

Degenerations in the additive categories of almost cyclic coherent Auslander-Reiten components

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Let A be a finite dimensional algebra over an algebraically closed field k and $\text{mod}_A(d)$ the affine variety of d -dimensional A -modules. The general linear group $\text{Gl}_d(k)$ acts on $\text{mod}_A(d)$ by conjugation, and the orbits correspond to the isomorphism classes of d -dimensional modules. We denote by $\mathcal{O}(M)$ the $\text{Gl}_d(k)$ -orbit of a module M in $\text{mod}_A(d)$. Then one says that a module N in $\text{mod}_A(d)$ is a *degeneration* of a module M in $\text{mod}_A(d)$ if N belongs to the Zariski closure $\overline{\mathcal{O}(M)}$ of $\mathcal{O}(M)$ in $\text{mod}_A(d)$, and we denote this fact by $M \leq_{\text{deg}} N$. Thus \leq_{deg} is a partial order on the set of isomorphism classes of A -modules of a given dimension. We consider also another partial order \leq_{ext} on the category $\text{mod } A$ of finite dimensional A -modules defined as follows:

$M \leq_{\text{ext}} N$ $:\Leftrightarrow$ there are modules M_i, U_i, V_i and short exact sequences $0 \rightarrow U_i \rightarrow M_i \rightarrow V_i \rightarrow 0$ in $\text{mod } A$ such that $M = M_1$, $M_{i+1} = U_i \oplus V_i$, $1 \leq i \leq s$, and $N = M_{s+1}$ for some natural number s .

For all modules M and N in $\text{mod}_A(d)$, we have $M \leq_{\text{ext}} N \implies M \leq_{\text{deg}} N$ but the converse implication is not true in general.

Recall that a connected component \mathcal{C} of the Auslander-Reiten quiver Γ_A of A is called *generalized standard* if $\text{rad}^\infty(X, Y) = 0$ for all modules X, Y in \mathcal{C} . Further, \mathcal{C} is called *almost cyclic* if all but finitely many modules of \mathcal{C} lie on oriented cycles (in \mathcal{C}). Moreover, \mathcal{C} is called *coherent* if every projective module P in \mathcal{C} is the starting module of an infinite sectional path and every injective module I in \mathcal{C} is the ending module of an infinite sectional path.

In the talk we describe when the partial orders \leq_{ext} and \leq_{deg} coincide for all modules of the same dimension from the additive category $\text{add}(\mathcal{C})$ of a generalized standard almost cyclic coherent component \mathcal{C} in Γ_A . Moreover, we also give a necessary and sufficient condition for the existence of degeneration $M \leq_{\text{deg}} N$ for arbitrary modules M, N of the same dimension from $\text{add}(\mathcal{C})$.