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40 ways to simulate liquid argon

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40 Ways to Simulate Liquid Argon

A case study in optimization and parallelization

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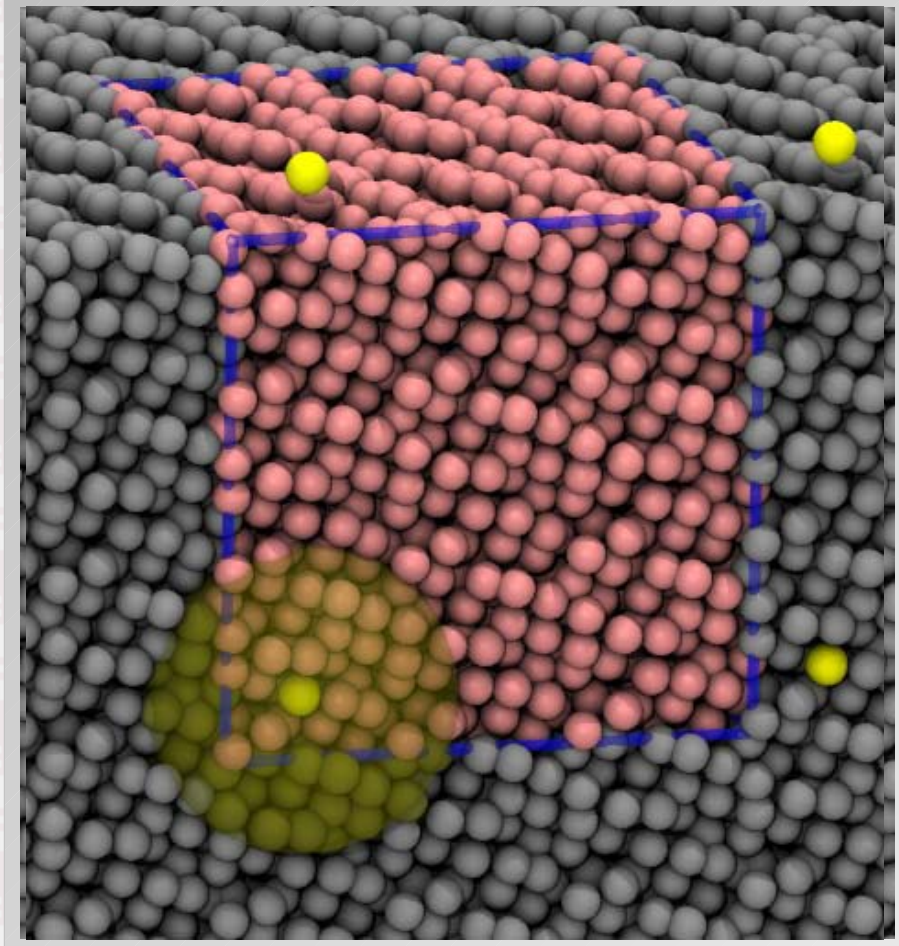
Today's Show

0) Overture: The physics of the model
1) First Act: Writing and optimizing a serial code
2) Intermezzo: Improve scaling with system size
3) Second Act: MPI parallelization
4) Third Act: OpenMP parallelization
5) Finale: GPU acceleration
6) Encore: Hybrid MPI/OpenMP parallelization
7) Last dance: Lessons learned

0) The Model for Liquid Argon



- Cubic box of particles with a Lennard-Jones type pairwise additive interaction potential

$$V = \sum_{i,j} \begin{cases} 4\epsilon\left[\left(\frac{\sigma}{r_{ij}}\right)^{12} - \left(\frac{\sigma}{r_{ij}}\right)^{6}\right], & r_{ij} < r_c \\ 0, & r_{ij} \geq r_c \end{cases}$$

- Periodic boundary conditions to avoid surface effects

Newton's Laws of Motion

- We consider our particles to be classical objects so Newton's laws of motion apply:
- 1. In absence of a force a body rests or moves in a straight line with constant velocity
- 2. A body experiencing a force **F** experiences an acceleration **a** related to **F** by **F** = m**a**, where m is the mass of the body.
- 3. Whenever a first body exerts a force **F** on a second body, the second body exerts a force **−F** on the first body

Velocity Verlet Algorithm

- The velocity Verlet algorithm is used to propagate the positions of the atoms

$$\vec{x}_i(t+\Delta t) = \vec{x}_i(t)+\vec{v}_i(t)\Delta t+\frac{1}{2}\vec{a}_i(t)(\Delta t)^2$$

$$\vec{v}_i(t+\frac{\Delta t}{2}) = \vec{v}_i(t)+\frac{1}{2}\vec{a}_i(t)\Delta t$$

$$\vec{a}_i(t+\Delta t) = -\frac{1}{m}\nabla V(\vec{x}_i(t+\Delta t))$$

$$\vec{v}_i(t+\Delta t) = \vec{v}_i(t+\frac{\Delta t}{2})+\frac{1}{2}\vec{a}_i(t)\Delta^2$$

L. Verlet, Phys. Rev. 159, 98 (1967); Phys. Rev. 165, 201 (1967).

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$$\vec{a}_i(t+\Delta t) = -\frac{1}{m}\nabla V(\vec{x}_i(t+\Delta t))$$

$$\vec{v}_i(t+\Delta t) = \vec{v}_i(t+\frac{\Delta t}{2})+\frac{1}{2}\vec{a}_i(t)\Delta^2$$

$$\begin{cases} 4\epsilon\left[-12\left(\frac{\sigma}{r_{ij}}\right)^{13}+6\left(\frac{\sigma}{r_{ij}}\right)^{7}\right], & r_{ij}<r_c \\ 0, & r_{ij}\geq r_c \end{cases}$$

L. Verlet, Phys. Rev. 159, 98 (1967); Phys. Rev. 165, 201 (1967).

What Do We Need to Program?

