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Nuclear reactor dynamics - II & III

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PART II

Because real life is harder…

- Need of numerical methods
- Factorization approach
- Improvements

Solution of the transport problem for realistic configurations

The usual "engineering" procedure for too complicated deterministic physico-mathematical problems:

1. Approximate the model (physics is distorted), e.g. transport → diffusion
2. Solve equations of approximate model by algorithms (numerically induced effects are introduced … discretizations, truncations … further distortions of physics)

Basics of reactor calculations

Split the full problem (too complicate) into a succession of problems trying to separate specific aspects and treat them separately (multi-scale)

Well-known technique in engineering

Basics of reactor calculations: dynamics

Handle numerical stiffness (very important for fast structures)

Reduce the complication of the full problem

Challenges in the simulation of neutron dynamics

- The Boltzmann equation is a very challenging problem

Example: 3D calculation of a nuclear reactor
- Space: ~ $(10^2)^3 = 10^6$ meshes
- Angle: ~ 10^2 directions (S_8 in 3D)
- Energy: ~ 10^1 - 10^2 groups

→ ~ 10^9 - 10^{10} unknowns for a steady-state calculation
- Time: $\Delta t \sim 10^{-6}$ s

→ ~ 10^6 pseudo-stationary calculation per second in time-dependent evaluation

It yields too much physical detail

In real systems only integral quantities can be observed

Challenges in the simulation of neutron dynamics

- ***Need to construct simplified models (multigroup, diffusion...) based on physical assumptions***

- ***Need of numerical algorithms (discretizations, expansions)***

Development of approximate models and algorithms

important: establish adequateness of approximations for the problem considered (benchmarks)

Models and methods for neutron dynamics

- Point kinetics
  - Derivation of the model and physical interpretation
- Quasi-static method
  - Improved quasi-statics (originally developed for fast reactors)
  - Predictor-Corrector quasi-statics
- Multipoint kinetics
  - Features of MPK approach

Point kinetics

the neutron distribution is factorized in an amplitude (time-dependent) and a shape (time independent)

$$n(r, E, \Omega, t) = P(t)\varphi(r, E, \Omega; \cancel{t})$$

<u>Critical systems</u>

Shape: fundamental eigenfunction of the model

$$\left(\hat{\mathbf{L}}_0 + \frac{1}{k}\hat{\mathbf{M}}_0\right)\varphi = 0$$

<u>Subritical systems</u>

Shape: steady-state solution, dominated by the source

$$\left(\hat{\mathbf{L}}_0 + \hat{\mathbf{M}}_0\right)\varphi + S_0 = 0$$

Point kinetics

The factorized form is introduced into the balance equations

$$\begin{cases} P\cancel{\dfrac{\partial \varphi}{\partial t}} + \varphi \dfrac{dP}{dt} = P\hat{\mathrm{B}}\varphi + \displaystyle\sum_{i=1}^{6} \lambda_i \left(\frac{\chi_i}{4\pi} C_i \right) + S \\ \dfrac{\partial(\chi_i C_i / 4\pi)}{\partial t} = P\hat{\mathrm{M}}_i \varphi - \lambda_i \left(\dfrac{\chi_i}{4\pi} C_i \right) \end{cases}$$

and is projected on a weighting function w:

$$\begin{cases} \langle w \mid \varphi \rangle \dfrac{dP}{dt} = \left\langle w \mid \hat{\mathbf{B}}\varphi \right\rangle P + \displaystyle\sum_{i=1}^{6} \lambda_i \left\langle w \mid \left(\frac{\chi_i}{4\pi} C_i \right) \right\rangle + \langle w \mid S \rangle \\ \left\langle w \mid \dfrac{\partial (\chi_i C_i / 4\pi)}{\partial t} \right\rangle = \left\langle w \mid \hat{\mathbf{M}}_i \varphi \right\rangle P - \lambda_i \left\langle w \mid \left(\dfrac{\chi_i}{4\pi} C_i \right) \right\rangle \end{cases}$$

Point kinetics

Weight w → solution of the adjoint steady-state problem

<u>Critical systems</u>

$$\left(\hat{\mathbf{L}}_0^\dagger + \frac{1}{k}\hat{\mathbf{M}}_0^\dagger\right) N_0^\dagger = 0$$

<u>Subritical systems</u>

$$\left(\hat{\mathbf{L}}_0^\dagger + \hat{\mathbf{M}}_0^\dagger\right) N_0^\dagger + S_0^\dagger = 0$$

The procedure is standard for critical reactors, while for subcritical source-driven systems the question on the adjoint source arises

definition can be given on the basis of

physical consideration and *variational principles*

Point kinetics

Integral quantities are evaluated and the differential equations for the amplitudes are derived:

$$
\left\{
\begin{array}{l}
\left\langle N_0 \right. \left\{
\begin{array}{l}
\dfrac{dP(t)}{dt} = \dfrac{\rho(t) - \tilde{\beta}}{\Lambda} P(t) + \displaystyle\sum_{i=1}^{6} \lambda_i \tilde{C}_i(t) + \tilde{S} \Big) \Big\rangle + \Big\langle N_0^\dagger \,|S\rangle \\
\dfrac{dC_i(t)}{dt} = \dfrac{\tilde{\beta}}{\Lambda} P(t) - \lambda_i \tilde{C}_i(t) \qquad \dfrac{\chi_i}{4\pi} C_i \Big) \Big\rangle
\end{array}
\right.
\end{array}
\right.
$$

having introduced the definition of the kinetic parameters

$$
\rho(t) = \frac{\left\langle N_0^+ \right| \delta \hat{\mathrm{K}} \varphi \Big\rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}} \varphi \Big\rangle} \qquad \tilde{\beta}_i = \frac{\left\langle N_0^+ \right| \hat{\mathrm{M}}_i \varphi \Big\rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}} \varphi \Big\rangle} \qquad \Lambda = \frac{\left\langle N_0^+ \right| \varphi \rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}} \varphi \Big\rangle}
$$

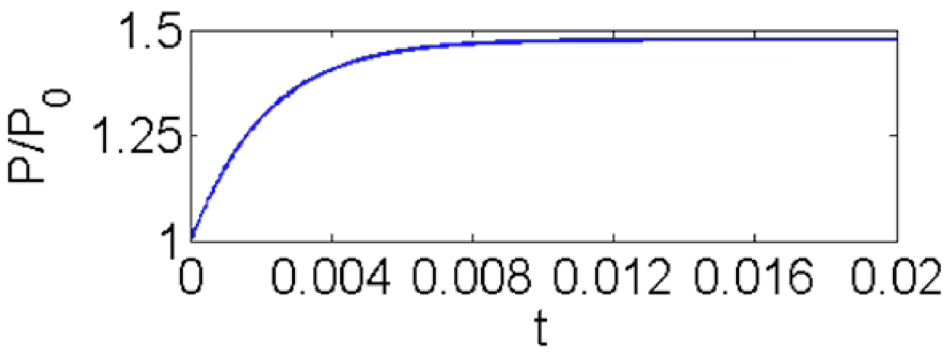
Point kinetics

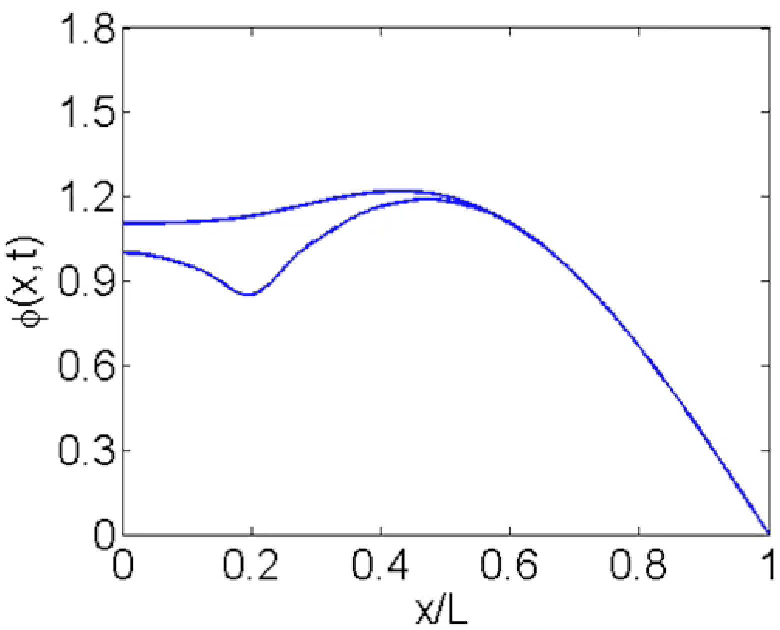
- Characteristics of the **point kinetic** approximation:
  - no space distortion during the transient
  - the evolution is space-time separable
  - any point is representative of the whole system

