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A model for image formation

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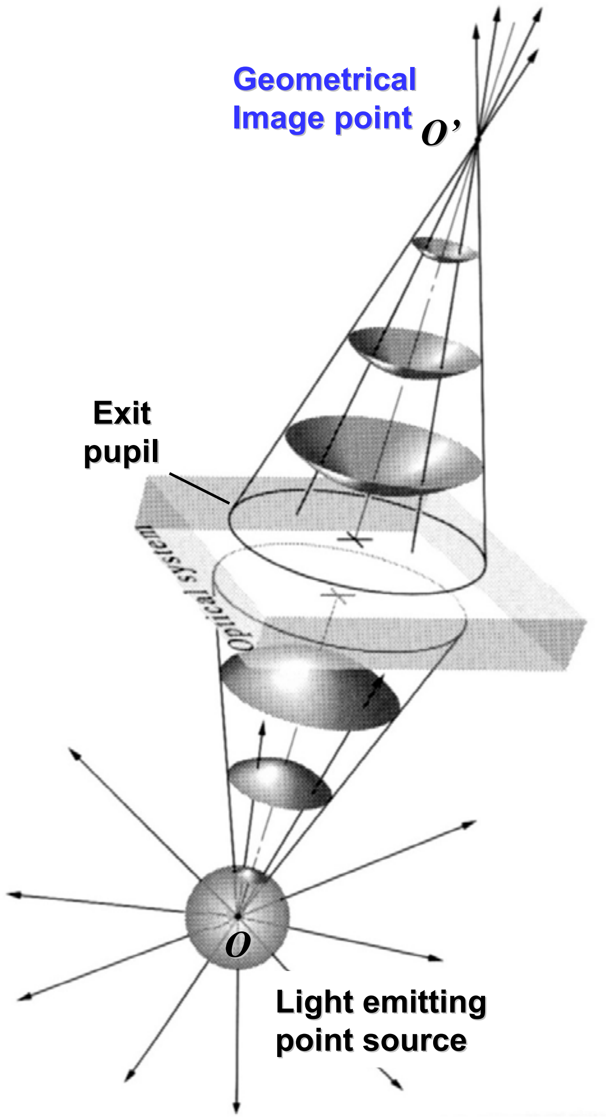
A model for image formation

Image formation

Forming an image of an object requires the establishment of bi-univocal relationship between two R_3 domains named *object space* and *image space*. In optical systems this process is accomplished by reversal of wavefront curvature.

However, the optical system intercepts (a portion of) the spherical wave emitted by the point source *S*, and forms a converging spherical wavefront, which is diffracted at the (circular) exit pupil.

Consequently light is not focussed precisely at the geometrical image *O* but rather into small volume around *O*. Knowledge of the three-dimensional light distribution near focus, i.e. the point spread function (PSF) is of particular importance for instrumental optics.



Modeling the effect of an optical system: 3D input-output relationship

$i(x, y, z)$: **Input light distribution**

$s(x, y, z)$: **Impulse response (3D PSF): describes a process causing the output distribution to be different from the original scene.**

$o(x, y, z)$: **Output light distribution**

Model equation:

$$\boxed{o(x,y,z) = i(x,y,z) \otimes s(x,y,z)}$$

The 3D convolution operation is defined as:

$$o(x,y,z) = \iiint_{u,v,w} i(u,v,w)\, s(x-u, y-v, z-w)\, du \quad dv \quad dw$$

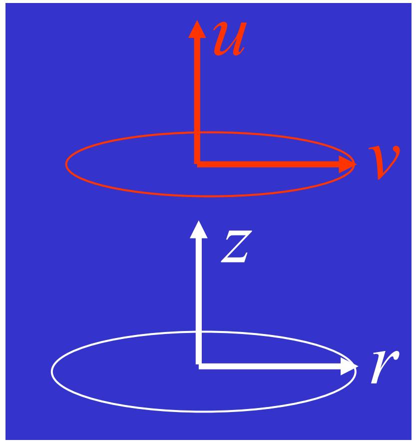
We need to determine the impulse response (3D PSF), i.e. output light distribution in image space for a point source input in object space.

