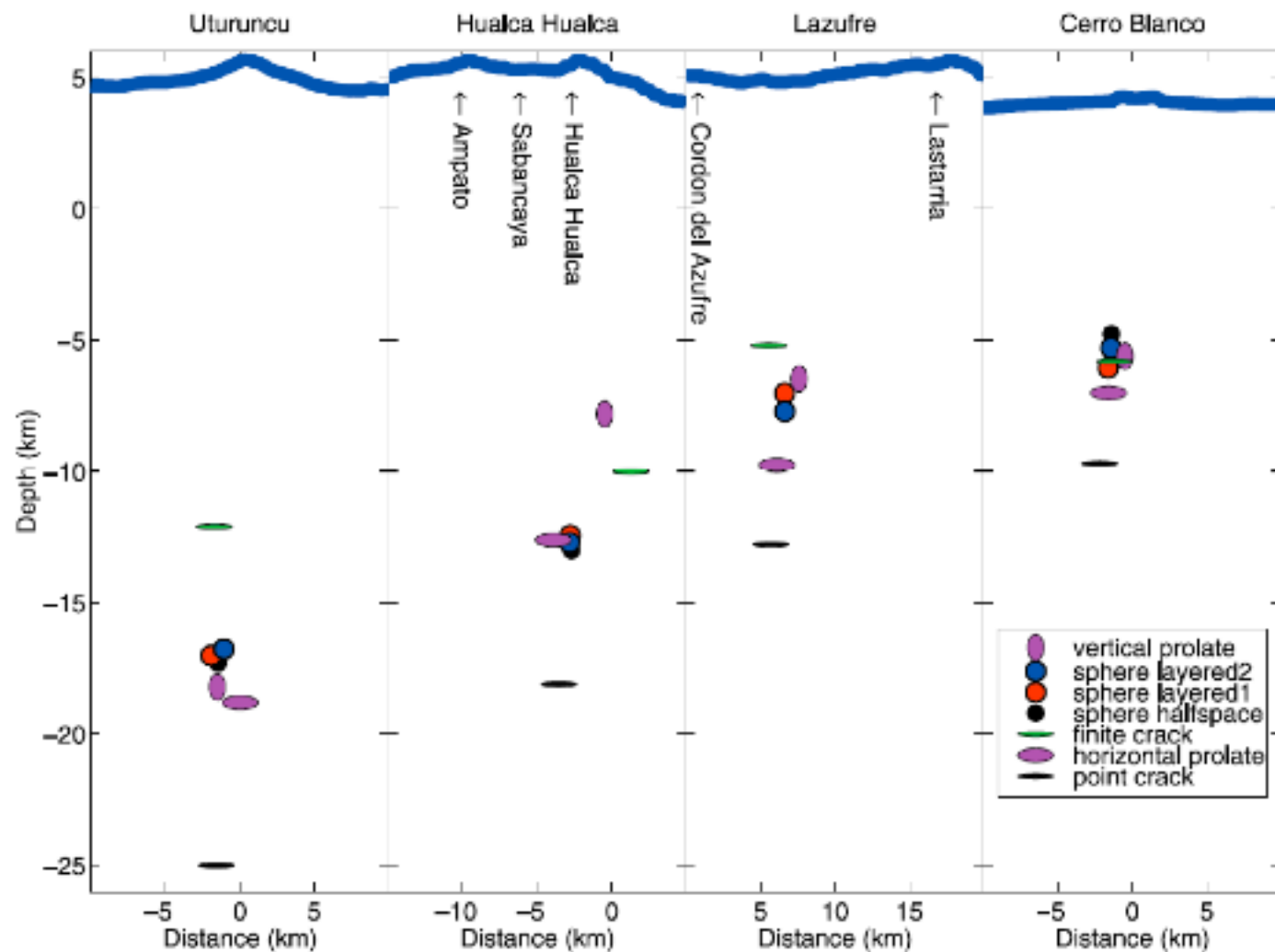
# Limitations of Simple Models used in Volcano Geodesy

- Ambiguity of source geometries
- Isotropic, Elastic Half-Space Assumption
- Volumes/Pressures dependent on assumptions about geometries or magma/rock properties
- Models are purely kinematic
- Magma is hot → visco-elastic effects?
- How to interpret signals?

# Source Model Ambiguity: Uturuncu



# Source Geometry Ambiguity: Effect on Depth Estimates





# InSAR vs GPS for geohazards

	InSAR	GPS
Frequency of Observation	Every 10-40 days for each satellite.	Up to 50 Hz. Typically get daily positions for continuous sites.
Spatial coverage	Continuous (in coherent areas)	Points
Dimensionality	1D (Line of Sight) Displacement	3D Displacements
Cost	Data is cheap/free for most satellites	Cost of instruments vary
Field requirements	None	Installation and servicing on the ground
Accuracy	~1 cm for single interferogram. Better for time-series/stacks	~1mm/yr for continuous sites

Dzurisin, D. (2007), *Volcano Deformation: Geodetic Monitoring Techniques*.

Fialko, Y. et al. (2001), *Geophys. J. Int.*, 146, 181-190