The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2060-6

Advanced School on Non-linear Dynamics and Earthquake Prediction

28 September - 10 October, 2009

The Random Walk and the Problem of Insurance of the real Estate Property

Michele Caputo

Università La Sapienza
Rome
Italy

mcaputo@g24ux.phys.uniroma1.it

The Random Walk and the problem of insurance of the real estate property.

Michele Caputo

Department of Physics, Università La Sapienza, Rome, Italy and Department of Geology and Geophysics andTexas A&M University, College Station , Texas, USA.

Email: mcaputo@g24ux.phys.uniroma1.it

Summary. We deal with the problem of the economic aspect on the consequences of an earthquake on the buildings, that is the insurance of the buildings; it has an innovative aspect since the risk for insurance companies could tentatively be estimated with the *random walk* (RW) method. The method is addressed to the case when only forward steps are possible and the probability density function (*pdf*) of their occurrence, that is the waiting time *pdf*, is formulated to lead to a Mittag Leffler function. It is seen that when *pdf* of the sizes of the steps is a negative power law then the integral equation leading to the solution of the problem is reduced to a fractional order differential equation with two different orders of fractional differentiation, The solution is then found in closed form using the Laplace Transform.. It is finally shown how the theory may tentatively model the risk of real estate insurance companies to cover the cost of the damage caused by earthquakes and consequently guide the estimate of the premiums for the risk. The method may also be used for the estimate of the regional stress in the crust and the regional accumulation of seismic moment (Caputo 2009).

Keywords: Random-walk, fractional derivative, non markovian, insurance company, earthquake damage.

1. A random walk model for the estimate of the cost of insuring real estate property for seismic risk.

In many circumstances mathematics has proved its effectiveness in modelling a variety of physical, chemical, biological and social phenomena. Often a single mathematical theory has proved its universality in modelling phenomena in quite different fields; an emblematic example is that of the *random walk* (RW) which has been successfully applied to study diffusions of various kind, to model financial phenomena and phase transitions (Weiss 1995) and also regional earthquake flow .

Most recent discussions of random walk, also implying the use of modern computers, are in Hayes (2008) and Pemantle (2007). In the following we will tentatively apply the RW theory to the modelling of the risk of insurance companies when covering the damage caused by earthquakes in seismic regions.

It is accepted generally that the seismic moment is a measure of the severity of an earthquake; it is expressed in units dyne cm and the scale covers from zero to the largest value estimated instrumentally which was that of the May 22 1960 Chilean earthquake with a moment of 5.5 10^{30} dyne cm (5.5 10^{23}Nm) with magnitude estimated 9.5, duration 2.5 minutes occurred on a fault about 1000 km long. The lower limit of the seismic moment scale is purely theoretical since earthquakes with moment less than 10^{16} dyne cm (10^{9} N m) have not been recorded and studied, they would correspond to an earthquakes caused by a slip of 1cm occurring on a fault of area 30x30 cm or with magnitude about zero (note that magnitude is a logarithmic scale and zero or negative magnitude does not mean that no earthquake occurred).

We know that small earthquakes, for sake of clarity say earthquakes with moment less than 10^{21} dyne cm, generally, will cause no damage to the real estate properties and that this damage is increasing with increasing seismic moment. There no theoretical or empirical acceptable relation between magnitude (or seismic moment) and intensity of earthquakes (Caputo 1984), mostly depending on the fact that the intensity depends on many factor which have little do to with the severity of the shaking of the ground caused by the earthquakes. There is no rigorous general relation between the economic damage caused by earthquakes and their magnitude of seismic moment and it would be very difficult to prove it since the damage depends strongly on the region. Even locally such relation is difficult to establish. The appropriate parameter to associate to the damage suffered by buildings during earthquakes seems the acceleration of the ground; however also this parameter depends of the type of ground where the buildings are located i.e.: soft soils are prone to cause more damage than rocks. As a first tentative approximation we will here assume that the damage D caused by an earthquake on real estate property be following linear relation

$$D = g\,(x - x_1) \qquad (1)$$

where x is the scalar seismic moment of the earthquake. x_1 is the threshold of seismic moment below which the earthquake will cause no damage to real estate property and g is a factor to be determined from the estimate of the damage caused by previous earthquakes of known seismic moment and from the average amount of damaged real estate property insured against earthquake damage..

