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# PSEUDOCHAOS AND STABLE-CHAOS IN STATISTICAL MECHANICS AND QUANTUM PHYSICS

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# **BOOK OF ABSTRACTS**

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# PREFACE

By pseudochaos it is usually meant the irregular behavior characterizing the unpredictable evolution of dynamical systems in the absence of the Lyapunov instability mechanism. Far from being a mere mathematical curiosity, pseudochaos has appeared in an astonishingly wide range of models, including spatially extended systems (e.g., coupled map lattices), finite and infinite polygonal billiards, cellular automata, neural networks etc. The many progresses made in the understanding of pseudochaos have unveiled its importance for the foundations of Statistical Mechanics (validity of transport equations, role of ergodicity), for the crossover from discrete to continuous descriptions, and even for the semi-classical limit of quantum mechanics.

In the last decade an increasing community of researchers had to deal with pseudochaos while trying to tackle problems in basic science as well as in applications and experiments. Actually, beyond its conceptual interest, pseudochaos provides an effective tool for analyzing and understanding many practical problems like transport in nanosystems, neural synchronization and plasticity, cryptography and adaptive computational procedures.

The main goal of this Workshop is surveying the relevance of pseudochaos for various research domains in mathematics and physics.

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- F. Vivaldi Queen Mary, University of London, U.K

This book is also available in electronic format at the WEB site:

# **ABSTRACTS OF LECTURES**

(in alphabetical order of lecturer)

## Quantum Transport

Italo Guarneri Università degli Studi dell'Insubria

Connections between transport and spectral properties. Transport for mixed systems and accelerator modes. Decay from classically stable islands.

#### From normal to anomalous deterministic diffusion

Rainer Klages

Queen Mary University of London School of Mathematical Sciences, London, UK www.maths.qmul.ac.uk/~klages

In my two lectures I will review how both normal and anomalous diffusion can be understood on the basis of microscopic deterministic chaos [1,2]. The first lecture is about 'normal' deterministic diffusion in chaotic dynamical systems, where the mean square displacement of an ensemble of particles grows linearly in time. Starting from the paradigmatic Bernoulli shift I will briefly remind of some fundamental chaos quantities and their relation to each other. I will then outline the state of the art about diffusion in simple deterministic random walk models constructed by using such maps. Analytical results yield a diffusion coefficient that is a complicated function of control parameters displaying both linear and fractal parameter dependencies. Random walk behavior is only recovered on coarse parameter scales. Computer simulations predict analogous results for Hamiltonian particle billiards like the periodic Lorentz gas, which can be related to single-molecule diffusion in zeolite nanopores.

The second lecture will start by introducing a nonlinear generalization of the Bernoulli shift that displays intermittency. This map exhibits weak chaos, and crosslinks to the new mathematical field of infinite ergodic theory will be outlined. I will review stochastic continuous time random walk theory by applying this fundamental concept to a random walk version of this model. The result predicts subdiffusion, that is, anomalous diffusion with a mean square displacement that grows less than linearly in time [2,3], as is confirmed by computer simulations. In a scaling limit a fractional diffusion equation will be derived. However, as in case of normal diffusion there are complicated fractal, possibly discontinuous parameter dependencies deviating from the predictions of stochastic theory. I will conclude by outlining applications of these ideas to theory and experiment on anomalous biological cell migration [4].

[1] R.Klages, Microscopic Chaos, Fractals and Transport in Nonequilibrium Statistical Mechanics (World Scientific, Singapore, 2007).

[2] R.Klages, From Deterministic Chaos to Anomalous Diffusion (book chapter for Reviews of Nonlinear Dynamics and Complexity Vol. 3, H.G.Schuster (Ed.), Wiley-VCH, Weinheim, to appear).

[3] R. Klages, G.Radons, I.M.Sokolov (Eds.), Anomalous transport (Wiley-VCH, Weinheim, 2008).

[4] P.Dieterich, R.Klages, R.Preuss, A.Schwab, PNAS 105, 459 (2008).

### STABLE CHAOS: AN INTRODUCTION Antonio Politi CNR – Institute of Complex Systems

#### Abstract

The phenomenon of irregular dynamics in the presence of negative (or non-positive) Lyapunov exponents is reviewed in different classes of models (ranging from coupled maps, to neural networks). The most appropriate tools for its identification and characterization are introduced and discussed. In particular, since *stable chaos* is a transient phenomenon that ends up in a periodic dynamics, I focus on the divergence rate of the transient for increasing the system size.

The analogy with chaotic cellular automata allows to clarify the transient and yet stationary character of *stable chaos*. The possibility and limits of a rigorous mapping between the two phenomena is illustrated with reference to a chain of periodically forced Duffing oscillators. Such an analogy suggests the idea of characterizing *stable chaos* in terms of the propagation velocity of perturbations of finite amplitude (the only ones that can be defined in the context of cellular automata). As a result, it is possible to give a broader definition of *stable chaos* as a phenomenon characterized by finite perturbations that propagate faster than infinitesimal ones.

