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Unified Scaling Law for Earthquakes and Seismic Hazard Assessment

Vladimir Kossobokov

International Institute of Earthquake Prediction Theory and Mathematical Geophysics
Moscow
Russia
Institute de Physique du Globe de Paris
France

volodya@mitp.ru/volodya@ipgp.jussieu.fr

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Vladimir Kossobokov

International Institute of Earthquake Prediction Theory and Mathematical Geophysics,
Russian Academy of Sciences,
84/32 Profsouznaya Ulitsa, Moscow 117997, Russian Federation

Institute de Physique du Globe de Paris,
4 Place Jussieu, 75252 Paris, Cedex 05, France

E-mail: volodya@mitp.ru or volodya@ipgp.jussieu.fr

Outline

- What is lacking in the Gutenberg-Richter relation, log N = A + B·(8 - M) ? Space.
- The Unified Scaling Law for Earthquakes:

  The first results and conclusions
- Revisiting the ABC problem on a global scale:

  The Global Seismic Hazard maps that display at 100-km scale the A, B, and C's for the recurrence of earthquakes
- Implications for assessing seismic hazard and risks at a given location, e.g., in megacities, or maximum intensity maps

What is lacking in the Gutenberg-Richter relation, $\log_{10}N = A + B\cdot(8 - M)$?

- Being a general law of similarity the GR relation establishes the scaling distribution of earthquake sizes in a given space time volume
- **…but gives no explanation to the question how the number, N, changes when you zoom the analysis to a smaller size part of this volume.**

The answer is not obvious at all.

Seismic activity is self similar:

Since the pioneering works of Keiiti Aki and M. A. Sadovsky

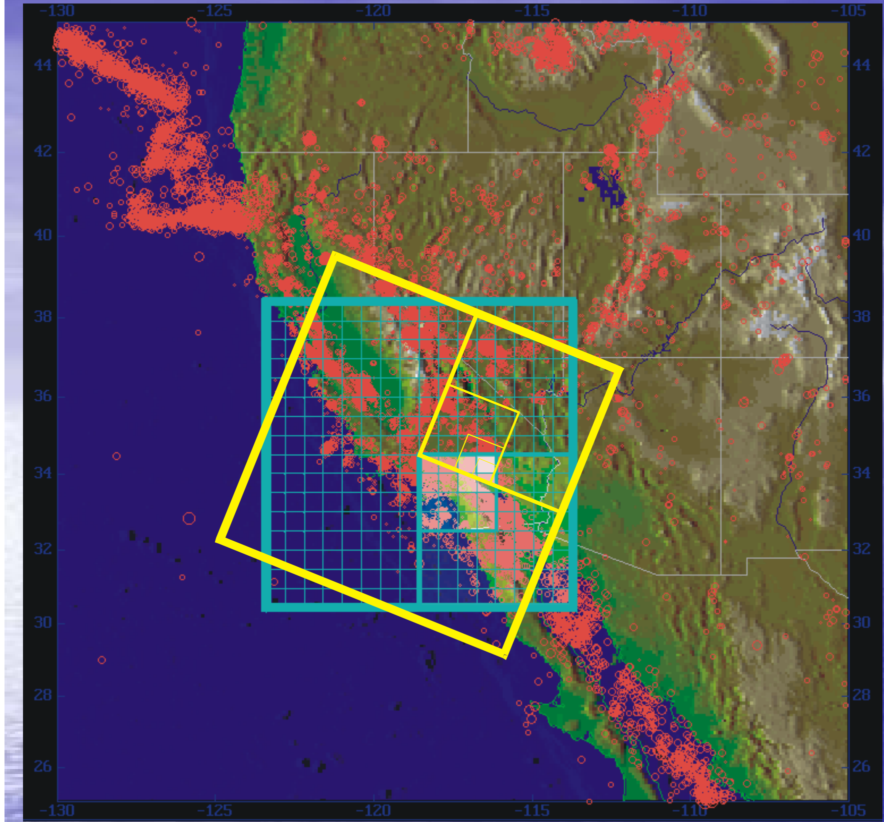
Okubo, P.G., K. Aki, 1987. Fractal geometry in the San Andreas Fault system. *J. Geophys. Res.*, **92** (B1), 345-356;
Садовский М.А., Болховитинов Л.Г., Писаренко В.Ф., 1982. О свойстве дискретности горных пород. *Изв. АН СССР. Физика Земли*, № 12, 3-18;
Садовский, М.А., Т.В. Голубева, В.Ф. Писаренко, и М.Г. Шнирман, 1984. Характерные размеры горной породы и иерархические свойства сейсмичности. *Известия АН СССР. Физика Земли*, 20: 87–96 .

the understanding of the fractal nature of earthquakes and seismic processes keeps growing.

The Unified Scaling Law for Earthquakes that generalizes Gutenberg-Richter relation suggests -

$$\log_{10} N = A + B\cdot(5 - M) + C\cdot\log_{10} L$$

where N = N(M, L) is the expected annual number of earthquakes with magnitude M in an earthquake-prone area of linear dimension L.



The scheme for box-counting

The counts in a set of cascading squares, **“telescope”**, estimate the natural scaling of the spatial distribution of earthquake epicenters and provide evidence for rewriting the G-R recurrence law.

The box-counting algorithm

(*Kossobokov and Mazhkenov, 1988*)

For each out of m magnitude ranges and for each out of h levels of hierarchy the following numbers $N_{j,i}$ are found:

$$N_{j,i} = \sum n_i (Q_i)^2 / N_j ,$$

where i = 0,1….h-1, j = 1,2….m, $n_j(Q_i)$ is the number of events from a magnitude range M_j in an area Q_i of linear size L_i; N_j is the total number of events from a magnitude range M_j.

The A, B, C's are derived by the least-squares method from the system

$$\log_{10} N_{j,i} = A + B \cdot (5 - M_j) + C \log_{10} L_i .$$

An interpretation of the box-counting

Number N_{ji} can be considered as the empirical mean recurrence rate of events in the magnitude range M_j, calculated over their locus in an area at the i-th level of spatial hierarchy.

Specifically, if we denote a "telescope" a set of h+1 embedded squares $W = \{w_0, w_1, \dots, w_h\}$, so that each w_i belongs to the I-th level of hierarchy. Note that each "telescope" grows uniquely from the lowest level. Assume that the M_j epicenter set is defined by a sample catalog of earthquakes $X_j = \{x_1, \dots, x_{Nj}\}$. Each earthquake x_k defines the "telescope" $W(x_k)$ that grows from $w_h(x_k)$, to which x_k belongs. Consider the set of "telescopes" $\{W(x_k)\}$ that corresponds to the catalog X_j. Denote $n_j(w_i)$ as the number of events from X_j that fall within w_i.

Then, the mean number of events in an area of i-th level of hierarchy over X_j is $N_{ji} = \Sigma_{\{k=1,\dots,Nj\}}\, n_j(w_i(x_k)) \,/\, N_j$.

Substituting summation over X_j by summation over the areas $w_i(x_k)$ from the i-th level, we obtain the formula of the USLE.

The first results (*Kossobokov and Mazhkenov, 1988*)

The method was tested successfully on artificial catalogs with prefixed A, B and C and applied in a dozen of selected seismic regions from the two hemispheres of the Earth to a certain intersection of faults.

12 KOSOBOKOV AND MAZHKENOV: SIMILARITY IN SPATIAL DISTRIBUTION OF SEISMICITY

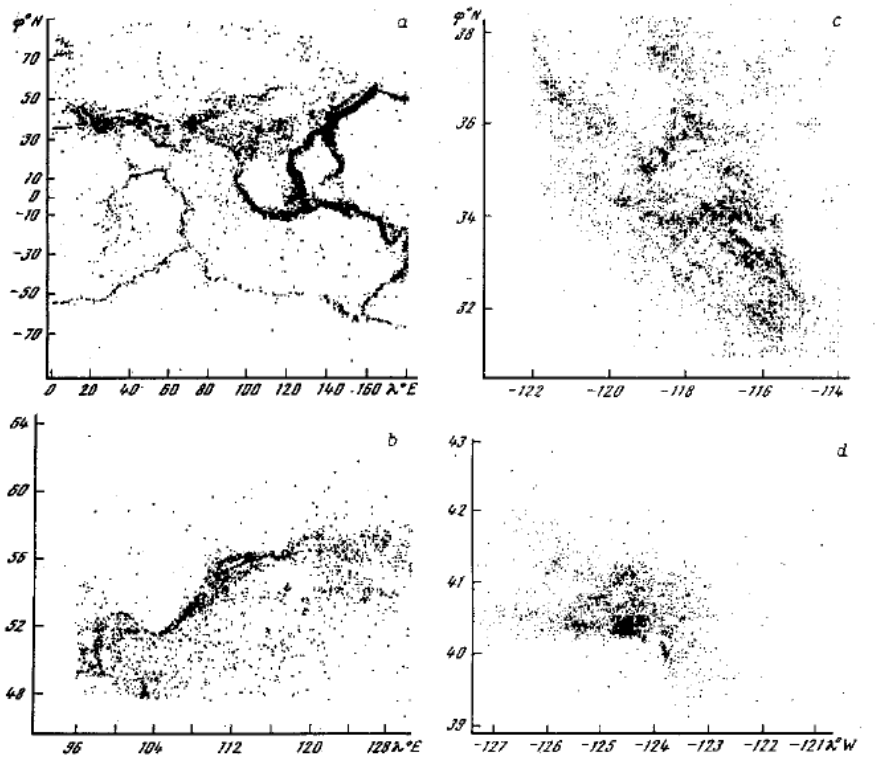


Fig. 2. Examples of spatial distribution of epicenters from catalogs of mainshocks. (a) Eastern Hemisphere. (b) Lake Baikal area. (c) Southern California. (d) The Cape Mendocino vicinity.