The approximation is poor when localized phenomena (e.g. control rod insertion) are concerned

Point kinetics - results

- Transient following extraction of a control device in a critical system
  - Simplified 1D system
  - Exact solution vs point kinetic results





Initial shape

movie

Point kinetics

- Results produced with point kinetics underestimate real power evolution

⇓

not reliable for safety assessment

- Spatial/spectral effects are neglected

- Need for a more sophisticated method, able to take into account these effects…

Quasi-static method

A digression on perturbation theory

Is there any connection between reactivity (the driving force of the transient) and the change induced to the multiplication eigenvalue of the system by the perturbation ?

A digression on perturbation theory

Original eigenvalue problem (quite general)

$$\mathcal{L}\varphi = \omega \mathcal{M}\varphi$$

Perturbed problem

$$(\mathcal{L} + \delta\mathcal{L})(\varphi + \delta\varphi) = (\omega + \delta\omega)(\mathcal{M} + \delta\mathcal{M})(\varphi + \delta\varphi)$$

A digression on perturbation theory

Expanding:

$$(\mathcal{L}\varphi) + \delta\mathcal{L}\varphi + [\mathcal{L}\delta\varphi]$$

$$= (\omega\mathcal{M}\varphi) + [\omega\mathcal{M}\delta\varphi] + \omega\delta\mathcal{M}\varphi + \delta\omega\mathcal{M}\varphi$$

Question: is it possible to evaluate $\delta\omega$

Without evaluating $\delta\varphi$?

Adjoint – neutron importance

Solve auxiliary problem:

$$\mathcal{L}^+\varphi^+ = \omega\mathcal{M}^+\varphi^+$$

Project perturbed equation on ajoint, retain only first-order terms and notice that contributions of terms involving flux perturbations vanish:

$$\begin{aligned}(\varphi^+, \mathcal{L}\delta\varphi) - (\varphi^+, \omega\mathcal{M}\delta\varphi) &= (\mathcal{L}^+\varphi^+, \delta\varphi) - (\omega\mathcal{M}^+\varphi^+, \delta\varphi) \\ (\mathcal{L}^+\varphi^+ - \omega\mathcal{M}^+\varphi^+, \delta\varphi) &= 0,\end{aligned}$$

Reactivity

Explicit expression for perturbation of eigenvalue:

$$\delta\omega = \frac{(\varphi^+, \delta\mathcal{L}\varphi) - \omega\,(\varphi^+, \delta\mathcal{M}\delta\varphi)}{(\varphi^+, \mathcal{M}\varphi)}.$$

$$\delta\omega = \delta\left(\tfrac{1}{k}\right) = -\tfrac{1}{k}\frac{\delta k}{k} = -\tfrac{1}{k}\rho$$

Conclusions…

- Reactivity introduced by point kinetics has a perturbative meaning
- Perturbation methods are very powerful and very useful in sensitivity analyses
- In fast reactor physics, beside kinetics, PM may be used for:

  Evaluation of control rod worth

  Evaluation of nuclide evolution

  Evaluation of self and mutual shielding of control system

A further question

Can perturbation analysis be applied to other integral quantities, rather than eigenvalues ?

Consider the problem:

$$\mathcal{B}\varphi = S$$

$$I = (\mathfrak{D}, \varphi)$$

Generalized perturbation theory

Perturbed problem:

$$(\mathcal{B} + \delta\mathcal{B})(\varphi + \delta\varphi) = S + \delta S$$

$$\mathcal{B}\varphi + \delta\mathcal{B}\varphi + \mathcal{B}\delta\varphi = S + \delta S$$

Generalized perturbation theory

What is the “best” adjoint problem for the projection?

$$\mathcal{B}^{+}\varphi^{+} = \mathfrak{D}.$$

The perturbation is obtained as:

$$\delta I = (\varphi^{+}, \delta S) - (\varphi^{+}, \delta \mathcal{B} \varphi)$$

Quasi-statics

- The factorization procedure is generalized as:

$$n(r,E,\Omega,t) = P(t)\varphi(r,E,\Omega;t)$$

No approximation introduced

Amplitude: fast evolving phenomena

Shape: slowing evolving phenomena

inserted into the t-d model and projected on a weight

$$\begin{cases} \left\langle w \mid \dfrac{\partial\varphi}{\partial t} \right\rangle P + \langle w \mid \varphi \rangle \dfrac{dP}{dt} = \left\langle w \mid \hat{\mathbf{B}}\varphi \right\rangle P + \sum_{i=1}^{6} \lambda_i \left\langle w \mid \left(\dfrac{\chi_i}{4\pi} C_i \right) \right\rangle + \langle w \mid S \rangle \\ \left\langle w \mid \dfrac{\partial\left(\chi_i C_i / 4\pi\right)}{\partial t} \right\rangle = \left\langle w \mid \hat{\mathbf{M}}_i \varphi \right\rangle P - \lambda_i \left\langle w \mid \left(\dfrac{\chi_i}{4\pi} C_i \right) \right\rangle \end{cases}$$

Quasi-statics

- Again, the weight is the solution of the adjoint model:

$$\begin{cases} \cancel{\frac{d}{dt}\left\langle N_0^\dagger \mid \varphi \right\rangle} P + \left\langle N_0^\dagger \mid \varphi \right\rangle \frac{dP}{dt} = \left\langle N_0^\dagger \mid \hat{\mathbf{B}}\varphi \right\rangle P + \sum_{i=1}^{6} \lambda_i \left\langle N_0^\dagger \mid \left(\frac{\chi_i}{4\pi} C_i \right) \right\rangle + \left\langle N_0^\dagger \mid S \right\rangle \\ \left\langle N_0^\dagger \mid \frac{\partial \left(\chi_i C_i / 4\pi \right)}{\partial t} \right\rangle = \left\langle N_0^\dagger \mid \hat{\mathbf{M}}_i \varphi \right\rangle P - \lambda_i \left\langle N_0^\dagger \mid \left(\frac{\chi_i}{4\pi} C_i \right) \right\rangle \end{cases}$$

and a normalization condition is introduced to make the factorization unique

$$\frac{d}{dt}\left\langle N_0^\dagger \mid \varphi \right\rangle = 0$$

Quasi-statics

- The final form of the equation for the apmlitude is the well-known point model:

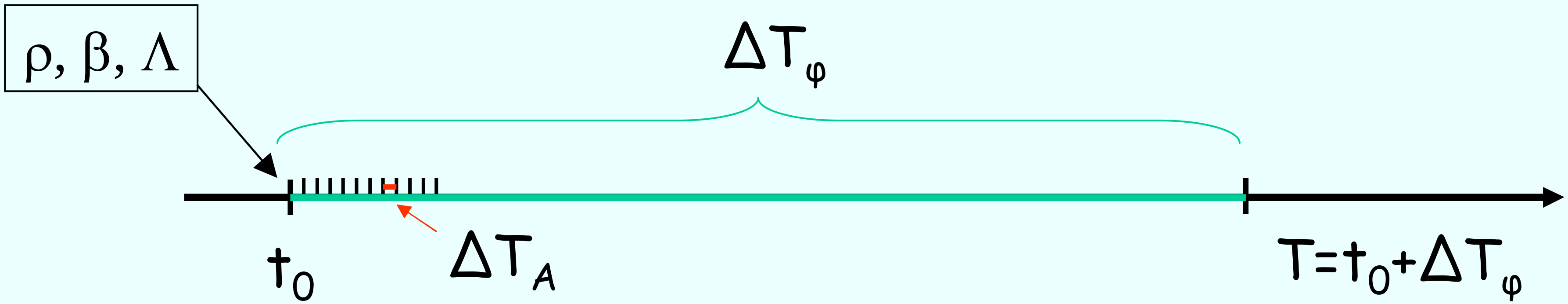
$$\begin{cases} \dfrac{dP(t)}{dt} = \dfrac{\rho(t) - \tilde{\beta}}{\Lambda} P(t) + \displaystyle\sum_{i=1}^{6} \lambda_i \tilde{C}_i(t) + \tilde{S} \\ \dfrac{dC_i(t)}{dt} = \dfrac{\tilde{\beta}}{\Lambda} P(t) - \lambda_i \tilde{C}_i(t) \end{cases}$$

but the kinetic parameters depend on the shape function, which is the other unknown of the problem