The damage to real estate property caused by earthquakes, estimated using the factor *g* in equation (1), depends also on the distance from the ipocenter, on the type of construction, on the material used, on the age of the building, on the soil properties where the building is located and therefore it would be possible to accept the proportionality between seismic moment and damage only in regions where exist compulsory standard codes of constructions and all buildings are erected according to that code. For all these reasons Italy is would be a region where the proportionality of equation (1) could tentatively used only accepting space averaging and in the long time run.

A critical point is the relation between the scalar seismic moment and the damage caused by the earthquake; if it is not linear, as assumed in equation (1), the total scalar seismic moment, accumulated after a given period if time, may be redistributed according to its *pdf* and the total damage estimated using the non linear relation between scalar seismic moment and damage.

In California instead most of the one family houses and the high rises have been built in the century after the 1906 San Francisco great earthquake taking into account the risk of earthquake damage, which generates an almost ideal situation for issuing insurance against the risk of damage caused by earthquakes.

2. The Random Walk model. Concerning the RW model which we use in this note, it considers a walker stepping on a straight line and always in the same direction The steps are separated by time intervals of duration ruled by a given probability density function (*pdf*) , this time separation being called *waiting-time* or *posing-time*; moreover the steps have variable size according to a given *pdf* called *jump pdf.* The theory gives the probability that at a given time t the walker has reached the distance x.

Concerning the risk of insurance companies, using the earthquake catalogue we use the *pdf* of x, assuming that x is the scalar seismic moment, and that the *pdf* of the waiting times be that of the interarrival time of the earthquakes of the region, defining a non markovian process (Markovian would be that with a memoryless evolution). Then the RW theory gives the probability that at a given time t the accumulation of seismic moment of the earthquakes, which have exceeded the threshold x_1, shown in equation (1), has reached a value x. Finally using the relation (1), remembering its linearity, one could tentatively estimate the amount of damage which the insurance companies would have to repair. The matter would be further complicated by the type of insurance, the deductible portion, the amount covered and all the details which generally are in the fine prints of the insurance forms, but this is not pertinent in the present note.

In a general we may state that the formula (1) and the RW model of this note may be considered only as a first approximation and for global risk analysis and not for zoning; in other words more detailed work would be needed for a practical application.

For our purpose we begin considering the continuous-time random-walk equation introduced by Montroll and Weiss (1965) in the Fourier-Laplace domain and use it space x time t domain in the form obtained on the basis of probabilistic considerations as done by Gorenflo et al. (2004) assuming that the particle, or walker, is initially ($t = 0$) at the origin ($x = 0$)

$$p(x,t) = \delta(x)M(t) + \int_0^t m(t-t')\left[\int_{-\infty}^{\infty} l(x-x')p(x',t')dx'\right]dt' \qquad (2)$$

with

$$M(t) = \int_t^{\infty} m(t')dt' = 1 - \int_0^t m(t')dt, \quad \int_0^{\infty} m(t')dt' = 1 \qquad (3)$$

where $p(x,t)$ is the probability to find the walker at x at the time t and, obviously, $m(t)$ is the *pdf* of the survival probability function $M(t)$ at the initial point position. The function $l(x)$ is the *pdf* of the steps. In this note instead of the steps of the walker we will consider earthquakes occurring randomly as the steps of the walker. In metrology it would be the length of the invar bars used to measure the distance between two given points (Caputo 1954), in the case of finance it would be the index of the stock exchange (Raberto et al. 2002), in economy x would be the amount of each transaction, in the case of seismology x is the scalar seismic moment of earthquakes (Caputo 2009).

Our assumptions will allow to transform the constitutive equation from the integro-fractional differential form into a simpler differential equation of fractional order simplifying the formulation of closed form solutions.

