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Exactly quantized sliding of a confined solid lubricant under shear

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Exactly Quantized Sliding of a Confined Solid Lubricant under Shear

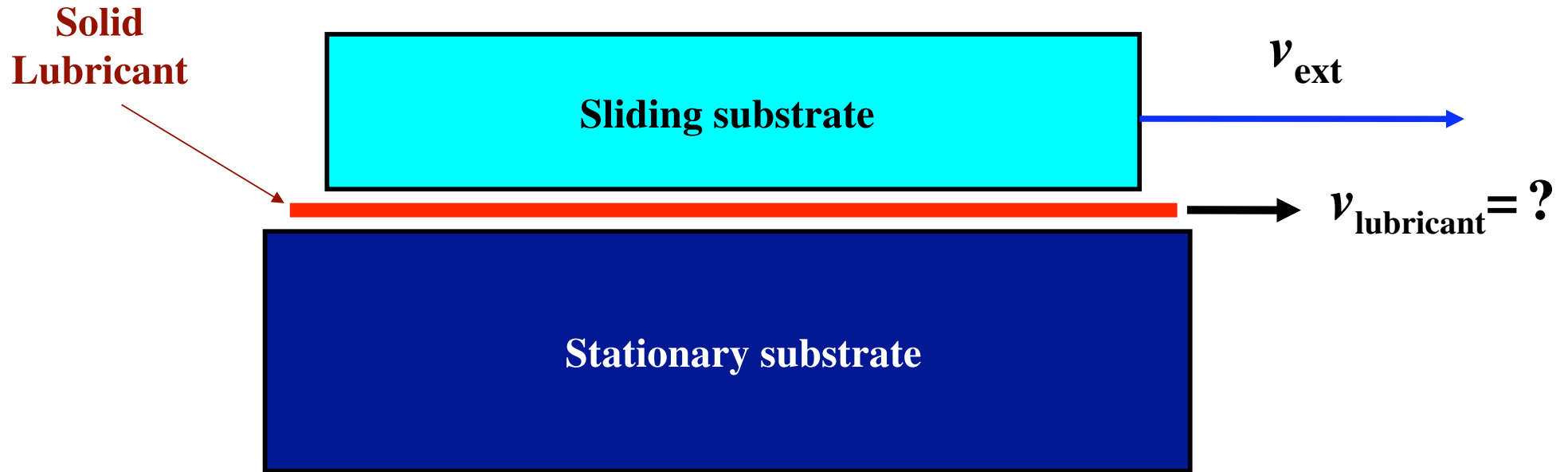
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in collaboration with:

- **Nicola Manini, Ivano E. Castelli** (*Physics Dept, Univ. Milano*)
- **Andrea Vanossi** (*SISSA Trieste/CNR-INFM S3 Modena*)
- **Giuseppe E. Santoro and Erio Tosatti** (*SISSA Trieste*)

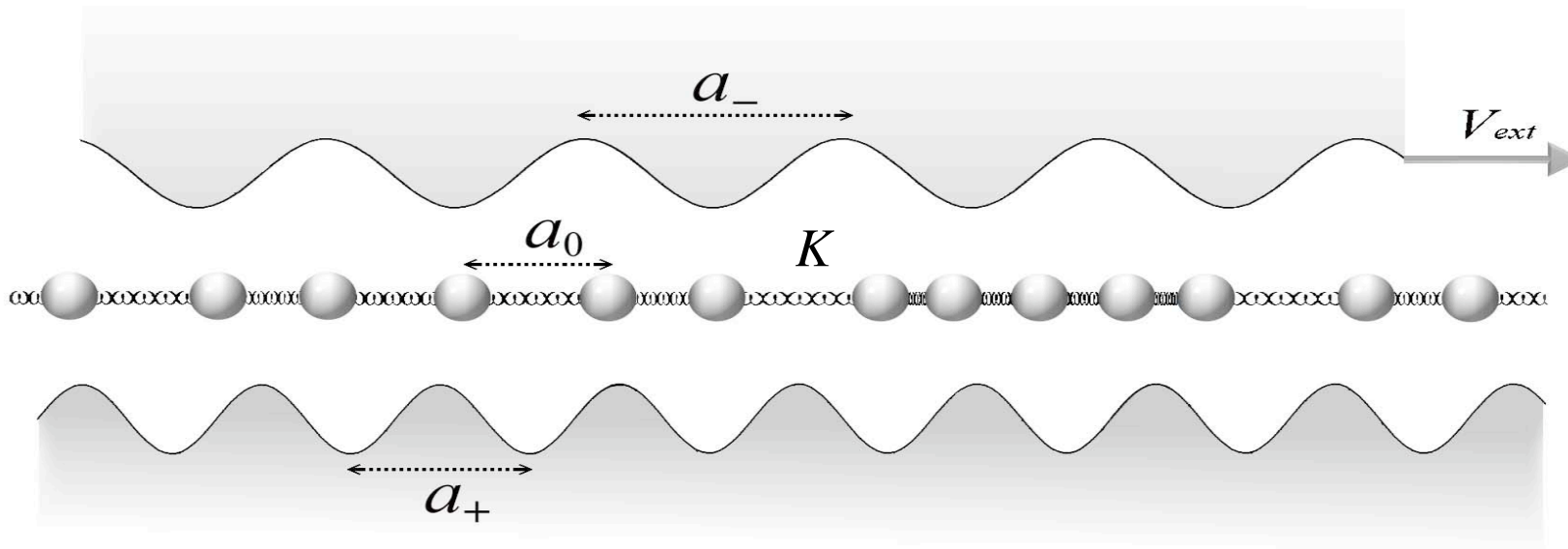
Lubricated Friction



- Macroscopic expectation: $v_{\text{lubricant}} = v_{\text{ext}}/2$
- **At nanoscale** (boundary-lubrication regime - solid lubricant layer): How do **substrate/lubricant periodicities** & **hardness of lubricant** influence $v_{\text{lubricant}}$?
 - 1D model
 - 2D model (multilayers)

1D Microscopic Model

Simplest scheme: a [generalized Frenkel-Kontorova](#) simulating the classic dissipative dynamics of a 1D chain of harmonically interacting atoms confined between two sinusoidal substrate potentials, with the top substrate driven at constant velocity v_{ext}



$$m\ddot{x}_i = K(x_{i+1} + x_{i-1} - 2x_i) - \frac{1}{2} \left[F_+ \sin \frac{2\pi}{a_+} x_i + F_- \sin \frac{2\pi}{a_-} (x_i - v_{\text{ext}} t) \right] - 2\gamma \left(\dot{x}_i - \frac{v_{\text{ext}}}{2} \right)$$

γ = phenomenological viscous damping ([underdamped](#) regime)

