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**Phase-field-crystal model for pinning a sliding of adsorbed layers**

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# Phase-field crystal model for pinning and sliding of adsorbed layers

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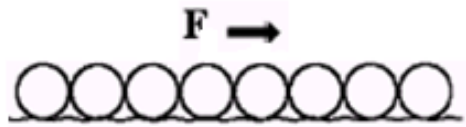
- J. A.P. Ramos, National Institute for Space Research - *INPE*, Brasil
- C.V. Achim, *Heinrich-Heine-Universitat Dusseldorf, Germany*.
- S.C. Ying, Brown University, USA
- K.R. Elder, Oakland University, USA
- M. Karttunen, University of Western Ontario, Canada
- T. Ala-Nissila, Helsinki University of Technology, Finland.

# Outline

- Driven adsorbed monolayer: results from particle models
- Phase-field crystal modelling
- Phase-field crystal model of an adsorbed monolayer

# Driven monolayer on a periodic potential

Persson, PRL 71, 112 (1993).



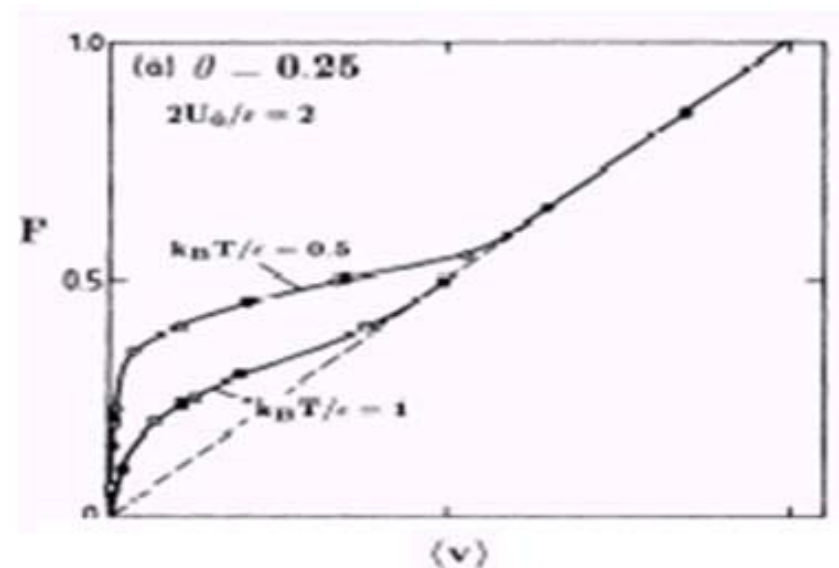
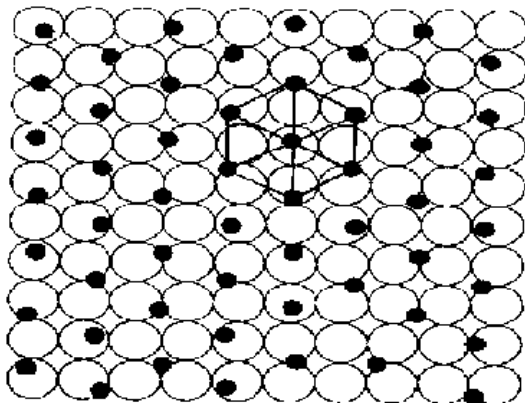
- Lennard-Jones interactions  $V(r)$
- square-lattice pinning potential  $U(r)$

- Langevin dynamics

$$m \frac{d^2 r_i}{dt^2} + m \eta \frac{dr_i}{dt} = - \frac{\partial V}{\partial r_i} - \frac{\partial U}{\partial r_i} + F + f_i$$

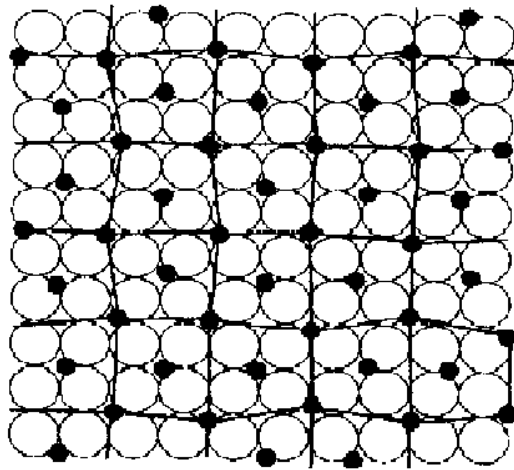
$$\langle f_i(t) f_j(t') \rangle = 2m\eta k_B T \delta_{i,j} \delta(t-t')$$

incommensurate



# Dynamic melting and freezing

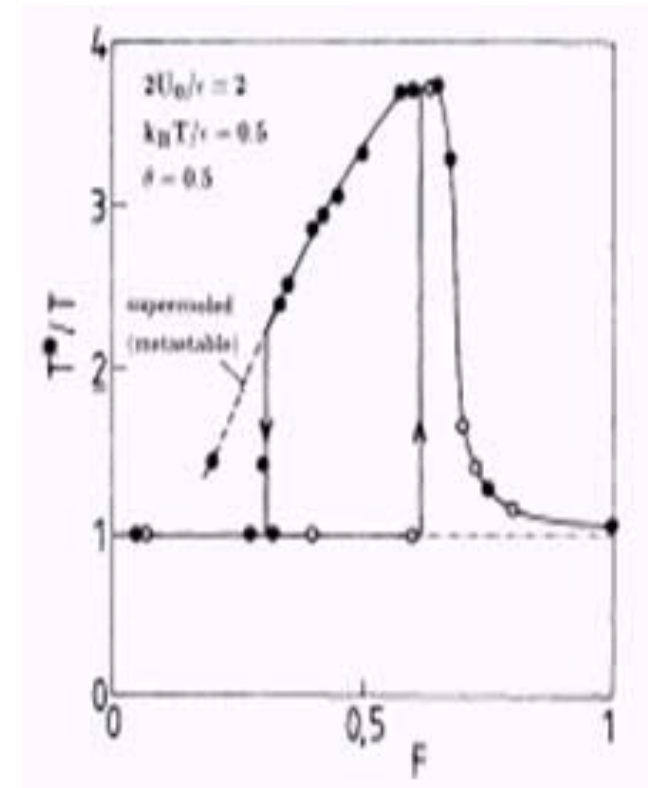
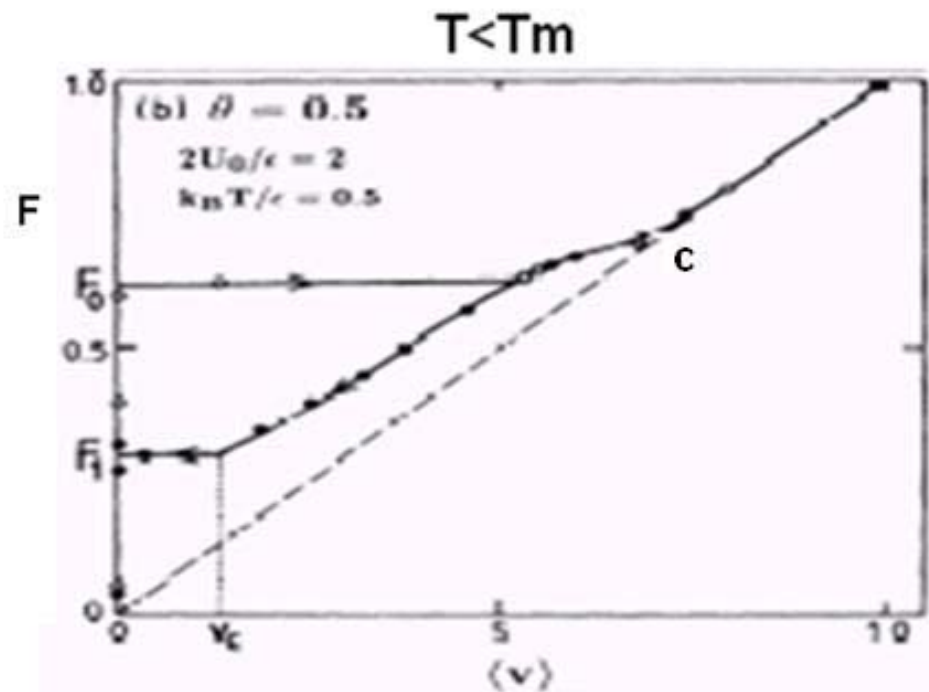
Persson, J.Chem.Phys.103,3449 (1995)