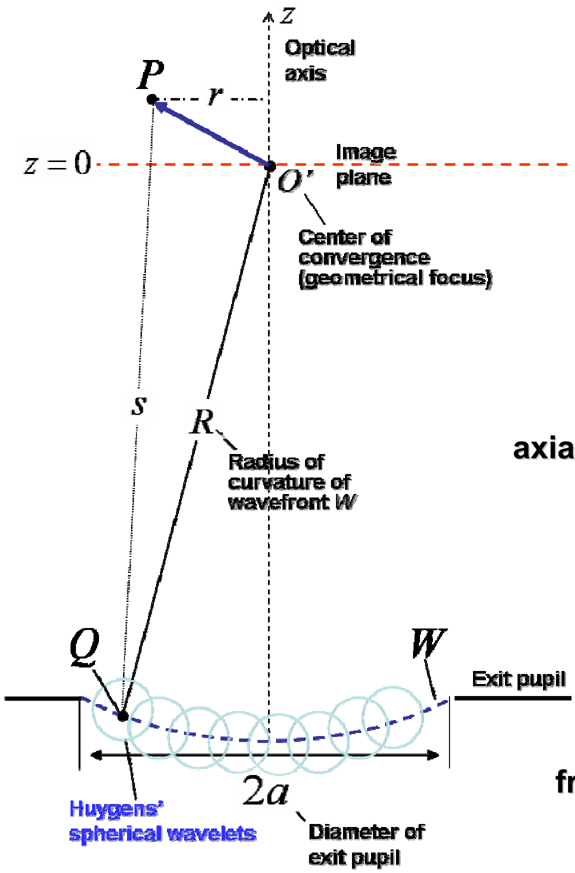
The 3D light intensity distribution near focus (PSF)

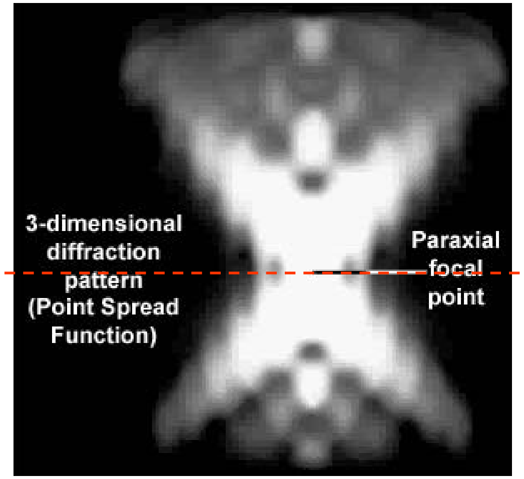
Irradiance at geometrical focus

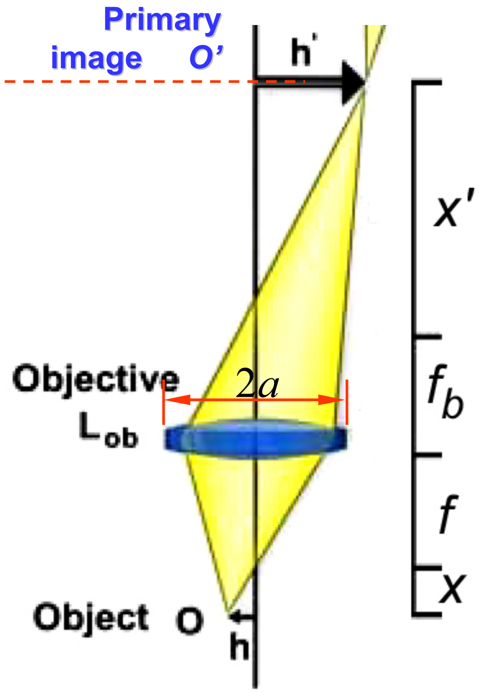
Bessel function (of the first kind) of order zero

$$s(u,v) = I(v,u)/I_o = \left| 2\int_0^1 J_o(v\rho)\ \exp\left(-j\tfrac{1}{2}u\rho^2\right)\rho\, d\rho \right|^2$$