⇒ **Two time-scales solution**

Improved quasi-statics

- The solution is obtained on a two-scale frame:
  - Evaluation of the kinetic parameters with the shape at time t_0 (if t_0=0, the initial shape is used)
  - Solution of the point model on time interval $[t_0,T]$ with a fine time mesh ΔT_A
  - Solution of the shape model (computationally expensive) on $\Delta T_\varphi = T-t_0$ to update shape function



Improved quasi-statics

- Characteristics of the algorithm:
  - The model is non linear

$$\begin{cases} \left\langle N_0^\dagger \mid \varphi \right\rangle \dfrac{dP}{dt} = \left\langle N_0^\dagger \mid \boxed{\hat{\mathbf{B}}\varphi} \right\rangle \boxed{P} + \displaystyle\sum_{i=1}^{6} \lambda_i \left\langle N_0^\dagger \mid \left(\frac{\chi_i}{4\pi} C_i\right) \right\rangle + \left\langle N_0^\dagger \mid S \right\rangle \\ \left\langle N_0^\dagger \mid \dfrac{\partial(\chi_i C_i/4\pi)}{\partial t} \right\rangle = \left\langle N_0^\dagger \mid \hat{\mathbf{M}}_i \varphi \right\rangle P - \lambda_i \left\langle N_0^\dagger \mid \left(\frac{\chi_i}{4\pi} C_i\right) \right\rangle \end{cases}$$

  - The normalization condition needs to be fulfilled

Iterations on the solution of the shape model are performed

Improved quasi-statics

- Iterative procedure for the shape update (1)
  - Solution of the shape model with known P and dP/dt:

$$\frac{\varphi^{(n+1)}-\varphi^{(n)}}{\Delta T_\varphi}P(T)+\varphi^{(n+1)}\left.\frac{dP}{dt}\right|_T=\hat{\mathbf{B}}\varphi^{(n+1)}P(T)+\sum_{i=1}^{6}\lambda_i\left(\frac{\chi_i}{4\pi}C_i^{(n+1)}\right)+S^{(n+1)}$$

$$\frac{\left(\chi_i C_i^{(n+1)}/4\pi\right)-\left(\chi_i C_i^{(n)}/4\pi\right)}{\Delta T_\varphi}=\hat{\mathbf{M}}_i\varphi^{(n+1)}P(T)-\lambda_i\left(\frac{\chi_i}{4\pi}C_i^{(n+1)}\right)$$

  - Renormalization of the shape

$$\tilde{\varphi}^{(n+1)}=\frac{\left\langle N_0^\dagger \mid \varphi_0\right\rangle}{\left\langle N_0^\dagger \mid \varphi^{(n+1)}\right\rangle}\varphi^{(n+1)}$$

Check on error on shape

Improved quasi-statics

- Iterative procedure for the shape update (2)
  - Computation of kinetic parameters with $\tilde{\varphi}^{(n+1)}$
  - Modification of P (continuity of total power)

$$\left\langle \hat{\mathbf{M}}^{(n+1)} \tilde{\varphi}^{(n+1)} \right\rangle P^{(n+1)} = \left\langle \hat{\mathbf{M}}^{(n)} \varphi^{(n)} \right\rangle P^{(n)}$$

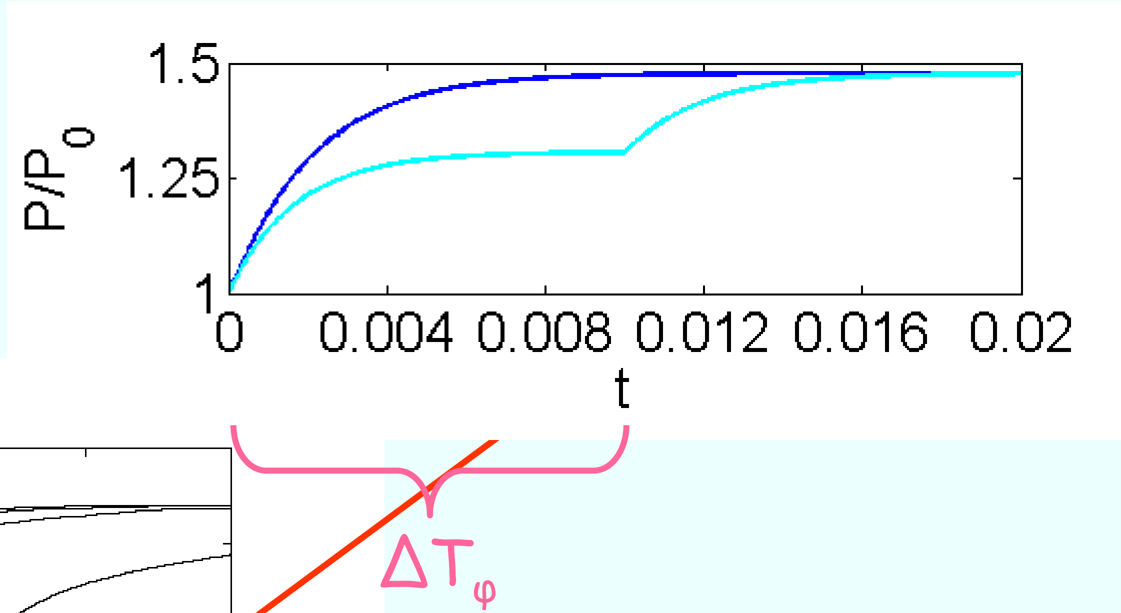
  - Modification of dP/dt (fulfillment of point model with updated kinetic parameters)

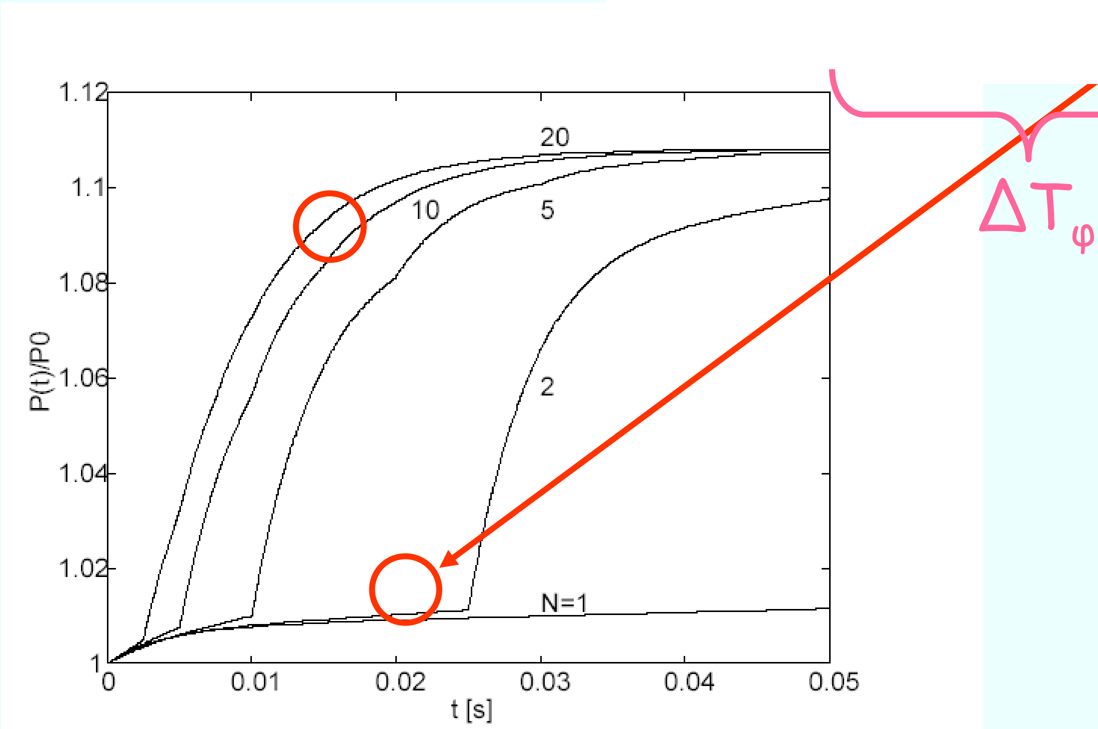
$$\begin{cases} \left.\dfrac{dP}{dt}\right|^{(n+1)} = \dfrac{\rho^{(n+1)} - \tilde{\beta}^{(n+1)}}{\Lambda} P^{(n+1)} + \displaystyle\sum_{i=1}^{6} \lambda_i \tilde{C}_i + \tilde{S} \\ \left.\dfrac{dC_i}{dt}\right|^{(n+1)} = \dfrac{\tilde{\beta}^{(n+1)}}{\Lambda} P^{(n+1)} - \lambda_i \tilde{C}_i \end{cases}$$