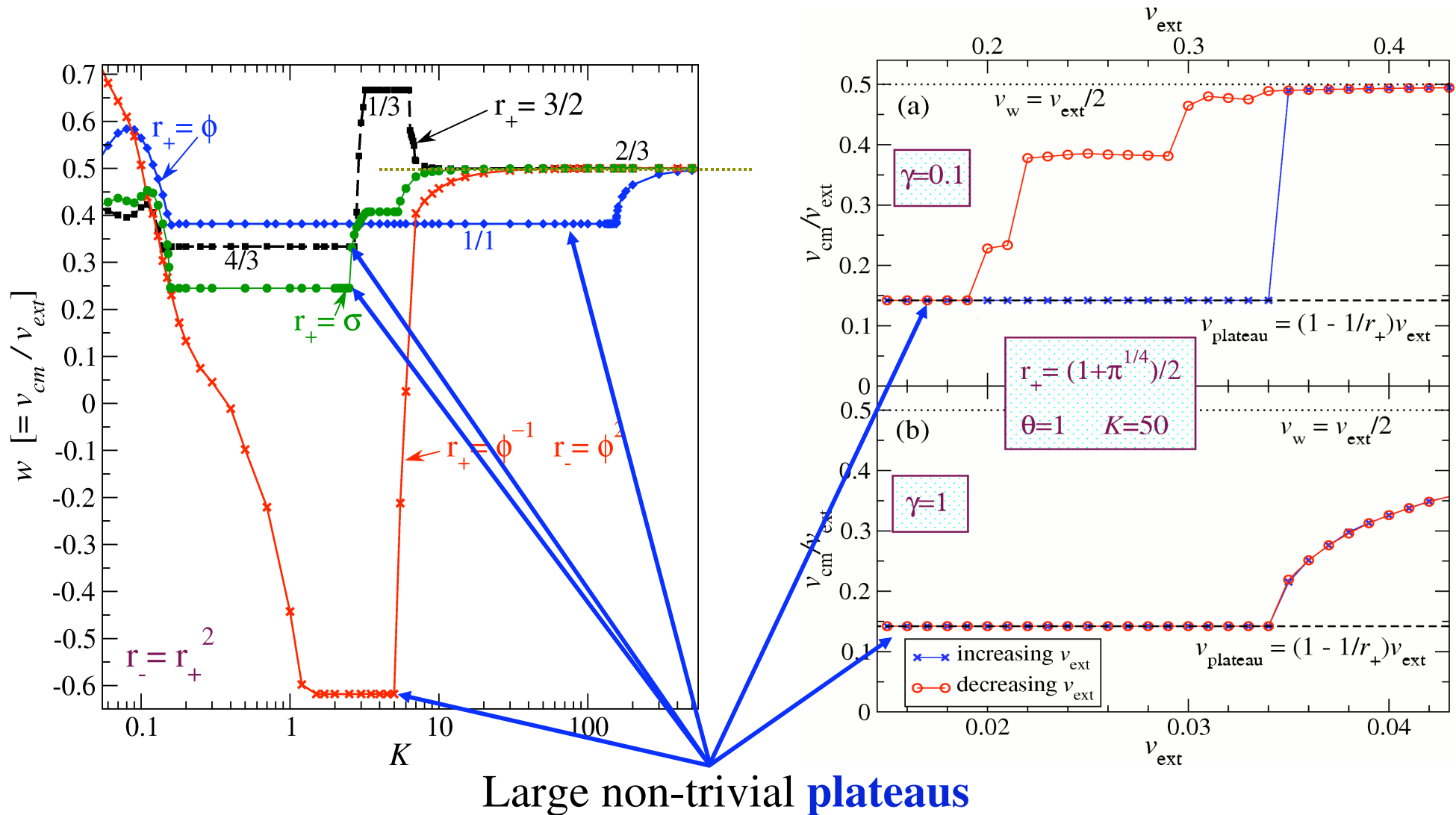
a_0 = average distance between the chain of atoms

$r = a / a_0$ = two independent **relevant length-ratios**

A. Vanossi, N. Manini, G. Divitini, G.E. Santoro, and E. Tosatti, PRL **97**, 056101 (2006)

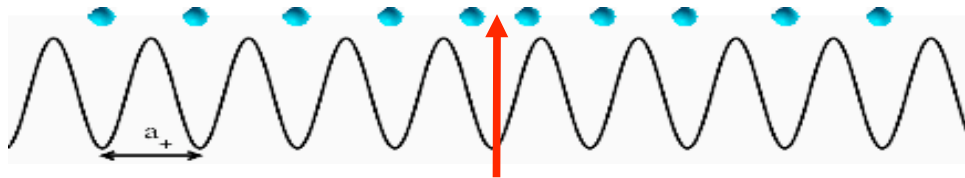
A. Vanossi, N. Manini, F. Caruso, G.E. Santoro, and E. Tosatti, PRL **99**, 206101 (2007);

Results for the CM lubricant sliding velocity



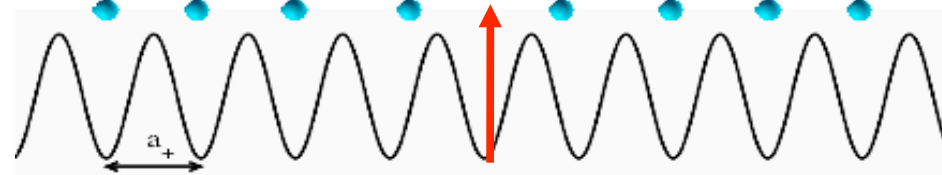
- ◆ *asymmetrical*
- ◆ *infinitely flat*
- ◆ *negative plateaus with backwards motion*
- ◆ $w = v_{plateau} / v_{ext}$ “universal” function of r_+ only (independent of K , v_{ext} , F_+ , F_- , ...)

Velocity-plateau mechanism: kinks and anti-kinks



Extra particle: **kink**

$$a_0 < a_+ \quad (r_+ = 1 + d)$$



Extra “hole”: **anti-kink**

$$a_0 > a_+ \quad (r_+ = 1 - d)$$

kinks/antikinks explain the **geometric** “quantized” plateau velocity

$$\rho_{sol} = d / a_+ = \frac{r_+ - 1}{a_+} \quad \rho_{part} = 1 / a_0 = \frac{r_+}{a_+}$$