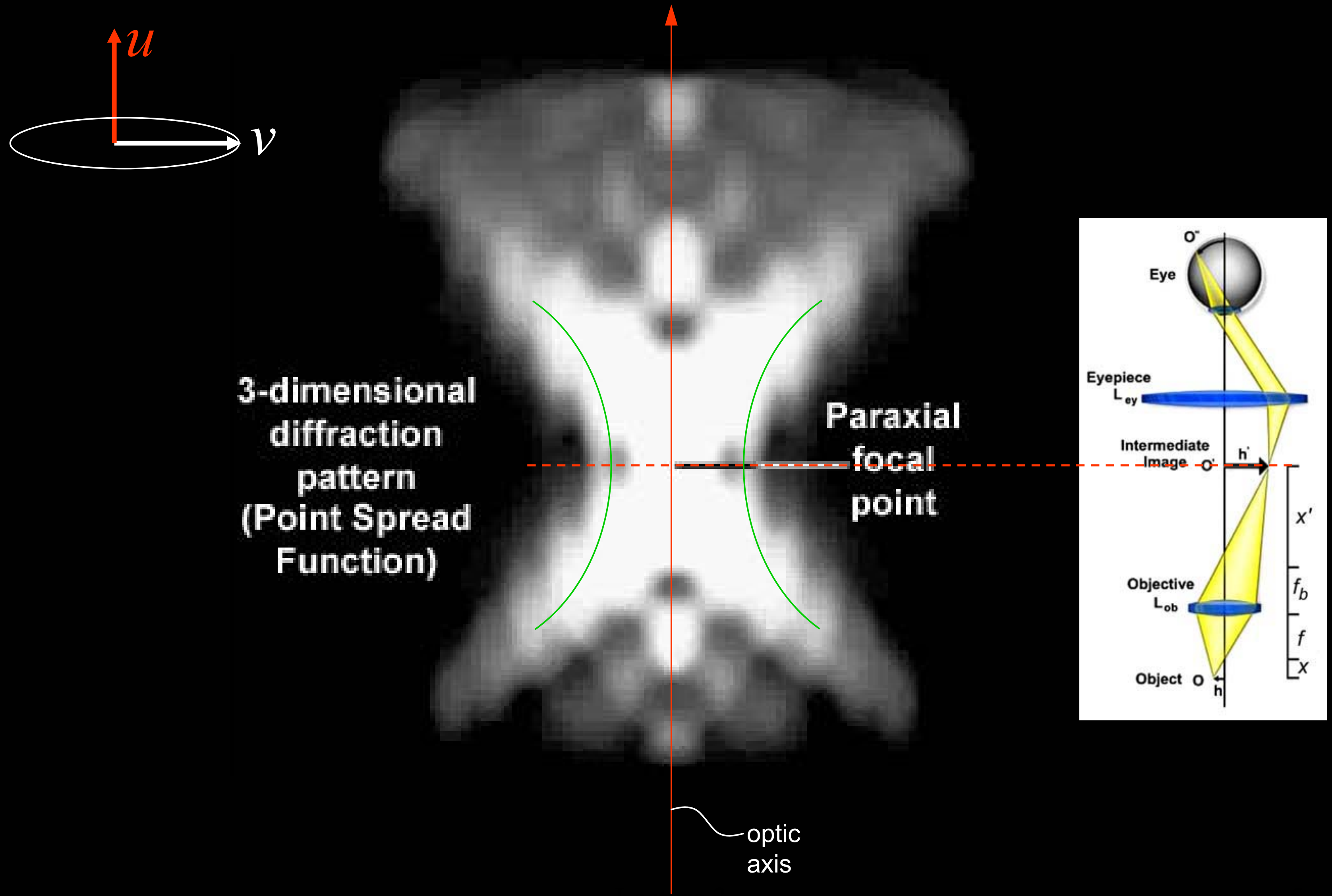
axial coordinate (**defocus**)
$$u = \frac{2\pi}{\lambda}\left(\frac{a}{R}\right)^2 z$$