14 KOSOBOKOV AND MAZHKENOV: SIMILARITY IN SPATIAL DISTRIBUTION OF SEISMICITY

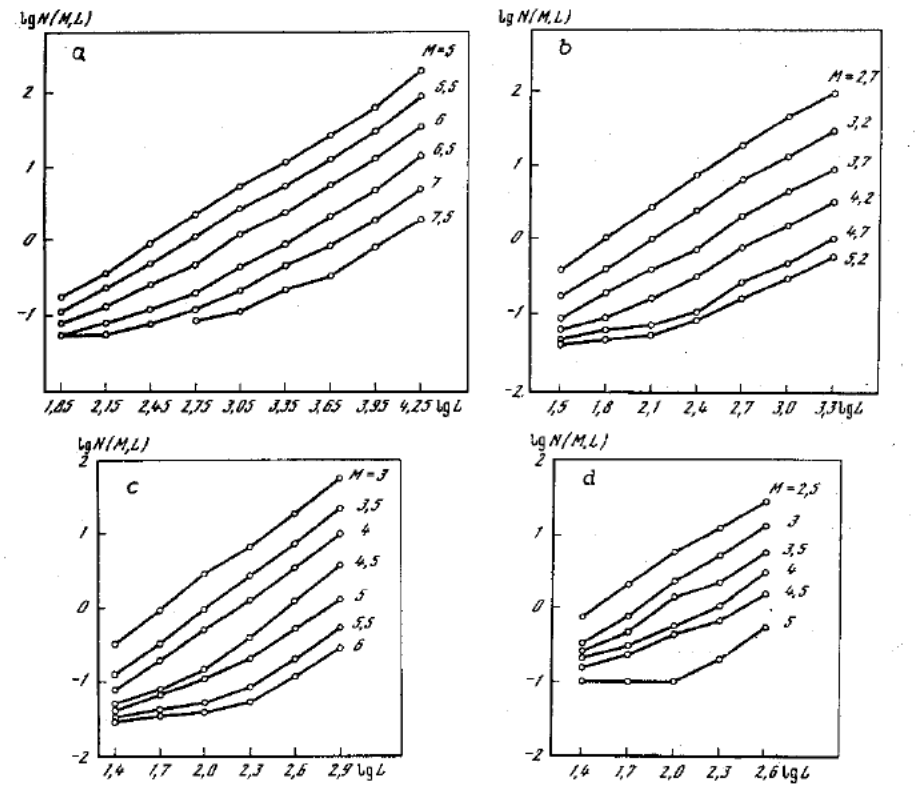


Fig. 3. Examples of log $N(M, L)$ graphs. (a) Eastern Hemisphere. (b) Lake Baikal area. (c) Southern California. (d) The Cape Mendocino vicinity.

The Unified Scaling Law for Earthquakes

We revisited the problem after Per Back et al. suggested the Unified Scaling Law for Earthquakes in a different formulation (with substitutes of 1/N = T and M = $\mathbf{Log_{10}}$ S),

"To understand the Unified Law for Earthquakes, it is essential to see what the value of x represents. The quantity $L^{df} \cdot S^{-b}$ in the scaling function represents the average number of earthquakes per unit time, with seismic moment greater than S occurring in the area size L × L. Therefore, x is a measure of the number of earthquakes happening within a time interval T. The Unified Law states that the distribution of waiting times between earthquakes depends only on this value."

Bak, P., K. Christensen, L. Danon, and T. Scanlon, 2002.
Unified Scaling Law for Earthquakes.
Phys. Rev. Lett. 88: 178501-178504

What Per Bak et al. (2002) done?...

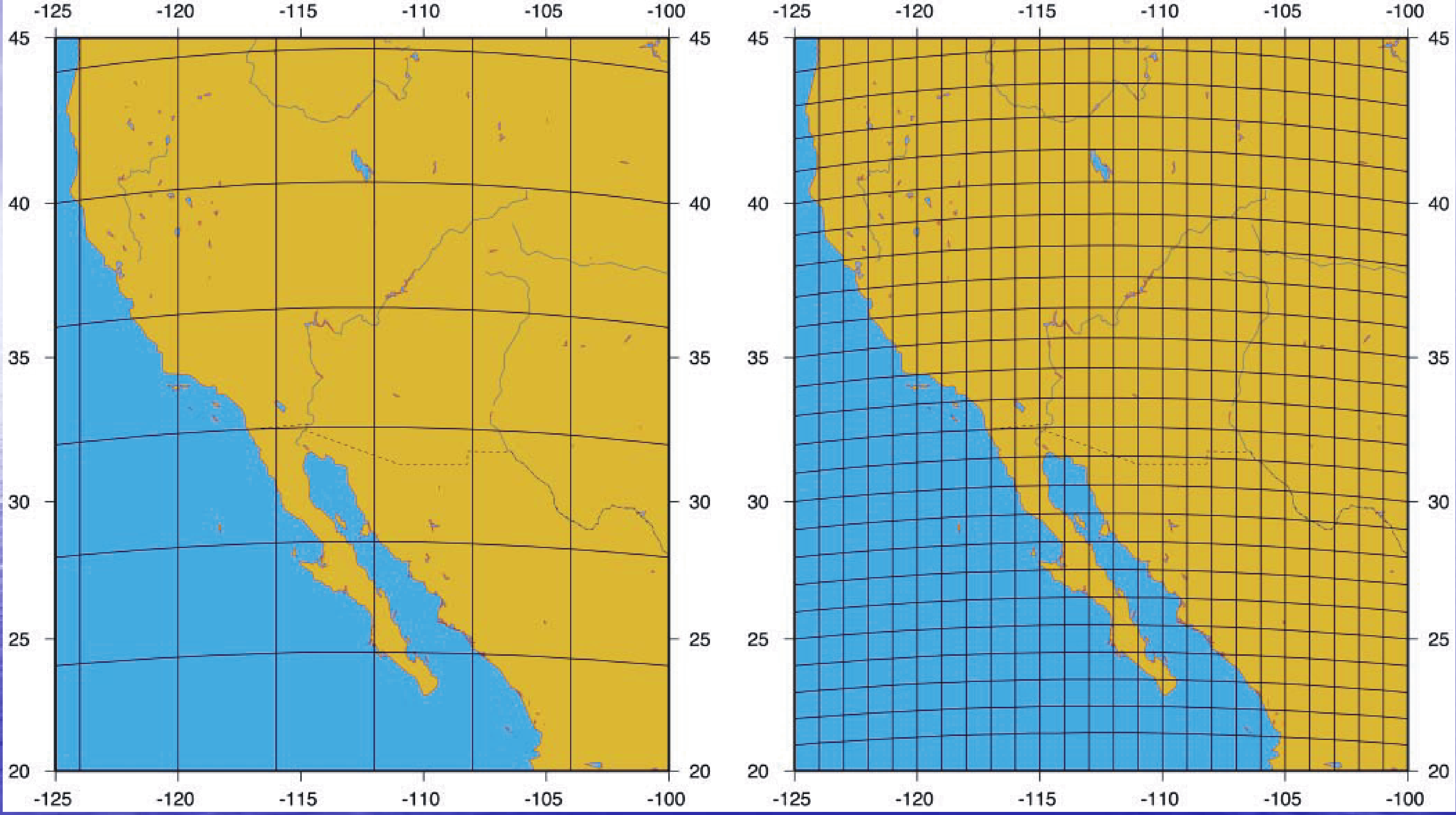


Fig. 1. Southern California seismic region covered by a grid of cells of 4° × 4° (*Left*) and cells of 1° × 1° (≈ 111 × 111 km^2) (*Right*).

What Per Bak et al. (2002) done?...

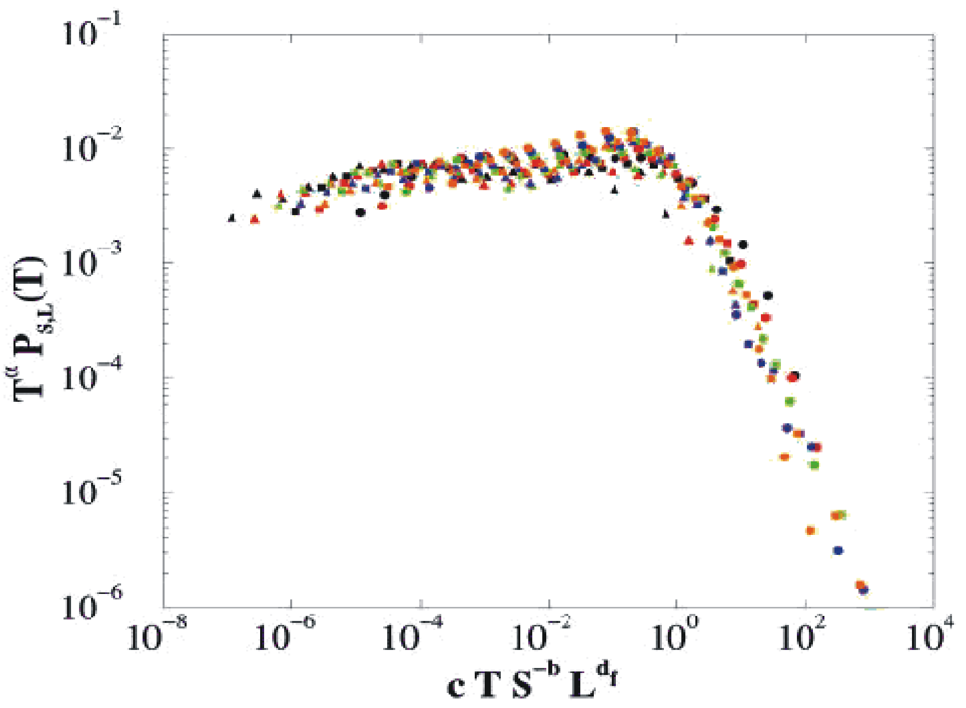
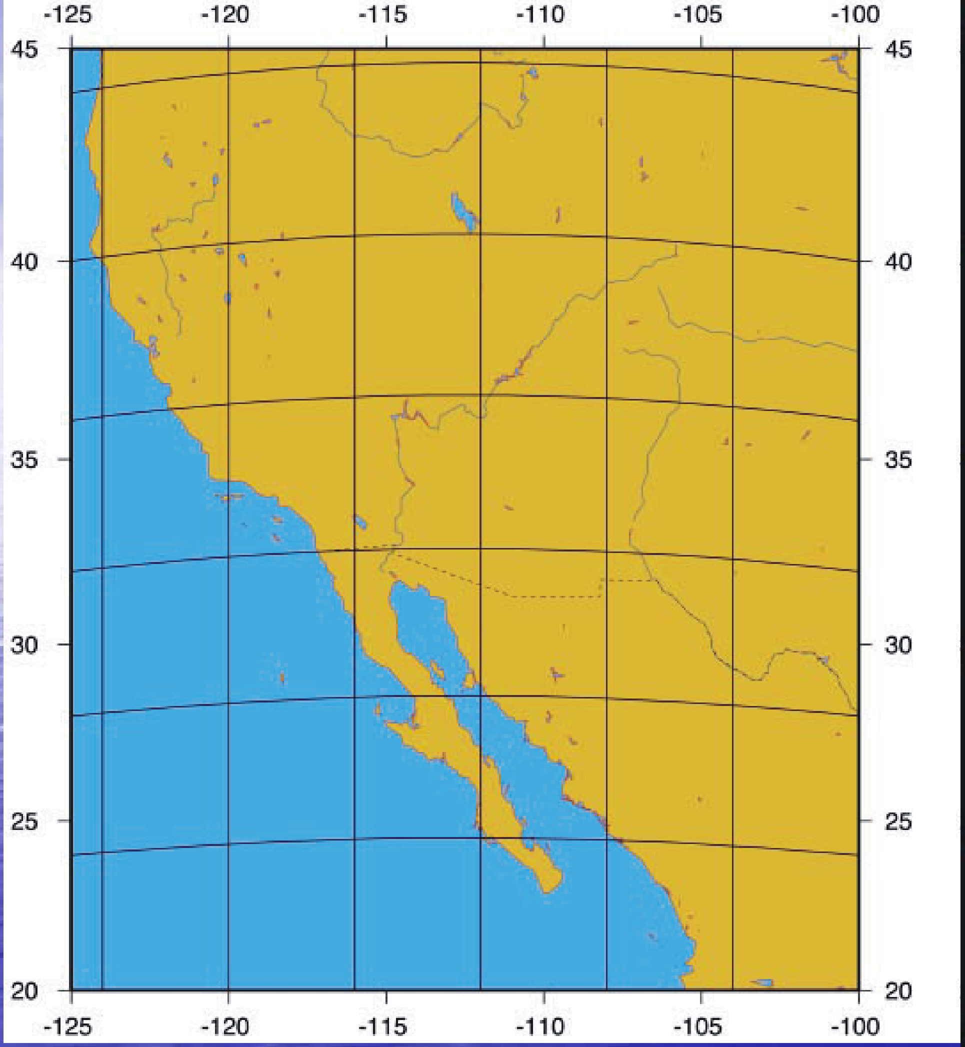