Classical tools like fractal dimensions and embedding theory are also introduced to show the relevant differences with standard space-time chaos. Moreover, I discuss the role of fluctuations of the Lyapunov exponents (multifractal analysis) as a way to resolve the paradox of an irregular and yet *linearly stable* dynamics.

Finally, the possible mechanims that may be generically responsible for the generation of *stable chaos* are reviewed. I conjecture that this is due to the presence of strong and localized nonlinearities (in phase space) and I will argue about the reason for a possible generic existence of such features.

#### Quantum chaos and unpredictability in quantum systems

#### Uzy Smilansky

Various quantum observables display random-like features which can be attributed and analyzed by invoking the properties of their classically chaotic counterparts. I shall demonstrate this phenomenon by discussing various examples which relate to spectral properties (both spectra and wave functions) and scattering data. Analogous phenomenae are observed also for "quantum graphs" (where the underlying classical dynamics is not standard) and the origin of the unpredictable behavior is somewhat different.

# **ABSTRACTS OF TALKS**

(in alphabetical order of speaker)

#### Novelties in the Fermi-Pasta-Ulam problem

#### Giancarlo Benettin

More than fifty years after the celebrated "numerical experiment" by Fermi, Pasta and Ulam in 1954, the FPU problem — in essence, the interplay between dynamics and statistical mechanics for a system of many weakly coupled oscillators — is still not understood in many fundamental aspects and in the possible physical consequences. Recently, thanks to the increased power of computers on one hand, and to significant theoretical developments in nonlinear waves on the other hand, a little progress has been made. The aim of the talk is to revisit the FPU problem, in the light of some recent results and work in progress, with special attention to the main question, namely the behavior of the system, and specifically the time scale for statistical equilibrium and the possible presence of metastable states, in the thermodynamic limit. Both one–dimensional and two–dimensional models will be considered.

## WHERE TO PLACE A HOLE TO ACHIEVE THE FASTEST ESCAPE Leonid A. Bunimovich Georgia Institute of Technology, USA

A natural question of how the survival probability depends upon a position of the hole was seemingly never addressed in the theory of open dynamical systems. This dependence occurred occurred to be very essential for some classes of dynamical systems. It is closely related to the distribution of periodic orbits. It seems obvious that the bigger the hole is the faster is the escape through that hole. However, generally it is not true, and some properties of dynamics may play a role comparable to the size of the hole. The main result is valid for all finite times (starting with some moment) which is unusual in dynamical systems theory where typically statements are asymptotic when time tends to infinity. These finite time results open up a completely new perspective for the studies of dynamics. Clearly the finite time predictions may have many applications as the problem in the title has by itself. One of the natural applications could be in the quantum chaos business as our results reveal some new properties of the periodic orbits.

#### Transport in chaotic and non-chaotic systems

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#### Abstract

After reviewing some chaotic and non-chaotic models able to display macroscopic transport properties I will focus on two deterministic models for Brownian motion. The first model consists of a heavy hard disk immersed in a rarefied gas of smaller and lighter hard disks acting as a thermal bath. The second is the same except for the shape of the particles, which is now square. The basic difference in these two systems lies the fact that hard-core elastic collisions make the dynamics of the disks chaotic whereas that of squares is not. Remarkably, this difference does not reflect on the transport properties of the two systems: simulations show indeed that the diffusion coefficients, velocity correlations and response functions of the heavy impurity are in agreement with kinetic theory for both the chaotic and non-chaotic model. The relaxation to equilibrium instead is very sensitive to the kind of interactions.

These observations together with previous results are used to think back and discuss some issues connected to chaos, statistical mechanics and diffusion.

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#### On classical and semiclassical properties of the triangle map

Mirko Degli Esposti, Università di Bologna

In this talk we aim to discuss, both analytically and numerically, the classical and quantum properties of the triangle map, a two-parameter family of piecewise parabolic automorphism of the two-dimensional torus. The dynamics is studied numerically by means of two different symbolic encoding schemes, both relying on the geometrical and topological properties of the map.

The ergodic properties of the triangle map are presented and discussed in terms of the Markov transition matrices associated to the above schemes and furthermore compared to the spectral properties of the Koopman operator in L2 (T2). Finally, semicalssical properties, such as the statistics of quantum energy levels v.s. random matrix theory are discussed.