1. Read in parameters and initial status and compute what is missing (e.g. accelerations)
2. Integrate Equations of motion with Velocity Verlet for a given number of steps
   a) Propagate all velocities for half a step
   b) Propagate all positions for a full step
   c) Compute forces on all atoms to get accelerations
   d) Propagate all velocities for half a step
   e) Output intermediate results, if needed

1) Initial Serial Code: Velocity Verlet

```
void velverlet(mdsys_t *sys) {
    for (int i=0; i<sys->natoms; ++i) {
        sys->vx[i] += 0.5*sys->dt / mvsq2e * sys->fx[i] / sys->mass;
        sys->vy[i] += 0.5*sys->dt / mvsq2e * sys->fy[i] / sys->mass;
        sys->vz[i] += 0.5*sys->dt / mvsq2e * sys->fz[i] / sys->mass;
        sys->rx[i] += sys->dt*sys->vx[i];
        sys->ry[i] += sys->dt*sys->vy[i];
        sys->rz[i] += sys->dt*sys->vz[i];
    }

    force(sys);

    for (int i=0; i<sys->natoms; ++i) {
        sys->vx[i] += 0.5*sys->dt / mvsq2e * sys->fx[i] / sys->mass;
        sys->vy[i] += 0.5*sys->dt / mvsq2e * sys->fy[i] / sys->mass;
        sys->vz[i] += 0.5*sys->dt / mvsq2e * sys->fz[i] / sys->mass;
    }
}
```

Initial Code: Force Calculation

```
for(i=0; i < (sys->natoms); ++i) {
   for(j=0; j < (sys->natoms); ++j) {
      if (i==j) continue;

      rx=pbc(sys->rx[i] - sys->rx[j], 0.5*sys->box);
      ry=pbc(sys->ry[i] - sys->ry[j], 0.5*sys->box);
      rz=pbc(sys->rz[i] - sys->rz[j], 0.5*sys->box);
      r = sqrt(rx*rx + ry*ry + rz*rz);
```

Compute distance between atoms i & j

```
      if (r < sys->rcut) {
        ffac = -4.0*sys->epsilon*(-12.0*pow(sys->sigma/r,12.0)/r
                          +6*pow(sys->sigma/r,6.0)/r);
        sys->epot += 0.5*4.0*sys->epsilon*(pow(sys->sigma/r,12.0)
                                -pow(sys->sigma/r,6.0));
        sys->fx[i] += rx/r*ffac;
        sys->fy[i] += ry/r*ffac;
        sys->fz[i] += rz/r*ffac;
      }}
```

Compute energy and force

Add force contribution of atom j on atom i

How Well Does it Work?

- Compiled with:
  `gcc -o ljmd.x ljmd.c -lm`
  Test input: 108 atoms, 10000 steps: 49s
  Let us get a profile:

% time	cumulative seconds	self seconds	calls	self ms/call	total ms/call	name
73.70	13.87	13.87	10001	1.39	1.86	force
24.97	18.57	4.70	346714668	0.00	0.00	pbc
0.96	18.75	0.18				main
0.37	18.82	0.07	10001	0.01	0.01	ekin
0.00	18.82	0.00	30006	0.00	0.00	azzero
0.00	18.82	0.00	101	0.00	0.00	output
0.00	18.82	0.00	12	0.00	0.00	getline

Step One: Compiler Optimization

- Use of pbc() is convenient, but costs 25%
  => compiling with -O3 should inline it
- Loops should be unrolled for superscalar CPUs
  => compiling with -O2 or -O3 should do it for us

  Time now: 39s (1.3x faster) Only a bit faster
- Now try some more optimization options:
  -ffast-math -fexpensive-optimizations -msse3

  Time now: 10s (4.9x faster) Much better!
- Compare to LAMMPS: 3.6s => need to do more

Now Modify the Code

- Use physics! Newton's 3rd : $F_{ij} = -F_{ji}$

```
for(i=0; i < (sys->natoms)-1; ++i) {
  for(j=i+1; j < (sys->natoms); ++j) {
    rx=pbc(sys->rx[i] - sys->rx[j], 0.5*sys->box);
    ry=pbc(sys->ry[i] - sys->ry[j], 0.5*sys->box);
    rz=pbc(sys->rz[i] - sys->rz[j], 0.5*sys->box);
    r = sqrt(rx*rx + ry*ry + rz*rz);
    if (r < sys->rcut) {
      ffac = -4.0*sys->epsilon*(-12.0*pow(sys->sigma/r,12.0)/r
                                +6*pow(sys->sigma/r,6.0)/r);
      sys->epot += 0.5*4.0*sys->epsilon*(pow(sys->sigma/r,12.0)
                                -pow(sys->sigma/r,6.0));
      sys->fx[i] += rx/r*ffac;       sys->fx[j] -= rx/r*ffac;
      sys->fy[i] += ry/r*ffac;       sys->fy[j] -= ry/r*ffac;
      sys->fz[i] += rz/r*ffac;       sys->fz[j] -= rz/r*ffac;
}}}
```