Fig. 6. The data in Fig. 3 with $T > 38$ s replotted with $T^{\alpha} P_{S,L}(T)$ as a function of the variable $x = cTS^{-b}L^{df}$, $c = 10^{-4}$. The Omori Law exponent $\alpha \approx 1$, Gutenberg–Richter value $b \approx 1$, and fractal dimension $d_f \approx 1.2$ have been used to collapse all of the data onto a single, unique curve $f(x)$. The curve is constant for $x < 1$, corresponding to the correlated, Omori Law regime but decays fast for $x > 1$, associated with uncorrelated events.

Christensen *et al.*

... and what they overlooked.



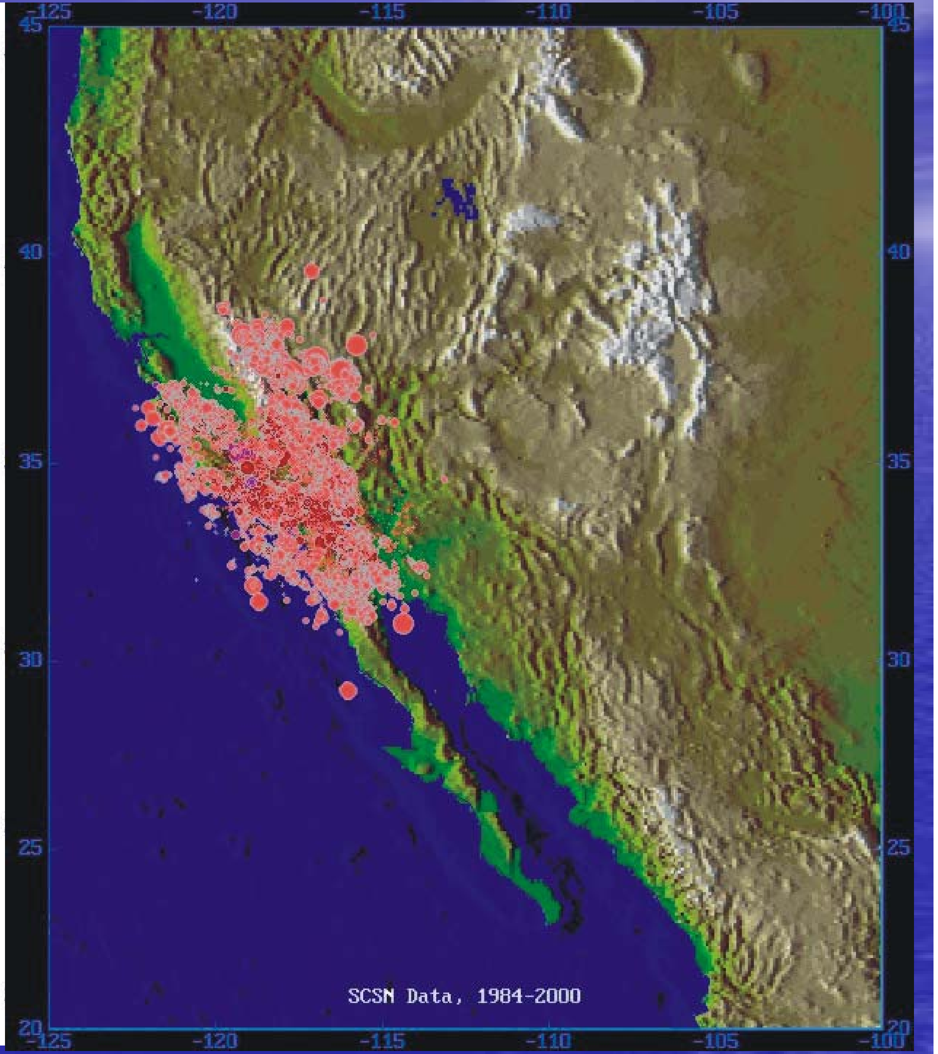


Fig. 1. Southern California seismic region covered by a grid of cells of 4° × 4° (*Left*) and cells of 1° × 1° (≈ 111 × 111 km^2) (*Right*).

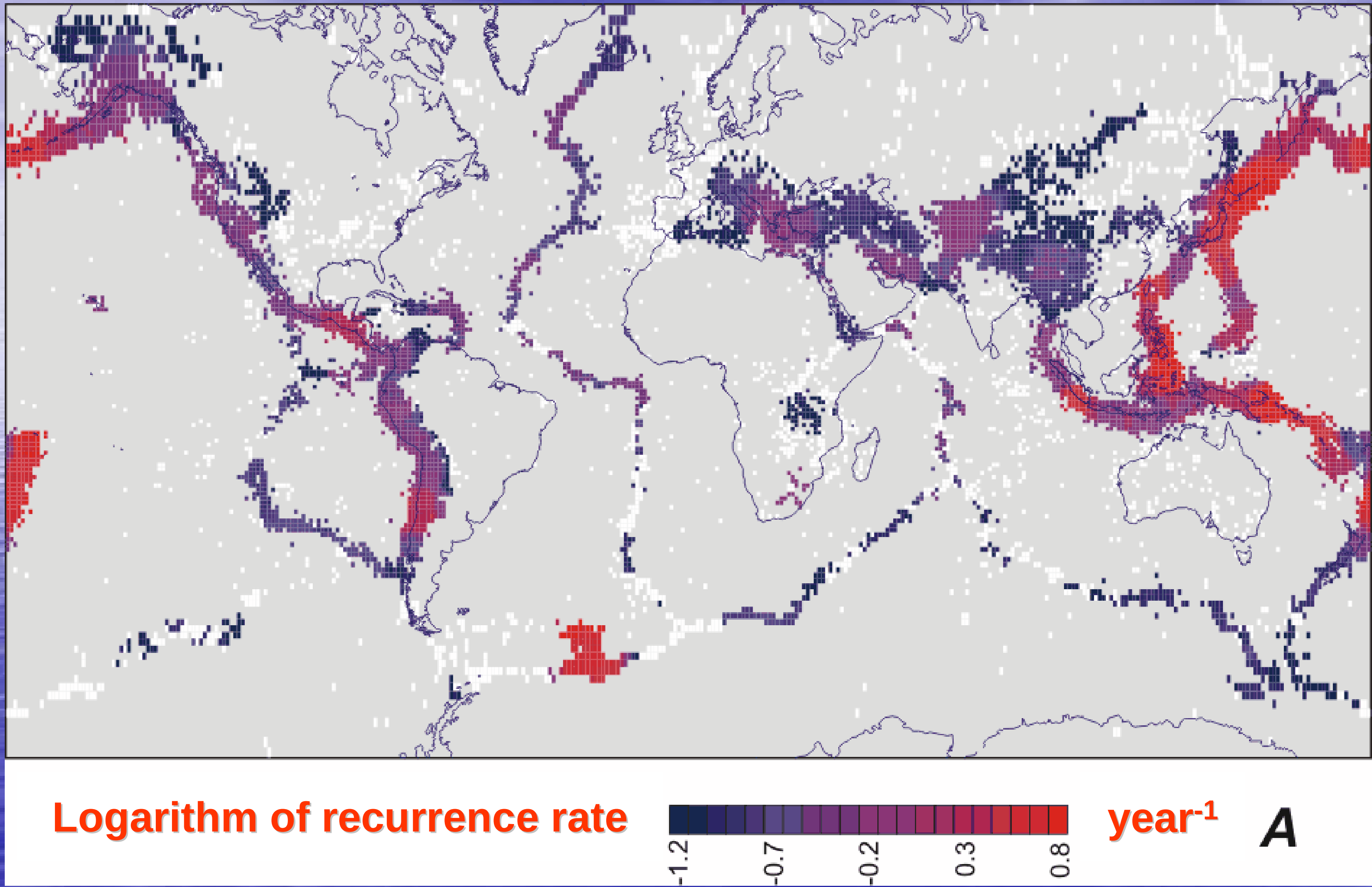
Revisiting the ABC problem on a global scale: **The Global Seismic Hazard maps display the A, B, and C's for earthquakes**