kinks dragged at full velocity v_{ext}

$$\rho_{sol} v_{ext} = \rho_{part} v_{plateau}$$

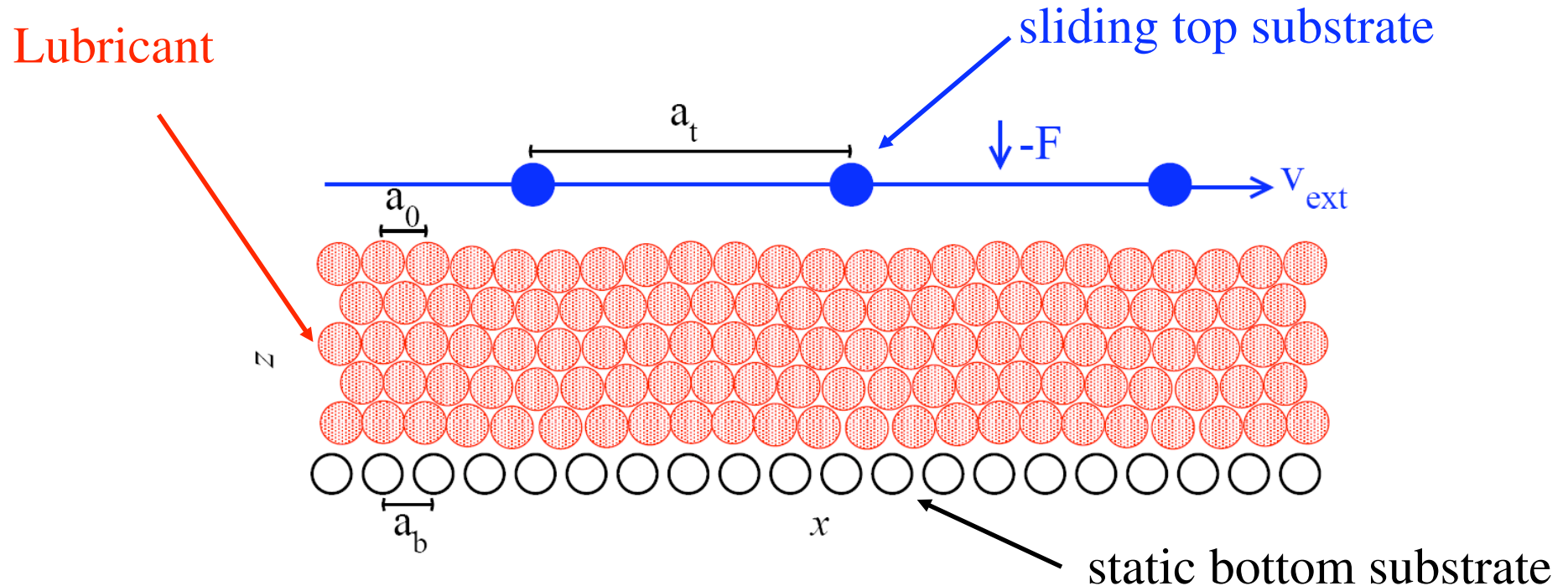


$$w = \frac{v_{plateau}}{v_{ext}} = 1 - \frac{1}{r_+}$$

only dependent on r_+ !

A more realistic model

2D (x,z) lubricant, Lennard-Jones potential, PBC



$$\Phi_a(r) = \epsilon_a \left[\left(\frac{\sigma_a}{r} \right)^{12} - 2 \left(\frac{\sigma_a}{r} \right)^6 \right]$$

$\epsilon_{tl} = \epsilon_{ll} = \epsilon_{bl} = \epsilon$ same LJ interactions

$\sigma_{tl} = a_t$, $\sigma_{bl} = a_b$, $\sigma_{ll} = a_0$, for simplicity, the 3 LJ radii σ_a coincide with the characteristic spacings a_t , a_b , a_0

The **2 relevant length-ratios** are defined by $\lambda_b = \frac{a_b}{a_0}$, $\lambda_t = \frac{a_t}{a_0}$,
with λ_b closer to unity

$$\vec{F}_j = -\frac{\partial}{\partial \vec{r}_j} \left[\sum_{i=1}^{N_t} \Phi_{tl}(|\vec{r}_j - \vec{r}_{ti}|) + \sum_{\substack{j'=1 \\ j' \neq j}}^{N_l} \Phi_{ll}(|\vec{r}_j - \vec{r}_{j'}|) + \sum_{i=1}^{N_b} \Phi_{bl}(|\vec{r}_j - \vec{r}_{bi}|) \right]$$

Total force acting on the
j-th **lubricant** particle

$r_{ti x}(t) = i a_t + v_{\text{ext}} t$, $r_{ti z}(t) = r_{t z}(t)$, where:

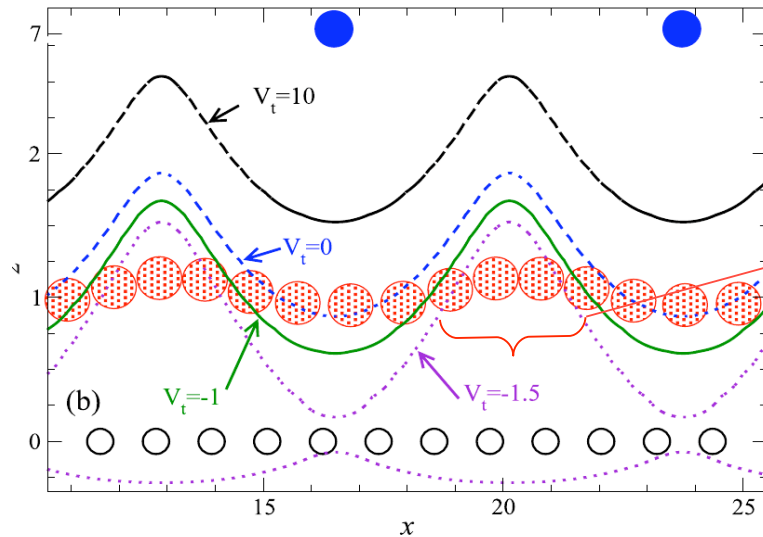
$$N_t m \ddot{r}_{t z} = - \sum_{i'=1}^{N_t} \sum_{j=1}^{N_l} \frac{\partial \Phi_{tl}}{\partial r_{ti' z}} (|\vec{r}_{ti'} - \vec{r}_j|) - N_t F$$

$$\vec{f}_{\text{damp } i} = -\eta \dot{\vec{r}}_i - \eta (\dot{\vec{r}}_i - \dot{\vec{r}}_t)$$

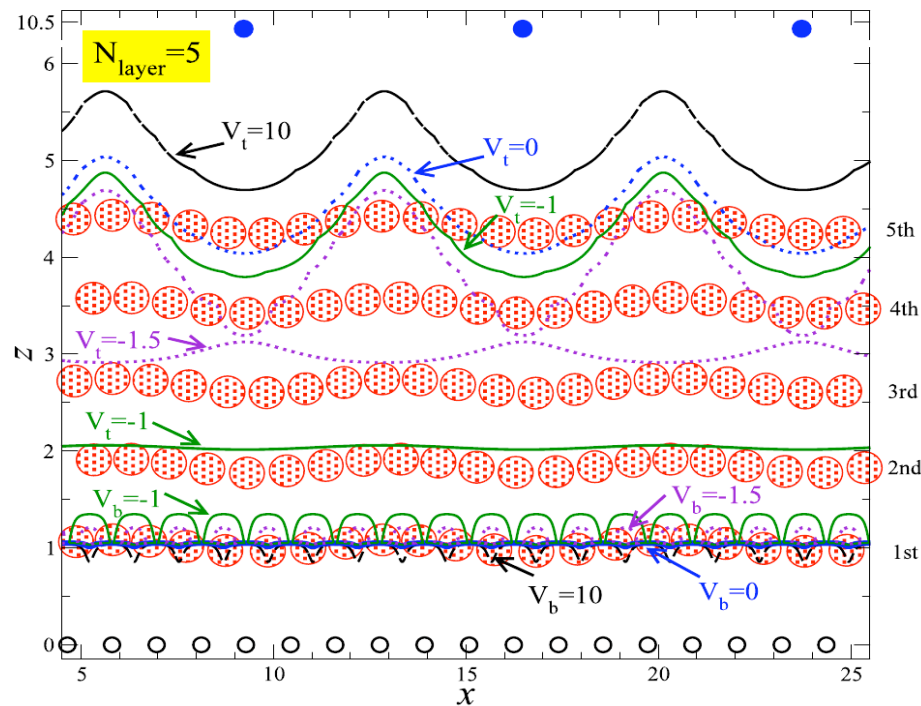
$\langle f_{j x}(t) f_{j x}(t') \rangle = 4\eta k_B T \delta(t - t')$ Standard **Langevin dynamics**, with the addition
of a dissipative damping term plus a Gaussian
random force to the Newton equations

$$\eta \sum_i^{N_l} (\dot{\vec{r}}_i - \dot{\vec{r}}_t) = \eta N_l (\vec{v}_{\text{cm}} - \dot{\vec{r}}_t)$$