Time now: 5.4s (9.0x faster) Another big improvement

More Modifications

- Avoid expensive math: pow(), sqrt(), division

```
c12=4.0*sys->epsilon*pow(sys->sigma,12.0);
c6 =4.0*sys->epsilon*pow(sys->sigma, 6.0);
rcsq = sys->rcut * sys->rcut;
for(i=0; i < (sys->natoms)-1; ++i) {
  for(j=i+1; j < (sys->natoms); ++j) {
    rx=pbc(sys->rx[i] - sys->rx[j], 0.5*sys->box);
    ry=pbc(sys->ry[i] - sys->ry[j], 0.5*sys->box);
    rz=pbc(sys->rz[i] - sys->rz[j], 0.5*sys->box);
    rsq = rx*rx + ry*ry + rz*rz;
    if (rsq < rcsq) {
      double r6,rinv; rinv=1.0/rsq;  r6=rinv*rinv*rinv;
      ffac = (12.0*c12*r6 - 6.0*c6)*r6*rinv;
      sys->epot += r6*(c12*r6 - c6);
      sys->fx[i] += rx*ffac;  sys->fx[j] -= rx*ffac;
      sys->fy[i] += ry*ffac;  sys->fy[j] -= ry*ffac;
      sys->fz[i] += rz*ffac;  sys->fz[j] -= rz*ffac;
}}}
```

=> 108 atoms: 4.0s (12.2x faster) still worth it

Improvements So Far

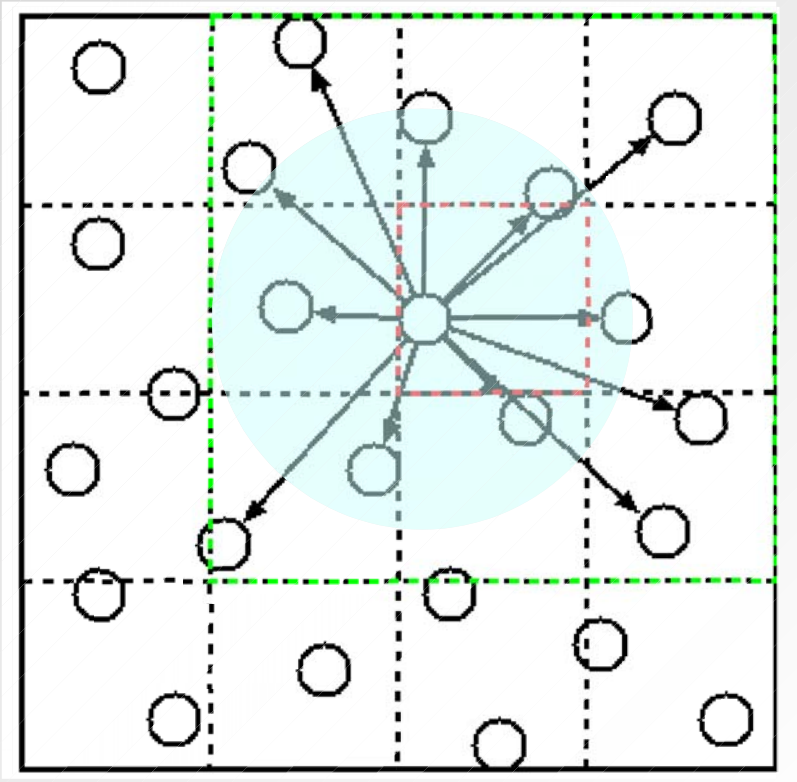
- Use the optimal compiler flags => ~5x faster
  but some of it: inlining, unrolling could be coded
- Use our knowledge of physics => ~2x faster
  since we need to compute only half the data.
- Use our knowledge of computer hardware
  => 1.35x faster. (there could be more: SSE)

  We are within 10% (4s vs. 3.6s) of LAMMPS.
- Try a bigger system: 2916 atoms, 100 steps
  Our code: 13.3s LAMMPS: 2.7s => Bad scaling with system size

2) Making it Scale with System Size

- Lets look at the algorithm again:
  We compute all distances between pairs
- But for larger systems not all pairs contribute and our effort is $O(N^2)$
- So we need a way to avoid looking at pairs that are too far away

  => Sort atoms into cell lists, which is O(N)