radial coordinate
$$v = \frac{2\pi}{\lambda}\frac{a}{R} r$$

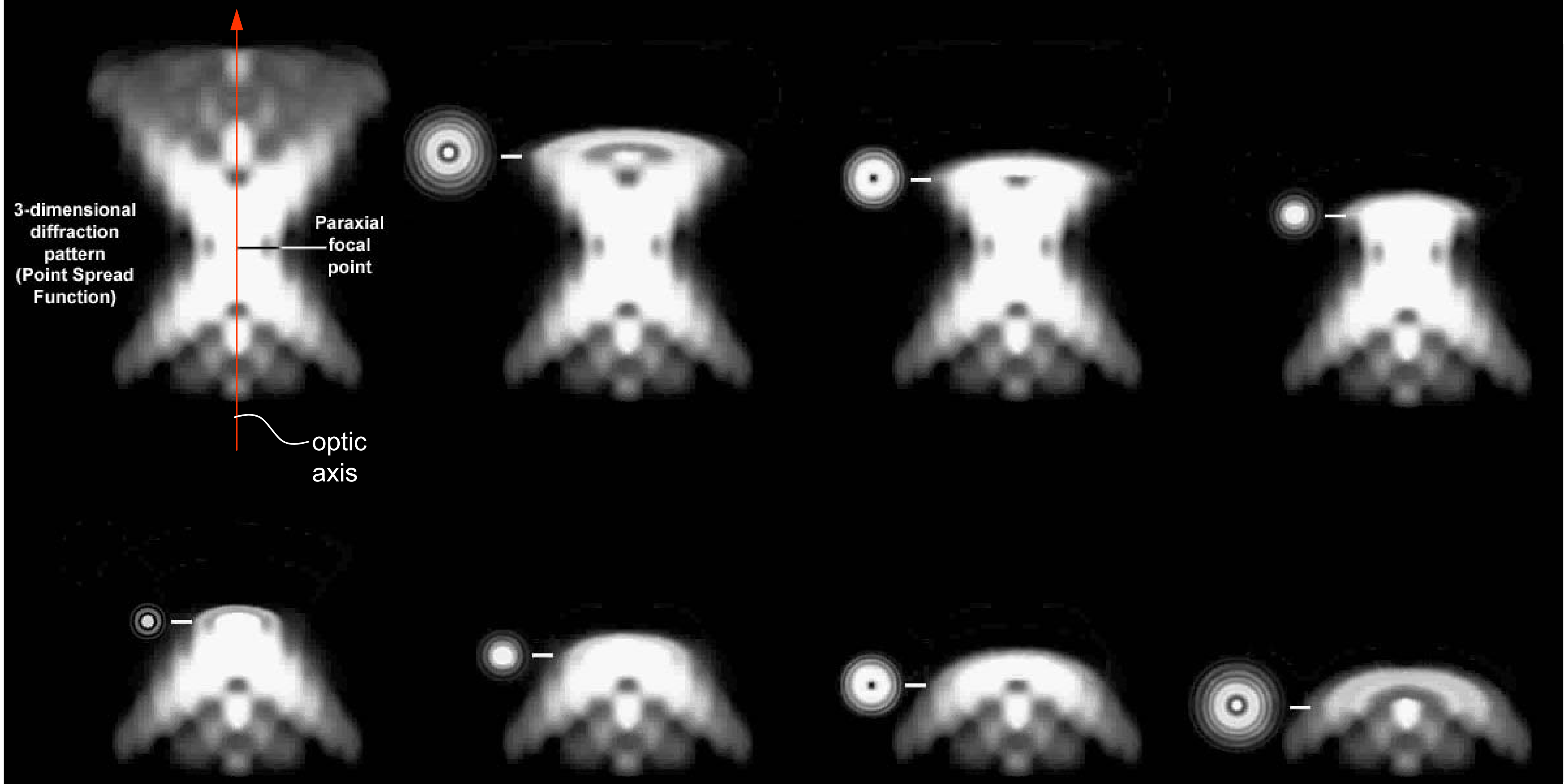
distance of P from optical axis
$$r = \sqrt{x^2 + y^2}$$

Linfoot EH, Wolf E (1956) *Proc Phys Soc B* 69:823-832

The 3D PSF is symmetrical across the geometrical image plane and has a tubular structure in the bright central portion



Sections of the 3D PSF in planes normal to the optic axis are concentric rings of alternating bright and dark fringes