The data from the US GS/NEIC hypocenter data base permitted us to investigate systematically regions from a wide range of seismic activity, A (that differ by a factor up to 30 or more).
We found, for earthquakes with hypocenters above 100 km –

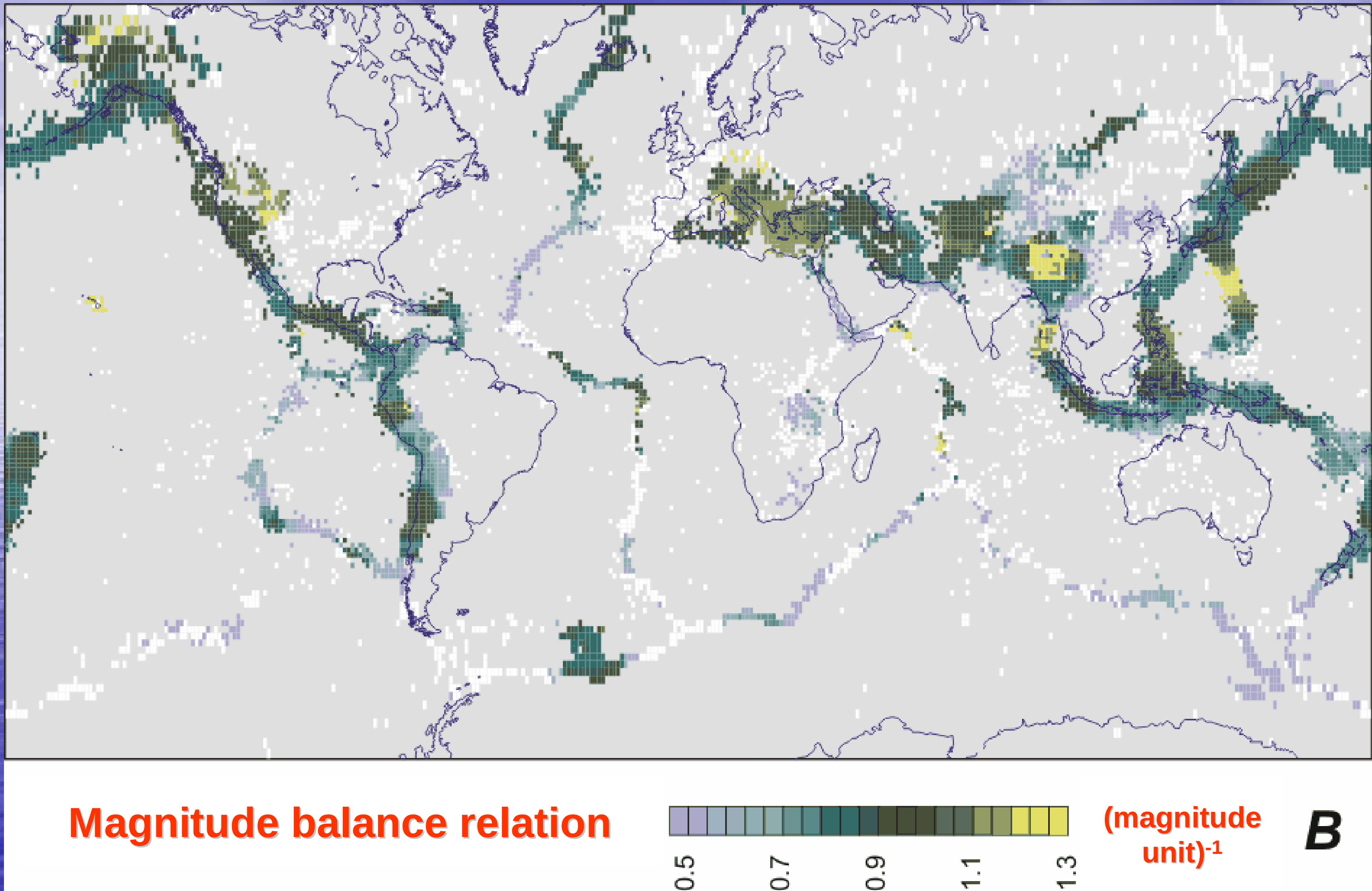
- the balance between magnitude ranges, B, varies mainly from 0.6 to 1.1 with a sharp maximum of density at 0.9, while

- the fractal dimension, C, changes from under 1 to 1.6 with a maximal density within 1.2-1.3.

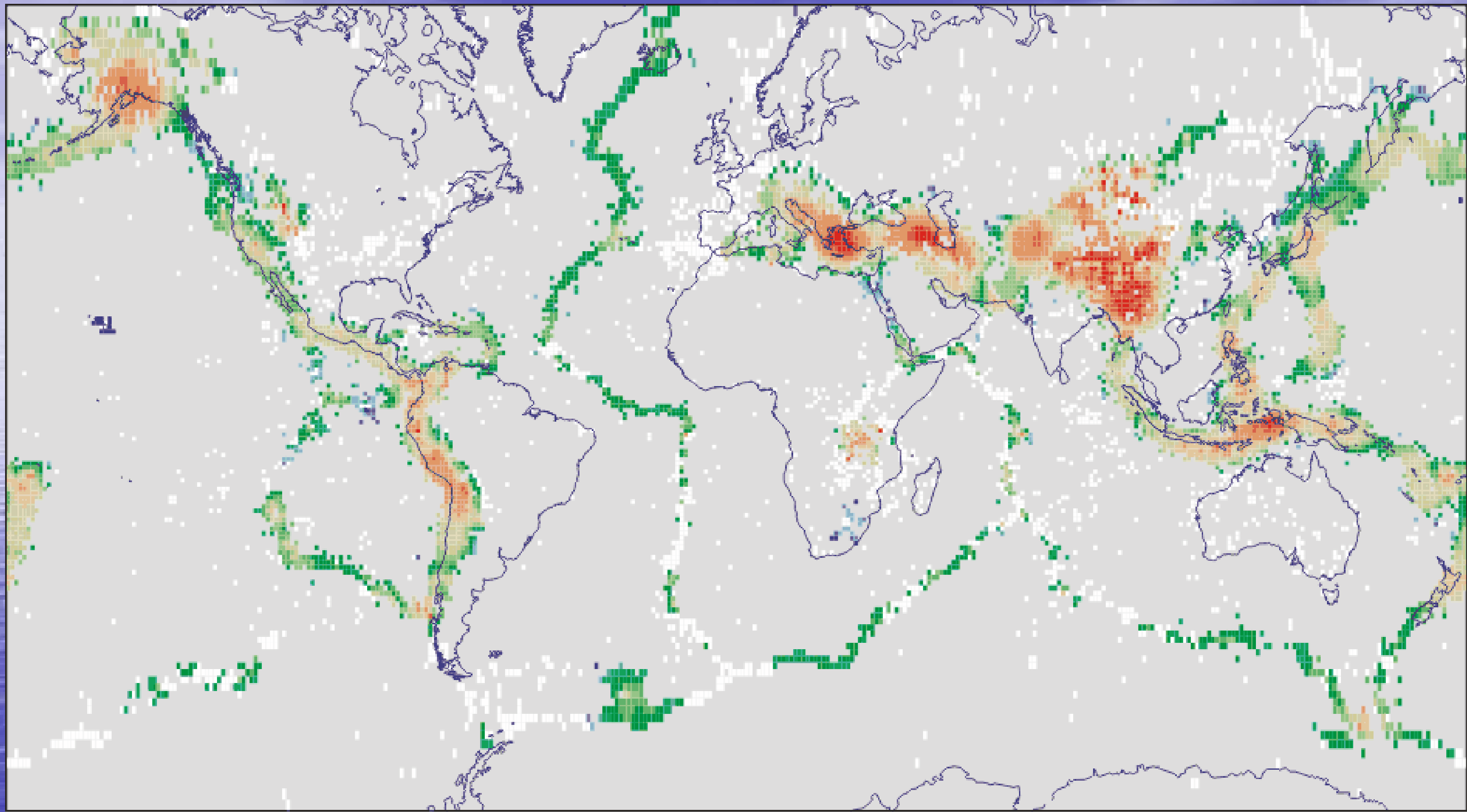
The Global Seismic Hazard map: Coefficient A



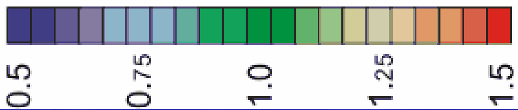
The Global Seismic Hazard map: Coefficient B