# Open billiards: Regular, chaotic and pseudo-chaotic

C. P. Dettmann, joint work with L. A. Bunimovich, E. G. D. Cohen and O. Georgiou

Open billiards are dynamical systems in which a particle moves in straight lines, except for collisions with the boundary and absorption at one or more holes which are typically subsets of the boundary. We can ask how the survival probability P(t) given an initial equilibrium distribution depends on time t and on the size and position of the hole(s), and how this relates to the dynamics of the corresponding closed system. The present work involves analytical and numerical explorations of four examples composed from straight and/or circular arcs: the diamond [1], the circle [2], the stadium [3], and extended Lorentz (circle, chaotic) and Ehrenfest (square, pseudo-chaotic) models [4]. In most cases we consider the limit of long times and small holes, in that order. The diamond is strongly chaotic, with exponential decay of the survival probability at long times at a rate proportional to the size of the hole(s) to leading order. We present a formalism to express corrections to this, including the interaction between two holes, in terms of correlation sums. The circle is integrable;  $P(t) \sim C/t$  at long times, with the coefficient C inversely proportional to the size of the hole(s) to leading order, with corrections written in terms of the Riemann zeta function or generalisations. There are connections to the Riemann Hypothesis, and also scaling behaviour observed at finite times. The stadium is intermittent, with  $P(t) \sim C/t$  also at long times; for holes in the straight segment the coefficient C can be determined exactly, for finite (not necessarily small) holes. The extended billiards exhibit varied behaviour depending on details of the dynamics. Numerical results confirming and illuminating each case are provided. Three features of importance are the observation of scaling behaviours, the role of the periodic orbits, and the interaction between holes. The results are of interest for the mathematical understanding of a wide class of open dynamical systems beyond billiards. They also motivate particular directions to follow in physical experiments, specifically the use of escape properties to probe the hyperbolicity, correlation functions and periodic orbits of the system.

#### References

- [1] L. A. Bunimovich and C. P. Dettmann, Peeping at chaos: Nondestructive monitoring of chaotic systems by measuring long-time escape rates, EPL, **80** 40001 (2007).
- [2] L. A. Bunimovich and C. P. Dettmann, Open circular billiards and the Riemann hypothesis, Phys. Rev. Lett. 94 100201 (2005).
- [3] C. P. Dettmann and O. Georgiou, Survival probability for the stadium billiard, arxiv:0812.3095
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#### Anderson Localization for the Nonlinear Schroedinger Equation (NLSE): results and puzzles

#### Shmuel Fishman Department of Physics, Technion, Haifa

The NLSE is relevant for the explorations of Bose-Einstein Condensates and for Nonlinear Classical Optics. A natural question is whether Anderson Localization survives the effect of nonlinearities in one dimension. Relevant experimental, numerical, heuristic and rigorous results will be presented. A perturbation expansion in the nonlinear term was developed and used to obtain a rigorous bound on the spreading for short times. In particular it is found that exponential localization holds at least for time scales inversely proportional to the square of the nonlinearity. The perturbation theory was used to obtain the values of the wave functions for short times. Conjectures on the long time behavior will be presented. Some results for a generalized version of the NLSE will be presented as well. The work reported in the talk was done in collaboration with Avy Soffer, Yevgeny Krivolapov and Hagar Veksler.

# Synchronization and Self-Organized Criticality in Neuronal Systems

T. Geisel\*

Max Planck Institute for Dynamics and Self-Organization & Faculty of Physics, University of Göttingen & Bernstein Center for Computational Neuroscience Göttingen

Numerous articles have been devoted to numerical simulations of more or less realistic and detailed models of complex neurons since it was hypothesized that the observed synchronization of distant neurons in the visual cortex might be related to the problem of feature binding in visual information processing. Such simulations, however, fall short of rigorously answering questions as e.g. on the stability or the speed of synchronization as a function of the network topology. I will review some of our work on abstract mathematical models of complex networks consisting of pulse-coupled neurons, where analytic approaches are feasible. We have shown, e.g., that the dynamics of synchronization is governed by unstable attractors in the case of excitatory coupling in the presence of delays. For inhibitory coupling there is stable nonchaotic irregular activity.

Considering networks with dynamical synapses, e.g., where the synaptic coupling exhibits fatigue under repeated presynaptic firing we found self-organized critical avalanches of activity. We show that in a range of interaction parameters this adaptation mechanism drives the network into a self-organized critical regime by adjusting the average coupling strengths to a critical value. We derive analytic expressions in a mean-field approach, which allow us to characterize the self-organization mechanism and explain recent experimental results in multi-electrode recordings of cortical slice cultures. In networks that include depressing and facilitating synapses we find phase transitions towards criticality with the coexistence of a SOC phase and a subcritical phase that are connected by a cusp bifurcation.

\*work in collaboration with A. Levina, M. Herrmann, M. Timme, and F. Wolf

### **References:**

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#### Stable or non-extensive chaos?

Peter Grassberger University of Calgary, Canada

We point out that the notion of instability and the associated notion of Lyapunov exponents is not unique for spatially extended systems with infinitely many degrees of freedom. In particular, one can associate positive Lyapunov exponents and entropies to "stable" chaotic systems, as is most obvious for "chaotic" cellular automata which can be considered as "superstable" extended dynamical systems in the conventional notation. The main difference between "stable" and bona fide unstable extended chaotic systems is then that the entropy is extensiv in the latter, while is is not extensiv in the former. For bounded (sub-)systems, the entropy is in all known examples related to the surface of the (sub-)system. We finally discuss whether there can exist also more general cases where the entropy scales with a power intermediate beween volume and surface.