Kinks in the 2D model

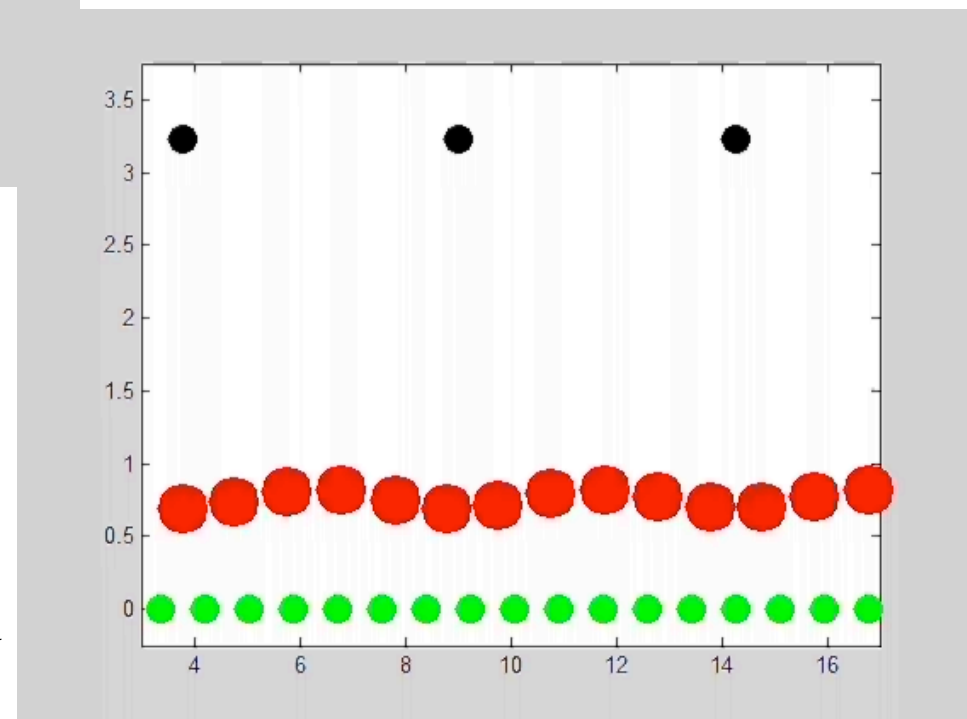
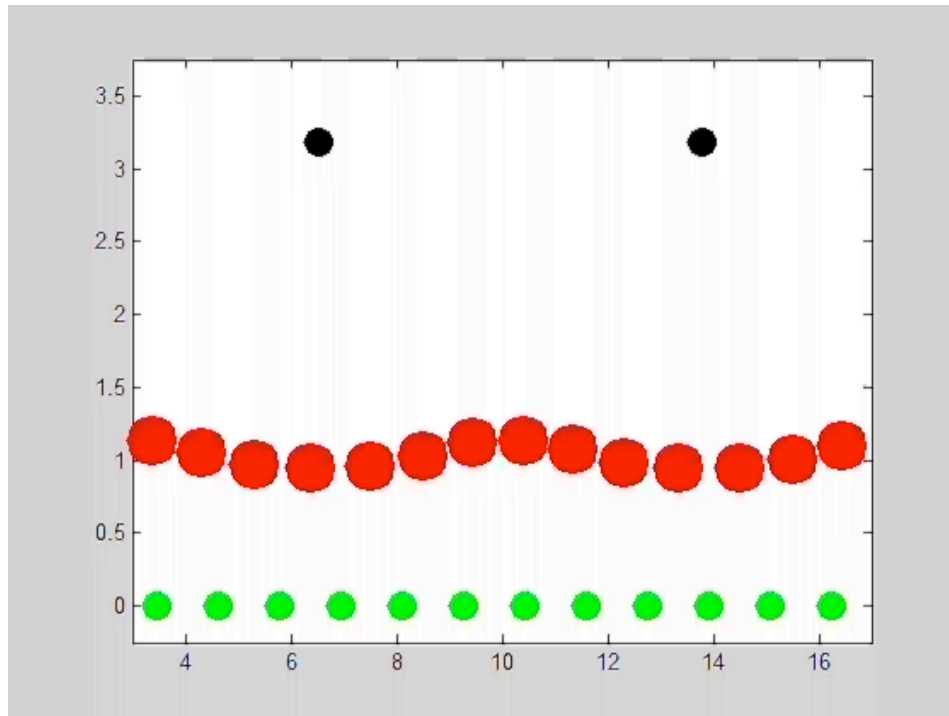


Kinks are visible like more dense and **vertically displaced** regions (non-uniform distribution of the vertical load)



Kinks are seen only in the horizontal displacement of the two lowest layers, while vertical undulations of all lubricant layers are apparent.

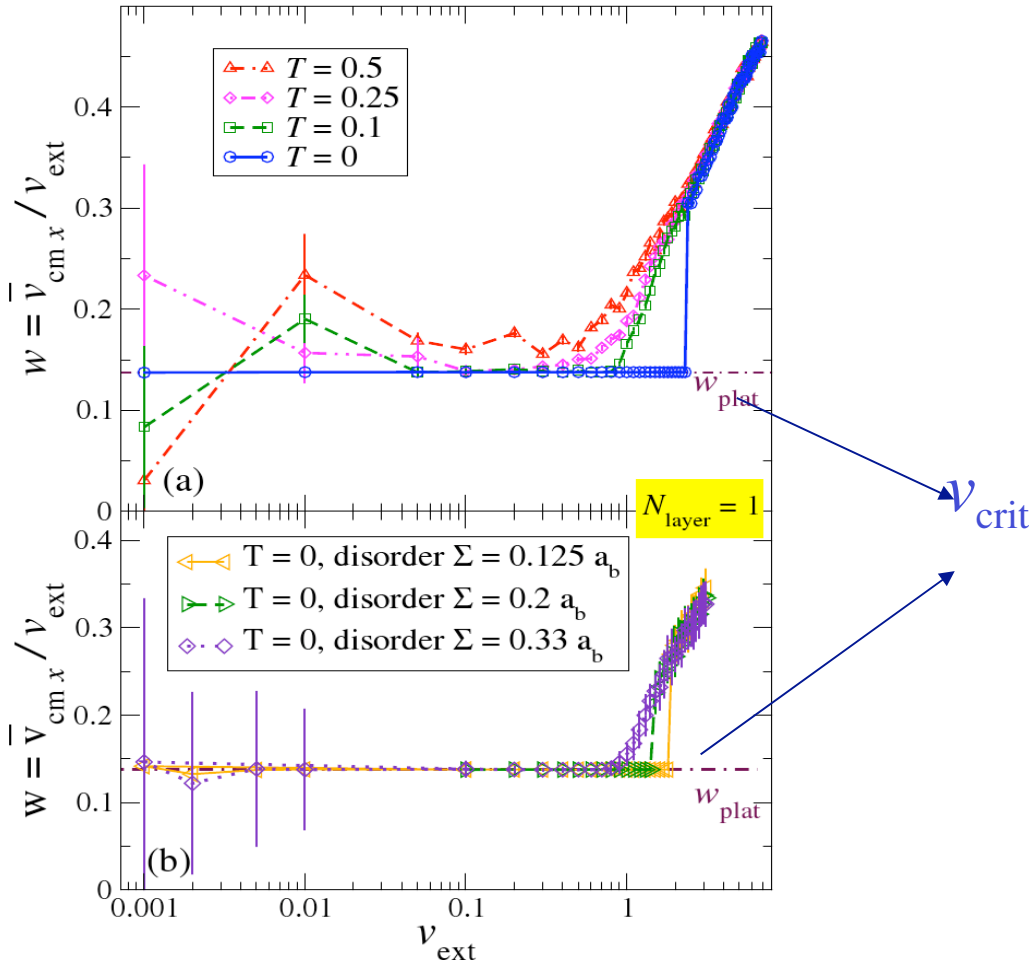
Results: kink ($a_0 < a_b$) & anti-kink ($a_0 > a_b$) dragging