The Global Seismic Hazard map: Coefficient C

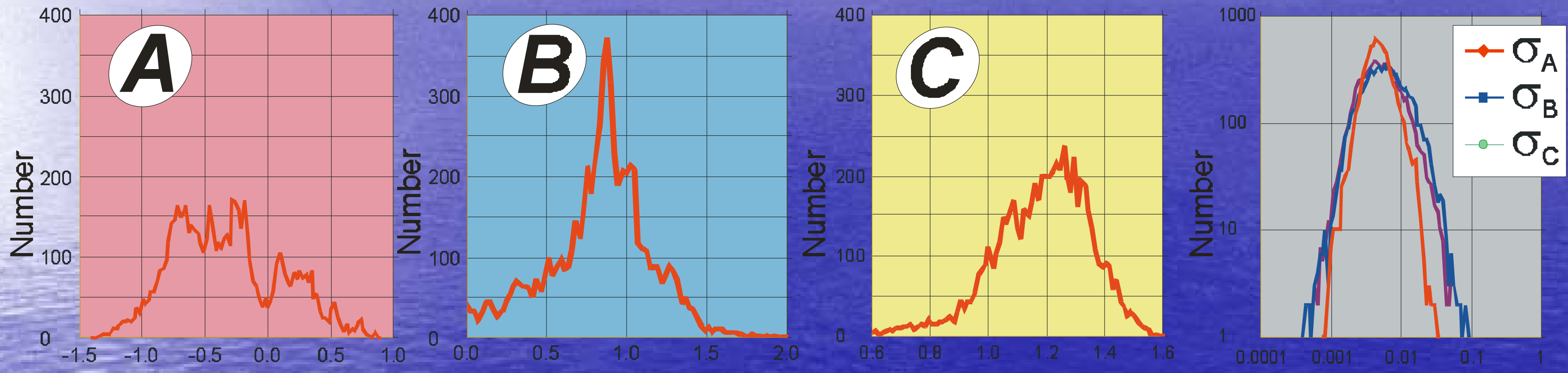


Fractal dimension of seismic epicenters



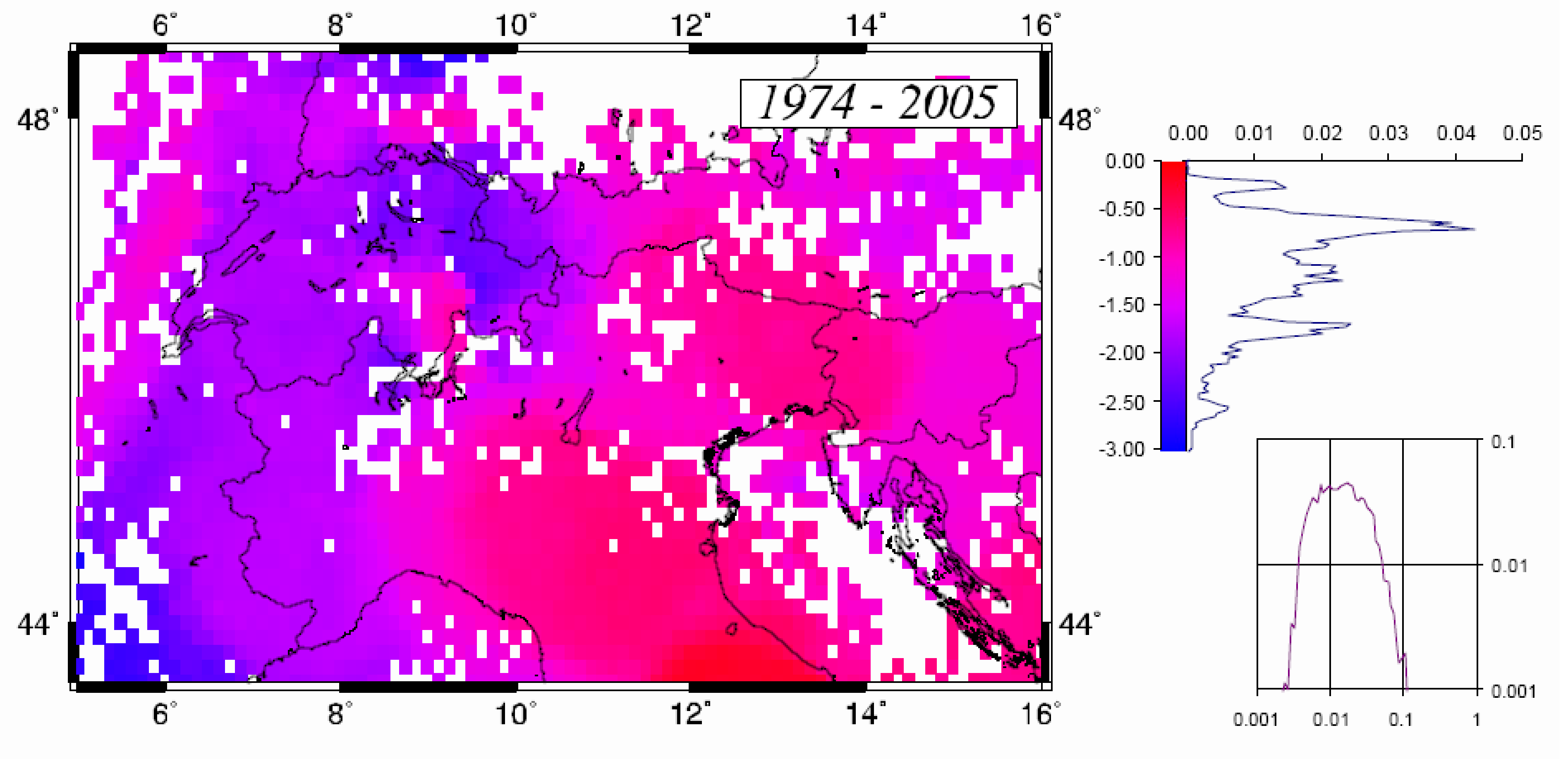
C

Histograms of A, B, C and σ's

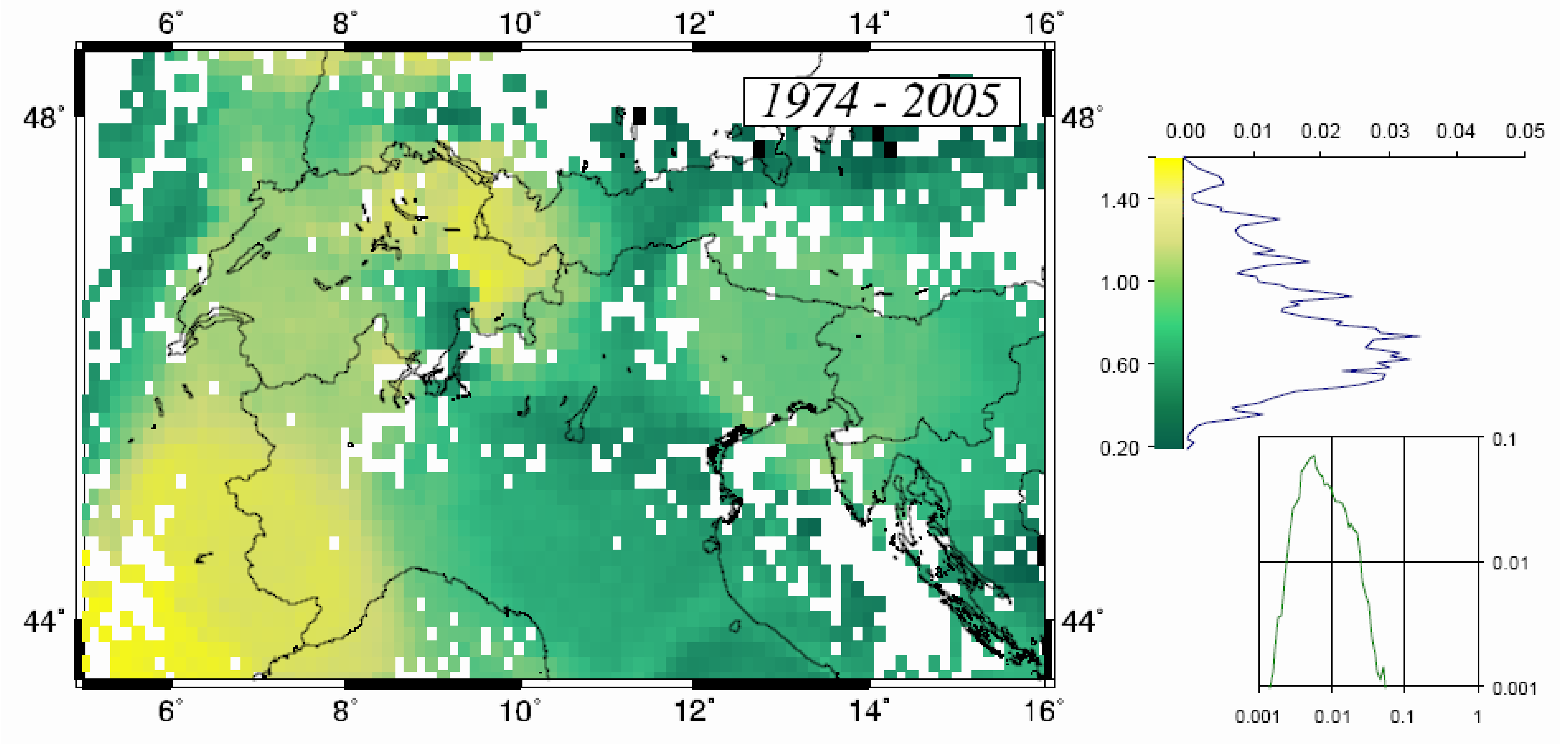


Note: The histogram of the coefficients' value errors, σ's, given in logarithmic scales. It suggests high degree of overall agreement with the assumption of self-similarity used in computations.