#### On recurrence and transience for infinite periodic polygonal surfaces

#### Eugene Gutkin

Abstract. We study the dynamics of billiard orbits in noncompact periodic polygons and related flat surfaces. For instance, we investigate the recurrence and transience of trajectories in the rectangular Lorenz gas.

# Energy transmission in oscillator chains with disorder

Stefano Lepri Istituto dei Sistemi Complessi ISC-CNR

The joint effect of disorder and nonlinearity on energy transport is still poorly understood. We present some numerical results on propagation of energy in chains of linear and nonlinear oscillators with quenched disorder. Two types of perturbation are considered, either an external periodic driving or an initally localized kick. The existence of transmission thresholds is highlighted.

#### **Complexity of the Dynamics in Discrete-Random Systems**

#### Giorgio Mantica Università degli Studi dell'Insubria

I will describe quantitatively the combined effects of discretization and randomness in the generation of dynamical complexity. I will present numerical techniques and results for classical lattice systems and quantum maps on the torus. Finally, I will discuss the relevance of this research to the so-called decoherence approach to quantum chaos.

#### Nonlinear destruction of Anderson localization in disordered nonlinear lattices

<u>Arkady Pikovsky</u>, Karsten Ahnert, Mario Mulansky, Steffen Tietsche Department of Physics and Astronomy, University of Potsdam, Karl-Liebknecht-Str. 24/25, D-14476 Potsdam-Golm, Germany Dima Shepelyansky

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We discuss what happens to Anderson localization in a disordered lattice if a nonlinearity is present. This situation is relevant for lattices of coupled oscillators, for a Bose-Einstein condensate (described by a nonlinear Gross-Pitaevsky equation) in a disordered potential, and to light propagation in a disordered nonlinear medium. Our main model is a discrete Anderson chain with a nonlinear energy shift, or, equivalently, a discrete nonlinear Schrödinger lattice with disorder. We discuss three problems for this model: (i) How an initially localized wave packet spreads [1]; (ii) How a regular wave is transmitted through a nonlinear disordered layer [2]; and (iii) How a thermalization in a finite disordered nonlinear lattice occurs [3]. In all cases nonlinearity leads to a delocalization.

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[2] S. Tietsche and A. Pikovsky. Chaotic destruction of Anderson localization in a nonlinear lattice. *Europhysics Lett.*, 84(1):10006, 2008.

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# Hard-point gas in one dimension: from ergodicity to thermoelectricity

#### Tomaž Prosen

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Several conjectures and numerical results on ergodic properties of a gas of differentmass point particles with hard-point interaction in one dimension shall be discussed [1,2,3,4]. In particular I will focus on the connection between mixing property of such systems and some of their transport properties, such as thermal conductivity, thermo-power, etc. At the end, I will outline a simple exactly solvable onedimensional dynamical model of a heat engine and discuss dynamical conditions for approaching the Carnot's efficiency [5,6].

#### References

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#### Complexity of matter transport in chaotic and non-chaotic particle systems

#### L. Rondoni, Politecnico di Torino

The question of the onset of diffusion in systems of both interacting and noninteracting particles, confined within polygonal channels, is addressed from the point of view of space and time scales relevant for the existence of local thermodynamic equilibrium (LTE). The fact that chaos, in the sense of positive lyapunov exponents, is neither necessary nor sufficient for LTE, while particle interactions, large numbers and proper separation of scales are necessary, is illustrated by several examples, and by comparisons with the predictions of kinetic theory. Anomalous time scales, and the consequent unpredictable peculiar macroscopic behaviour of most systems here considered lead to a notion of "non-asymptotic" "transport complexity", meant to characterize physically interesting situations, in which LTE cannot be established. In these situations, differently from the cases in which standard thermodynamics applies, the properties of the systems of interest sensitively depend on external parameters.

## Google matrix, dynamical attractors

## and Ulam networks

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Abstract: The studies of the properties of the Google matrix generated by a coarsegrained Perron-Frobenius operator of the Chirikov typical map with dissipation are reported. The finite size matrix approximant of this operator is constructed by the Ulam method. This method applied to the simple dynamical model creates the directed Ulam networks with approximate scale-free scaling and characteristics being rather similar to those of the World Wide Web. The simple dynamical attractors play here the role of popular web sites with a strong concentration of PageRank. A variation of the Google parameter  $\alpha$  or other parameters of the dynamical map can drive the PageRank of the Google matrix to a delocalized phase with a strange attractor where the Google search becomes inefficient. The properities of the Google matrix in other networks are also discussed.