backward lubricant motion at speed $w=1-1/\lambda_b = -0.19$

Velocity quantization ($N_{\text{layer}} = 1$)

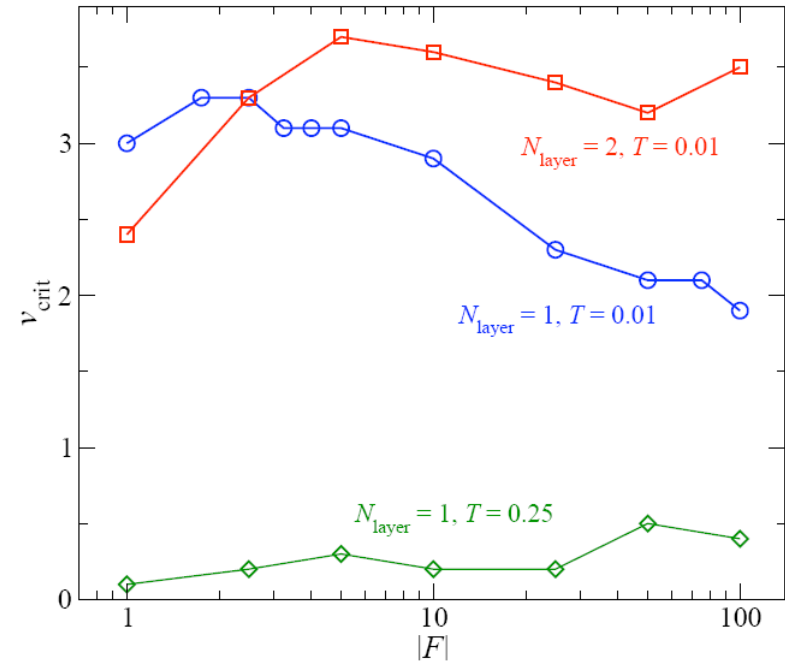
varied v_{ext}



The plateau attractor tends to affect the dynamics even in the presence of large thermal fluctuations and disordered substrates.

$$w_{\text{plat}} = \frac{\overline{v_{\text{cm} x}}}{v_{\text{ext}}} = 1 - \frac{1}{\lambda_b}$$

varied load $|F|$



Competition between the beneficial role of limiting thermal fluctuations, and the detrimental effect of hampering the vertical lubricant movements.

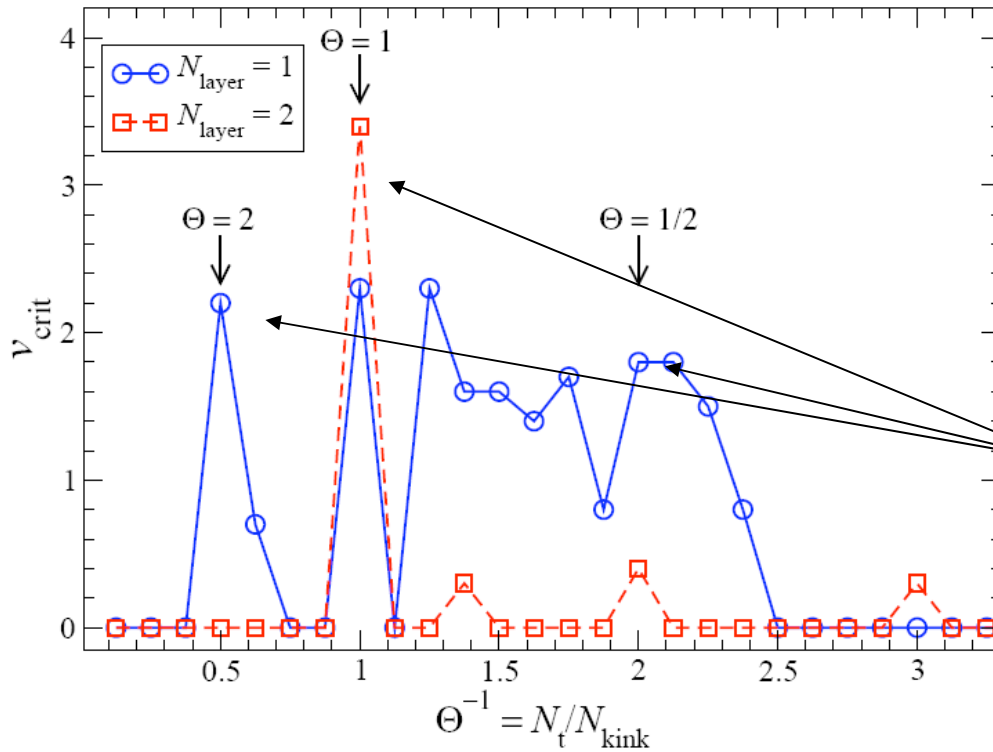
Role of kink coverage $\theta = N_{\text{kink}}/N_t$

λ_t , not affecting the velocity plateau value, crucially sets the kink coverage:

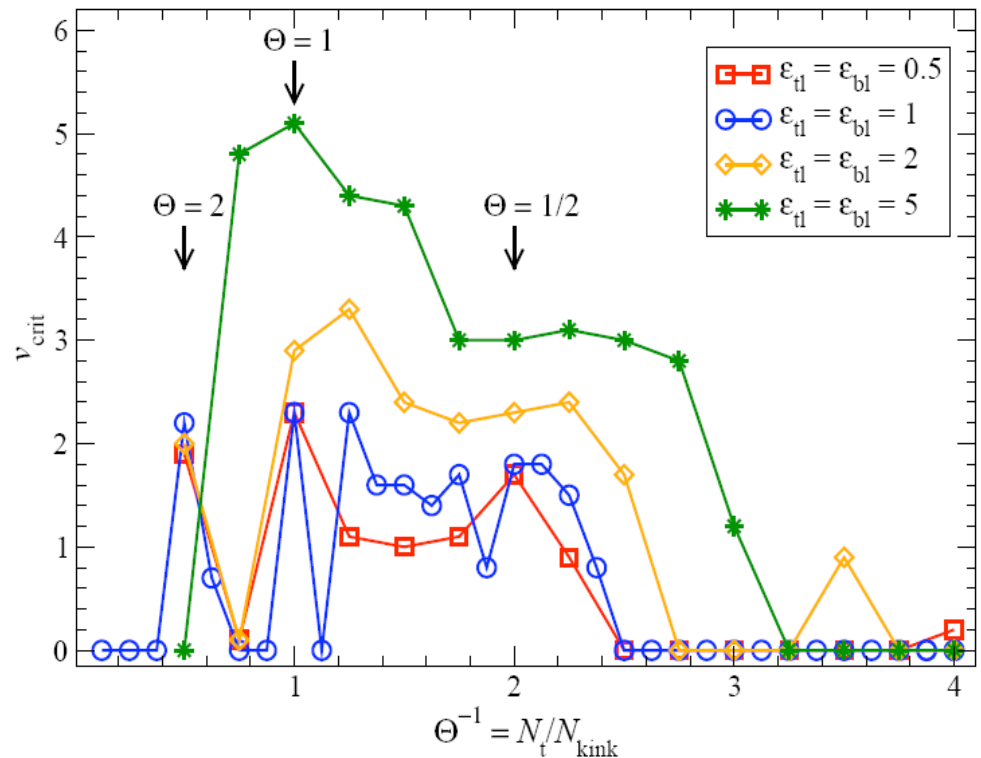
$$\Theta = N_{\text{kink}}/N_t = (1 - \lambda_b^{-1}) \lambda_t$$

and affects the plateau robustness, v_{crit}

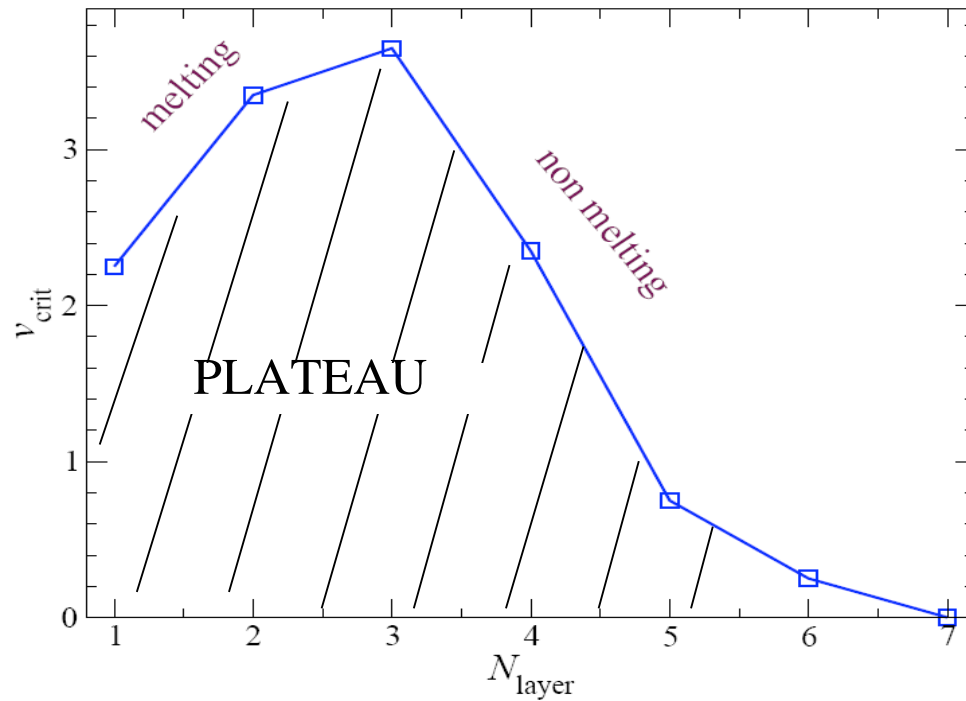
v_{crit} -maxima at well-commensurate coverage θ



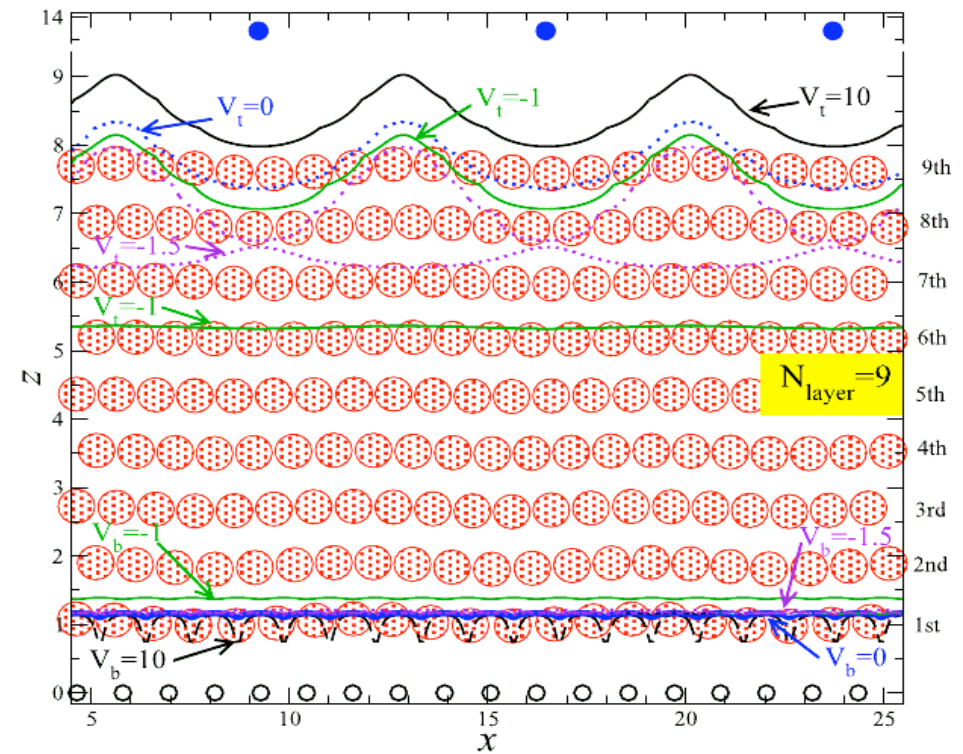
Stronger substrate interactions favour in-register lubricant regions, with narrow kinks being easily dragged by the top corrugations