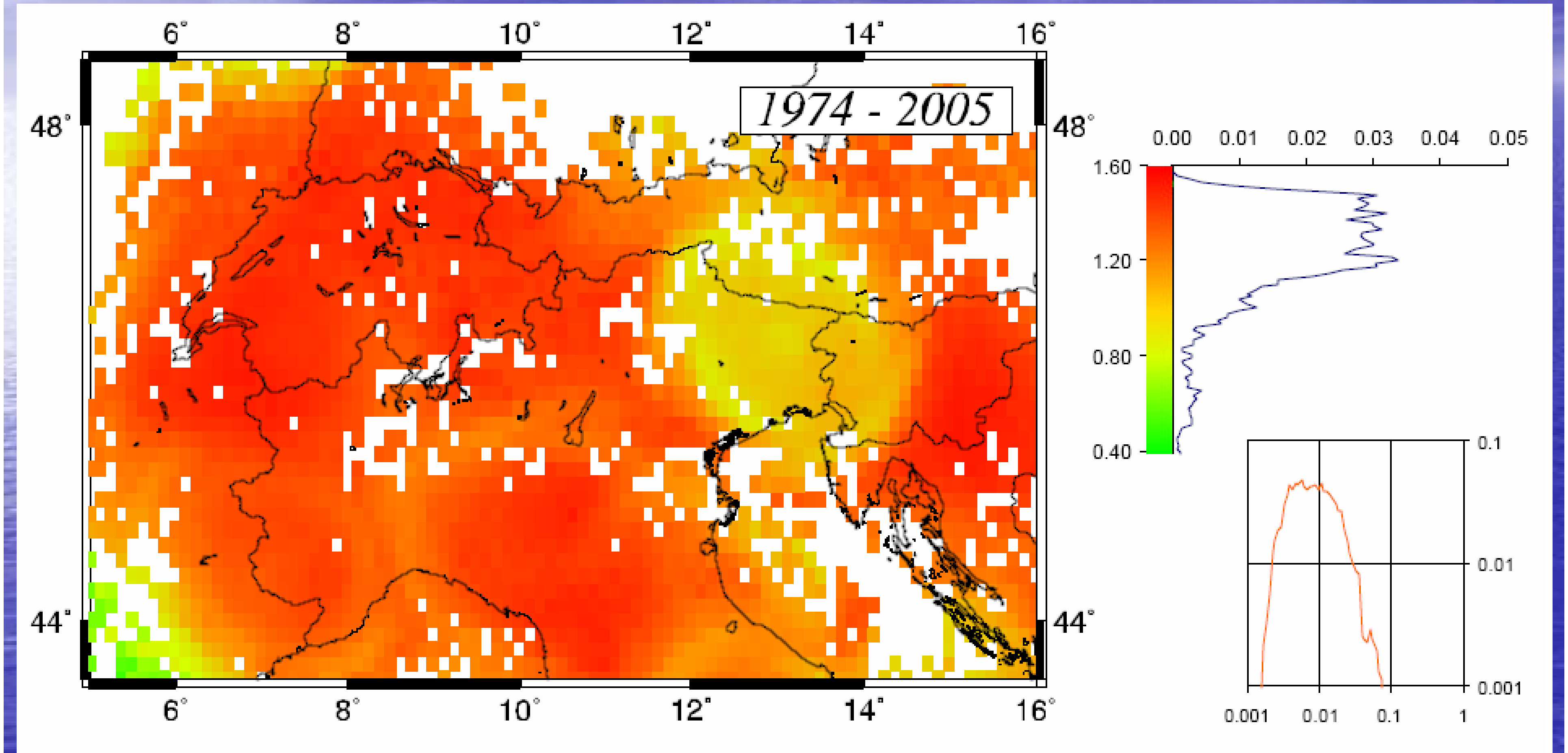
The Regional Seismic Hazard Map: Northern Italy

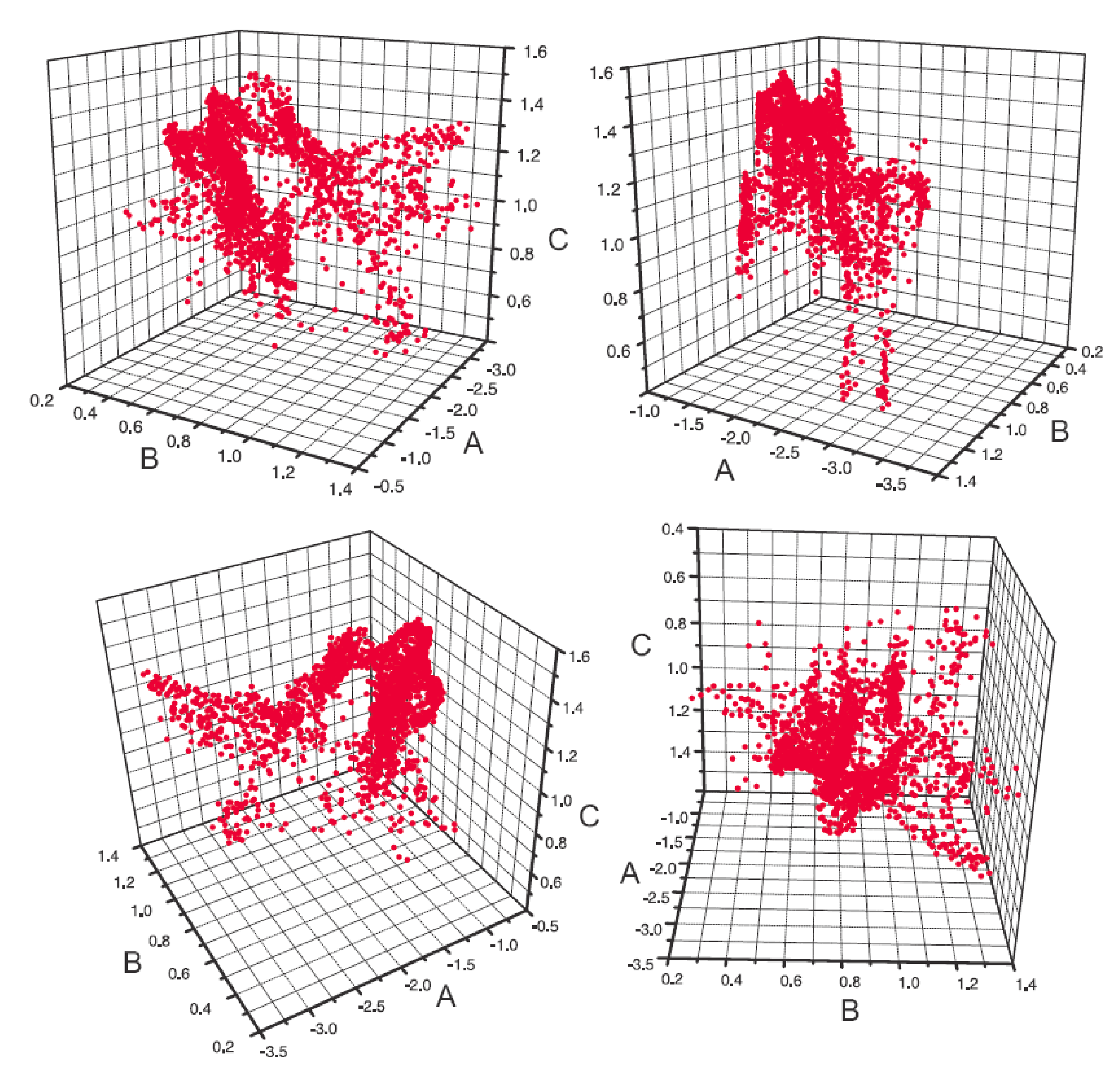


The Regional Seismic Hazard Map: Northern Italy



The Regional Seismic Hazard Map: Northern Italy





Sample 3-D views of the 2352 combinations of *A*, *B*, *C* coefficients in Italy and surroundings, 1870-2005.

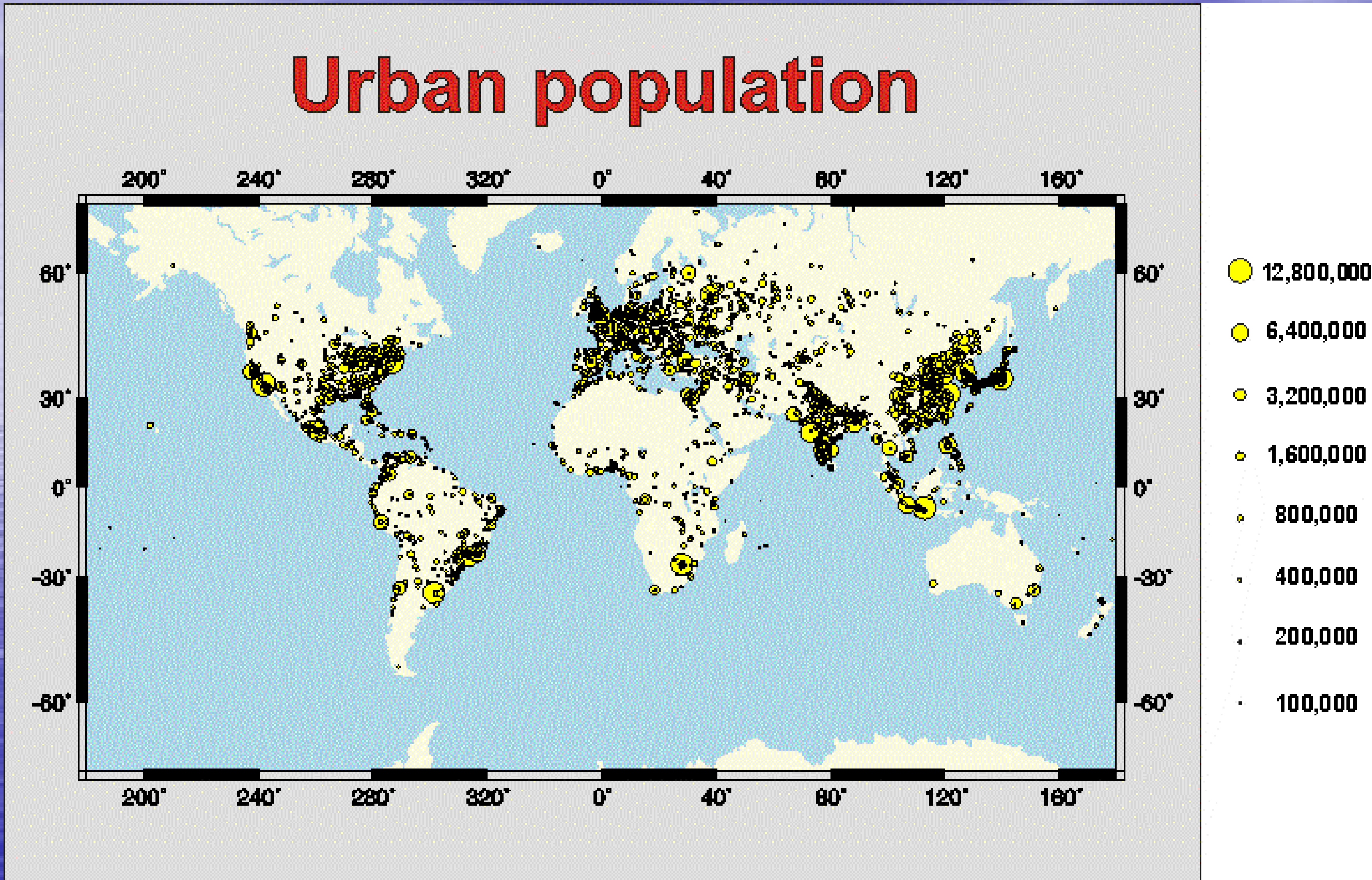
Direct implications for assessing seismic hazard at a given location (e.g., in a mega city)

The estimates for Los Angeles (SCSN data, 1984-2001) -
A = -1.28; B = 0.95; C =1.21 (σ_{total} = 0.035)
- imply a traditional assessment of recurrence of a large earthquake in Los Angeles, i.e., an area with L about 40 km,
from data on the entire southern California, i.e., an area with L about 400 km,
being **underestimated by a factor of** $10^2 / 10^{1.21} = 10^{0.79}$ **> 6 !**

Similarly, the underestimation is about a factor of
6.4 for San Francisco (A = -0.38, B = 0.93, C = 1.20, σ_{total}=0.07),
4.6 for Tokyo (A = 0.14, B = 0.94, C = 1.34, σ_{total}=0.05),
8 for Petropavlovsk-Kamchatsky (A = -0.01, B = 0.83, C = 1.22, σ_{total}=0.05),
10 for Irkutsk (A = -1.12, B = 0.80, C = 1.05, σ_{total}=0.03),
etc.