  - Substitution into the shape model and…

Improved quasi-statics - Results

- Improvement of the dynamic simulation of the transient





Improved quasi-statics

- Characteristics of the method:
  - Spatial and spectral effects can be taken into account
  - Solution converges to reference when ΔT_φ is reduced
  - The method <u>can</u> allow to obtain high quality results with reduced computational time

BUT

  - The definition of the interval ΔT_φ largely influences the quality of the results (need of adaptive procedure)
  - The convergence of the shape is not always ensured
  - The iterative procedure of the shape update can be time consuming when large modifications of the shape are involved
  - The procedure can become too expensive computationally

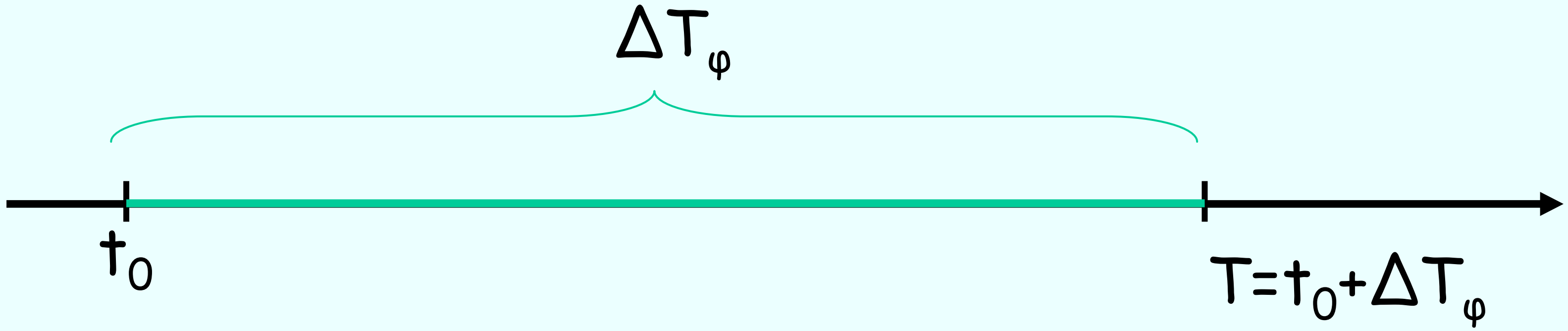
Needs for alternative numerical schemes
to avoid the non-linearity of the problem

Predictor-corrector quasi-statics

Predictor-Corrector quasi-statics

- Scheme for the solution of the quasi-static equation, avoiding the non linearity of the model
  - Solution of the balance model on the time mesh $\Delta T\varphi$

$$\begin{cases} \dfrac{\partial n}{\partial t} = \hat{\mathbf{B}}n + \displaystyle\sum_{i=1}^{6} \lambda_i \frac{\chi_i}{4\pi} C_i + S \\ \dfrac{\partial \left(\chi_i C_i / 4\pi\right)}{\partial t} = \hat{\mathbf{M}}_i n - \lambda_i \dfrac{\chi_i}{4\pi} C_i \end{cases}$$

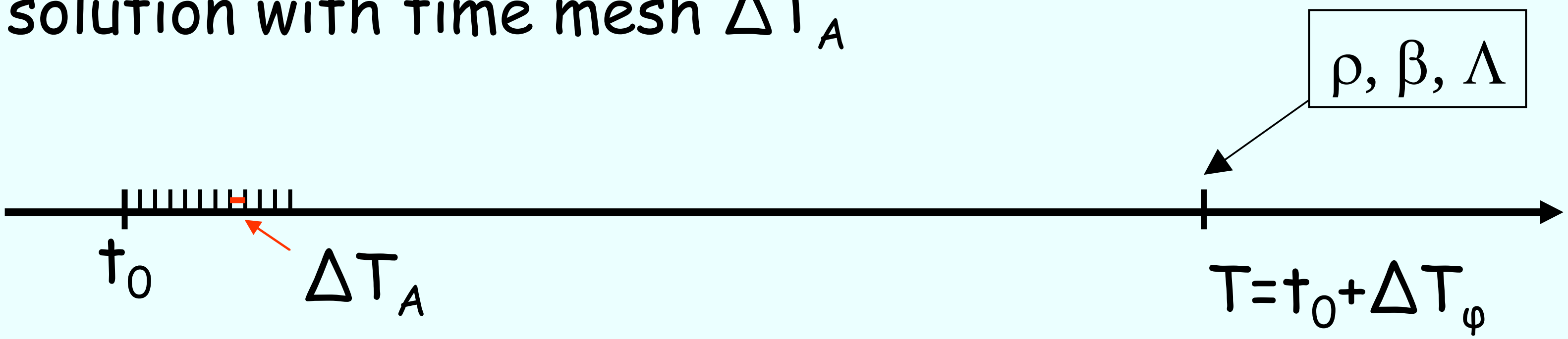


Predictor-Corrector quasi-statics

- Scheme for the solution of the quasi-static equation, avoiding the non linearity of the model (2)
  - Renormalization if the flux in order to obtain a proper shape function

$$\varphi^{(n+1)} = \frac{\left\langle N_0^\dagger \mid \varphi_0 \right\rangle}{\left\langle N_0^\dagger \mid n^{(n+1)} \right\rangle} n^{(n+1)}$$

  - Evaluation of kinetic parameters and point kinetic solution with time mesh ΔT_A