Lubricant Multilayers

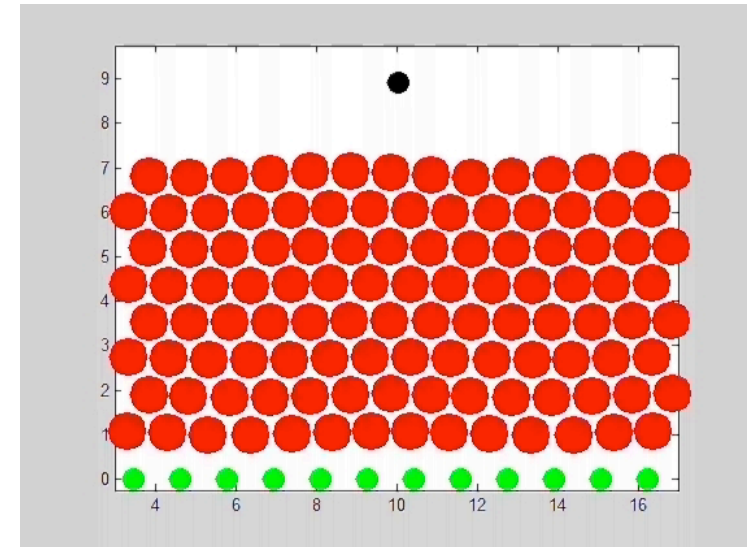
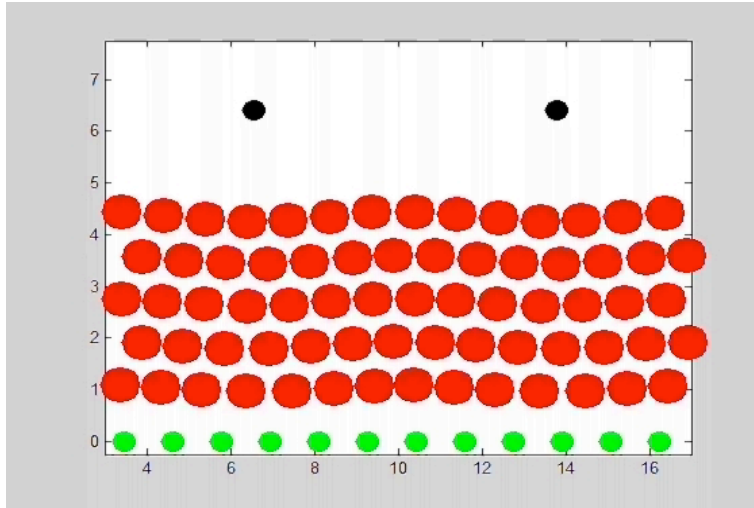


v_{crit} dependence on N_{layer} in fully commensurate $\theta = 1$ condition

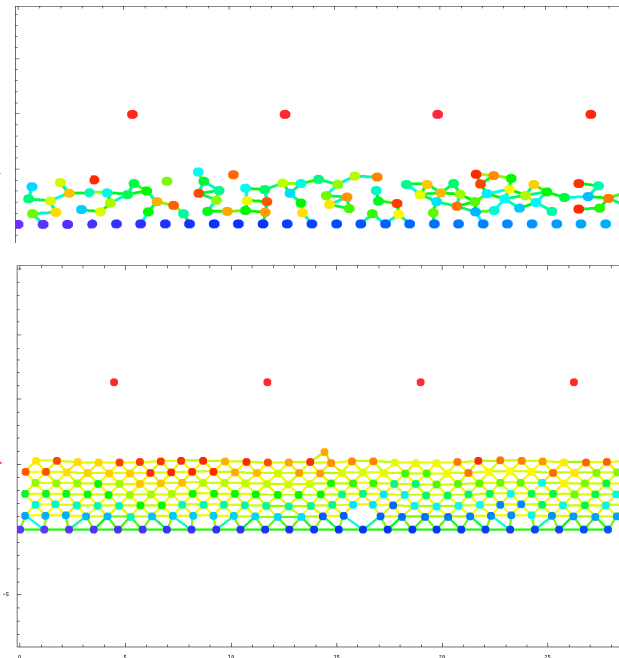
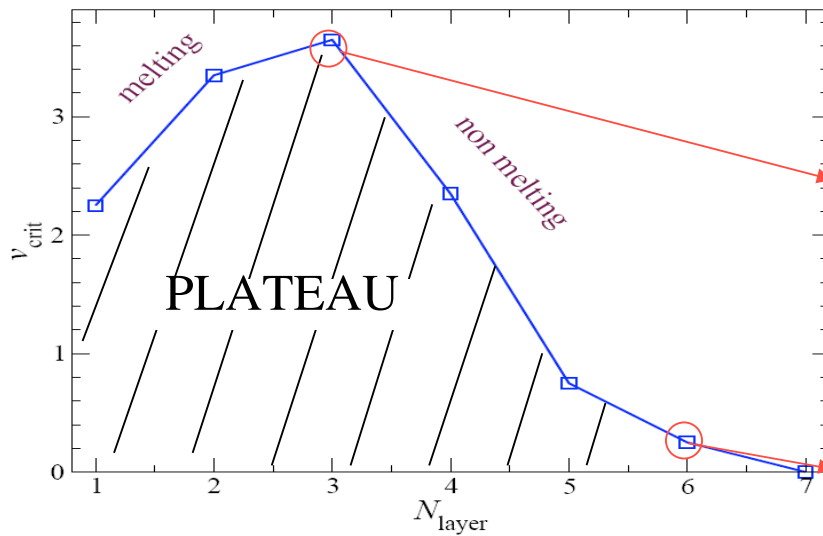


Lubricant vertical corrugation induced by the kinks favors the perfect velocity quantization.

By increasing N_{layer} , z -displacements decay rapidly, and such a mechanism becomes less effective to support the dynamically pinned state.