The Cell-List Variant

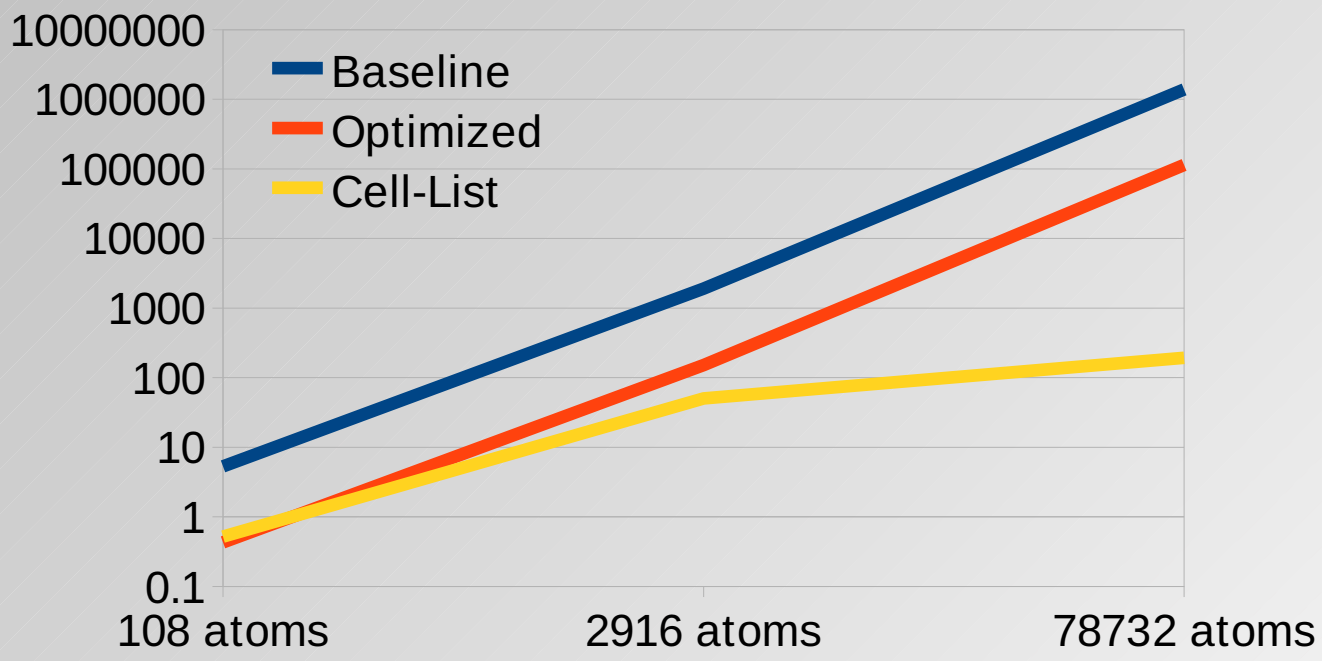
- At startup build a list of lists to store atom indices for atoms that “belong” to a cell
- Compute a list of pairs between cells which contain atoms within cutoff. Doesn't change!
- During MD sort atoms into cells
- Then loop over list of “close” pairs of cells *i* and *j*
- For pair of cells loop over pairs of atoms in them
- Now we have linear scaling with system size at the cost of using more memory and an O(N) sort

Cell List Loop

```
for(i=0; i < sys->npair; ++i) {
    cell_t *c1, *c2;
    c1=sys->clist + sys->plist[2*i];
    c2=sys->clist + sys->plist[2*i+1];

        for (int j=0; j < c1->natoms; ++j) {
            int ii=c1->idxlist[j];
            double rx1=sys->rx[ii];
            double ry1=sys->ry[ii];
            double rz1=sys->rz[ii];

            for(int k=0; k < c2->natoms; ++k) {
                double rx,ry,rz,rsq;
                int jj=c2->idxlist[k];
                rx=pbc(rx1 - sys->rx[jj], boxby2, sys->box);
                ry=pbc(ry1 - sys->ry[jj], boxby2, sys->box);
                ...
```

- 2916 atom time: 3.4s (4x faster), LAMMPS 2.7s

Scaling with System Size



- Cell list does not help (or hurt) much for small inputs, but is a huge win for larger problems
  => Lesson: always pay attention to scaling

3) What if optimization is not enough?

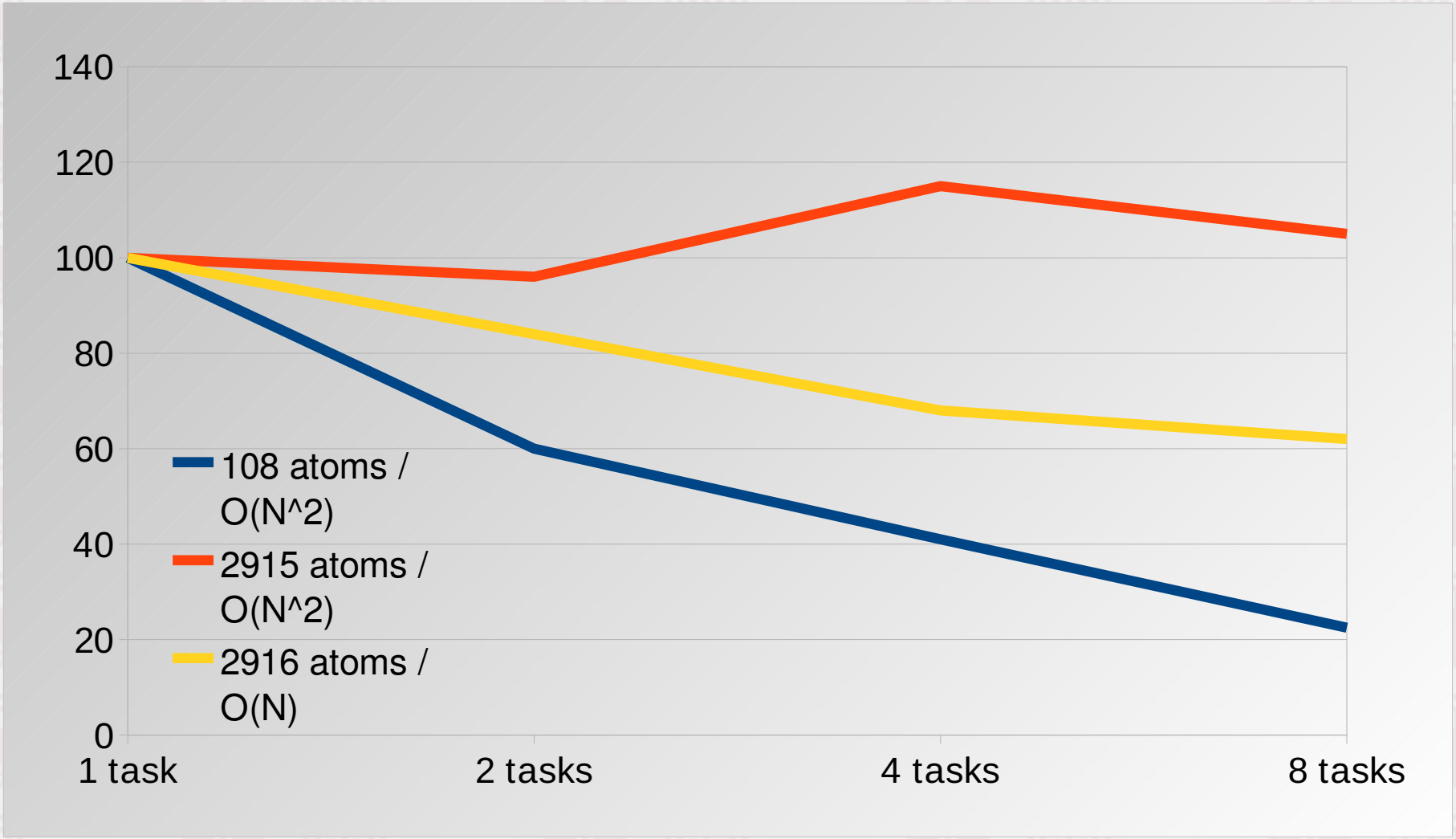
- Having linear scaling is nice, but twice the system size is still twice the work
  => Parallelization
- Simple MPI parallelization first
  - MPI is “share nothing” (replicated or distributed data)
  - Run the same code path with the same data but insert a few MPI calls
    - Broadcast positions from rank 0 to all before force()
    - Compute forces on different atoms for each rank
    - Collect (reduce) forces from all to rank 0 after force()