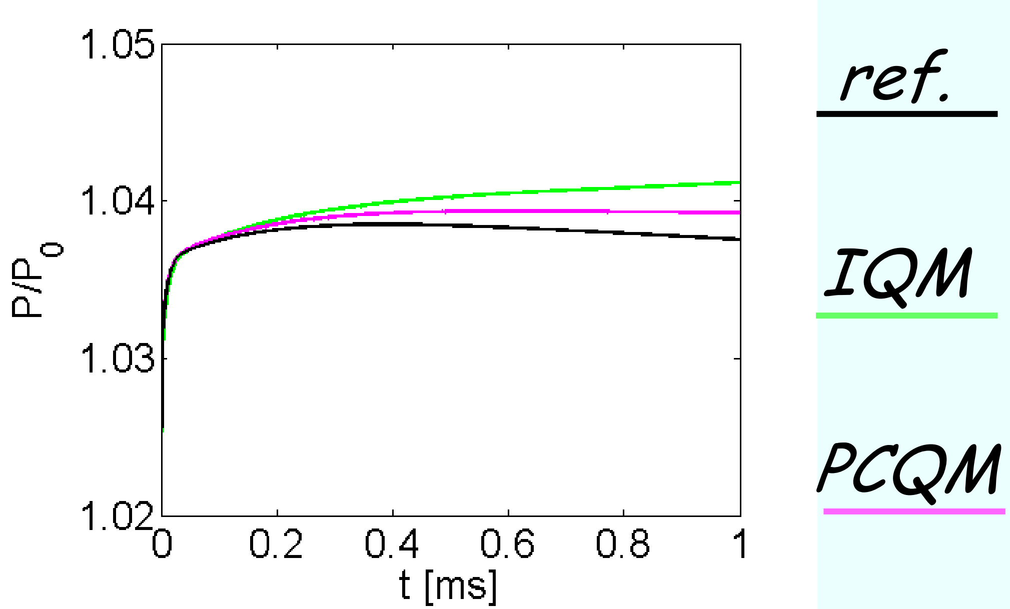
Predictor-Corrector quasi-statics

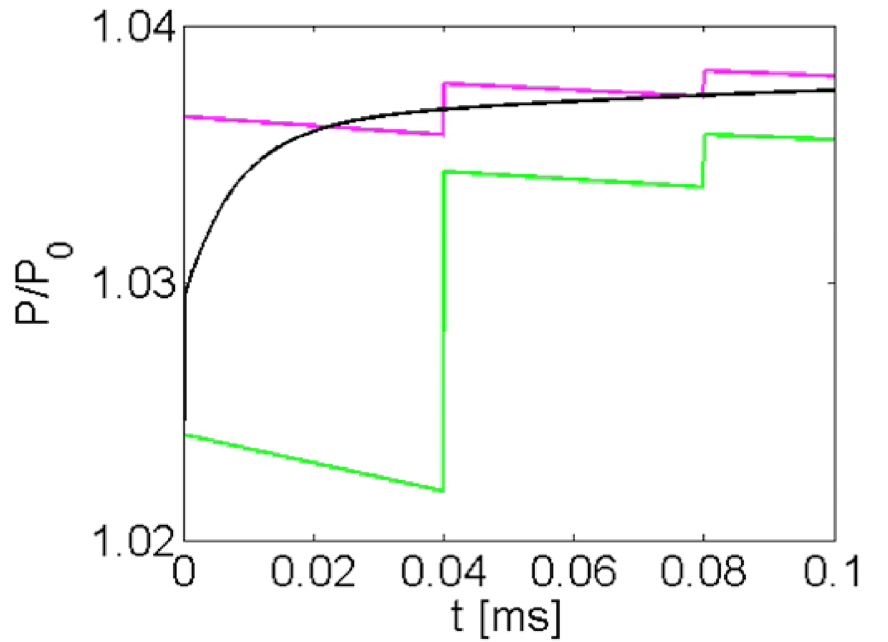
- Characteristics of PC quasi-statics
  - No iterations to fulfil normalization are required
  - Kinetic parameters used for point-kinetic calculations are more suitable to describe the transient during $\Delta T\varphi$ and can provide more accurate results
  - The computational effort can be effectively reduced with respect to IQM

  - When transients with large power effects and small shape modifications are involved, PCQM can fail in reducing computational time (point-like transients)

P-C quasi-statics - Results

Convergence of IQM and PCQM to the reference solution





Different performance of kinetic parameters generated with IQM and PCQM

movie

Further improvements

- The factorization procedure can be improved, subdividing the domain in several regions of the phase space
- This approach can be very effective when loosely coupled systems are concerned

Multipoint method

The multipoint method

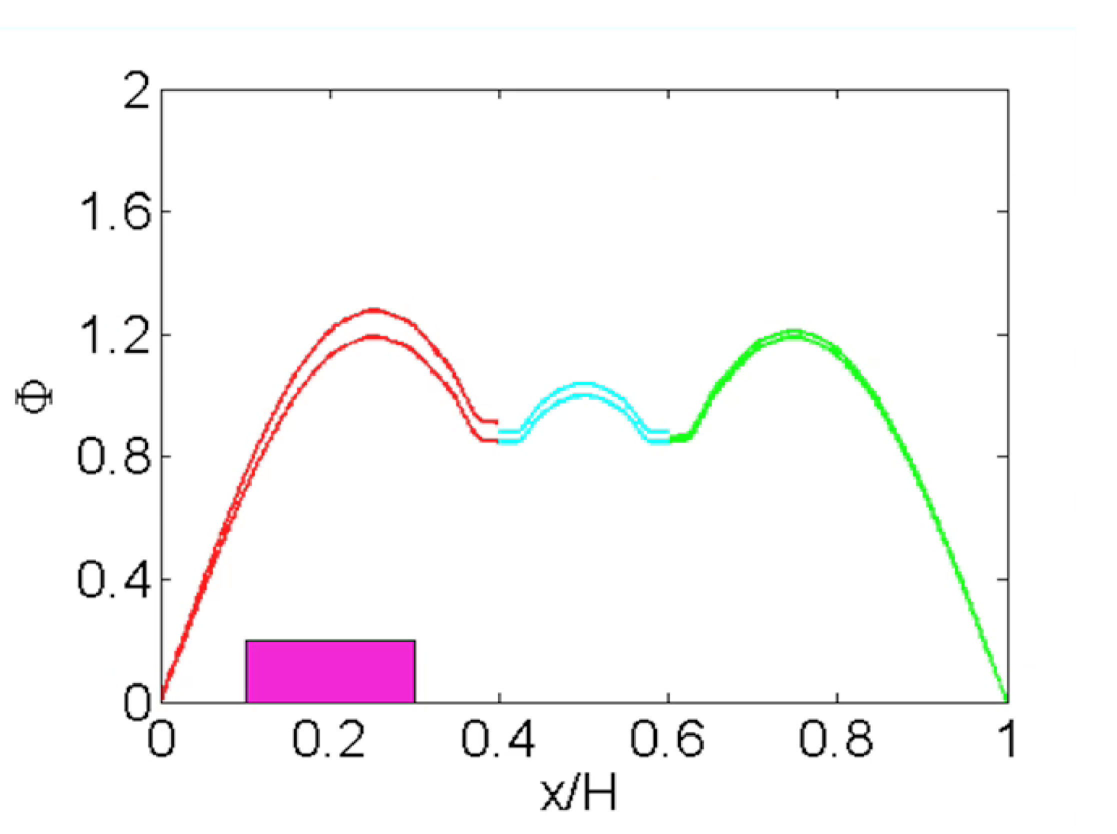
- The method can be viewed as an extension of the point kinetic model
- The domain considered in the phase space is subdivided in K (reasonably small) regions (points)
- The neutron density in each region is factorized in a product of amplitude and shape

⇩

- K point-like systems for the amplitudes P_K are obtained, all coupled by integral coefficients obtained by the projection technique

The multipoint method

- Example of the multipoint phylosophy



Initial neutron density

Localized perturbation

Different regions are affected differently by the perturbation…

… and are simulated with different amplitude functions

movie

The multipoint method

Balance equations in discretized form and phase-space subdivision:

$$
\begin{cases}
\dfrac{1}{v_m}\dfrac{d\phi_{nm}}{dt} = \displaystyle\sum_{n'}\sum_{m'} k_{nm,n'm'}\phi_{n'm'} + \\
\qquad\qquad \displaystyle\sum_{i=1}^{6} \lambda_i \chi_{i,m} C_{i,n} + S_{nm} \\
\dfrac{dC_{i,n}}{dt} = \beta_i \displaystyle\sum_{m'} f_{nm'}\phi_{nm'} - \lambda_i C_{i,n} \qquad i = 1, 2, ..., 6
\end{cases}
$$

$$
\phi_{nm}(t) = \phi(\mathbf{r}_n, V_m, t) \qquad C_{i,n}(t) = C_i(\mathbf{r}_n, t)
$$

$$
\phi_{nm}(t) = A_{NM}(t)\varphi_{nm}(t) \qquad \mathbf{r}_n, V_m \in \Gamma_{NM}
$$

The multipoint method

Regionwise inner products $$\langle w|\, g\rangle = \left[\sum_n \sum_m\right]_{NM} w_{nm} g_{nm}$$