#### Chaotic synchronizations of spatially extended systems as non-equilibrium phase transitions

A. Torcini

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The general mechanisms responsible for the synchronization of chaotic extended systems with short-range interactions in one spatial dimension are briefly summarized [1, 2], moreover recent results concerning bidimensional lattice [10] as well as power-law coupled systems [8] are presented. In particular, chaotic synchronization of replica of coupled map lattices is considered: two replicas of spatially extended chaotic systems synchronize to a common spatio-temporal chaotic state when coupled above a critical strength. As a prototype of each single spatio-temporal chaotic system a lattice of maps is considered. Furthermore, each unit in the one dimensional chain is linked to the corresponding one in the replica via a local coupling [2]. The spatial extension allows to interpret the synchronization transitions (STs) as nonequilibrium critical phenomena: namely, as absorbing phase transitions [3–6]. Within this framework two different kind of continuous transitions have been identified for nearest-neighbour coupling : one ruled by linear mechanisms and one by nonlinear effects [7, 9]. In the first case the ST belongs to the multiplicative noise (MN) universality class, while if nonlinear effects prevail the critical exponents coincide with those measured for directed percolation (DP). More recently spatially extended chaotic systems with algebraically decaying interactions have been considered. Also in this situation the STs appear to be continuous, while the critical indexes vary with continuity with the power law exponent characterizing the interaction. Moreover, for discontinuous maps strong numerical evidences indicate that the transition belongs to the "anomalous directed percolation" family of universality classes previously found for Levy-flight spreading of epidemic processes [8]. As a last aspect STs in two spatial dimensions will be discussed with the aim to propose a framework for an experimental measurements of MN critical exponents, since experimental evidence of this universality class are still lacking [10].

- L. Baroni, R. Livi, and A. Torcini, in *Dynamical Systems: from Cristal to Chaos*, eds J.M. Gambaudo, P. Hubert, P. Tisseur and S. Vaienti (World Scientific, Singapore, 2000) p. 23; L. Baroni, R. Livi, and A. Torcini, Phys. Rev. E 63, 036226 (2001).
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#### PSEUDO-CHAOS IN PIECEWISE ISOMETRIC SYSTEMS

Franco Vivaldi Queen Mary, University of London

After a long gestation, intermittent progress, and repeated discoveries of the same basic phenomena, the study of piecewise isometries has now blossomed into an active area of research, fuelled by its geometrical appeal and important applications, particularly to digital data processing.

In this class of dynamical systems the geometry of phase space is shaped by discontinuity rather than nonlinearity, resulting in strikingly complex behaviour from minimal ingredients. Piecewise isometries have zero entropy, and they may be thought of as a discontinuous analogue of smooth symplectic maps with divided phase space, combining stable motions and pseudo-chaos.

I will survey recent results and open problems in the two-dimensional case, considering scaling, unstable periodic orbits, near-resonant behaviour, invariant curves. The mathematical tools needed to study these systems combine elements of dynamical systems and ergodic theory, algebraic number theory, fractal geometry, and diophantine approximations.

# **ABSTRACTS OF POSTERS**

(in alphabetical order of presenting author)

#### Chaotic Behavior of the Q-state Potts Model on Recursive Lattice: Q < 2

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We considered Q-state Potts model on Bethe lattice in presence of external magnetic field for 0 < Q < 1and 1 < Q < 2 separately by means of a recursion relation technique. This allows studying the phase transition mechanism in means of the obtained one dimensional rational mapping. The convergence of Feigenabaum  $\alpha$  and  $\delta$  exponents for the aforementioned mapping is investigated. We regarded the Lyapunov exponent as an order parameter for the characterization of the model and showed numerical that it is positive indeed in the chaotic regime for the obtained rational mapping. Arnold tongues with winding numbers w = 1/2 and w = 2/4 are constructed for Q < 2. The dependence of the mapping's behavior on the value of Q is investigated. We also proposed an approximate method for constructing Arnold tongues via  $\delta$  Feigenbaum exponent.

#### Time reversal focusing as a novel Loschmidt echo procedure in wave dynamics

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The focalization of acoustic waves through the time reversal mirror technique [1] in a non-homogeneous medium was an experimental deed showing that an initially localized pulse can be accurately reconstructed by an array of receiver-emitter transducers that reinject the recorded signal. These experiments provoked a natural surprise while yielding reconstructions, that albeit not perfect, were highly faithful. In this time-reversal focusing (TRF) procedure the play back signal builds up in the region of the original excitation, in the form of a reversed wave amplitude and can be viewed as the wave version of the Loschmidt echo [2]. A salient feature of the TRF experiments is that the refocusing improves when the wave-propagation occurs in a disordered medium or in a chaotic cavity, as compared with the homogeneous or integrable case. In this work we develop a semiclassical theory [3] of the TRF in order to explain this robustness and the role played by chaos. Considering the ergodic properties in the cavities, we propose different models for the perturbation and analyze those regimes where the focusing takes place. Finally, we compare the obtained analytical results with numerical simulations.

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#### On universality of algebraic decays in Hamiltonian systems

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Hamiltonian systems with a mixed phase space typically exhibit an algebraic decay of correlations and of Poincare' recurrences, with numerical experiments over finite times showing system-dependent power-law exponents. We conjecture the existence of a universal asymptotic decay based on results for a Markov tree model with random scaling factors for the transition probabilities. Numerical simulations for different Hamiltonian systems support this conjecture and permit the determination of the universal exponent.