Replicated Data MPI Version

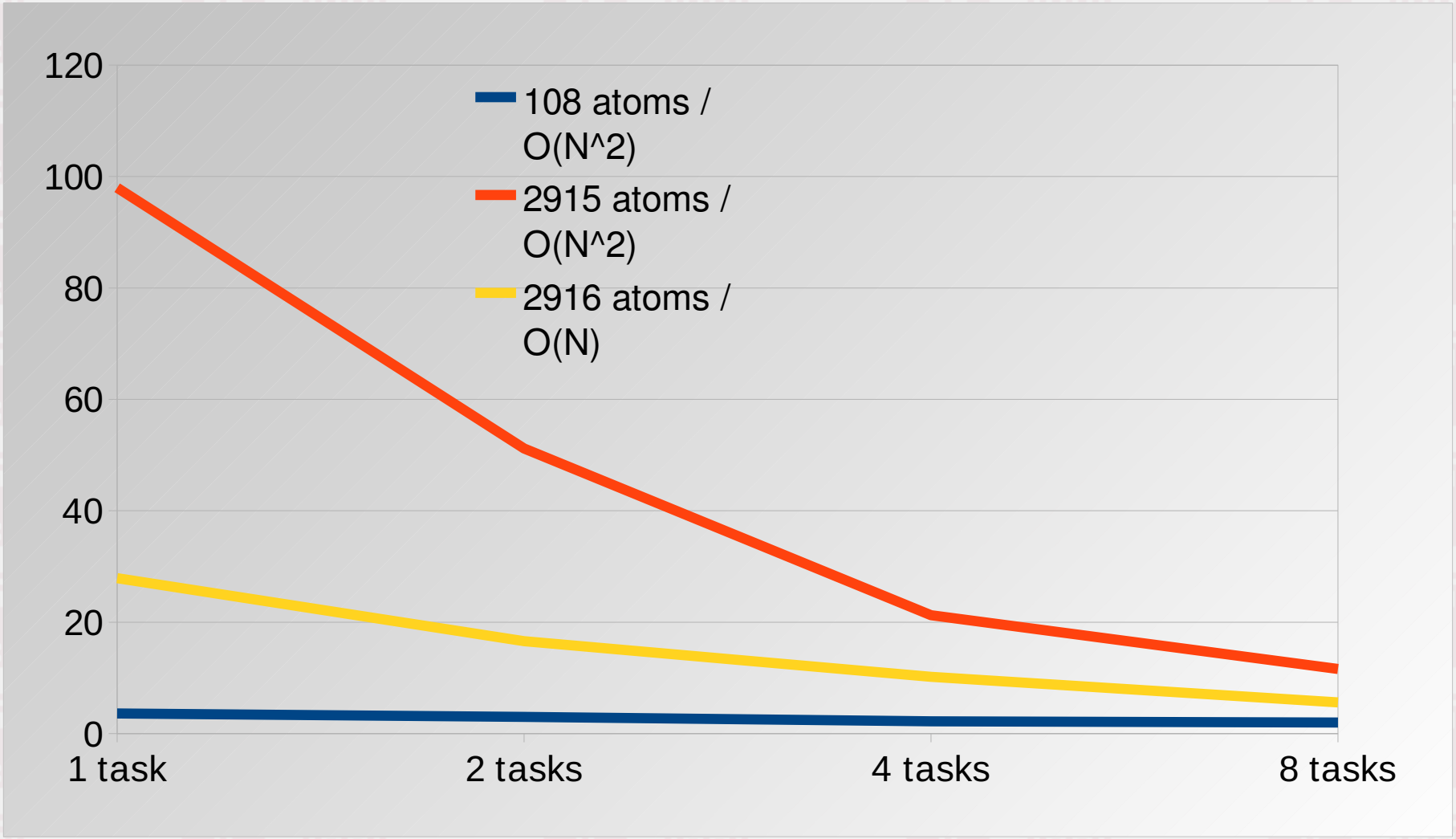
```
static void force(mdsys_t *sys) {
   double epot=0.0;
   azzero(sys->cx,sys->natoms); azzero(sys->cy,sys->natoms); azzero(sys->cz,sys->natoms);
   MPI_Bcast(sys->rx, sys->natoms, MPI_DOUBLE, 0, sys->mpicomm);
   MPI_Bcast(sys->ry, sys->natoms, MPI_DOUBLE, 0, sys->mpicomm);
   MPI_Bcast(sys->rz, sys->natoms, MPI_DOUBLE, 0, sys->mpicomm);
   for (i=0; i < sys->natoms-1; i += sys->nsize) {
      ii = i + sys->mpirank;
      if (ii >= (sys->natoms - 1)) break;
      for (j=i+1; i < sys->natoms; ++j) {
     [...]
            sys->cy[j] -= ry*ffac;
            sys->cz[j] -= rz*ffac;
   } }
   MPI_Reduce(sys->cx, sys->fx, sys->natoms, MPI_DOUBLE, MPI_SUM, 0, sys->mpicomm);
   MPI_Reduce(sys->cy, sys->fy, sys->natoms, MPI_DOUBLE, MPI_SUM, 0, sys->mpicomm);
   MPI_Reduce(sys->cz, sys->fz, sys->natoms, MPI_DOUBLE, MPI_SUM, 0, sys->mpicomm);
   MPI_Reduce(&epot, &sys->epot, 1, MPI_DOUBLE, MPI_SUM, 0, sys->mpicomm);
}
```