commensurate

Effective temperature

$$kT^* = m(\langle v^2 \rangle - \langle v \rangle^2)$$



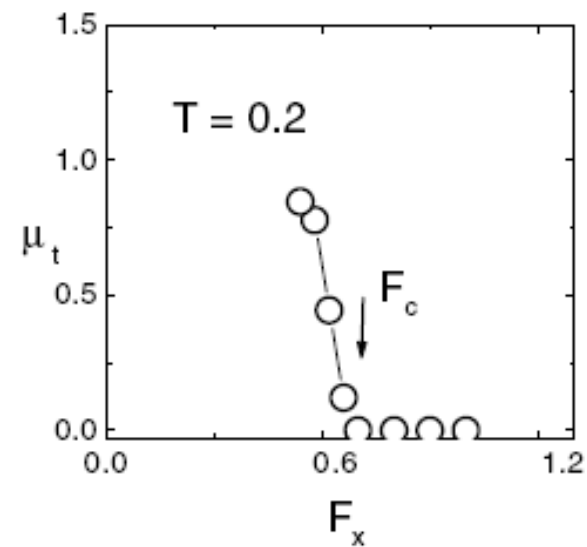
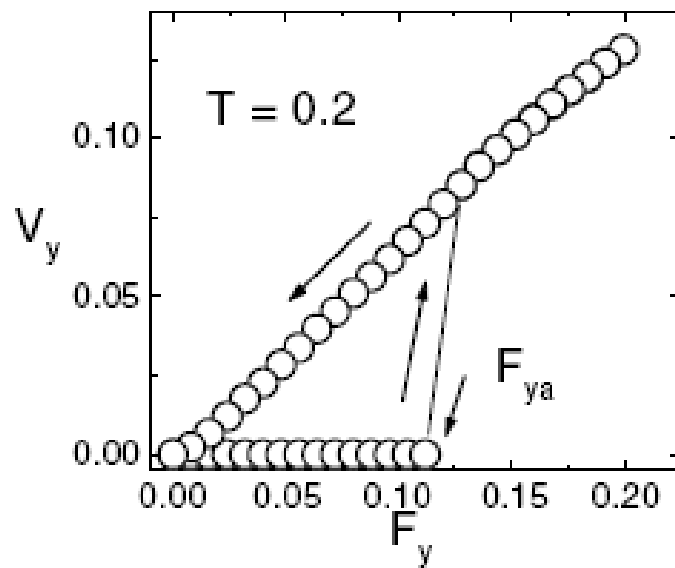
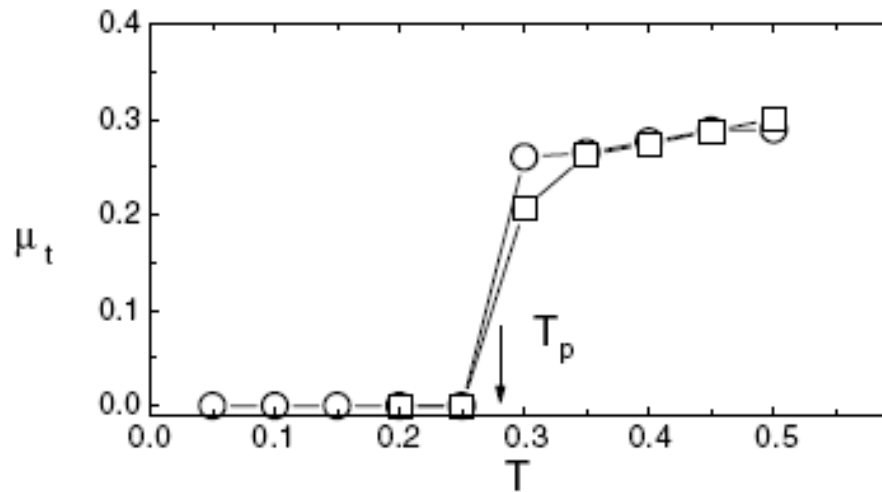
# Transverse pinning

Granato, Ying, PRL85, 5368 (2000)

- Response to additional transverse force in the sliding state

$$F \gg F_c$$

$$\mu_t = \lim_{f_y \rightarrow 0} \frac{V_y}{f_y}$$



# Phase-field modelling

- instead of particles, represent the system by continuous fields
- describe solid, liquid and gas phases
- choose a free-energy that is a functional of the fields
- the dynamics of the fields should minimize the free-energy functional:

simplest free-energy functional

$$F[\psi(x)] = \int d\vec{x} \left[ \frac{\lambda}{2} (\nabla \psi(x))^2 + \frac{r}{2} \psi^2 + \frac{u}{4} \psi^4 \right]$$

