WEIGHTED PROJECTIVE LINES AND INVARIANT SUBSPACES OF NILPOTENT OPERATORS

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This is a report on a joint work with Helmut Lenzing and Hagen Meltzer. Let $p \geq 2$ be an integer. We establish a close link between the category $\mathcal{S}(\widetilde{p})$ of invariant subspaces of (graded) linear operators of nilpotency index p, studied by Claus Ringel and Markus Schmidmeier [1], and the category $\mathrm{vect}(\mathbb{X})$ of vector bundles over a weighted projective line $\mathbb{X} = \mathbb{X}(2,3,p)$ of weight type $(2,3,p)$.
The category $\mathcal{S}(\widetilde{p})$ is known to be Frobenius in a natural way. There is a natural exact structure on $\mathrm{vect}(\mathbb{X})$ such that it is a Frobenius category with the system $\mathcal{L}$ of line bundles as the indecomposable projective-injective objects.

Theorem. *There is a (explicitly given) partition $\mathcal{L} = \mathcal{P} \sqcup \mathcal{F}$ into so-called persistent and fading line bundles, respectively, such that*

(1) *the factor category $\mathrm{vect}(\mathbb{X})/[\mathcal{F}]$ is still Frobenius, the indecomposable projective-injective objects given by the system $\mathcal{P}$ of persistent line bundles, and*
(2) *there is an equivalence of Frobenius categories between $\mathrm{vect}(\mathbb{X})/[\mathcal{F}]$ and $\mathcal{S}(\widetilde{p})$.*

Corollary. *There is an equivalence of triangulated categories between the stable categories $\underline{\mathrm{vect}}(\mathbb{X}) := \mathrm{vect}(\mathbb{X})/[\mathcal{L}]$ and $\underline{\mathcal{S}}(\widetilde{p})$.*

Furthermore, we will discuss several applications to the category $\underline{\mathcal{S}}(\widetilde{p})$, like the

- description of the shape of the Auslander-Reiten components;
- explicit construction of a tilting object in $\underline{\mathcal{S}}(\widetilde{p})$ whose endomorphism ring is the path algebra of the linear quiver $1 \xrightarrow{x} 2 \xrightarrow{x} \ldots \xrightarrow{x} 2p-3 \xrightarrow{x} 2(p-1)$ with relations $x^3 = 0$;
- computation of the fractional Calabi-Yau dimension of $\underline{\mathcal{S}}(\widetilde{p})$.

References

[1] C. M. Ringel and M. Schmidmeier, *Invariant subspaces of nilpotent linear operators. I.* J. Reine Angew. Math. 614 (2008), 1–52.