- Easy to implement, but lots of communication

MPI Parallel Efficiency



MPI Parallel Execution Times



4) OpenMP Parallelization

- OpenMP is directive based
  => code (can) work without them
- OpenMP can be added incrementally
- OpenMP only works in shared memory
  => multi-core processors
- OpenMP hides the calls to a threads library
  => less flexible, but less programming
- Caution: write access to shared data can easily lead to race conditions

Naive OpenMP Version

```
#if defined(_OPENMP)
#pragma omp parallel for default(shared) \
    private(i) reduction(+:epot)
#endif
    for(i=0; i < (sys->natoms)-1; ++i) {
        double rx1=sys->rx[i];
        double ry1=sys->ry[i];
        double rz1=sys->rz[i];
        [...]
                {
                    sys->fx[i] += rx*ffac;
                    sys->fy[i] += ry*ffac;
                    sys->fz[i] += rz*ffac;
                    sys->fx[j] -= rx*ffac;
                    sys->fy[j] -= ry*ffac;
                    sys->fz[j] -= rz*ffac;
                }
```

Each thread will work on different values of "i"

Timings (108 atoms):
1 thread: 4.2s
2 threads: 7.1s
4 threads: 7.7s
8 threads: 8.6s

Race condition:
"i" will be unique for each thread, but not "j"
=> multiple threads may write to the same location concurrently

Naive OpenMP Version

```
#if defined(_OPENMP)
#pragma omp parallel for default(shared) \
    private(i) reduction(+:epot)
#endif
    for(i=0; i < (sys->natoms)-1; ++i) {
        double rx1=sys->rx[i];
        double ry1=sys->ry[i];
        double rz1=sys->rz[i];
        [...]

#if defined(_OPENMP)
#pragma omp critical
#endif
                    {
                        sys->fx[i] += rx*ffac;
                        sys->fy[i] += ry*ffac;
                        sys->fz[i] += rz*ffac;
                        sys->fx[j] -= rx*ffac;
                        sys->fy[j] -= ry*ffac;
                        sys->fz[j] -= rz*ffac;
                    }
```

Each thread will work on different values of "i"

The "critical" directive will let only one thread execute this block at a time

Timings (108 atoms):

1 thread: 4.2s

2 threads: 7.1s

4 threads: 7.7s

8 threads: 8.6s

OpenMP Improvements

- Use **omp atomic** to protect one instruction
  => faster, but requires hardware support
  108: 1T: 6.3s, 2T: 5.0s, 4T: 4.4s, 8T: 4.2s
  2916: 1T: 126s, 2T: 73s, 4T: 48s, 8T: 26s
  => some speedup, but serial is faster for 108,
  at 2916 atoms we are often beyond cutoff
- Don't use Newton's 3rd Law => no race condition
  108: 1T: 6.5s, 2T: 3.7s, 4T: 2.3s, 8T: 2.1s
  2916: 1T: 213s, 2T: 106s, 4T: 53s, 8T: 21s
  => better scaling, but we lose 2x serial speed

MPI-like Approach with OpenMP

```
#if defined(_OPENMP)
#pragma omp parallel reduction(+:epot)
#endif
    {  double *fx, *fy, *fz;
#if defined(_OPENMP)
        int tid=omp_get_thread_num();
#else
        int tid=0;
#endif
        fx=sys->fx + (tid*sys->natoms); azzero(fx,sys->natoms);
        fy=sys->fy + (tid*sys->natoms); azzero(fy,sys->natoms);
        fz=sys->fz + (tid*sys->natoms); azzero(fz,sys->natoms);
        for(int i=0; i < (sys->natoms -1); i += sys->nthreads) {
            int ii = i + tid;
            if (ii >= (sys->natoms -1)) break;
            rx1=sys->rx[ii];
            ry1=sys->ry[ii];
            rz1=sys->rz[ii];
```

Thread Id is like MPI rank

sys->fx holds storage for one full fx array for each thread => race condition is eliminated.

