Otto Kerner, Modules of finite complexity over selfinjective algebras.

Let Λ be a finite dimensional algebra over some algebraically closed field K, and M be a finite dimensional Λ-module with minimal projective resolution

$$P^\bullet: \quad \cdots \to P^2 \xrightarrow{\delta^2} P^1 \xrightarrow{\delta^1} P^0 \xrightarrow{\delta^0} M \to 0.$$

The *complexity of* M is defined as

$$\operatorname{cx} M = \inf\{n \in \mathbb{N} | \dim P^i \leq c i^{n-1} \text{ for some positive } c \in \mathbb{Q} \text{ and all } i \geq 0\}$$

If no such n exists, then we say that the complexity of M is infinite. Important examples are group algebras $\Lambda = KG$ for finite groups G. In this case all finite dimensional Λ-modules have finite complexity. Using this fact, P. Webb described in 1982 the possible shapes of the stable Auslander-Reiten components $\mathcal{C}_s$ in the Auslander-Reiten quiver $\Gamma(KG)$.

In collaboration with Dan Zacharia we studied the case, where Λ is a selfinjective algebra. Under this assumption the complexity is constant for all indecomposable modules in the stable part $\mathcal{C}_s$ of an Auslander-Reiten component $\mathcal{C} \in \Gamma(\Lambda)$, and the complexity is 1, if $\mathcal{C}$ contains a τ-periodic indecomposable module. I will give a report on the main result of our joint project:

Theorem. *Let $\mathcal{C}$ be a non τ-periodic component of the Auslander-Reiten quiver of a selfinjective algebra Λ over an algebraically closed field, containing a non projective module of finite complexity. Then the stable component $\mathcal{C}_s$ is of type $\mathbb{Z}\Delta$, where Δ is either an extended Dynkin diagram of type $\tilde{A}_n, \tilde{D}_n$, $\tilde{E}_i$ $(i = 6, 7, 8)$, or of the type $\mathbb{Z}A_\infty, \mathbb{Z}A_\infty^\infty$ or $\mathbb{Z}D_\infty$. If the component is a regular component, then only the infinite Dynkin trees can occur.*