3. Model with Mittag Leffler (ML) *pdf* for the waiting time and a negative power law for the size of the steps. If the variable x is subject to the limitation $x \geq 0$, that is one considers possible only forward steps, then the Fourier Transform in equation (2) may be considered a Laplace transform (LT) and we write equation (2) as

$$p(x,t) = \delta(x)M(t) + \int_0^t m(t-t')\left[\int_0^{\infty} l(x-x')p(x',t')dx'\right]dt' \qquad (4)$$

Under suitable non restrictive conditions, as shown in the appendix A1, it may be seen following the procedure used by Gorenflo et al. (2004), that equation (1) may be written

$$p(x,t) = \int_0^\infty l(x - x')p(x',t)dx' - \int_0^t N(t - t')p'((x,t')dt' \qquad (5)$$

where the second term in the RHS of equation (5) is a fractional derivative of order b provided

$$\begin{aligned} &N(t) = wt^{-b} / \Gamma(1-b)\,, \quad 0< t\,, \quad b \in]0,1[\\ &N(t) = 0, \quad t \le 0 \end{aligned} \qquad (6)$$

which implies that, as shown in the appendix A2, *m(t)* is the ML function.

In this case equation (5) may be written more simply

$$p(x,t) = \int_0^t l(x - x')p(x',t')dx' - w(\partial^b / \partial t^b)\, p(x,t) \qquad (7)$$

where the fractional derivative of order b with respect to t is defined as

$$\partial^b p(x,t) / \partial t^b = (1/\Gamma(1-b))\int_0^\infty [\partial p(x.u)/\partial u]du/(t-u)^b$$

(Kiryakova 1994, Podlubny 1999) with LT

$$LT[\partial^b p(x.t)/\partial t^b] = LT[p(x,t)] - s^{b-1}p(x,0^+), \qquad s > 0$$

where s is the LT variable.

In order to solve equation (7) we take its double Laplace Transform (LT), with LT variable s for t and k for x respectively, finding

$$p(k,s) = ws^{b-1} /[1 - l(k) + ws^b] \qquad (8)$$

Assume now in equation (7) that

$$l(x) = zx^{-1-a} [H(x_1 - x) - H(x_2 - x)] \qquad (9)$$

$$z = a/(x_1^{-a} - x_2^{-a})$$

where the value of z ensures normalization and whose LT is

$$LT\, l(x) = z\, k^a \Gamma(-a)[exp(-x_1 k) - exp(-x_2 k)] \qquad (10)$$

The limitation set to *l(x)* means that we consider only earthquakes with seismic moment in the range $[x_1, x_2]$ where x_1 is the minimum seismic moment which could cause some damage and x_2 is represents a realistic upper limit of the scale of seismic moments of the region.

Substituting equation (10) in equation (7) we find

$$p(k,s) = w\, s^{b-1}/(ws^b + 1 - z\, k^a \Gamma(-a)[exp(-x_1 k) - exp(-x_2 k)]\,) \qquad (11)$$

It is readily verified that the RHS of equation (11) is nil when k and/or s go to infinity.

Concerning the double LT $^{-1}$ of equation (11) we take first that with respect to *s* and subsequently that with respect to *k*. Setting $s = u\exp(i\theta)$ and integrating along the Bromwich path we find

$$p(k,t) = (\sin b\pi / b\pi)$$

$$\int_0^\infty du \exp(-(u(1 - zk^a\Gamma(-a)[\exp(-x_1 k) - \exp(-x_2 k)]) / w)^{1/b}\, t) / (u^2 + 2u\cos b\pi + 1) \qquad (12)$$

The computation of the LT^{-1} with respect to *k* is done assuming $k = r\exp(i\varphi)$ and integrating again along the Bromwich path we find

$$p(x,t) = (\sin b\pi / b\pi)(1/2i\pi)$$

$$\int_0^\infty du \int_0^\infty [\exp(-rx - (u(1 - zr^a \exp(-i\pi a)[e^{x_1 r} - e^{x_2 r}]) / w)^{1/b}\, t) + \qquad (13)$$

$$-\exp(-rx - (u(1 - zr^a \exp(i\pi a)[e^{x_1 r} - e^{x_2 r}]) / w)^{1/b}\, t)]dr / (u^2 + 2u\cos b\pi + 1)$$