MPI-like Approach with OpenMP (2)

- We need to write our own reduction:

```
#if defined (_OPENMP)
#pragma omp barrier
#endif
    i = 1 + (sys->natoms / sys->nthreads);
    fromidx = tid * i;
    toidx = fromidx + i;
    if (toidx > sys->natoms) toidx = sys->natoms;

    for (i=1; i < sys->nthreads; ++i) {
        int offs = i*sys->natoms;
        for (int j=fromidx; j < toidx; ++j) {
            sys->fx[j] += sys->fx[offs+j];
            sys->fy[j] += sys->fy[offs+j];
            sys->fz[j] += sys->fz[offs+j];
        }
    }
```

Need to make certain, all threads are done with computing forces

Use threads to parallelize the reductions

More OpenMP Timings

- The **omp parallel** region timings
  108: 1T: 3.5s, 2T: 2.5s, 4T: 2.2s, 8T: 2.5s
  2916: 1T: 103s, 2T: 53s, 4T: 19s, 8T: 10s
  => better speedup, but serial is faster for 108,
  at 2916 atoms we are often beyond cutoff
- This approach also works with cell lists:
  108: 1T: 4.3s, 2T: 3.1s, 4T: 2.4s, 8T: 2.9s
  2916: 1T: 28s, 2T: 15s, 4T: 8.9s, 8T: 4.1s
  => 6.8x speedup with 8 threads.
  That is **62x** faster than the first serial version

5) GPU Version with CUDA

- GPU is threading with thousands of threads

  => One thread per loop iteration

  => Same issues as OpenMP, but more extreme
- Cannot use the best MPI-like threading strategy as it would need too much memory

  => Don't apply Newton's 3rd law
- Summing up of energy is a problem
  - Globally accessible memory is slow
  - Fast memory is only shared by groups of threads

CUDA Force Kernel Launch

- Original force routine becomes a wrapper
  - Move data between host and GPU
  - Pad position and forces to be multiple of block size

```
static void force(mdsys_t *sys) {
   cudaMemcpy(sys->g_pos, sys->pos, 3*sys->nwords*sizeof(double), cudaMemcpyHostToDevice);

   int nblocks = sys->nwords/BLKSZ;
   dim3 grid, block;
   block.x = BLKSZ;
   grid.x  = nblocks;

   g_force<<<grid,block>>>(sys->g_pos, sys->g_frc, sys->g_res, sys->g_sys);

   cudaMemcpy(sys->frc, sys->g_frc, 3*sys->nwords*sizeof(double), cudaMemcpyDeviceToHost);
}
```

CUDA Force Kernel (part 1)

- Derive atom index to work on from block and thread index number
- Use one large array for x-, y-, and z-data

```
__global__ void g_force(double *pos, double *frc, double *res,
gdata_t *sys)
{
    __shared__ double mye[BLKSZ];
    const int tid = threadIdx.x;
    const int idx = blockIdx.x*blockDim.x + tid;
    const int offs1 = sys->nwords;
    const int offs2 = 2*offs1;
    const double rx = pos[idx];
    const double ry = pos[idx+offs1];
    const double rz = pos[idx+offs2];
[...]
```

CUDA Force Kernel (part 2)

```
double fx, fy, fz;
fx = fy = fz = 0.0;
const int natoms = sys->natoms;
for (int j = 0; idx < natoms && j < natoms ; ++j) {
    const double rx2=g_pbc(rx - pos[j], boxby2, box);
    const double ry2=g_pbc(ry - pos[j + offs1], boxby2, box);
    const double rz2=g_pbc(rz - pos[j + offs2], boxby2, box);
    const double rsq = rx2*rx2 + ry2*ry2 + rz2*rz2;
    if (rsq > 0.1 && rsq < rcsq) {
        const double rinv=1.0/rsq;
        const double r6=rinv*rinv*rinv;
        const double ffac = (12.0*c12*r6 - 6.0*c6)*r6*rinv;
        mye[tid] += 0.5*r6*(c12*r6 - c6);
        fx += rx2*ffac;
        fy += ry2*ffac;
        fz += rz2*ffac;
}}
frc[idx] = fx;
frc[offs1+idx] = fy;
frc[offs2+idx] = fz;
```

CUDA Force Kernel (Part 3)

- Pre-summing the Energy into shared memory
  - Reduce amount of data to be transferred
  - Reduce computation on CPU
  - Cascaded sum uses some threading
  - Need to synchronize threads, but is “cheap” on GPU

```
/* tree reduction */
for (int i=BLKSZ/2; i > 0; i >>= 1) {
    __syncthreads();
    if (tid < i)
        mye[tid] += mye[i+tid];
}
/* tid 0 has the sum over BLKSZ elements */
if (tid == 0) res[blockIdx.x] = mye[0];
```

CUDA Version Speed

- 108 atoms: 4x slower => not enough threads
- 2918 atoms: 5.4x faster for $O(N^2)$ algorithm
  1.5x faster than CPU with cell-list
- 78732 atoms: 12.0x faster for $O(N^2)$ algorithm
  but: 2.2x slower than CPU with cell-list
- Using single precision math (8x more on GPU):
- 2918 atoms: 11x faster (2x faster than DP)
- 78732 atoms: **75x** faster (6x faster than DP)

GPU Version Lessons

- Need enough work/data to use GPU efficiently
- Use single precision where possible, but remember to accumulate critical data in double (or use scaled 64-bit integers)
- Double precision only on new hardware
- Due to huge number of threads, computing more numbers can be faster if it offsets memory use and data transfer to and from the GPU
- Better scaling methods win over brute force

7) Hybrid OpenMP/MPI Version

- With multi-core nodes, communication between MPI tasks becomes a problem
  => all communication has to us one link
  => reduced bandwidth, increased latency
- OpenMP and MPI parallelization are orthogonal and can be used at the same time
  Caution: don't call MPI from threaded region
- Parallel region OpenMP version is very similar to MPI version, so that would be easy to merge