Introduce factorization (shape equations – known amplitudes):

$$\begin{cases} \dfrac{1}{v_m}\varphi_{nm}\dfrac{dA_{NM}}{dt} + \dfrac{1}{v_m}A_{NM}\dfrac{d\varphi_{nm}}{dt} = \\ \displaystyle\sum_{N'}\sum_{M'}\left[\sum_{n'}\sum_{m'}\right]_{N'M'} k_{nm,n'm'}\varphi_{n'm'}A_{N'M'} + \\ \displaystyle\qquad \sum_{i=1}^{6}\lambda_i\chi_{i,m}C_{i,n} + S_{nm} \\ \\ \dfrac{dC_{i,n}}{dt} = \beta_i \displaystyle\sum_{M'}\left[\sum_{m'}\right]_{M'} f_{nm'}\varphi_{nm'}A_{NM'} - \lambda_i C_{i,n} \\ \qquad i = 1, 2, ..., 6, \quad \mathbf{r}_n, V_m \in \Gamma_{NM} \end{cases}$$

The multipoint method

Project on weight (amplitude equation – known shape):

$$
\begin{cases}
\dfrac{dA_{NM}}{dt} = \sum_{N'} \sum_{M'} K_{NM,N'M'} A_{N'M'} + \\
\qquad \sum_{i=1}^{6} \lambda_i C_{i,NM} + S_{NM} \\
\dfrac{dC_{i,NM}}{dt} = \beta_i \sum_{M'} F_{i,NM,M'} A_{NM'} - \lambda_i C_{i,NM} \\
\quad i = 1, 2, \ldots, 6,
\end{cases}
$$

The multipoint method

Normalization condition (its application may require iteration):

$$\frac{d}{dt}\left[\sum_n \sum_m\right]_{NM} w_{nm}\frac{1}{v_m}\varphi_{nm}(t) = \frac{d}{dt}\gamma_{NM} = 0$$

Kinetic effective parameters and source are introduced

Multipoint effective terms

$$K_{NM,N'M'} = \frac{1}{\gamma_{NM}}\left[\sum_n \sum_m\right]_{NM}\left(w_{nm}\left[\sum_{n'} \sum_{m'}\right]_{N'M'} k_{nm,n'm'}\varphi_{n'm'}\right)$$

coupling terms

effective source

$$S_{NM} = \frac{1}{\gamma_{NM}}\left[\sum_n \sum_m\right]_{NM} w_{nm}S_{nm}$$

$$C_{i,NM} = \frac{1}{\gamma_{NM}}\left[\sum_n \sum_m\right]_{NM} w_{nm}\chi_{i,m}C_{i,n}$$

effective delayed concentration

delayed fission term

$$F_{i,NM,M'} = \frac{1}{\gamma_{NM}}\left[\sum_n \sum_m\right]_{NM} w_{nm}\chi_{i,m}\beta_i\left[\sum_{m'}\right]_{M'} f_{nm'}\varphi_{nm'}$$

Multipoint features

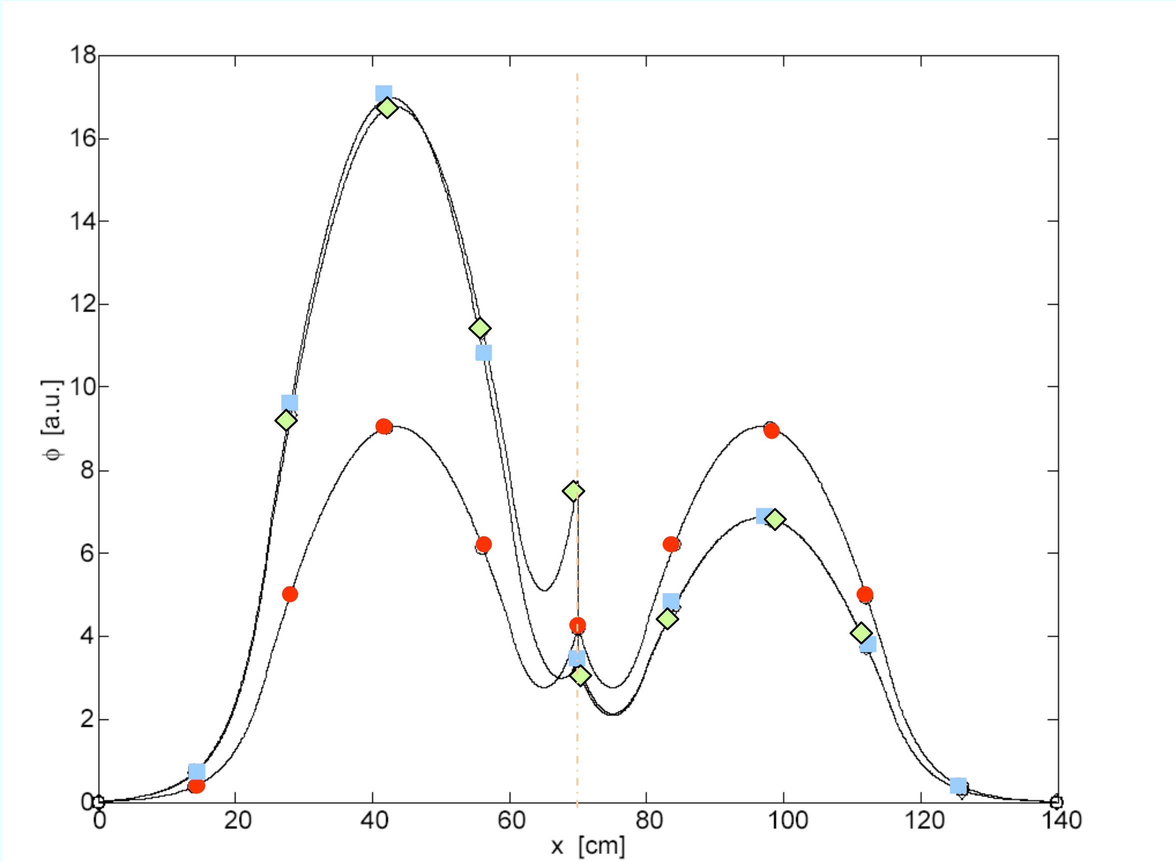
Multipoint can be used in quasi-statics

Graph to show features of multipoint:

Circle (●) : PK

Square (■): exact

Diamond (◇): 2-point



Effect of choice of points

Different subdivision of the phase space has influence on the accuracy of the results