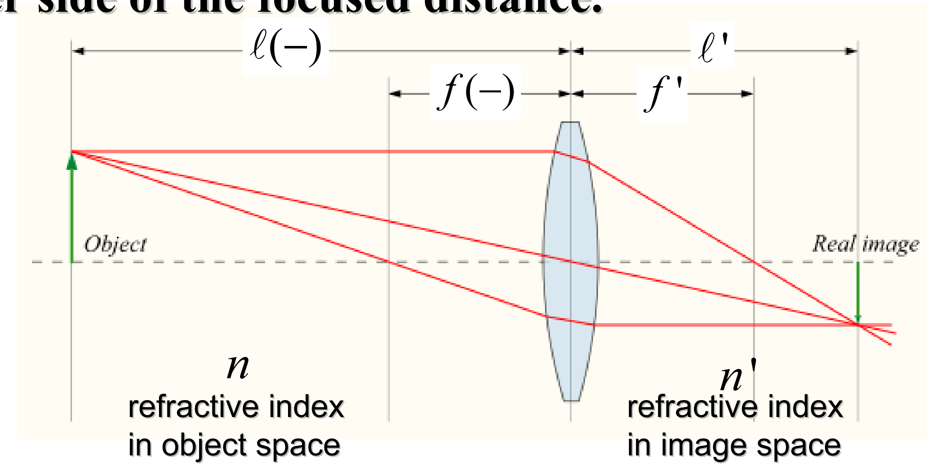


Depth of field

Although a lens can precisely focus at only one distance, the decrease in sharpness is gradual on either side of the focused distance.

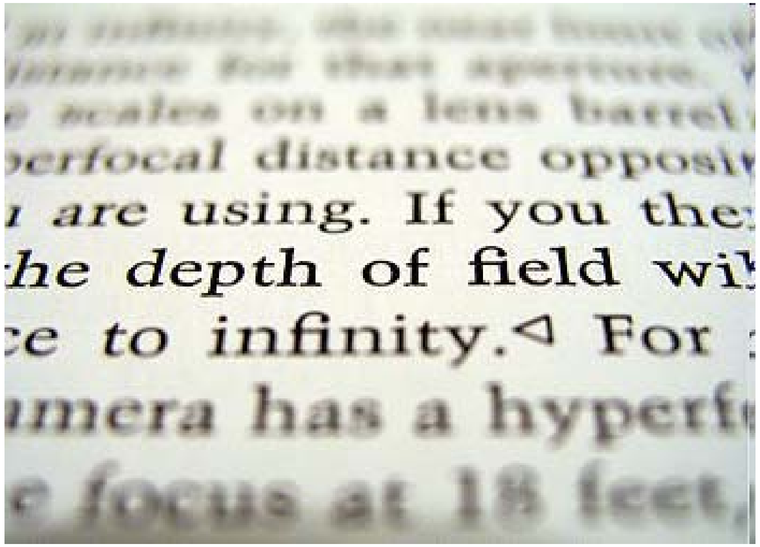
$$-\frac{n}{\ell}+\frac{n'}{\ell'}=\frac{1}{f'}$$

