# Generalized epimorphisms of $K$-categories and reflective subcategories of their module categories

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Let $K$ be any commutative ring. It is a well-known fact that the following conditions are equivalent for a homomorphism $f : A \longrightarrow B$ of associative unital $K$-algebras:

1. $f : A \longrightarrow B$ is an epimorphism (= right cancellative) in the category of associative unital algebras

2. The multiplication map $B \otimes_A B \longrightarrow B \longrightarrow B$ is an isomorphism of $B$-bimodules

3. The restriction of scalars $f_* : Mod - B \longrightarrow Mod - A$ is fully faithful.

It is traditional to look at a small $K$-category $\mathcal{A}$ as a $K$-algebra 'with several objects' and at the functors $\mathcal{A}^{op} \longrightarrow Mod - K$ as (right) $K$-modules. In this context conditions 1-3 above have natural substitutes, but the substitute of 1 is not equivalent any more to the substitutes of 2 and 3.

In this talk we shall clarify the precise relationship between these substitutes and the corresponding facts for algebras with enough idempotents. As it is common rule nowadays, we take the substitutes of 2 and 3 as definition of epimorphism of $K$-categories, and then extend a theorem of Gabriel and de la Peña. Namely, given a small $K$-category $\mathcal{A}$, we show a bijection between the sets (!) of equivalence classes of surjective on objects epimorphisms of $K$-categories with domain $\mathcal{A}$ and the fully exact reflective subcategories of $Mod - \mathcal{A}$.

The natural question that arises is whether, by appropriately replacing the notion of epimorphism, the above bijection can be extended to arbitrary reflective subcategories of $Mod-\mathcal{A}$, although these do not form a set in general. We show that it is essentially the case, with the 'epimorphisms with domain $\mathcal{A}$' being replaced by 'reflecting pairs for $\mathcal{A}$', a notion that concerns a general form of epimorphism that we call pseudoepimorphism. A functor $F : \mathcal{A} \longrightarrow \mathcal{B}$ is a (right) pseudoepimorphism when the restriction of scalars $F_* : Mod - \mathcal{B} \longrightarrow Mod - \mathcal{A}$ is fully faithful on the representable $\mathcal{B}$-modules.