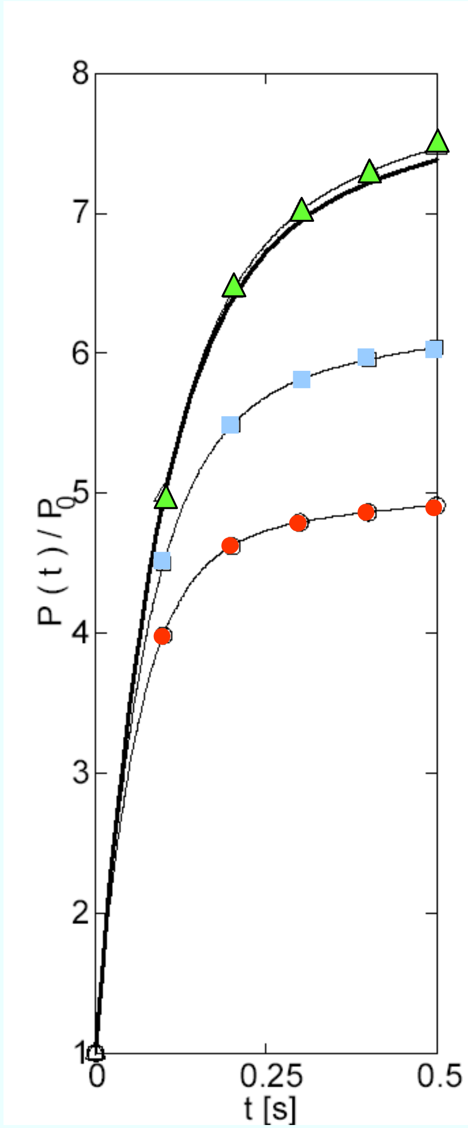
Bold: exact

Circle (●) : PK

Square (■): 2-point

Triangle (▲): 2-point

■ and ▲ are characterized by different subdivisions of the spatial domain



Effect of choice of points

- The different subdivision of the spatial domain are evidenced

  Bold: exact

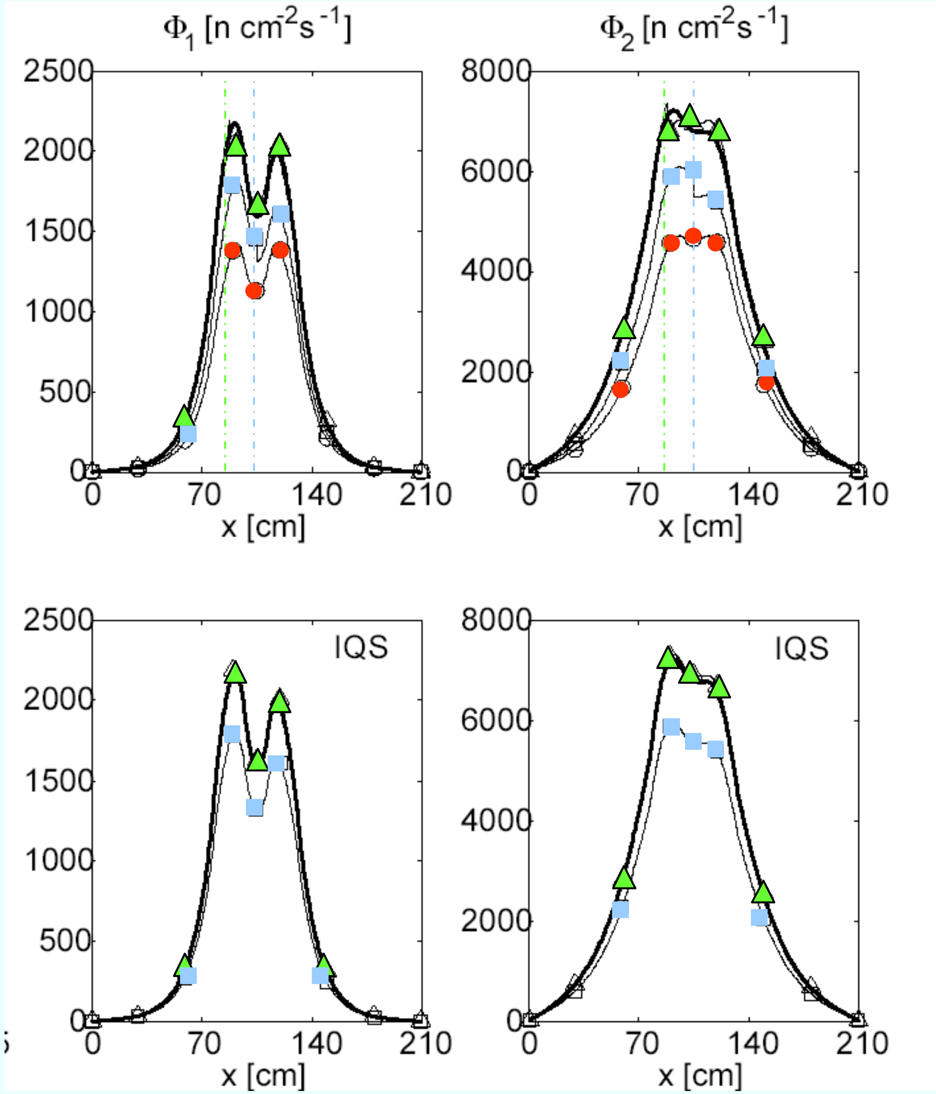
  Circle (•) : PK

  Square (▪): 2-point

  Triangle (▵): 2-point

- Update of shape functions through quasi-static procedure

Continuity of fluxes



Hard problems in kinetics

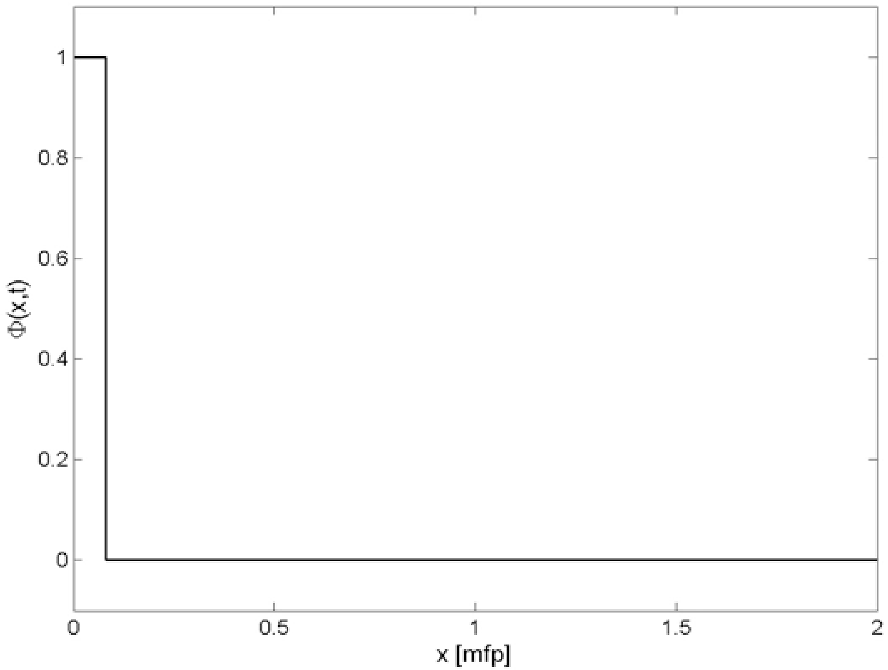
- Propagation phenomena or
- Is there anything better than diffusion ?
- …do we need something better than diffusion ?

Pulse propagation

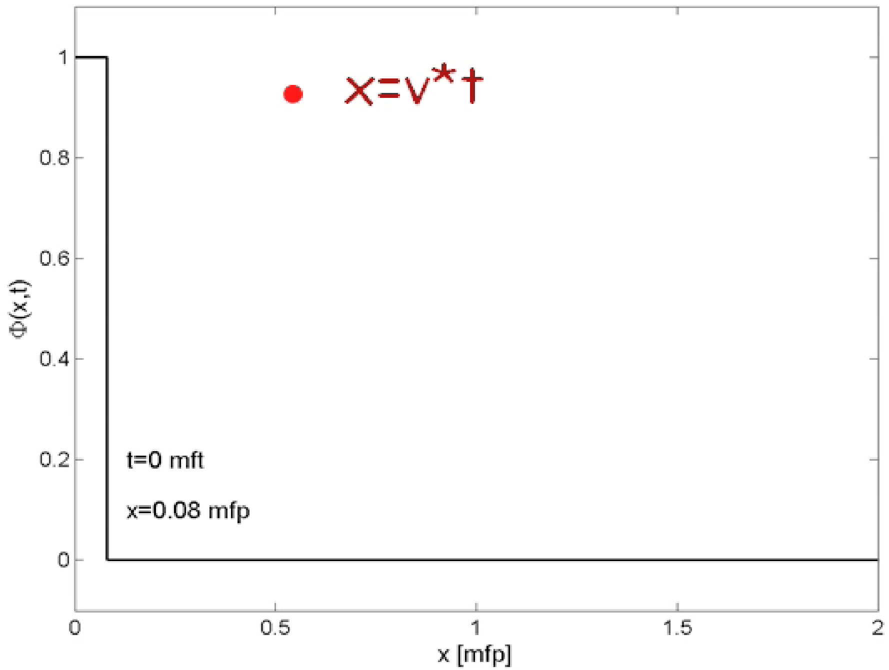
- Appearance of time-dependent ray effects connected to angular discretization
- Inadequateness of diffusion theory due to the infinite-velocity limit (no ray effects)
- Space and time ray effects in multi-D
- Spatial distortions due to space discretizations