- non conserved field  
Ginzburg-Landau  
Allen-Cahn equation

$$\frac{\partial \psi(x)}{\partial t} = -\gamma \frac{\partial F[\psi]}{\partial \psi(x)}$$

- conserved field  
Cahn-Hilliard equation

$$\frac{\partial \psi(x)}{\partial t} = \gamma \nabla^2 \frac{\partial F[\psi]}{\partial \psi(x)}$$

# Phase-field-crystal modelling

Elder, Katakowski, Haataja, Grant, PRL. 88, 245701 (2002)

- continuous phase field retaining some information on atomic scale

$$F[\psi] = \int d\vec{x} \left[ \frac{1}{2} \psi(\vec{x}) (r + \lambda(\nabla^2 + k_o^2)^2) \psi(\vec{x}) + \frac{u}{4} \psi(\vec{x})^4 \right]$$

$$\frac{\partial \psi(x)}{\partial t} = \gamma \nabla^2 \frac{\partial F[\psi]}{\partial \psi(x)}$$

- free-energy minimized by a periodic pattern : a phase-field crystal
- allows for elasticity, multiple crystal orientations, dislocations , etc.
- dynamics on diffusive times scales
- can be derived from classical density functional theory of freezing (DFT)
- phase field can be identified as deviations of the density from the uniform liquid:

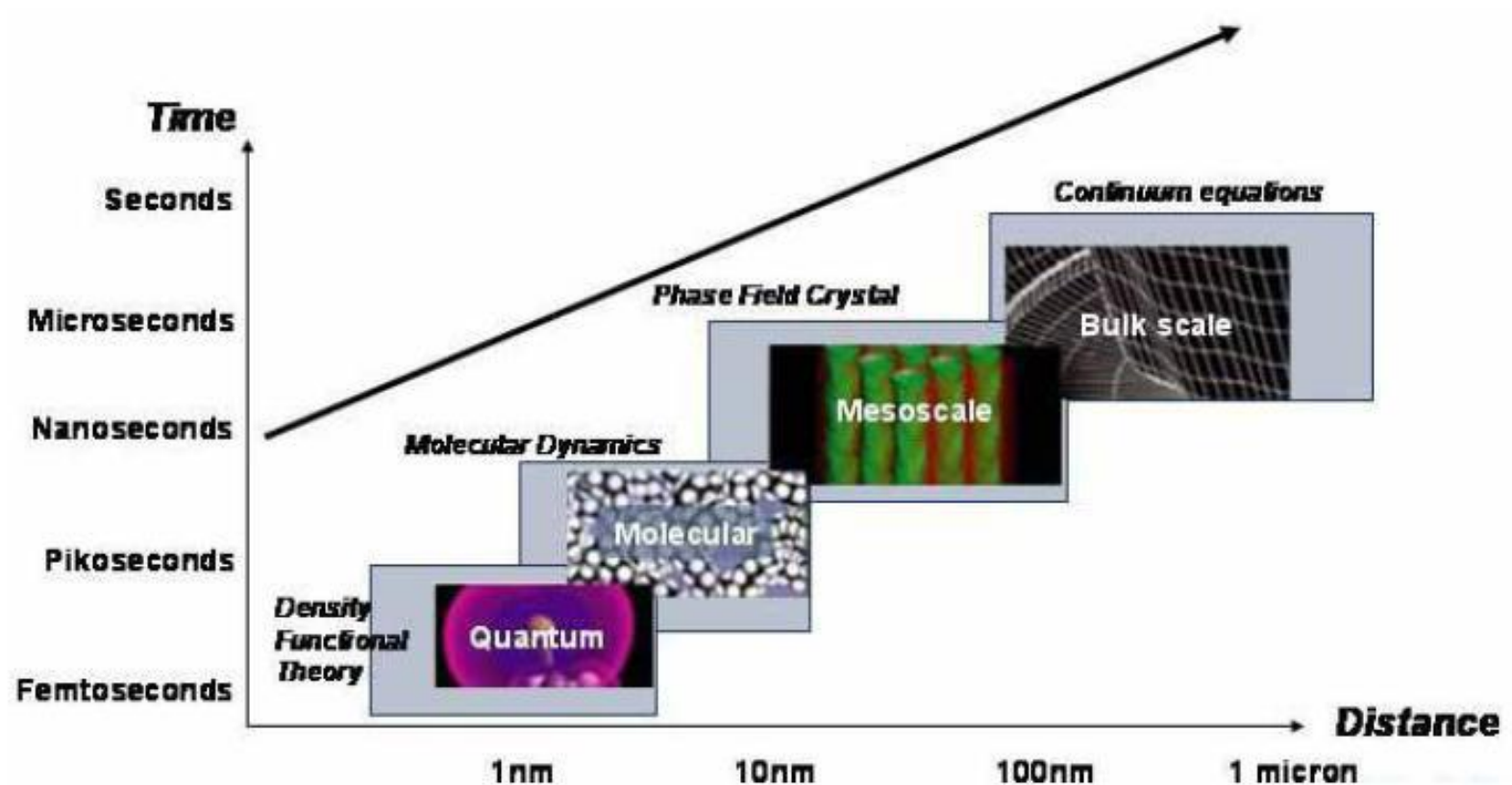
$$\psi(\vec{x}) = (\rho(\vec{x}) - \rho_l) / \rho_l$$

Elder, Provatas, Berry,  
P. Stefanovic, Grant, PRB 75, 064107 (2007).



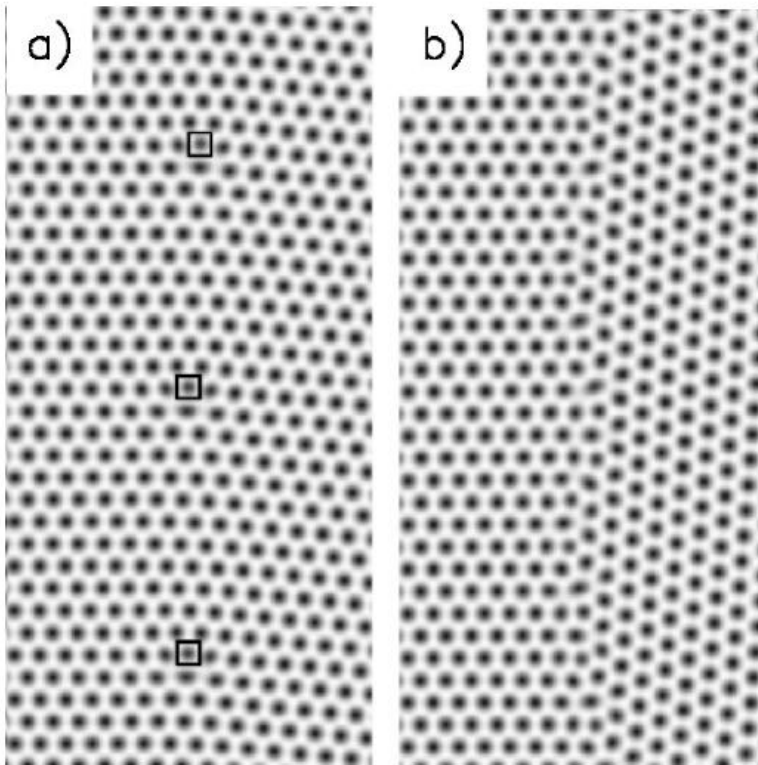
# Time and length scales

- phase-field crystal modelling provides a bridge between atomistic and continuous length scales