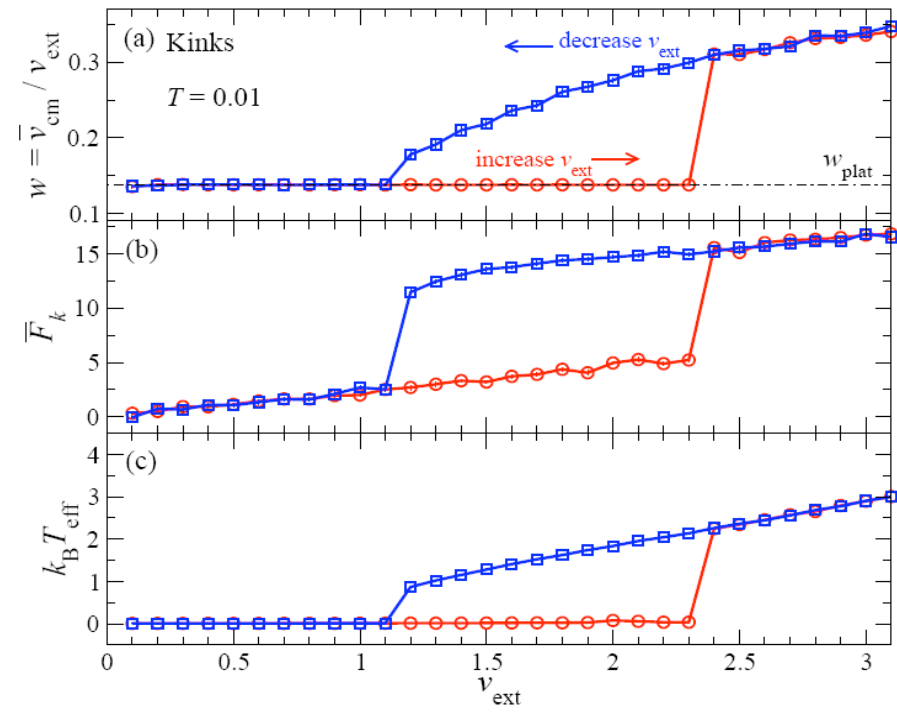
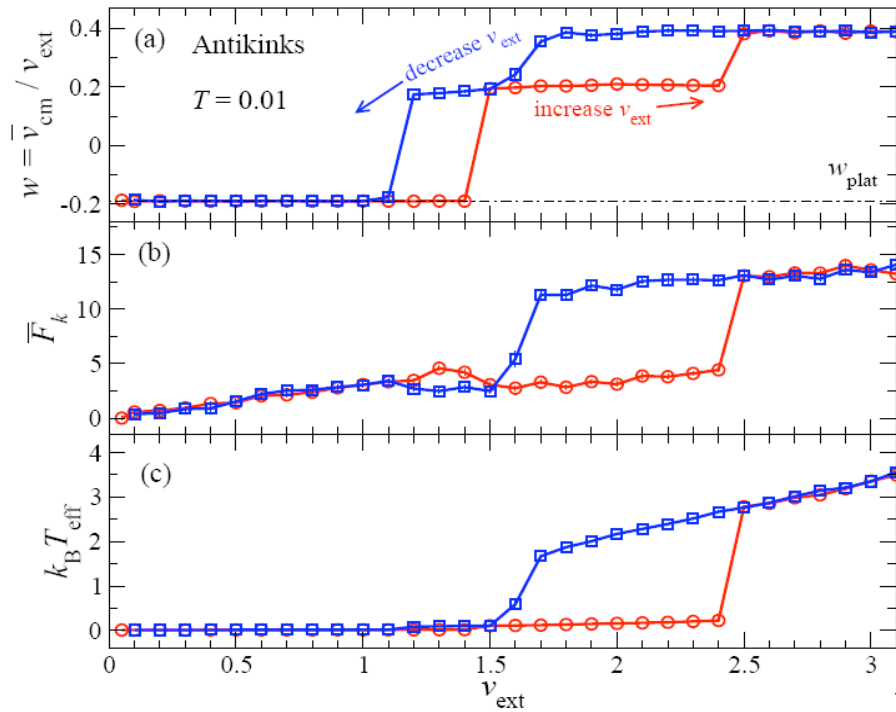
Lubricant Melting at the depinning transition



While the plateau occurs due to solitons being driven gently, the transition to the unpinned state may involve the lubricant melting, due to large shear-induced Joule heating.

Hysteresis & Friction

By cycling here up and down v_{ext} , an **hysteresis cycle** opens, even at finite temperature.



$$F_k = \text{kinetic friction force} \quad k_B T_{\text{eff}} = \frac{1}{N_1} \overline{E_{k \text{ cm}}}$$

$$\overline{F_k} = \frac{4\eta}{m v_{\text{ext}}} \underbrace{\left(\overline{E_{k \text{ cm}}} - N_1 k_B T \right)}_{\text{fluctuations of the particle velocities in the CM frame.}} + \frac{\eta N_1}{v_{\text{ext}}} \underbrace{\left[\overline{v_{\text{cm}}^2} + \overline{(\dot{\vec{r}}_t - \vec{v}_{\text{cm}})^2} \right]}_{\text{force contribution coming from the lubricant as "rigid body"}}$$

fluctuations of the particle
velocities in the CM frame.

force contribution coming from the
lubricant as “rigid body”

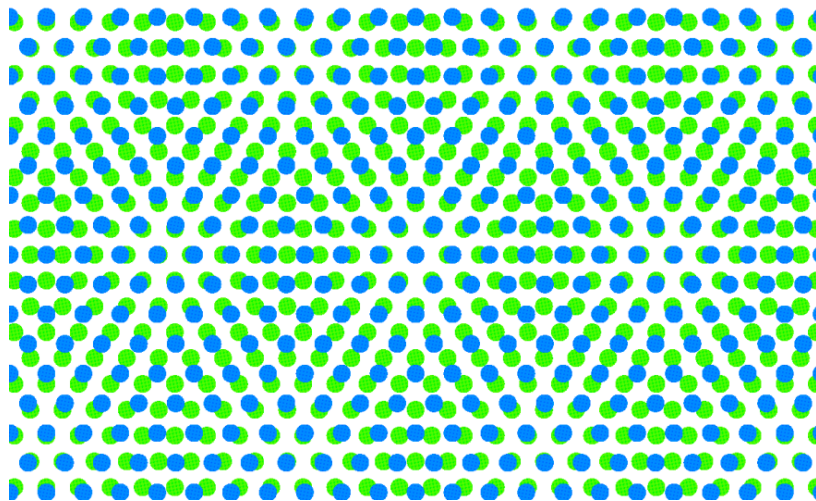
(..minimum if $v_{\text{cm} x} = \frac{1}{2} v_{\text{ext}}$)

I.E. Castelli, R. Capozza, A. Vanossi, G.E. Santoro, N. Manini, and E. Tosatti

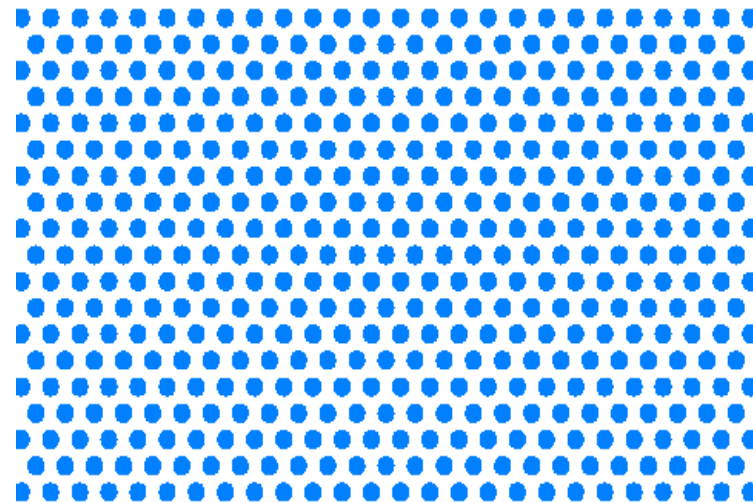
Accepted for publication in J.Chem. Phys

Work in progress

- Role of disorder (preliminary results) and substrate flexibility
- Realistic MD simulations: 3D model



dragging of a moiré pattern



Conclusions

The observed **dynamical pinning** of the solid lubricant onto a rigidly **quantized sliding state** has been understood in terms of solitons being set into motion by the shear due to the moving surfaces.

This kind of **soliton dragging** can be argued to represent a rather general mechanism, possibly at play in some realistic situations.

Thanks for Your Attention!!

all equal LJ energies ϵ ; $\sigma_{tp}=a_t$, $\sigma_{bp}=a_b$, $\sigma_{pp}=a_0$

model units		typical
length	a_0	0.3 nm
mass	m	50 a.m.u.
energy	ϵ	1 eV
force	ϵa_0^{-1}	0.5 nN
velocity	$\epsilon^{1/2} m^{-1/2}$	1000 m/s
time	$m^{1/2} \epsilon^{-1/2} a_0$	0.2 ps

standard MD (Newton e.o.m.)

4th-order Runge-Kutta integration

Nose-Hoover thermostat chain

PBC

drop an initial “thermalization” transient
($\sim 10^3$ time units)

pressure $-F$ applied to top