Homological properties of Hecke algebras and related algebras

One would like to know whether selfinjective algebras occuring 'in nature' satsify the finiteness conditions
which ensure that modules have varieties defined via Hochschild cohomology (as discussed in E-Holloway-Snashall-Solberg-Taillefer). For type A Hecke algebras, this is the case when
algebra is tame, and the proof uses results on symmetric Koszul algebras with radical cube zero (joint with O. Solberg).
In general, it is open whether Hecke algebras satisfy suitable finiteness.
We have however determined the
cohomology of the Hecke algebra when the characteristic is zero, and in particular it follows that
it is finitely generated (joint with D. Benson and A. Mikaelian).
Continuing in the direction of selfinjective algebras with radical cube zero, we describe minimal projective
resolutions of modules via Chebyshev polynomials (joint with S. Schroll) when the algebra is symmetric. There is also
a connection with generalized mesh algebras, as introduced in joint work with J. Bialkowski and A. Skowronski.

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