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#### Transport phenomena in the asymmetric quantum multibaker map

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By studying a modified unbiased quantum multibaker map, we were able to obtain a finite asymptotic quantum current without a classical analog. We show a characterization of the finite asymptotic current behavior with respect to the  $\hbar$  value, the shape of the initial conditions, and the features of the spectrum. We have considered different degrees of asymmetry in these studies and we have also analyzed the classical and quantum phase-space distributions for short times in order to understand the mechanisms behind the generation of the directed current. This result suggests a general method for the design of purely quantum ratchets and sheds light on the investigation of the mechanisms leading to net transport generation by breaking symmetries of quantum systems. Moreover, we propose the multibaker map as a resource to study directed transport phenomena in chaotic systems without bias. In fact, this is a paradigmatic model in classical and quantum chaos, but also in statistical mechanics.

#### **Stochastic Manifestation of Chaos in Complex Electromagnetic Cavities**

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In last decades, the theoretical and experimental investigation of quantum chaotic phenomena in microwave billiards have been receiving particular effort by both the physical and the engineering communities. Electromagnetic reverberation chamber (RC) is one further paradigm to study even complex wave interactions taking place inside a dynamic (mode-stirred) cavity.

It basically consist of an electrically large metallic cavity with (randomly) moving diffractors inside it (mode-stirrers), responsible for a boundary perturbation. If the wavelength is small compared to the chamber dimensions, we can take advantage of the semiclassical approximation and represent the field structure as a continuous sum of partial field components (modes or plane waves, related by the Gutzwiller's trace formula), stochastic for nature. Ensemble averaging over the movement or the chamber volume – i.e., in time or space – it is turned out that such components are *delta-correlated* with zero mean. In particular, those properties arise for the complex Cartesian field. Several approaches have been proposed for the quest of the wave-chaos footprints inside a reverberation chamber, based on ray model, Lyapunov's exponent or spectral statistics.

In this work, a simple but effective link between random (Gaussian) electromagnetic field and quantum theory is presented and the connections with chaotic behaviors discussed. In particular, the observable random (time) evolution of the complex Cartesian field in the Gauss plane is represented as a two dimensional random walk (Brownian movement). This is the consequence of the mechanical mode-stirrer action. Thus, the Langevin equations for both the real and the imaginary part of the electric field can be used with a forcing term related to the stirring process and the associated Fokker-Planck equation (fpe) developed. Its diffusion coefficient is found to be the correlation constant because of the cavity ergodicity. Upon the Floquet theory and the use of separable test solutions, the fpe can be written as a Schroedinger equation with a chaotic Hamiltonian, where the diffusion constant and the 2D random potential of the chamber are involved. Solution in terms of chaotic probability density functions are given and particular effort has been spent to provide a meaningful and physically consistent definition of the complex cavity 2D potential.

#### Towards the Carnot efficiency in a classical cavity

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A simple classical dynamical model of a thermoelectric heat engine is presented which promises to give some theoretical general guidelines for future heat engine designs. It is composed two thermochemical baths connected with two conductors that are modeled as classical deterministic scatterers. The general features and especially the efficiency of the heat engine are studied as function of the dynamical properties of the conductors. In specific simple cases of one- and two-dimensional wires analytic results were obtained showing that in a certain limit the near to Carnot efficiency is reachable at the price of a low-power output. The latter can be improved by increasing the temperature gradient between baths.

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#### Magnetic Properties and Lyapunov Exponent of the antiferromagnetic Ising and Q-state Potts models on a recurrent lattices

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We consider the Ising model with two-, and three-spin exchange interactions on a zigzag ladder in a strong magnetic field. Using the recursion relation method for the partition function the exact results for the magnetization and Lyapunov exponent have been found. The existence of the magnetization plateau at 1/3 in case of mutual two-, and three-spin exchanges at low temperatures and different values of exchange constants have been shown. In an external magnetic field, bifurcation points appear on the plateaus of the magnetization curve in the finite interaction. It is shown that for some values of two- and three-spin exchange parameters in the antiferromagnetic case the maximum of Lyapunov exponent approaches zero which means that no phase transition in this system. We have also described bifurcation structure, period doubling and chaos for the antiferromagnetic Q-state Potts model on the Bethe lattice by using the same method. The resulting one-dimensional rational mapping has a positive Lyapunov exponent in the region of the chaotic regime for the antiferromagnetic Q-state Potts (Q< 2) model.

### Pseudochaos in quantum many-body systems?

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We present a comparative statistical analysis of a generic chaotic and generic integrable many-body system (Bose-Hubbard model and hardcore bosons with nearest neighbor interactions, respectively). To our surprise physical properties of both systems were found to be quite similar.