The lens law



$$\frac{f}{f'}=-\frac{n}{n'}$$

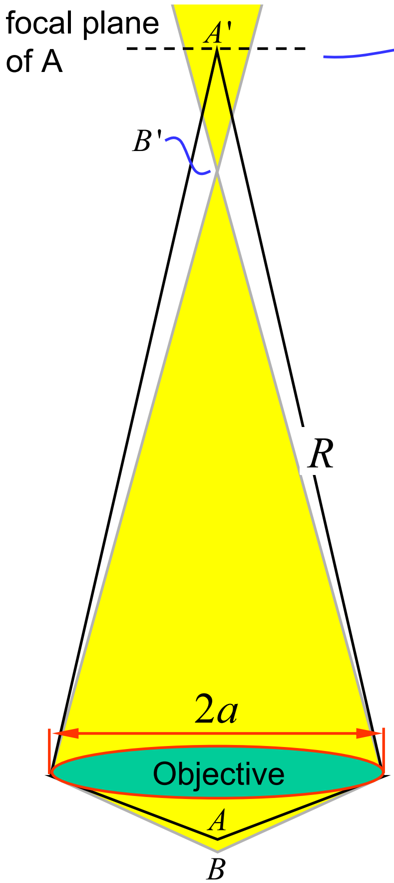
Focal length ratio



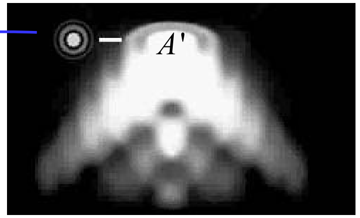
The *depth of field* (DOF) is the portion of a scene that appears sharp in the image.

However, when observing transparent samples that are not infinitely thin, fluorescent or light-scattering objects that are out of focus produce unwanted light that is collected by the objective and reduces the contrast of the signal from the region of focus, i.e. cause *axial blurring*. Consequently the ability to discriminate objects in different focal planes is seriously compromised.