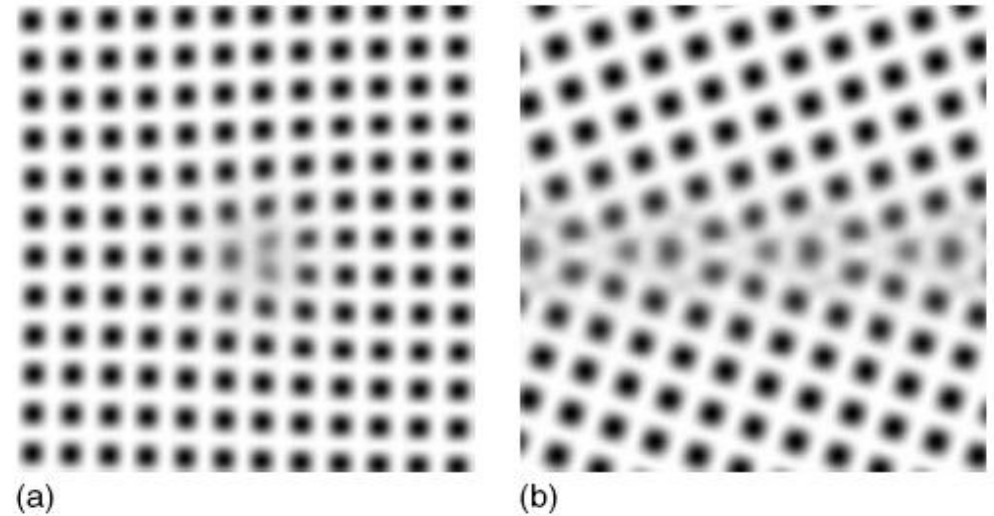
## Examples of defects in the phase-field crystal Grain boundaries

two dimensions  
hexagonal



Elder, Grant, PRE 70, 051605 (2004).

three dimensions  
bcc



Jaatinen, Achim, Elder, Ala-Nissila, PRE 80,  
031602 (2009)

# Phase-field-crystal model for an adsorbed monolayer

Achim, Karttunen, Elder, Granato, Ala-Nissila, Ying, PRE 74, 021104 (2006);

- add a pinning potential  $V(x,y)$

$$F = \int d\vec{x} \left[ \frac{1}{2} \psi (r + (\nabla^2 + 1)^2) \psi + \frac{1}{4} \psi^4 + V\psi \right]$$

- conserved dynamical equations

$$\frac{\partial \psi}{\partial \tau} = \nabla^2 \{ [r + (1 + \nabla^2)^2] \psi + \psi^3 + V \}.$$

- for  $V(x,y)=0$ , the minimum correspond to hexagonal lattice  $k_h \approx 1$
- lattice constant mismatch