$$p(x,t) = (\sin b\pi / b\pi)(1/2i\pi)$$

$$\int_0^\infty du \int_0^\infty \exp(-rx)\{\exp(-(u/w)^{1/b}(1 - zr^a \exp(-i\pi a)[e^{x_1 r} - e^{x_2 r}])^{1/b}\, t) +$$

$$-\exp(-(u/w)^{1/b}(1 - zr^a \exp(i\pi a)[e^{x_1 r} - e^{x_2 r}])^{1/b}\, t)\}dr / (u^2 + 2u\cos b\pi + 1)$$

$$p(x,t) = (\sin b\pi / b\pi^2)\int_0^\infty du \int_0^\infty dr$$

$$\exp(-rx)[\exp(-(u/w)^{1/b}\rho^{1/b} t\cos(\psi/b)]\sin((u/w)^{1/b}\rho^{1/b} t\sin(\psi/b)) / (u^2 + 2u\cos b\pi + 1)$$

where

$$\rho = \left| 1 - zr^a \exp(-i\pi a)[e^{x_1 r} - e^{x_2 r}] \right| =$$

$$= \left\{ (1 - zr^a[e^{x_1 r} - e^{x_2 r}]\cos\pi a)^2 + (zr^a \sin\pi a[e^{x_1 r} - e^{x_2 r}])^2 \right\}^{0.5} =$$

$$= \left\{ 1 - 2zr^a[e^{x_1 r} - e^{x_2 r}]\cos\pi a + (zr^a[e^{x_1 r} - e^{x_2 r}])^2 \right\}^{0.5}$$

$$\tan\psi = zr^a \sin\pi a[e^{x_1 r} - e^{x_2 r}] / (1 - zr^a \cos(\pi a)[e^{x_1 r} - e^{x_2 r}])$$

We should remember that the value of *x* represents the total moment released by the earthquakes of the region at the time *t* and that, since the effects caused by single earthquakes in terms of the moment *x* is assumed a linear function of *x* according to equation (1), we may estimate directly the amount of the total damage *D* caused by earthquakes at the time *t* using this equation which we recall for convenience

$D = g\,(x - x_1)$

where *g* is a factor to be determined from the damage caused by previous earthquakes of known seismic moment and from the average amount of insured real estate. The factor *g* may be a function of location since not all places have the same amount of insured property, as a first approximation the average value of the region may be adopted.

We note that writing in base 10 the log form of equation (9) and extending ourselves outside the range $]x_1, x_2[$ we obviously find the usual density distribution of the moments of the earthquakes of a seismic region

$$l(x) = z(x - D/g)^{-1-a} \qquad \log l(x) = \log (D/g)^{-1-a} z - (1+a)\log x \qquad (15)$$

that is $1 + a$ is the so called b_0*- value* of the region generally limited in the range [1.5-2].

In the problems of forecasting is of interest to discuss the meaning and the effects of the time variation of the parameter $a+1$. The decrease of $a+1$ increases the effect of memory since the kernel function of the fractional derivative weights more the past rates of the function, while an increase of $a+1$ has the contrary effect, that is, it decreases the effect of the memory. In fractal words, the decrease of $a+1$ causes decrease of the amplitude of the high frequencies relative to the low ones, that is it decreases the fractal dimension of the system and therefore stabilises it. However this conjecture should be used with great care since it may lead to different implications depending on the approach used.

4. Conclusions. The limitations of the models presented in the preceding sections are that the *pdf* of *t* be a ML function and that the *pdf* of *x* be a truncated negative power law

$$m(t) = LT^{-1}(1/(s^b + 1) =$$
$$= ((\sin \pi b)/b\pi)\int_0^\infty u^{1/b} \exp(-(u)^{1/b} t)\exp(/(u^2 + 2u\cos \pi b + 1) \qquad b \in]0,1[\qquad (16)$$

$$l(x) = zx^{-1-a}\,[H(x_1 - x) - H(x_2 - x)] \qquad a \in]0,1[\qquad (17)$$

Concerning the truncation of equation (9) an obvious case where the truncation would be necessary is in finance where smaller transactions are not technically considered and too large ones are not possible. The same may be applied to seismology since $m(t)$ is readily fitted from the generally monotonically decreasing curve enveloping the histogram of the waiting times in catalogues of the local earthquakes which may have different form in different regions and are necessarily truncated at least by the limited length covered by the catalogues of earthquakes.

