ICTP/FANAS Conference on trends in Nanotribology

19 - 24 October 2009

Pure adhesion in friction

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Pure Adhesion in Friction

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Joint ICTP/FANAS Conference on Trends in Nanotribology, October 2009
Friction

Friction Coefficient
- Macroscopic friction coefficient $\mu$
- Amontons law 1699
- Defined as ratio between the frictional force and the normal force

Idea
Can friction be (partly) explained by a simple adhesive model?
Friction

**Adhesion**
- Attractive force between two bodies
- Van der Waals force
- Lennard-Jones Potential
- Surface energy density $\gamma_{12}$

**JKR Theory**
- Hertz contact theory for contact between rigid surface and elastic sphere 1882
- Johnson, Kendall and Roberts 1971
- Improvement by inclusion of surface effects
- Surface energy $\gamma_{12} \to 0$ gives back Hertz theory
Geometrical Set up

\[ z_2(x) = Z_2 + \frac{(x-X_2)^2}{2R} \]

\[ z_1(x) = Z_1 - \frac{(x-X_1)^2}{2R} \]
Round Asperities

Parabolas

- Round spheres approximated to first order as parabolas

\[
\begin{align*}
  z_1(x) &= Z_1 - \frac{(x - X_1)^2}{2R} \\
  z_2(x) &= Z_2 + \frac{(x - X_2)^2}{2R}
\end{align*}
\]
Indentation Depth

**Indentation**

- Difference in $z$ coordinates
- Indentation

\[ \hat{d} \equiv z_1 \left( \frac{X_1 + X_2}{2} \right) - z_2 \left( \frac{X_1 + X_2}{2} \right) = Z_1 - Z_2 - \frac{(X_1 - X_2)^2}{4R} \]

- Actual indentation: Multiply by cosine of angle
- Approximated by unity
Uniform Distribution of Asperities

- Uniform distribution of asperities along $x$ axis
- Macroscopic length $l$

$$\psi_1(X_1) = \frac{1}{2l}, \quad \psi_2(X_2) = \frac{1}{2l}$$

- Stochastic averaging

$$\langle g \rangle_x = \int dX_0 \frac{1}{2l} g(X_0)$$

with

$$X_0 \equiv X_1 - X_2$$
Normal Distribution of Heights

- Gaussian distribution of heights
- Standard deviation $L$, macroscopic distance $Z_0$

$$\Phi_1(Z_1) = \frac{1}{\sqrt{2\pi L}} e^{-\frac{Z_1^2}{2L^2}}, \quad \Phi_2(Z_2) = \frac{1}{\sqrt{2\pi L}} e^{-\frac{(Z_2-Z_0)^2}{2L^2}}$$

- Inspired by Greenwood and Williamson 1966
- Stochastic averaging

$$\langle g \rangle_z = \int_{-\infty}^{+\infty} \frac{d\gamma}{\sqrt{4\pi}} \frac{dc}{L} e^{-\frac{dc^2}{4L^2}(\gamma+z_0)^2} g(\gamma)$$

$$\gamma \equiv \frac{Z_1 - Z_2}{dc}, \quad z_0 \equiv \frac{Z_0}{dc}$$
## Physics

### JKR Theory

#### Adhesion Theories

- Models for adhesion
- Johnson, Kendall and Roberts 1971
- Bradley 1932
- Derjaguin, Müller and Toporov 1975
- Tabor 1976
- Potential elastic energy
- Surface energy
**Formulation**

- Force $F$ due to elastic deformation and adhesion
- Indentation depth $d$
- Contact radius $a$

\[
F = \frac{4E^*a^3}{3R} - \sqrt{8\pi \gamma_{12}a^3 E^*} \\
d = \frac{a^2}{R} - \sqrt{\frac{2\pi \gamma_{12}a}{E^*}}
\]
**Effective Young’s modulus** \( E^* \) if two different materials interact

\[
\frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}
\]

\( \nu_i \) - Poisson number

**Radius** \( R \): harmonic mean if two round surfaces adhere

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}
\]
Critical Values for Detachment