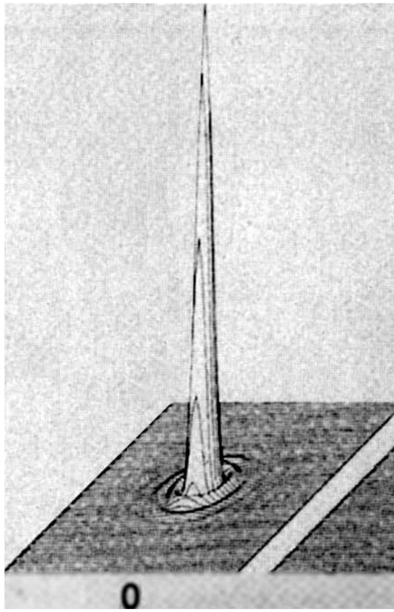
Axial blurring

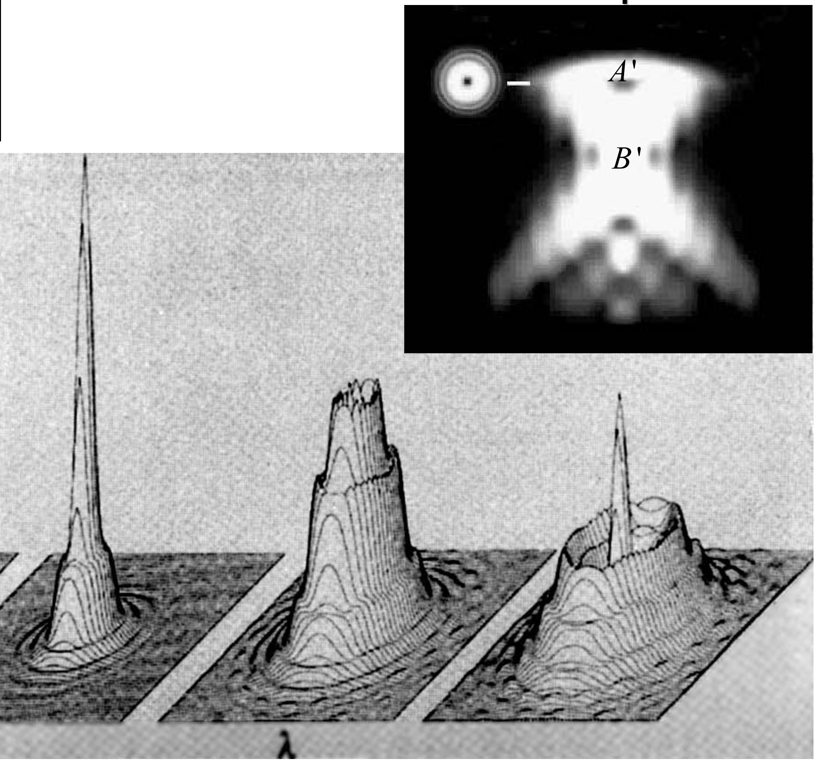


3D PSF of A
in the focal plane of A



3D PSF of B
in the focal plane of A

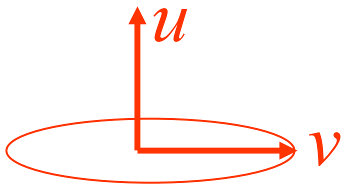


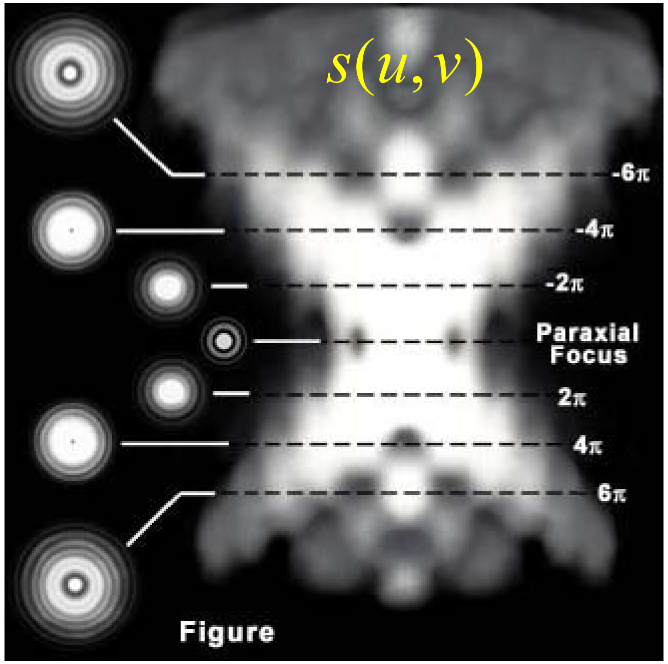


Effect of defocus on the intensity PSF

A more formal discussion of axial blurring: Fourier optics in 3D

Optical coordinates →





Normalized pupil function →

$$P(\rho) = \begin{cases} 1 & \rho \le 1 \\ 0 & \rho > 1 \end{cases}$$

Effect of defocus

Normalized point spread functions

$$\begin{cases} \tilde{h}(v,u) \equiv 2 \int_0^{+\infty} P(\rho)\ J_o(v\rho) \exp\left(-j\tfrac{1}{2}u\rho^2\right) \rho\, d\rho & \textbf{(cPSF)} \\ \tilde{s}(u,v) \equiv \left|\tilde{h}(u,v)\right|^2 & \textbf{(iPSF)} \end{cases}$$

Coherent

Incoherent

Note that → $\tilde{s}(v,u) = \tilde{s}(v,-u)$

The representation of the image formation process in the frequency domain under incoherent illumination is given by

$$F\left\{o\left(x,y,z\right)\right\} = F\left\{i\left(x,y,z\right)\right\}\cdot \mathrm{OTF}$$

where

$$\mathrm{OTF} = F\left\{s\left(x,y,z\right)\right\}$$

is the 3D optical transfer function of the system.

In terms of the spatial frequencies, the normalized OTF for circular pupil system can be represented as

Effect of defocus

$$\mathrm{OTF}(w,u) = F_v\left\{\tilde{s}(v,u)\right\} = F_v\left\{\left|2\int_0^{+\infty} P(\rho)\; J_o(v\rho)\exp\left(-\tfrac{1}{2}\,ju\rho^2\right)\rho d\rho\right|^2\right\}$$