The parameters *b* and *w* necessary for the determination of the waiting *pdf* are therefore obtained with a fitting to the available data in limited range. However the catalogues are usually complete in a too limited range of scalar seismic moment to allow a possible application of the model presented in this note; that is the determination of the parameters *z* and *a* could present difficulties.. In fact in the case of Italy a catalogues of earthquake with intensity $I \geq X$ (Caputo 2000) cover nine centuries mostly using the

Intensity to characterize the single events and allow to retrieve the function (6) $N(t) = wt^{-b} / \Gamma(1-b)$ finding $b = 0.78$, $w = 17.4$ where t is in units of the average time of return 11.63 years. We note however that the *pdf* of I is not rigorously that of M_0 ; however we should also note that the time distributions of the large events considered is independent from the *pdf* of the moments. In fact we may shuffle the time series of the events changing the *pdf* of the waiting times without changing the *pdf* of their seismic moment (Caputo 1976).

Concerning the other necessary *pdf* however, since the relations between seismic moment and intensity are unreliable for Italy as well for the world (Caputo 1983), with the data available, it is impossible to formulate it. Concerning the parameter g a comprehensive discussion of the data recorded by the insurance companies is needed.

A critical point is the linear relation between the scalar seismic moment and the damage caused by the earthquake; assumed in equation (1), which could be accepted only in a first very crude approximation. If this relation is non linear, the total scalar seismic moment, accumulated after a given period if time, may be redistributed according to its *pdf* and the total damage estimated using the non linear relation between scalar seismic moment and damage.

It may be objected that the total seismic moment accumulated after a given period of time may be obtained from the *pdf* of the seismic moments extrapolating in time that from the data catalogue, however this simple method would not take into account the *pdf* of the waiting times of the events and give no probability..

With reference to insurance business we note that the *pdf* of the scalar moment M_O of earthquakes given by equation (16) and the ML function suggested for the waiting time, (the inter-arrival time between earthquakes) determined from the catalogues of earthquakes, would confirm that the time-fractional RW equation may tentatively be applied in the study of the risk which insurance companies may face when they cover with insurance real estate properties against the damage caused by the regional earthquake flow.

As a final comment the system would need calibration. In fact there is need of further developments concerning the equation (1) relating the seismic moment to the damage caused by the earthquake since the damage depends on the distance between the epicenter of the earthquake and the location of the property insured and also on the geology of the location.

Appendix A. In order to simplify equation (3) and to introduce a derivative of fractional order we rewrite it here

$$p(x,t) = \delta(x)M(t) + \int_0^t m(t-t')[\int_o^\infty l(x-x')p(x',t')dx']dt' \qquad \text{(A1)}$$

and, with double LT, we obtain

$$p(k,s) = M(s)/(1-l(k)m(s)) \qquad \text{(A2)}$$

Remembering that the LT of equation (3) is

$$M(s) = [1 - m(s)]/s \qquad \text{(A3)}$$

we find, substituting it in equation (A2)

$$[(1-m(s))/sm(s)][s\, p(k,s)-1] = [l(k) -1]p(k,s) \qquad \text{(A4)}$$

Set now

$$N(s) = [1-m(s)]/s\, m(s) = M(s)/m(s) = s^{b-1} \qquad \text{(A5)}$$

Whose LT is

$$\mathrm{LT}^{-1}\, N(s) = \mathrm{LT}^{-1}(1-m(s))/sm(s) = t^{-b}/\Gamma(1-b) \qquad \text{(A6)}$$

and substituting in equation (A4) we find

$$N(s)[s\, p(k,s)-p(k,0)] = [l(k) -1]p(k,s)$$

or

$$s^{b-1}[s\, p(k,s)-p(k,0)] = [l(k) -1]p(k,s) \qquad \text{(A7)}$$

Taking the LT^{-1} of equation (7) with respect to s we obtain

$$\int_0^t N(t-t')\partial p'(x,t')/\partial t' dt' = -p(x,t) + \int_0^\infty l(x-x')p(x',t)dx'$$