- Maximal negative force $F_c$
- Associated contact radius $a_c$
- "Indentation" depth $d_c$ - length of neck

\[
F_c = - \frac{3}{2} \pi \gamma_{12} R
\]
\[
a_c = \sqrt[3]{\frac{9 \pi \gamma_{12} R^2}{8 E^*}}
\]
\[
d_c = - \frac{3}{64} \sqrt[3]{\pi \gamma_{12}^2 R^2}
\]
**Dimensionless Quantities**

- **Dimensionless relations**
  \[\tilde{a} \equiv \frac{a}{|a_c|}\]
  \[\tilde{F} \equiv \frac{F}{|F_c|} = \tilde{a}^3 - 2 \tilde{a}^2\]
  \[\tilde{d} \equiv \frac{d}{|d_c|} = 3 \tilde{a}^2 - 4 \tilde{a}\]

- Implicit \(F - d\) - relation: Solvable by Cardano’s formula
- Instead: Approximation by power law

\[\tilde{F} \approx \alpha (\tilde{d} + 1)^{\beta} - 1, \quad -1 \leq \tilde{d} \leq 10\]

\[\alpha \approx \frac{1}{9}, \quad \beta \approx \frac{5}{3}\]
Forces

Decomposition of Forces

**Decomposition**

- Adhesive force $F$ acts along the line connecting the sphere centers
- Decomposition necessary
- Small angles - first order approximation
- Normal force $F_N = F$
- Friction force $F_T = F \frac{x_0}{2R}$
First Contact

- First Contact

Intersection of parabolas

\[ z_1 = z_2 \Rightarrow Z_1 - \frac{(x - X_1)^2}{2R} = Z_2 + \frac{(x - X_2)^2}{2R} \]

\[ x_{1,2} = \frac{X_1 + X_2}{2} \pm \sqrt{R (Z_1 - Z_2) - \frac{(X_1 - X_2)^2}{4}} \]

Condition for just one intersection point

\[ x = \frac{X_1 + X_2}{2}, \quad X_1 - X_2 = \pm 2 \sqrt{R (Z_1 - Z_2)} \]
**Contact**

- First contact at intersection of parabolas
- **Breaking of contact** delayed by adhesion
- Scope of interaction extended by critical indentation $d_c$
- Boundary

\[
X_{0,\text{min}} = -2\sqrt{R}(Z_1 - Z_2)
\]

\[
X_{0,\text{max}} = 2\sqrt{R}(Z_1 - Z_2) + Rd_c
\]
Normal and tangential forces are horizontally averaged quantities

\[
\langle \tilde{F}_N \rangle_x = \frac{1}{2l} \int_{X_{0,\min}}^{X_{0,\max}} dX_0 \left[ \alpha \left(1 + \frac{1}{d_c} \left(Z_1 - Z_2 - \frac{X_0^2}{4R}\right)\right)^\beta - 1 \right]
\]

\[
\langle \tilde{F}_T \rangle_x = \frac{1}{2l} \int_{X_{0,\min}}^{X_{0,\max}} dX_0 \left[ \alpha \left(1 + \frac{1}{d_c} \left(Z_1 - Z_2 - \frac{X_0^2}{4R}\right)\right)^\beta - 1 \right] \frac{X_0}{2R}
\]
Horizontal Averaging

Frictional force

**Horizontally averaged frictional force**

- Integration of the frictional force $F_T$ easily performed

$$\langle \tilde{F}_T \rangle_x = \frac{1}{2} \frac{d_c}{l} \left( \frac{\alpha}{1 + \beta} - 1 \right)$$

- Frictional force proportional to $d_c$ but independent of $R$
Normal Force

**Horizontally averaged normal force**

- Normal force

\[
\langle \tilde{F}_N \rangle_x = \sqrt{\frac{R}{d_c}} \frac{d_c}{l} \left[ \frac{\alpha}{2} (1 + \gamma)^{\beta + \frac{1}{2}} \left[ B\left(\frac{1}{2}, \beta + 1\right) + B\left(\frac{\gamma}{\gamma + 1}, \frac{1}{2}, \beta + 1\right) \right] - \sqrt{\gamma} - \sqrt{\gamma + 1} \right]
\]

- (Incomplete) Beta function

\[
B(z, a, b) \equiv \int_0^z t^{a-1} (1 - t)^{b-1} \, dt
\]

\[
B(1, a, b) = B(1, a, b) = \frac{\Gamma(a) \Gamma(b)}{\Gamma(a + b)}
\]