Hybrid OpenMP/MPI Kernel

- MPI tasks are like GPU thread blocks
- Need to reduce forces/energies first across threads and then across all MPI tasks

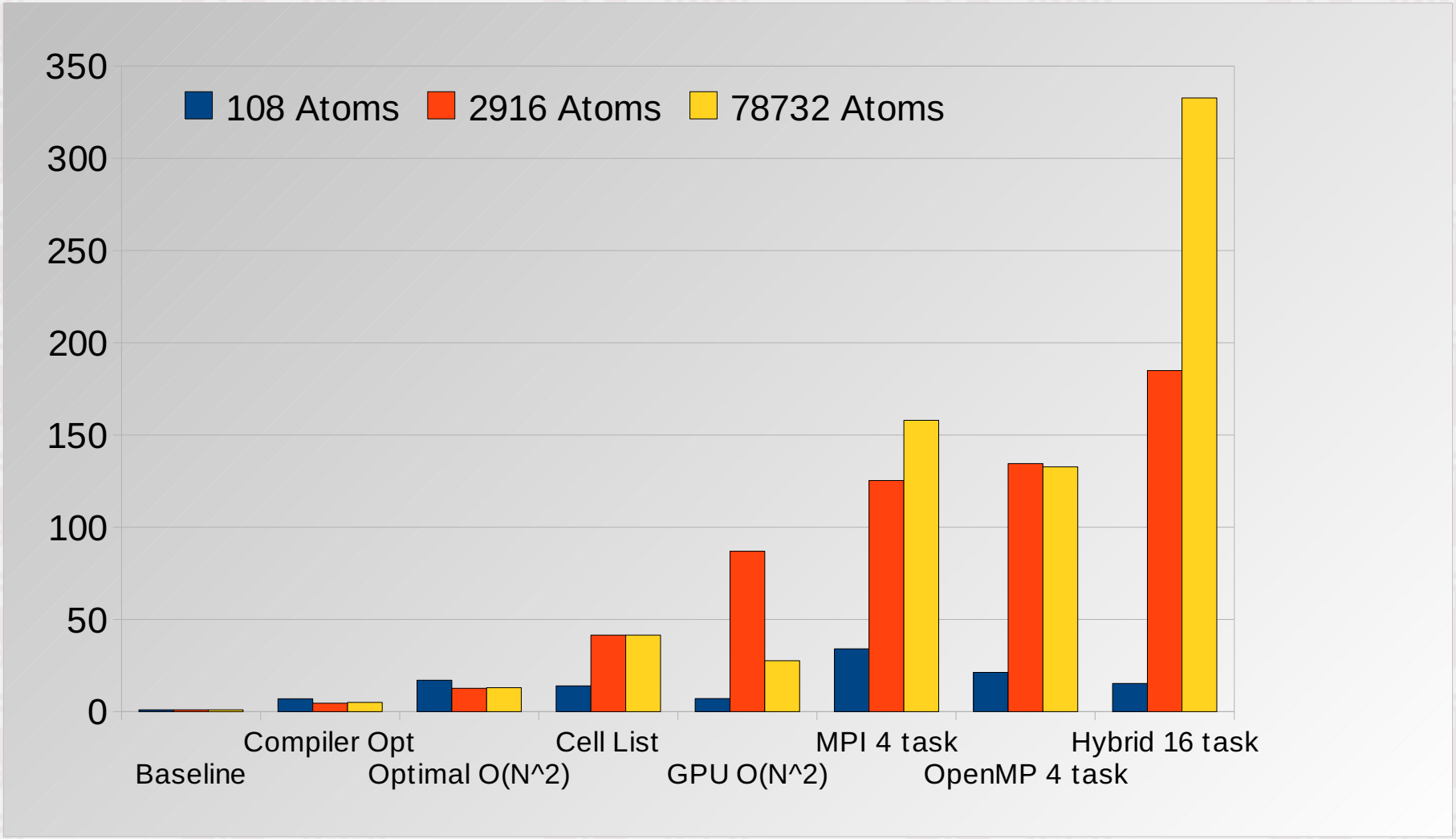
```
[...]
      incr = sys->mpisize * sys->nthreads;
      /* self interaction of atoms in cell */
      for(n=0; n < sys->ncell; n += incr) {
          int i,j;
          const cell_t *c1;

          i = n + sys->mpirank*sys->nthreads + tid;
          if (i >= sys->ncell) break;
          c1=sys->clist + i;

          for (j=0; j < c1->natoms-1; ++j) {
 [...]
```

Hybrid OpenMP/MPI Timings

	2916 atoms system:	78732 atoms system:
Cell list serial code:	18s	50.1s
16 MPI x 1 Threads:	14s	19.8s
8 MPI x 2 Threads:	5.5s	8.9s
4 MPI x 4 Threads:	4.3s	8.2s
2 MPI x 8 Threads:	4.0s	7.3s
=> Best speedup:	4.5x	6.9x
=>Total speedup:	**185x**	**333x**

Two nodes with 2x quad-core

Total Speedup Comparison



Conclusions

- Make sure that you exploit the physics of your problem well => Newton's 3rd law gives a 2x speedup for free (but interferes with threading!)
- Let the compiler help you (more readable code), but also make it easy to the compiler
  => unrolling, inlining can be offloaded
- Understand the properties of your hardware and adjust your code to match it
- Strategies that help on GPU, help with threading