$$(1/\Gamma(1-b))\int_0^t (t-t')^{-b}\partial p'(x,t')/\partial t' dt' = -p(x,t) + \int_0^\infty l(x-x')p(x',t)dx' \qquad \text{(A8)}$$

where we see that the integrand of the LHS of equation (A8) is the fractional derivative of order b of $p(x,t)$ (Podlubny 1999) . Equation (A8) may finally be written

$$(\partial^b/\partial t^b)p(x,t) = -p(x,t) + \int_0^\infty l(x-x')p(x',t')dx' \qquad \text{(A9)}$$

which is the equivalent to the analogous formula obtained by Gorenflo et al. (2000) in the case when x covers all the real range and with the use of the FT instead of the LT.

It is seen from equation (A5) that $m(t)$ is

$$m(t) = LT^{-1}(1/(s^b+1)) = ((\sin\pi b)/\pi b)\int_0^\infty u^{1/b}\exp(-tu^{1/b})/(u^2+2u\cos(b\pi)+1) \qquad \text{(A10)}$$

which is the ML function. The ML function is monotonically decreasing, with $m(0) = \infty$, $m(\infty) = 0$, it is normalized i.e.

$$\int_0^\infty m(t)dt = 1 \qquad \text{(A11)}$$

and also, at $t = 0$, its integral converges to unity.

Appendix B. Since we have assumed in equation (9)

$$l(k) = z\ k^a, \quad l(x) = z\,x^{-1-a}/\Gamma(-a) \qquad \text{(B1)}$$

we may write equations (8)

$$\begin{aligned} p(k,s) &= ws^{b-1}p(k,0) + [l(k) - ws^b]p(k,s) = \\ &= ws^{b-1}p(k,0) + zk^{a-1}kp(k,s) - ws^b p(k,s) \end{aligned} \qquad \text{(B2)}$$

or, remembering the definition of fractional derivative,

$$p(x,s) = \int_0^x (x-x')^{-a}[\partial p(x',s)/\partial x']dx' + p(0,s)/x^a - s^b p(x,s) + s^{b-1}p(x,0) \qquad \text{(B3)}$$

which finally gives

$$p(x,t) = \partial^a p(x,t)/\partial x^a + p(0,t)/x^a\Gamma(1-a) - \partial^b p(x,t)/\partial t^b \qquad \text{(B4)}$$

Equation (B4) is the integro-differential RW equation transformed into a fractional order differential equation for the case of the *pdf* considered..

References.

Bear J., Dynamics of fluids in porous media, Dover Pubblications, New York, 1972.

Burgatti P., Teoria matematica dell'elasticità, Zanichelli Press, Bologna, 1931.

Caputo M., Sul problema delle traiettorie casuali, Tesi di Laurea, Università degli Studi di Bologna, Istituto di Fisica, Academical Year 1953-54, 1954.

Caputo M., Properties of Earthquakes Statistics. Annali di Matematica Pura ed Applicata, 4, 111, 185-193, 1976.

Caputo M., Keilis-Borok V., Oficerova E., Ranzman E., Rotwain I., Solovjeff I., Pattern recognition of earthquake-prone areas in Italy, *Phys. Earth Planetary Interior,* 21, 305-320, 1980.

Caputo M., A note on random stress model for seismicity statistics and Earthquakes prediction, *Geophys. Research Letters*, 8, 5, 485- 488, 1981.

Caputo M., Are there one to one relations between, magnitude, moment, intensity and acceleration of the ground ? *Geophysical J. R. astron. Soc*. 72, 83-92 1983.

Caputo M., Comparison of five independent catalogues of earthquakes of a seismic region, *Geophysical Journal International*, 143, 417-426, 2000.

Caputo M., Random walk modelling of extreme events occurrence, International J. Differential Equations, (in press), 2009.

Caputo M. and Faita G., Primo catalogo dei maremoti delle coste italiane, Atti Accademia Nazionale dei Lincei, *Memorie, Classe Scienze fisiche matematiche naturali,* VIII, XVII, 7, 211 – 356, 1984.