Pulse propagation

Diffusion



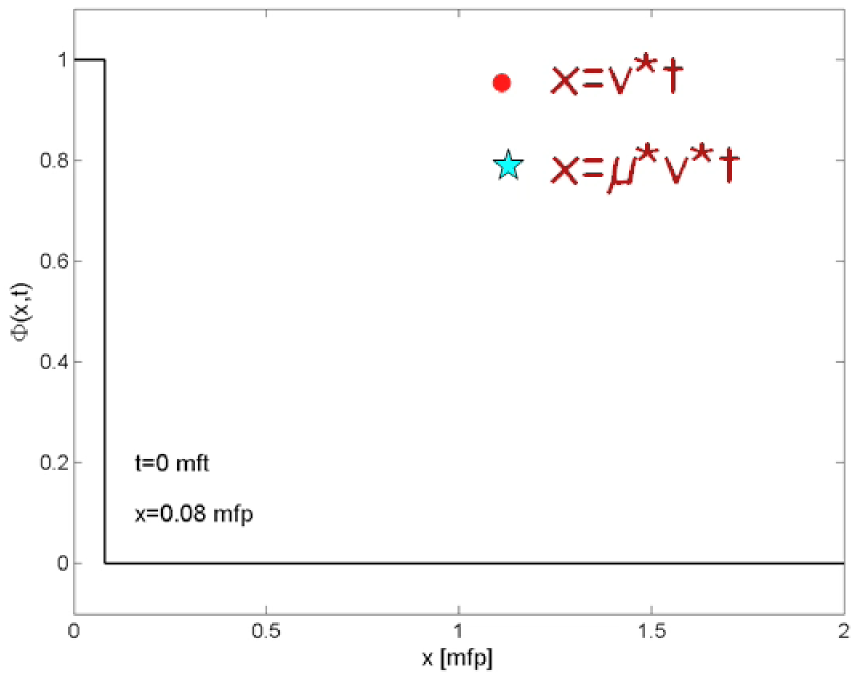
Exact transport



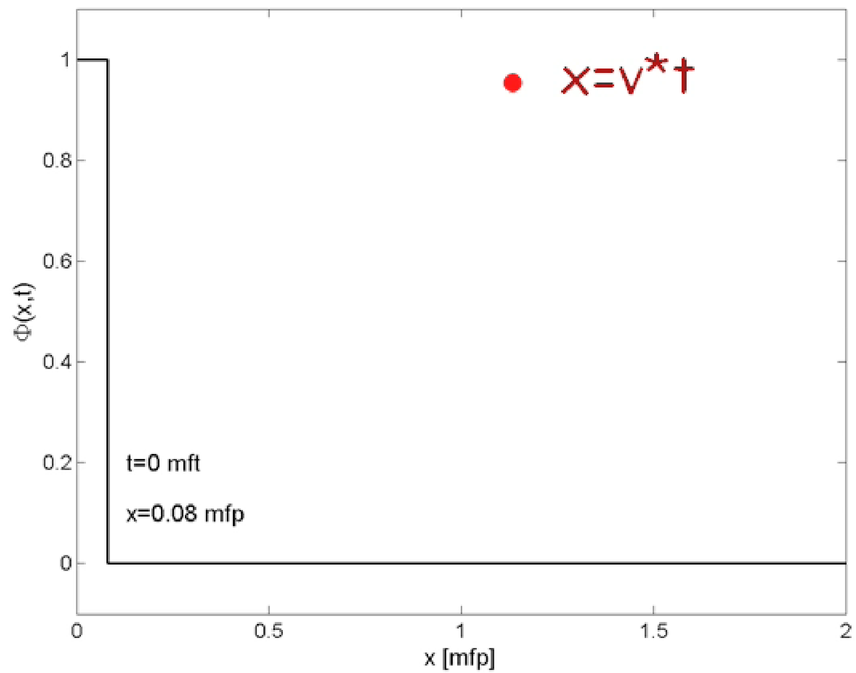
movie

Pulse propagation

P_1



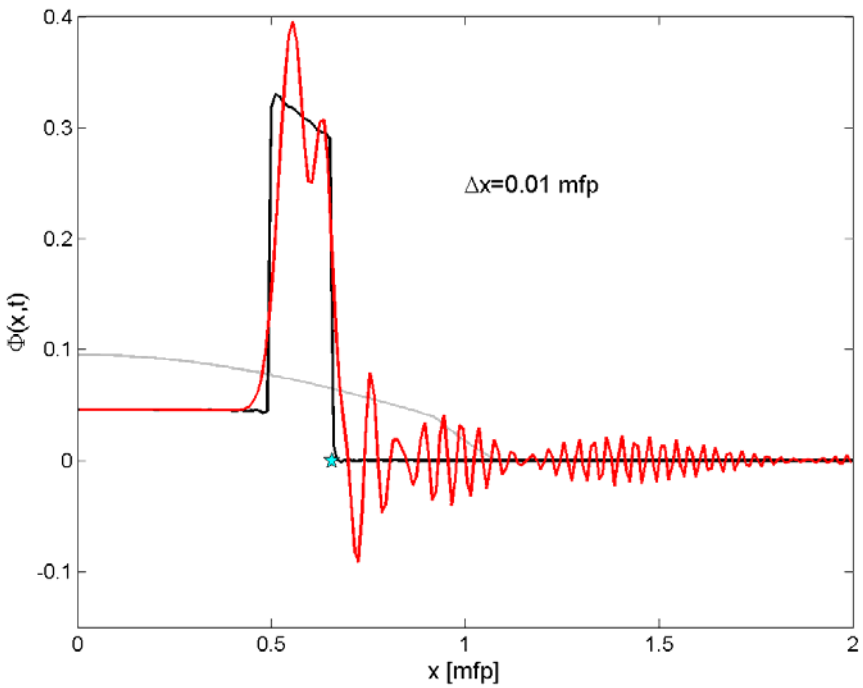
Exact transport



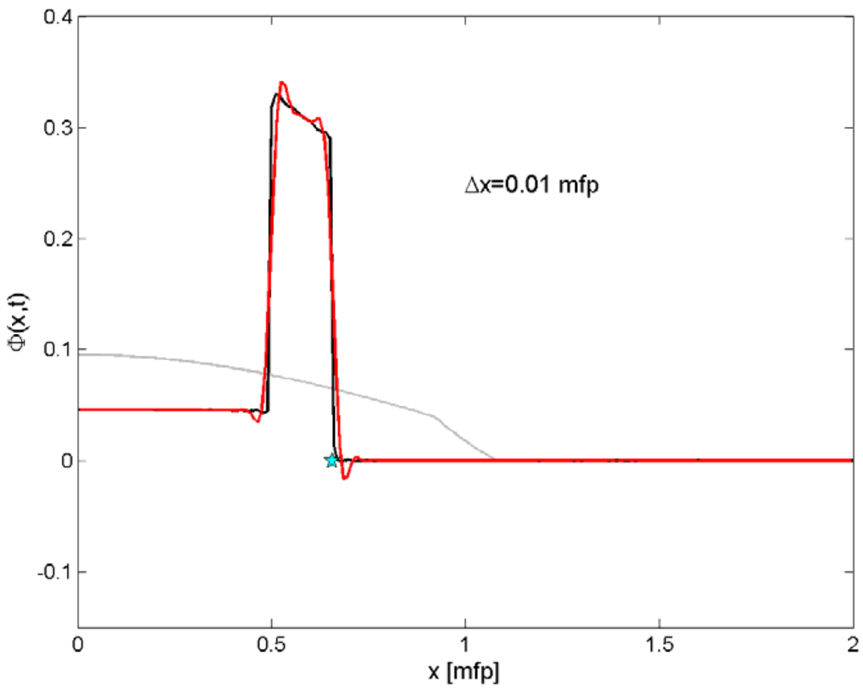
movie

Pulse propagation

P_1 – diamond difference



P_1 – linear discontinuous



Enhanced quasi-static schemes: Alternative factorizations

Rather than assuming the standard amplitude-shape factorization:

$$\phi(\mathbf{r}, E, \mathbf{\Omega}, t) \;=\; T(t)\,\psi(\mathbf{r}, E, \mathbf{\Omega}, t)$$

a privileged variable in phase space is identified and treated separately in the factorization

$$\phi(\mathbf{r}, E, \Omega, t) = \phi(\mathbf{X}, t) = T(x_1, t)\psi(\mathbf{X}, t)$$