**Averaging of frictional force**

- **Stochastic mean over the height coordinates** $Z_i$
- **Frictional force**

\[
\langle \langle \tilde{F}_T \rangle \rangle = \int_0^{\infty} \frac{d\gamma}{\sqrt{4\pi}} \frac{dc}{L} \ e^{-\frac{dc^2}{4L^2}(\gamma+z_0)^2} \ \frac{dc}{2l} \left( \frac{\alpha}{1+\beta} - 1 \right) \\
= \frac{F_c}{4} \ \frac{dc}{l} \left( \frac{\alpha}{1+\beta} - 1 \right) \ \text{erfc} \left[ \frac{dcz_0}{2L} \right]
\]

- **Complementary error function** \( \text{erfc} \)

\[
\text{erfc} \ (x) = \frac{2}{\sqrt{\pi}} \ \int_x^{\infty} \ dt \ e^{-t^2}
\]

- **Frictional force depends linearly on indentation depth** $d_c$
Height Averaging

**Height Mean**

Averaging of normal force

- Normal force

\[
\langle\langle \tilde{F}_N \rangle\rangle = -\frac{\sqrt{d_c} R}{l} \int_0^\infty \frac{d\gamma}{\sqrt{4\pi}} \frac{d_c}{L} e^{-\frac{d_c^2}{4L^2} (\gamma + z_0)^2} \left[ \sqrt{\gamma + 1 + \sqrt{\gamma}} - \frac{\alpha}{2(1+\gamma)^{\frac{1}{2}+\frac{1}{2}}} \left[ B \left( \frac{1}{2}, \beta+1 \right) + B \left( \frac{\gamma}{\gamma+1}, \frac{1}{2}, \beta+1 \right) \right] \right]
\]

- No closed analytic expression
- Numerically evaluated
Normal Force $\tilde{F}_N$ as function of the normalized macroscopic distance $Z_0/L$ and the normalized indentation depth $d_c/L$
Normal Force

- Normal force turns from negative to positive
- Negative: Adhesive effects prevail
- Positive: Compression takes over
Frictional force

Normalized friction force $\tilde{F}_T$ as function of the normalized macroscopic distance $Z_0/L$ and the normalized indentation depth $d_c/L$. 

![Frictional force graph](image.png)
Macroscopic friction coefficient as statistical mean

\[ \mu = \frac{\langle \langle F_T \rangle \rangle}{\langle \langle F_N \rangle \rangle} \]

\[ \mu = \frac{\sqrt{\frac{\pi}{2}} \sqrt{\frac{L^2}{Rd_c}} \left( 1 - \frac{\alpha}{1+\beta} \right) \text{erfc} \left[ \frac{Z_0}{2L} \right]}{\int_0^\infty d\gamma e^{-\left(\frac{d\gamma+Z_0}{2L}\right)^2} \left[ \sqrt{1+\gamma+\sqrt{\gamma}} - \frac{\alpha}{2}(1+\gamma)^{\beta+\frac{1}{2}} \left[ B\left(\frac{1}{2},\beta+1\right) + B\left(\frac{\gamma}{\gamma+1},\frac{1}{2},\beta+1\right) \right] \right]} \]

Numerically evaluated integral
Diagram of Friction Coefficient

Friction coefficient $\mu$ as function of the normalized macroscopic distance $Z_0/L$ and the normalized critical indentation depth $d_c/L$
Discontinuity

- Discontinuity in friction coefficient
- Reason: Zero of normal force
- Not observed in experiments
Zero of Normal Force

Parametric plot of the values of the normalized macroscopic distance $Z_0/L$ and the critical indentation depth $d_c/L$ for vanishing normal force.
Length Scales

Hierarchy

1. Macroscopic length $l$
2. Macroscopic separation $Z_0$
3. Curvature radius of asperities $R$
4. Stochastic length scale $L$
5. Indentation depth $d_c$
Conclusions and Outlook

- Too simplistic model
- Discontinuity in friction coefficient
- Vanishing of normal force at finite frictional force
- Inclusion of different physical effects necessary
Thanks

Thank you for your attention!