Gorenflo R., Mainardi F., Scalas E., and Raberto M., Fractional Calculus and Continuous-Time Finance III: the Diffusion Limit, *Mathematical Finance*, 171,180, Birkauser Verlag, Basel, 2001.

Hayes B., 2008. Wagering with Zeno*, American Scientist*, 96, 3, 194-199.

Kiryakova V., Generalized Fractional Calculus and Applications, Pitman Research Notes in Mathematics N° 301, Longman, Harlow.. N.Y., 1994.

Montroll E. & Weiss G.H., *Random Walks on Lattices* , Journal Mathematical Physics, 6, 2, 167-181, 1995.

Pemantle R., A survey of random processed with reinforcement, *Probability Survey,* 4, 1-79.

Podlubny I., Fractional differential equations*,* New York, Academic Press.

Rodkin M.V. and Pisarenko V. F., Damage from natural disaster: fast growth of losses or stable ratio?, Russian Journal of Earth Science, 10, ES 1004, doi:10.2205/2007ES000267, 2008.

Weiss G.H., 1995. *Aspects and Applications of the Random Walk*, North Holland, N.Y..

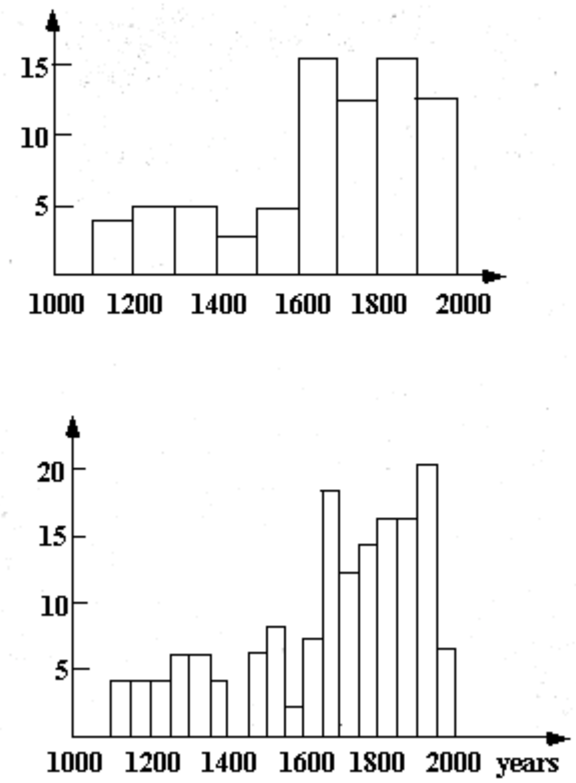

Fig. 1 Number of earthquakes with Intensity equal or larger than X in the Italian region. Is clear the increase of the activity around the mid of the XVIIth century. It may seem that in the past century we had a deficiency of large earthquakes.

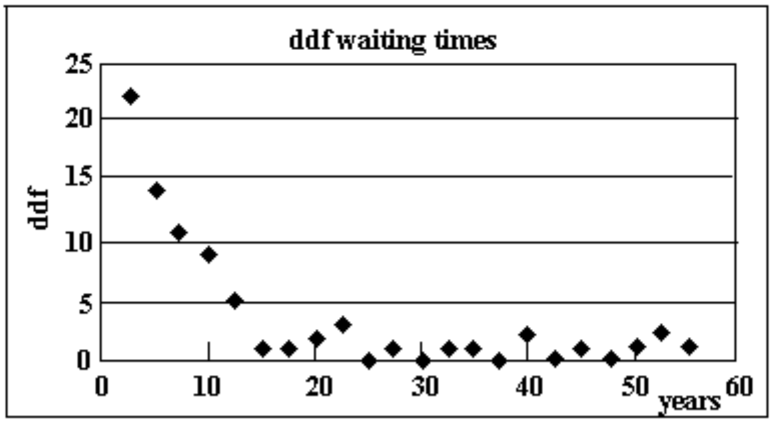

Fig. 2 Waiting time of Italian earthquakes with Intensity equal or larger that X. The density distribution is approximated by a negative power law. $N(t) = wt^{-b} / \Gamma(1-b)$ with $b =$ 0.78, $w =$ 17.4 where t is in units of the average time of return 11.6 years.