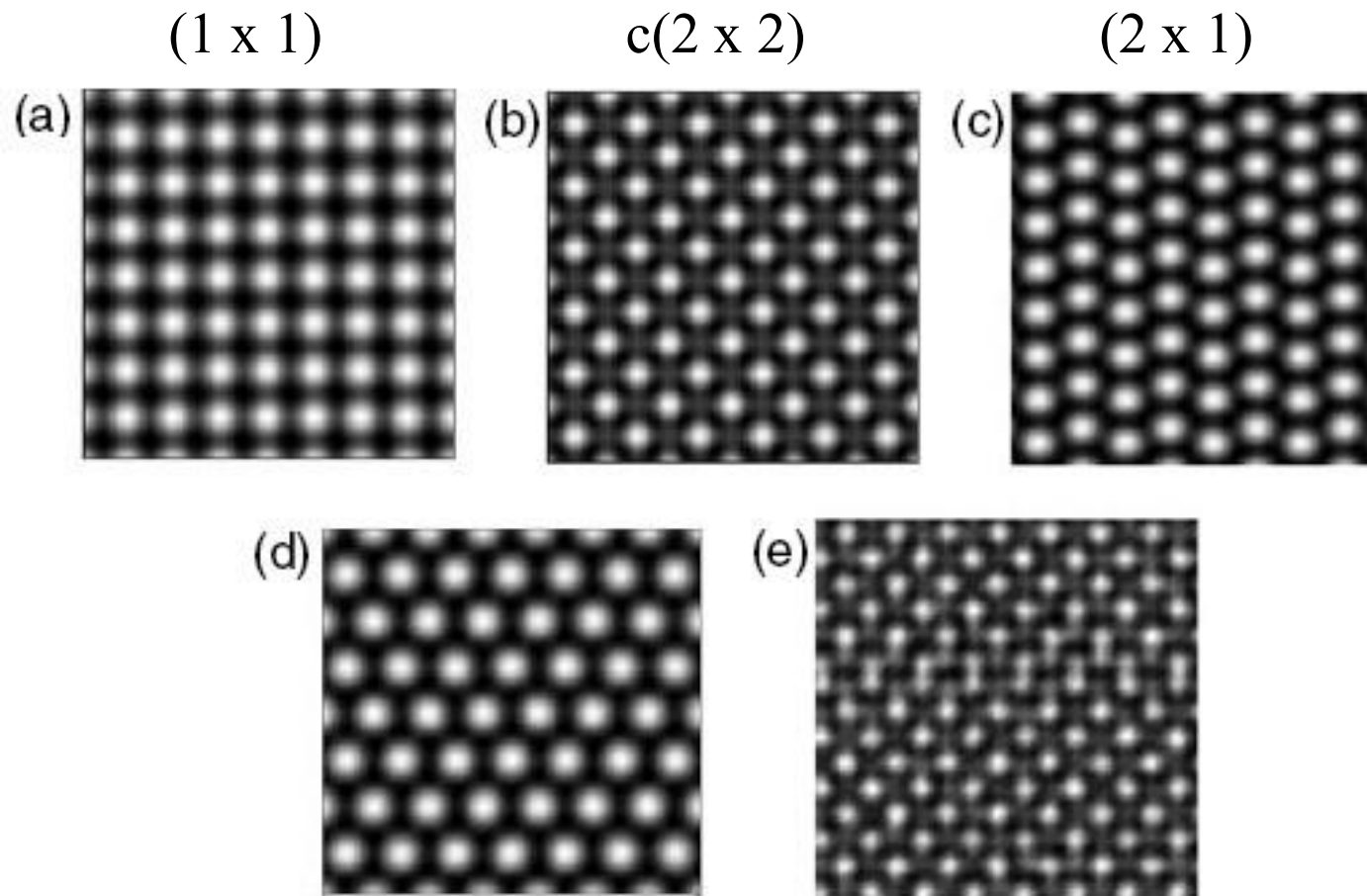
$$\delta_m = (k_h - k_s) / k_h$$

## **Fundamental requirements to model adsorbed layers**

- commensurability effects
- depinning by thermal fluctuations
- force-induced depinning

## Minimum free-energy structures

- square-lattice pinning potential  $V(x,y) = V_0[\cos(k_s x) + \cos(k_s y)]$ ,

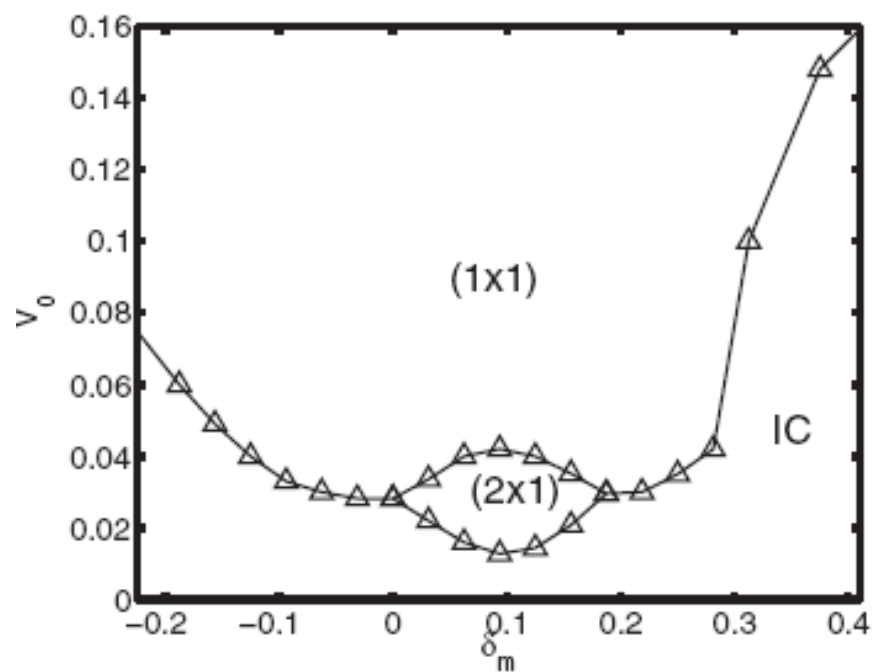


Hexagonal IC

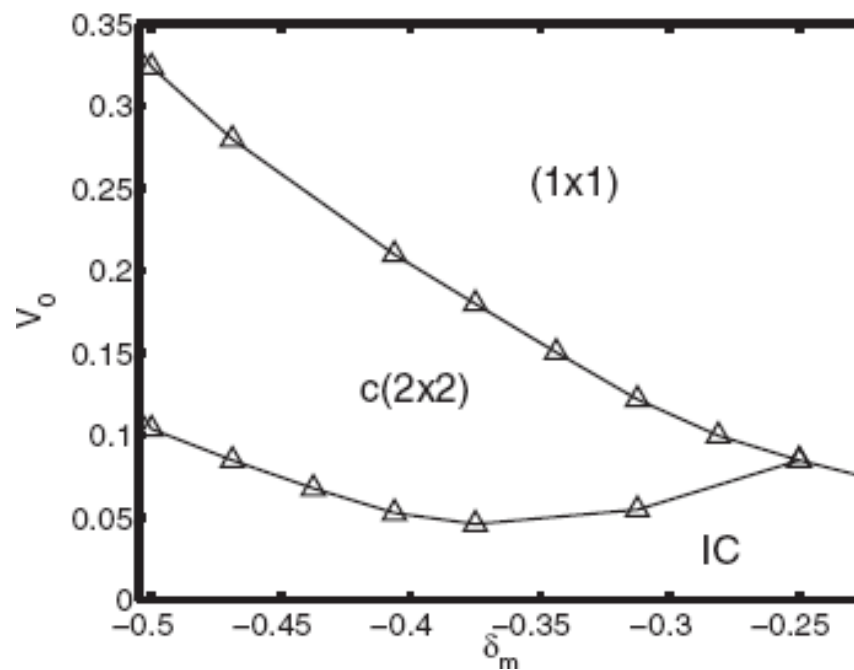
IC with domain walls near c(2 x 2)

## Potential-misfit phase diagram

near (1 x 1) phase



near c(2 x 2) phase



# Thermal fluctuations

Ramos, Granato, Achim, Ying, Elder, Ala-Nissila, PRE 78, 031109 (2008);

- to go beyond mean-field approximation  
regard free-energy functional as an effective Hamiltonian

$$H_{PFC} = \int d\vec{x} \left[ \frac{1}{2} \psi (r + (\nabla^2 + 1)^2) \psi + \frac{1}{4} \psi^4 + V\psi \right]$$

- fluctuations are taken into account by the partition function:

$$Z = \int \delta\psi e^{-H_{PFC}[\psi]/kT}$$

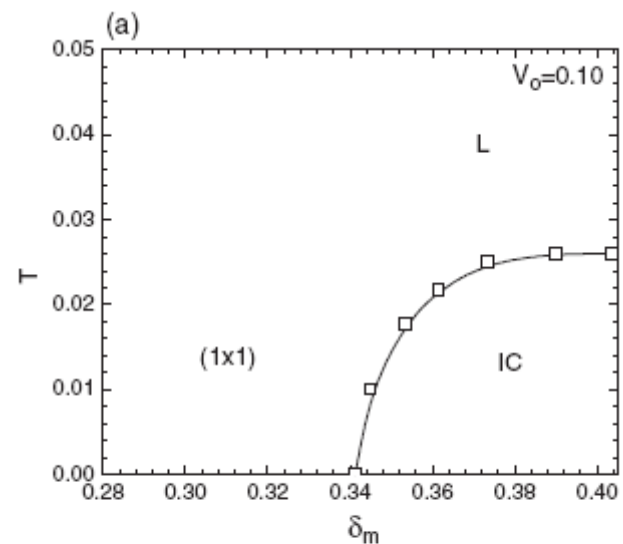
- equilibrium behavior obtained by:

- Langevin dynamics: 
$$\frac{\partial \psi(\vec{x})}{\partial t} = \gamma \nabla^2 \frac{\partial H_{PFC}}{\partial \psi(\vec{x})} + \zeta(\vec{x}, t)$$
$$\langle \zeta_i(\vec{x}, t) \zeta_j(\vec{x}', t') \rangle = 2 \gamma k_B T \nabla^2 \delta(\vec{x} - \vec{x}') \delta(t - t')$$

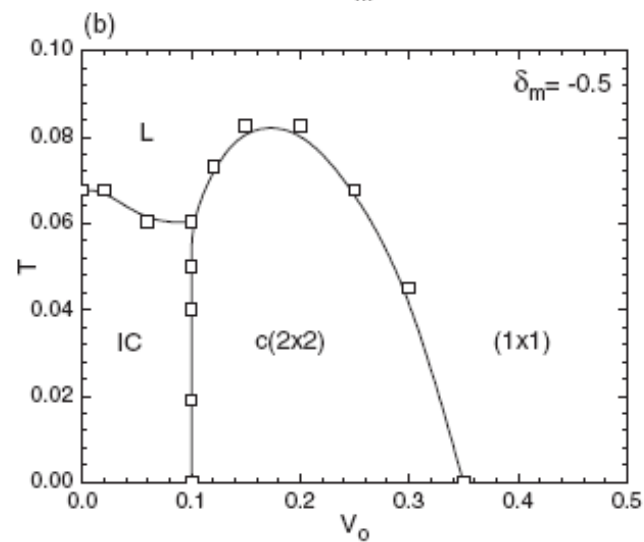
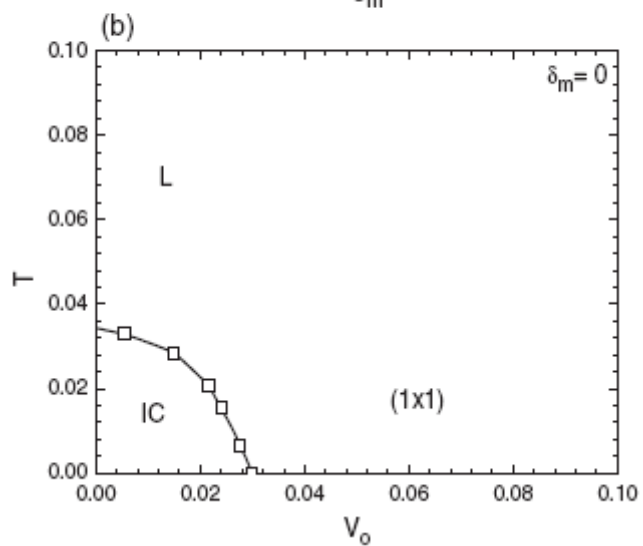
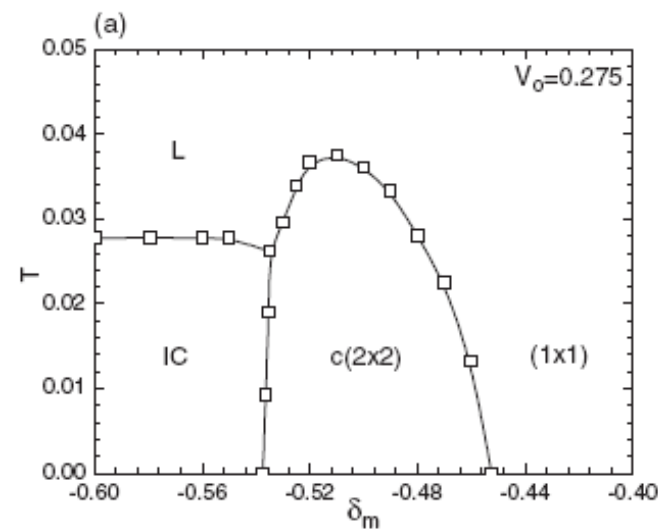
- Monte Carlo method: Metropolis , Parallel tempering

# Temperature phase diagram

near (1 x 1) phase



near c(2 x 2) phase

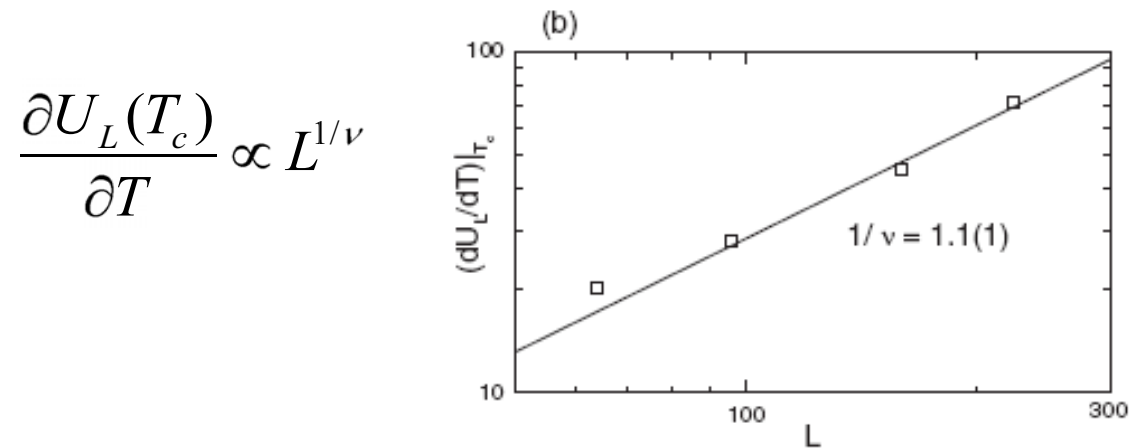
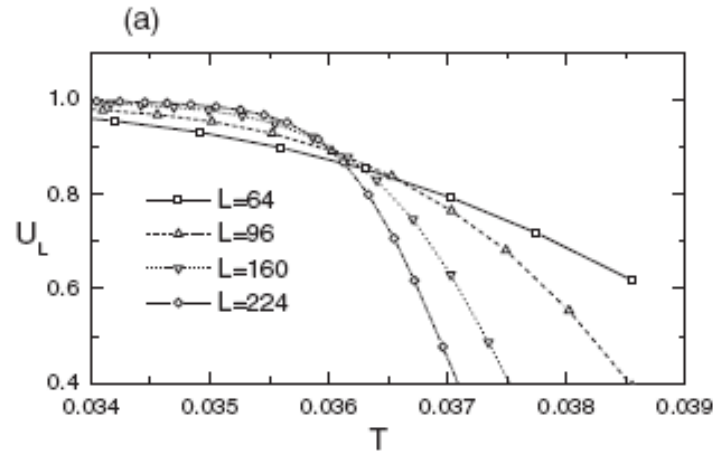