where the radial frequency w given by

$$w=\sqrt{f_X^2+f_Y^2}$$

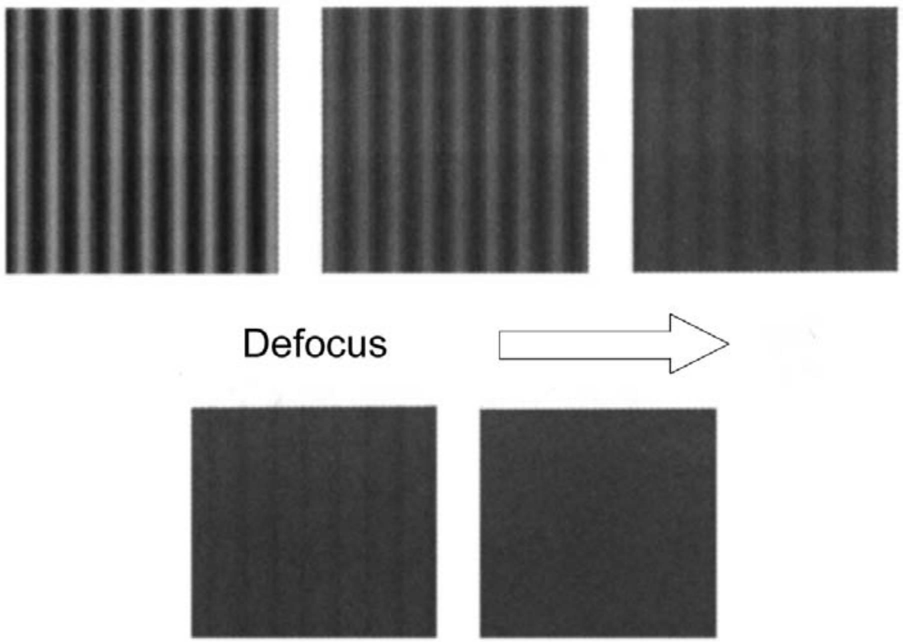
is conjugated to the radial optical coordinate

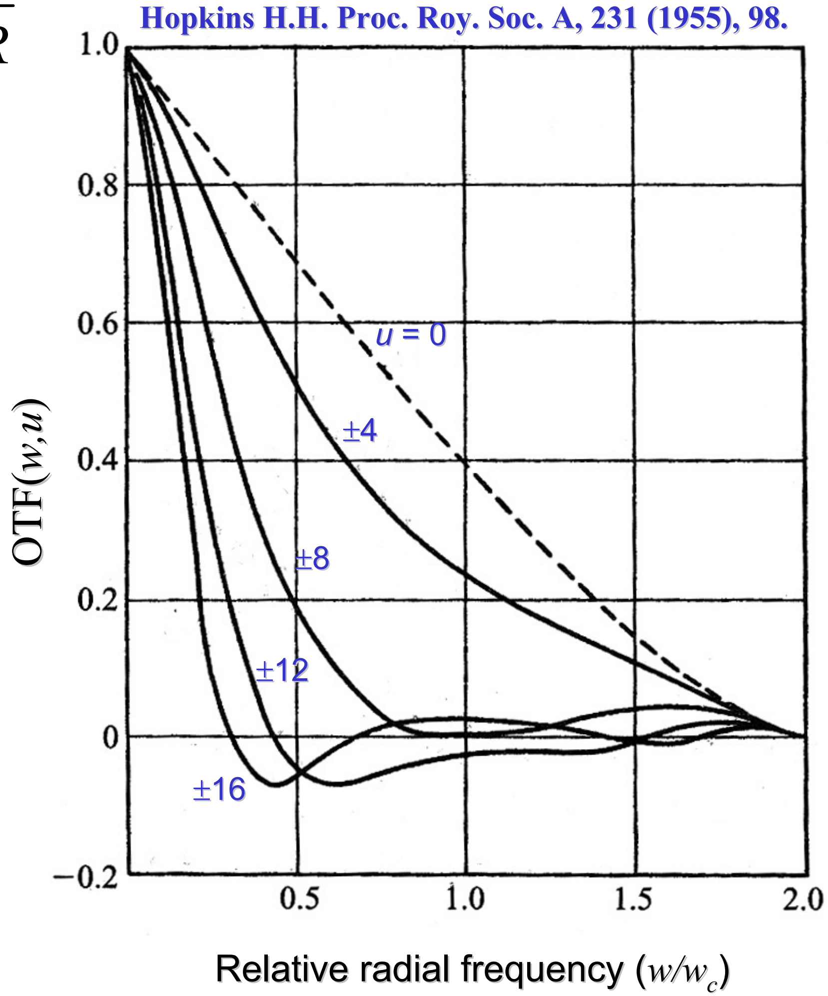
$$v = \frac{2\pi}{\lambda}\frac{a}{R}\sqrt{x^2+y^2}$$

λ is the wavelength,

and the cutoff frequency w_C is given by

$$w_C = \frac{a}{\lambda R}$$





Axial blurring explained:

low frequency components filter through even at large defocus!