Scaling for unified application of an earthquake prediction method.

Convolving Seismic Hazard with Object of Risk and its Vulnerability provides an estimation of Seismic Risk



To avoid misleading counterproductive interpretations, we have to emphasize that risk estimates presented here are rather synthetic, given for methodological purposes. The estimations addressing more realistic and practical kinds of seismic risk, not presented here, should involve experts in distribution of objects of risk of different vulnerability, i.e., specialists in earthquake engineering, social sciences and economics.

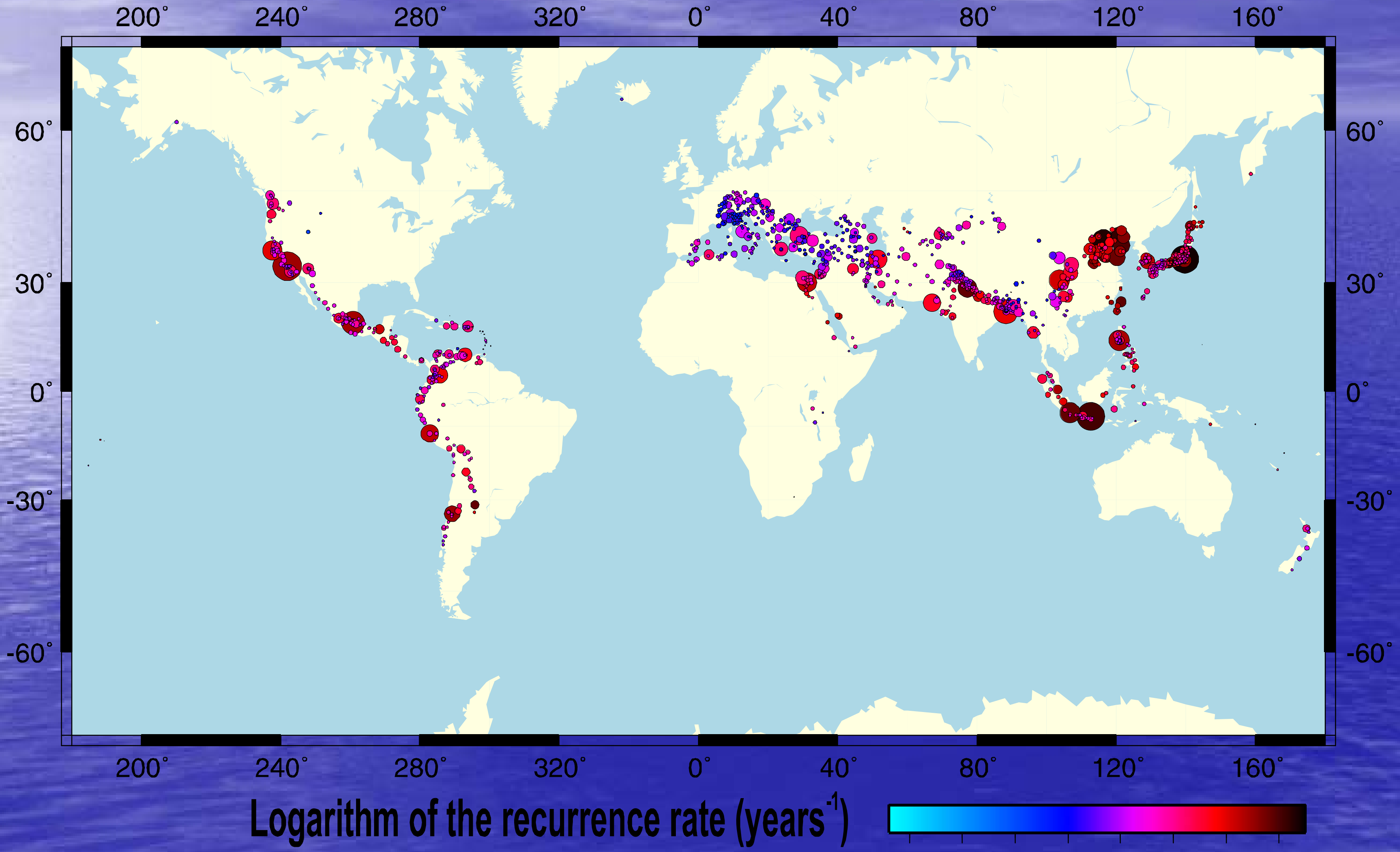
Recurrence rates

$10^{A} \times 10^{B\times(5-M)} \times (\text{City Area})^{C/2}$

City Area = 0.001×Population/(1°×1°)

Synthetic estimation for educational purposes only.

Strong, magnitude 6, earthquakes.



Synthetic estimation for educational purposes only.

Top ten* recurrence rates for strong (M6+) earthquakes

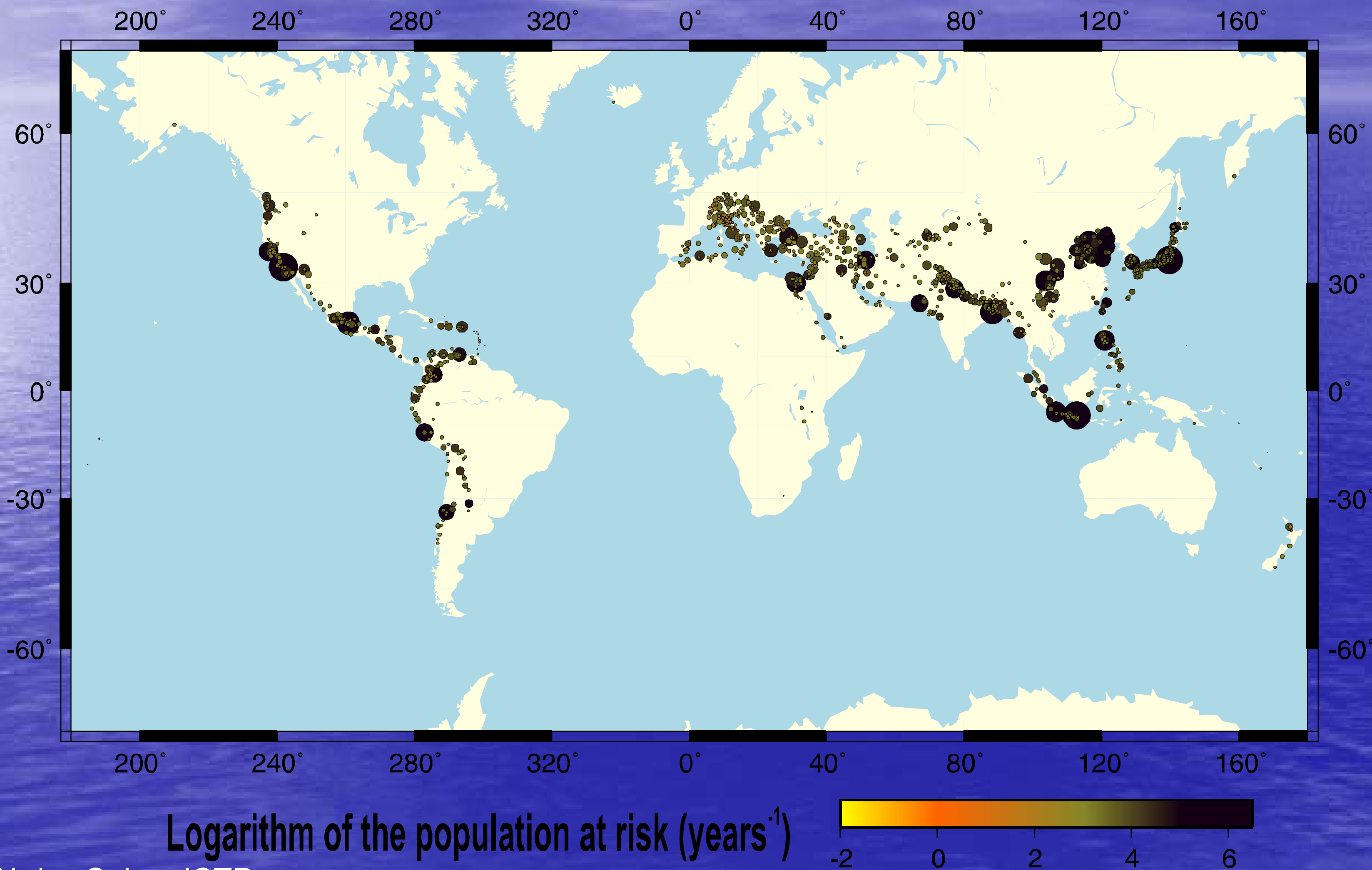
City	Country	Population	A	B	C	Recurrence rate, $years^{-1}$
Tokyo	Japan	11,906,331	0.14	0.94	1.34	0.15663
Taipei	China	1,769,568	0.22	0.80	1.15	0.08580
Jakarta	Indonesia	6,503,449	0.15	1.06	1.23	0.08349
Kobe	Japan	1,422,922	0.17	0.90	0.84	0.07368
Yokohama	Japan	3,049,782	0.15	0.95	1.32	0.06258
Kyoto	Japan	1,480,355	0.16	0.93	0.96	0.06177
Santiago	Chile	4,099,714	0.08	1.05	1.21	0.05579
Quanzhou	China	403,180	0.39	0.95	0.96	0.05310
Los Angeles	US	13,074,800	-0.34	0.95	1.19	0.05267
Gaoxiong	China	828,191	0.21	0.80	1.18	0.05165

Urban population at risk

Synthetic estimation for educational purposes only.