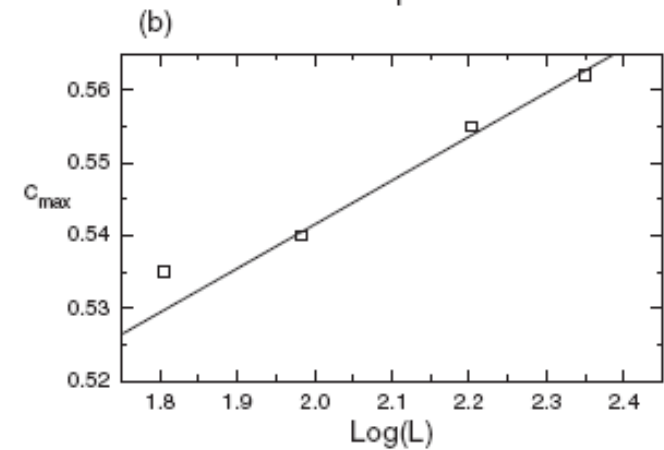
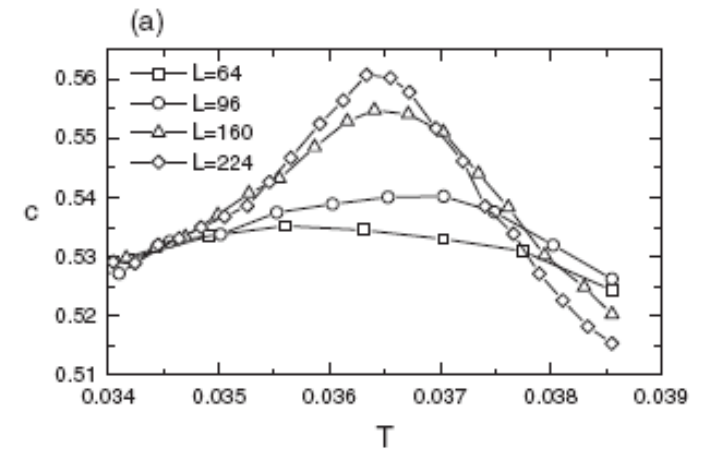
# Scaling analysis c(2x2) melting

Binder ratio

$$U_L = 2 - \frac{\langle |\rho_{k_0}|^4 \rangle}{(\langle |\rho_{k_0}|^2 \rangle)^2}$$



Specific heat  $C_{\max} \propto L^{\alpha/\nu}$



- results consistent with Ising universality class

# Velocity response at zero temperature force-induced depinning

Achim, Ramos, Karttunen, Elder, Granato, Ala-Nissila, Ying, PRE79, 011606 (2009);

$$H_{PFC} = \int d\vec{x} \left[ \frac{1}{2} \psi (r + (\nabla^2 + 1)^2) \psi + \frac{1}{4} \psi^4 + V \psi \right]$$

- add a convective gradient term proportional to external force in the dynamical equations:

$$\frac{\partial \psi(\vec{x})}{\partial t} = \nabla^2 \frac{\partial H_{PFC}}{\partial \psi(\vec{x})} + \vec{f} \cdot \vec{\nabla} \psi(\vec{x})$$

- velocity measurements:

- from the phase field

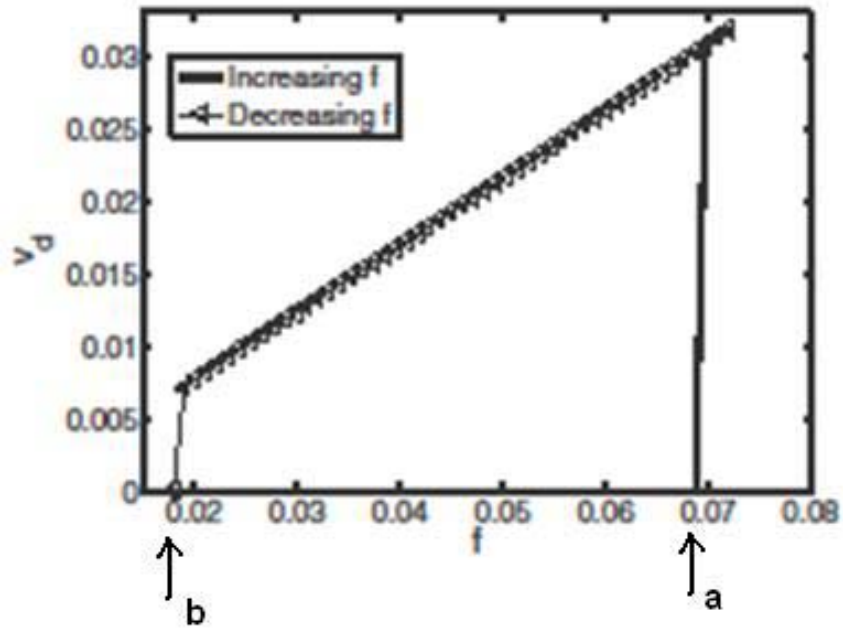
$$V = \langle \left\langle \left| \frac{\partial \psi}{\partial t} \right| \right\rangle_x / \left\langle \left| \frac{\partial \psi}{\partial x} \right| \right\rangle_x \right\rangle_t$$