# SIGNATURE OF CHAOS IN THE SEMI QUANTUM BEHAVIOR OF A CLASSICALLY REGULAR TRIPLE WELL SYSTEM

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In the Hamiltonian picture, the triple well  $\Phi^6$  potential system shows regular motion. But in the Time Dependent Variational Approach [1] using a generalized Gaussian, quantum fluctuative degrees of freedom are introduced to draw the dynamics. The extended classical system with fluctuation variables, non-linearly coupled to the average ones, exhibit energy dependent transitions between regular behavior and semi quantum chaos monitored by numerical indicators such as bifurcation diagram, Poincaré sections, phase portraits and Lyapunov exponents. The chaotic behavior is obtained for some values of the total energy of the system and in the regions bounded by the Plank's constant  $\hbar$  from classical( $\hbar = 0$ ) to quantum( $\hbar = 1$ ) limits. it is required to consider quantum corrections in a dynamical study. Also the tunelling process is underlined. So, even for classically regular system, the quantum counterpart is found to be chaotic, it is the break down of the Micheal Berry's **Quantum Chaology** [2].

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#### Self-organized integrability in systems with long range interactions ?

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Dynamics of many-body long-range interacting systems are investigated, using the XY-Hamiltonian mean-field model as a case study. We show that regular trajectories, associated with invariant tori of the single-particle dynamics emerge as the number of particles is increased. Moreover, the construction of stationary solutions as well as studies of the maximal Lyapunov exponent of the systems show the same trend towards integrability. This feature provides a dynamical interpretation of the emergence of long-lasting out-of-equilibrium regimes observed generically in long-range systems. This is alternative to a previous statistical mechanics approach to such phenomena which was based on a maximum entropy principle. Previously detected out-of-equilibrium phase transitions are also revisited within this framework.

#### Lyapunov Spectra of Coupled Chaotic Maps

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#### ABSTRACT

Largest Lyapunov exponents and Lyapunov spectra of desynchronous coupled chaotic maps with finite coupling strength are investigated by considering different map functions. For sufficiently large system size there exists a parameter region where the largest Lyapunov exponents are independent of system size and coupling strength. This parameter region is called the scaling region. Some scaling exponent and scaling function in the distribution of Lyapunov spectra are found. These scaling behaviors are model independent. All the above numerical observations are explained, based on heuristic physical understanding on the competition of intensity of chaoticity and strength of coupling and on an analogy of the discrete coupled maps with continuous extended systems.

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#### Wave chaos in ocean acoustics

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Long-range sound propagation in a range-dependent underwater sound channel is examined. This problem is equivalent, in mathematical sense, to motion of an undamped nonlinear oscillator driven by an external force. Longitudinal sound-speed perturbation due to ocean internal waves gives rise to strong chaos of rays propagating near the waveguide axis. The crucial role in the emergence of chaos is played by ray scattering on resonances with vertical oscillations of the perturbation. We study wavefield manifestations of chaotic ray dynamics when the perturbation is range-periodic. This problem is closely related to the well-known problem of quantum chaos. It is found that some of the Floquet modes of the waveguide reveal well-ordered chains of peaks inside the chaotic sea. Quite surprisingly, the periodic orbits corresponding to those peaks do exist only with a significantly reduced amplitude of the perturbation. We refer to this phenomenon as the recovery of periodic orbits [1].

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# Instability statistics and mixing rates

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In last years, there has been considerable interest in understanding of how finite order estimates of time or space averages converge to their asymptotic limit. In this work we claim that looking at probability distributions of finite time largest Lyapunov exponents, and more precisely studying their large deviation properties, yields an extremely powerful technique to get quantitative estimates of polynomial decay rates of time correlations and Poincaré recurrences in the -quite delicate- case of dynamical systems with weak chaotic properties.

#### SEMICLASSICAL STUDY OF STATISTICAL MEASURES OF SPECTRAL FLUCTUATIONS IN SOME PSEUDOINTEGRABLE BILLIARDS

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Abstract : We obtain close analytical form for many important spectral fluctuation measures e.g. Two-point correlation, Spectral Rigidity, Form Factor, Nearest neighbor distribution, for some Pseudo-Integrable billiards. Our results are based on the semi-classical periodic orbit theory, which establishes connection between quantum density of states and the classical periodic orbits. We obtain complete information regarding classical periodic orbits using modified form of the Interval Exchange Transformation maps for the systems under consideration. This includes classical length spectrum of the periodic orbits, band areas of different classes of periodic orbits and the growth rate of periodic orbits. Using information about lengths and band areas for different classes of periodic orbits, semi-classical form for various spectral fluctuations can be written as sum over all (infinite) periodic orbits which can be converted into the integral form using the growth rate of periodic orbits, resulting in the closed analytical form for these measures.

Our results are valid for both asymptotic and non-asymptotic region of the spectrum. We show that approach to asymptotic is much slower than that for the integrable systems. This indicates that one need to consider higher region of the spectra to infer about asymptotic behavior for non-integrable systems. We also provide quantitative criteria for such slow convergence to asymptotics. Two examples of the pseudo-integrable billiards considered by us belongs to subclass of almost integrable billiards where different bands of periodic orbits belonging to same class with correlated lengths collectively occupy complete phase space area. Therefore asymptotic form for various measures exactly belongs Poissonian statistic. This may not be true for pseudo-integrable systems which are not almost integrable as periodic orbits need not collectively occupy complete phase space area. However, as our approach is valid also for strict pseudo-integrable polygonal billiards one can conclude that these system should show intermediate statistic (between Poisson and GOE) for the spectral fluctuations.