Recurrence rate×Population

Strong, M6, earthquakes.



Synthetic estimation for educational purposes only.

Top ten* of the population at risk for strong (M6+) earthquakes

City	Country	Population	A	B	C	Population at risk, year^{-1}
Tokyo	Japan	11,906,331	0.14	0.94	1.34	1,864,928
Los Angeles	US	13,074,800	-0.34	0.95	1.19	688,671
Jakarta	Indonesia	6,503,449	0.15	1.06	1.23	543,000
Mexico	Mexico	8,831,079	-0.16	1.05	1.24	444,839
Manila	Philippines	6,720,050	0.03	1.16	1.35	325,408
Santiago	Chile	4,099,714	0.08	1.05	1.21	228,741
Lima	Peru	5,008,400	-0.26	0.86	1.36	204,522
Yokohama	Japan	3,049,782	0.15	0.95	1.32	190,865
San Francisco	US	5,877,800	-0.38	0.93	1.20	183,198
Taipei	China	1,769,568	0.22	0.80	1.15	151,830

Maximum intensity maps

One can use the long-term estimates of the USLE coefficients to characterize seismic hazard in traditional terms of maximum expected intensity. Specifically, consider the values of A, B, and C obtained for grid points of a regular *l*×*l* mesh.

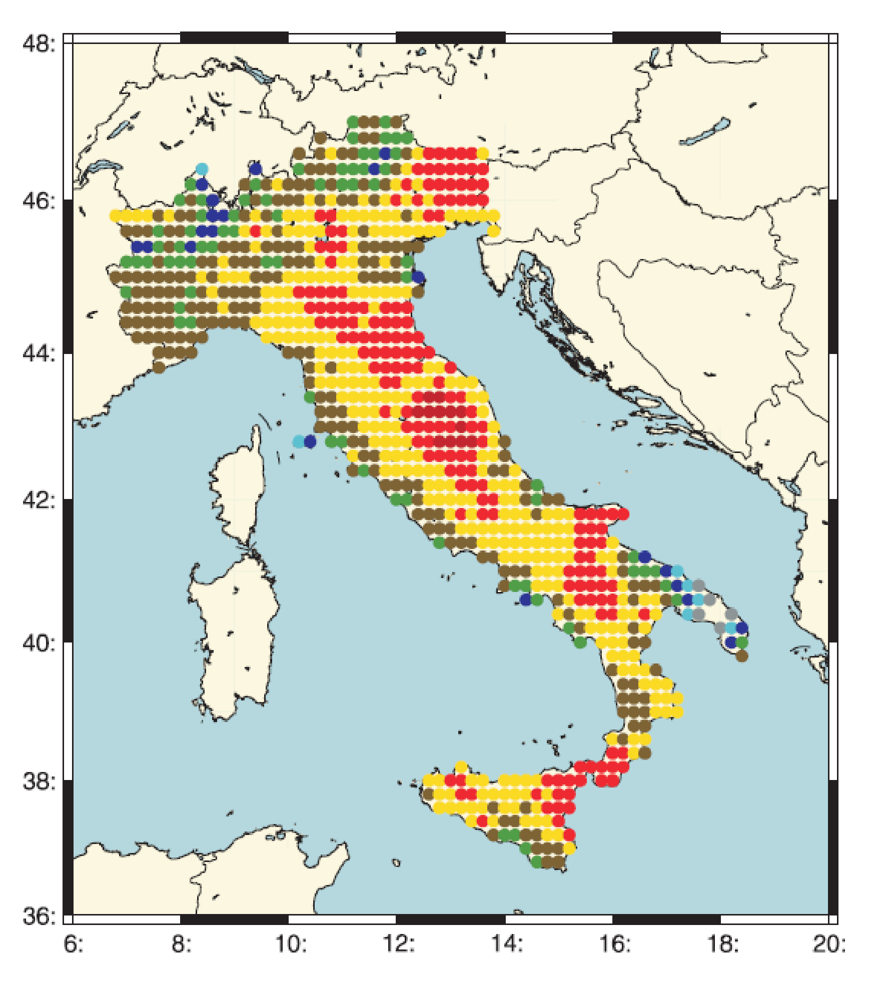
Using formula Log N(M,L) = A + B·(5 - M) + C·Log L, for magnitude ranges from M1 to M2 with 0.5-magnitude step we have calculated the expected number of events in T years N_T (M)= T × N(M).

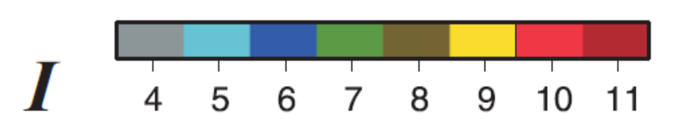
For each cell we find the maximum magnitude with the expected number N_T(M) = p% or greater and assign the intensity that corresponds to this maximum magnitude.

Presumably, the intensity assigned to a cell indicates the maximum one with probability of exceedance of p% in T years.

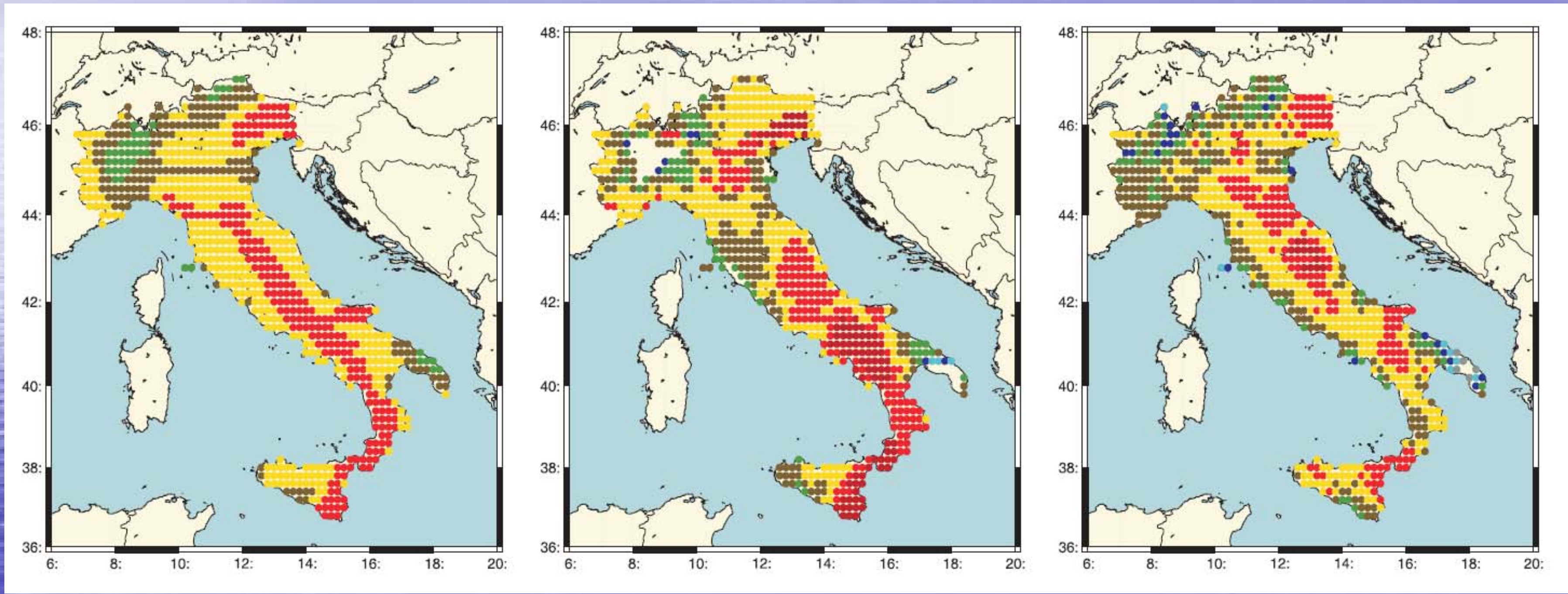
M	4	4.5	5	5.5	6	6.5	7.0
I	V	VI	VII	VIII	IX	X	XI

Italy, p=10%, T=50 years, *l*=0.2°

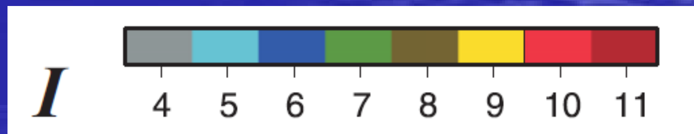




Comparison of traditional PSHA to neo-deterministic and USLE-based seismic hazard assessment



PSHA **NDPSH** **USLE**



Conclusions

The evident heterogeneity of patterns of seismic distribution and dynamics are apparently scalable according to the generalized Gutenberg-Richter recurrence law that accounts for the fractal nature of faulting. The results of our global and regional analyses imply

(i) the recurrence of earthquakes in a seismic region, for a wide range of magnitudes and sizes, can be characterized with the following law:

$$\text{Log } N(M,L) = A + B\cdot(5 - M) + C\cdot\text{Log } L,$$

where N(M,L) is the expected annual number of main shocks of magnitude M within an earthquake-prone area of liner size L

(ii) for a wide range of seismic activity, A, the balance between magnitude ranges, B, varies from 0.6 to 1.4, while the fractal dimension, C, changes from under 1 to 1.6

(iii) an estimate of earthquake recurrence rate depends on the size of the territory that is used for averaging and may differ dramatically when rescaled in traditional way to the area of interest.

The confirmed multiplicative scaling of earthquakes changes the traditional view on their recurrence, the catastrophic ones in particular, and has serious implications for estimation of seismic hazard, for the Seismic Risk Assessment, as well as for earthquake prediction.

The observed temporal variability of the USLE coefficients suggests investigating predictive power and efficiency of some *ABC* –related patterns with more data accumulated worldwide in the future. Such patterns, if confirmed in testing, might indicate the transient "lock-unlock" status of the faults in the system of blocks-and-faults, however, it appears yet premature to come out with an algorithmic formulation.

Finally, let me beware again on a synthetic character of seismic risk assessments presented in this talk for illustrative purposes only.

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