- \* from the local maxima positions

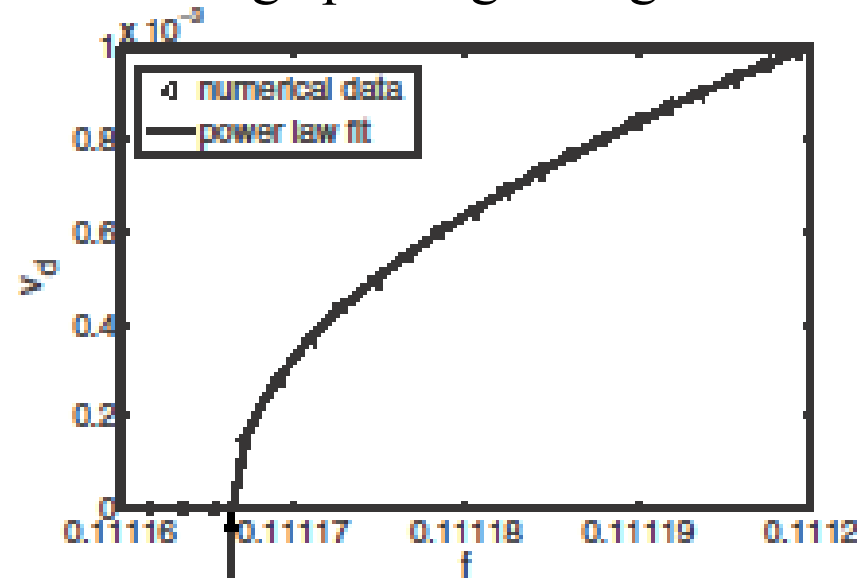
$$\vec{v}_d = \left\langle \frac{1}{N_p} \sum_{i=1}^{N_p} \vec{v}_i(t) \right\rangle$$

# Velocity response at T=0

Small pinning strength

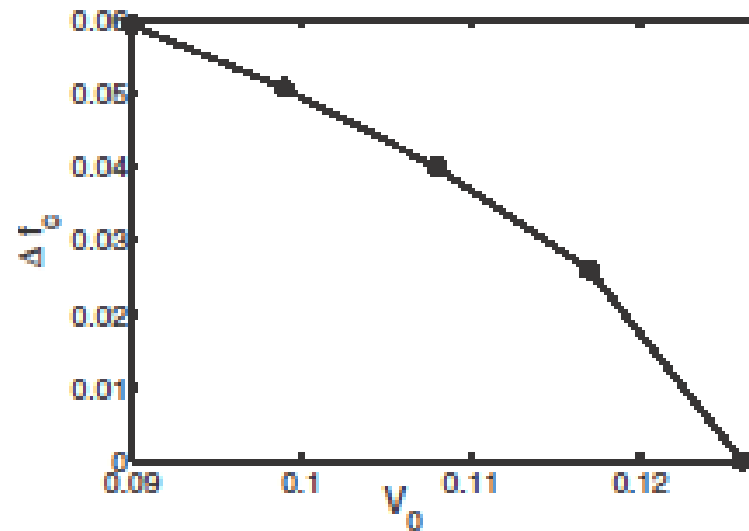


Large pinning strength



Different critical forces

$$\Delta f_c = f_a - f_b$$

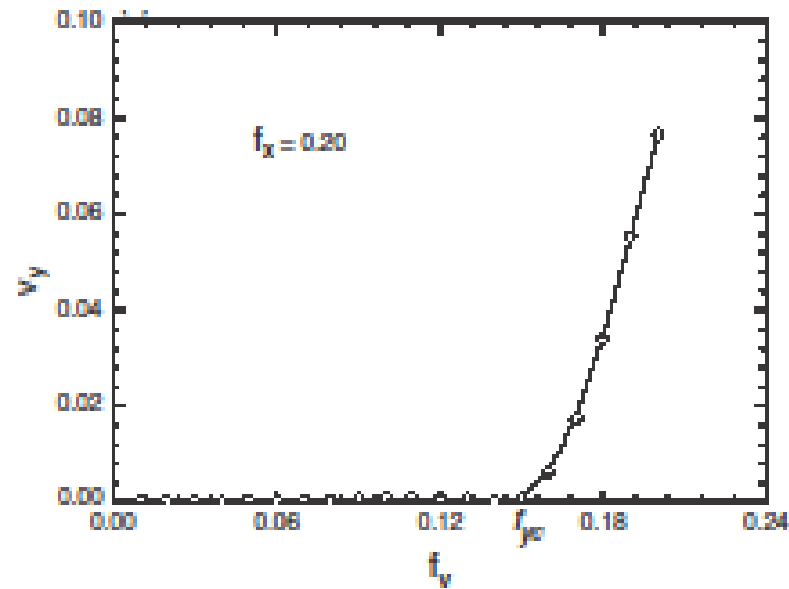


$$v_d \propto (f - f_c)^\xi$$

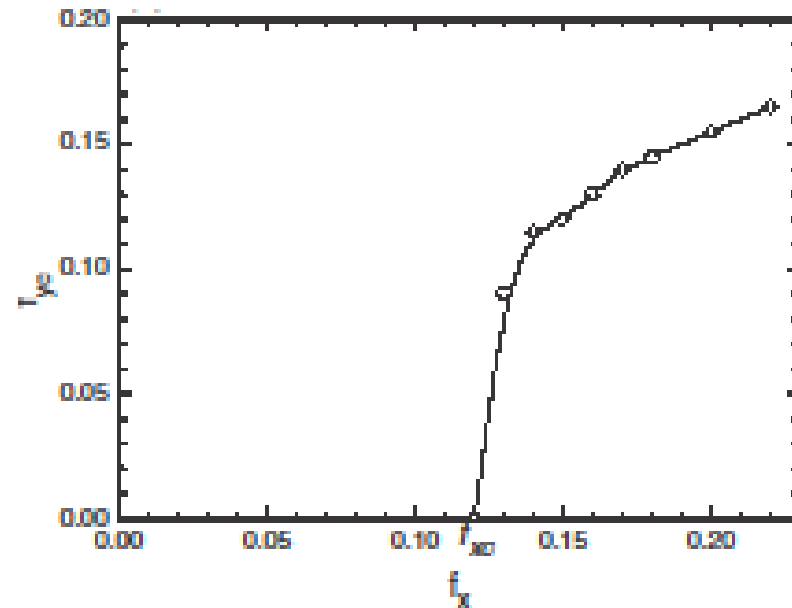
$$\xi = 0.50(3)$$

## Transverse response at large velocity

transverse velocity response



critical transverse force



velocity from peak positions

$$\vec{v}_d = \left\langle \frac{1}{N_p} \sum_{i=1}^{N_p} \vec{v}_i(t) \right\rangle$$

# Conclusions

- PFC model provides a density field description of adsorbed layers: commensurability effects, thermal depinning, force depinning and inertial effects.
- velocity response to an external force at low  $T$ , shows hysteresis with dynamical melting and freezing transitions at distinct critical values.
- main features of the nonlinear response are similar to the results obtained for atomistic models.
- dynamical melting and freezing mechanisms appear to be different
- It should be possible to describe realistic adsorbed layer systems by adjusting the parameters of the model.