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# Scattering in Mesoscopics Systems: Statistical Interpretation to the Impurities Non Localized by Quantum Uncertainty

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We study the scattering problem for a particle without spin, through out a one dimensional system of barriers delta, which is described dynamically by oscillating scatterers under a harmonic potential. The particle will find the scatterers like fixed points due to the mass relation, such as in the Born-Oppenheimer approximation. However the wave function is described depending on the probability distribution for the localization of the scatterers with a stochastically and no correlated description. Analytically the N deltas problem is resolved founding the recursion formulas for the transmission and reflexion coefficients. The probabilities amplitudes are calculated by integrals functions depending on the probabilities of localization of the barriers. This method show that for mobiles impurities the coherent quantum behavior is broken due to the resonances are not well defined a kind of interference pattern around the classical resonances will be shown by an algorithm designed for the calculus. That suggest that the complete wave function considering the ensemble will be a density matrix for the localization of the barriers and particle.

# Chaotic properties of the truncated elliptical billiard

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We examine, numerically and analitically, the dynamical properties of truncated elliptical billiards (TEB) [1]. This two parameter family of billiards is constructed by truncating the ellipse on opposite sides. The existence and linear stability of several periodic orbits are investigated in the full parameter space. Poincaré plots are computed and used for evaluation of the chaotic fraction of the phase space with the box-counting method [2,3]. The limit of the fully chaotic behaviour is identified with circular arcs. Above this limit, for flattened elliptical arcs, mixed dynamics with numerous stable eliptical islands is present, similarly as in the elliptical stadium billiards (ESB) [4]. Below this limit the full chaos extends over the whole region of elongated ellipses and the existing orbits are either unstable or neutral. This is different form the behaviour in the ESB where chaotic region is strictly bounded form both sides. To examine the mechanism of this difference, a generalisation to a novel three parameterfamily (GTESB) of stadium-like boundary with eliptical arcs is proposed.

Billiards are found to be useful models as 2D optical microlasers which are widely used in modern opto-electronic devices. We also analize the outoging intensities for certain parameters of truncated elliptical billiard.

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#### A Delay Model for Viral Infected Toxin Producing Phytoplankton and Zooplankton System

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An eco-epidemiological delay model is proposed and analysed for virally infected, toxin-producing phytoplankton (TPP) and zooplankton system. With delay differential equation model system. We have studied the effect of time delay on the stability behavior. It is shown that time delay can destabilize the otherwise stable nonzero equilibrium state. The co-existence of species in the form of stable periodic cycles is possible with suitable delay. The same investigation is also done for infection-free model. Numerical simulation suggests that the proposed model displays a wide range of complex behavior like limit cycles and chaos. The chaos is occurred for different values of parameter.

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#### Lattice oscillator stochastic dynamics as Gibbs path system and nonequilibrium phase transitions

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I show the existence of a long-range order and an order parameter with a help of a generalized Peierls argument for a gradient stochastic (Brownian) dynamics of lattice oscillators interacting via a pair quadratic potential with a special initial Gibbs state. This result can be interpreted as a synchronization and an absence of chaos and it is published in the Journal of Statistical Physics. Not many results are obtained in the mathematical theory of phase transitions and synchronization. The same technique is used by me for a proof of the existence of a long-range order in quantum lattice systems of oscillators with manybody interaction potentials.

# Hyperchaotic Synchronization of two coupled modified canonical Chua's circuits: Numerical and Experimental investigations

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#### Abstract

A modified canonical Chua's circuit exhibiting hyperchaos is designed and its dynamics is investigated in detail, both numerically and through hardware laboratory experiments. The system is found to exhibit the period three doubling and border collision bifurcation leading to hyperchaos. The hyperchaotic synchronization of two such unidirectionally coupled identical systems has been effected. It has been observed experimentally and confirmed through numerical and PSPICE simulations. The different states of synchronization, as the systems move from asynchronous state to complete synchronization as well as the different routes followed by these systems under change of coupling parameter are also investigated.

Keywords: Electronic circuits; border collision bifurcations; hyperchaos; synchronization;

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#### Hamiltonian ratchets with a wavelike perturbation

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Motion of non-interacting particles in a space-periodic potential well under a wavelike perturbation is studied. All particles are initially localized close to minima of the potential. We demonstrate three methods providing generation of the directed particle transport with small perturbation's amplitude. The first method is based on the usage of a perturbation consisted of a specific combination of plane waves [1]. The other two imply adiabatic variations of perturbation's phase or orientation. Adiabatic phase modulation results in pulse-like growth of mean particle flux, until the saturation is achieved. In the case of the adiabatic modulation of the orientation, the pulse-like dynamics is accompanied by occurence of particle jets with giant acceleration [2]. That phenomenon occurs due to forming of resonance channels in the underlying phase space.

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