Winter College on Optics and Energy

8 - 19 February 2010

Lighting and illumination engineering
I. Introduction

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15 February 2010

John Koshel
College of Optical Sciences - The Univ. of Arizona & Photon Engineering, LLC
Where is Illumination Design?

- **Academia:**
  - Universidad Politécnica de Madrid (Spain)
  - Philips-University Marburg (Germany)
  - Ben-Gurion University of the Negev (Israel)
  - University of California/Merced (USA)
  - Lighting Research Center at RPI (USA)

- **Industry:**
  - Light Prescription Innovators (USA, Spain, PacRim)
  - Optics and Energy Concepts (Germany)
  - Software Companies (USA & Europe): Photon Engineering (FRED), ORA (LightTools), BRO (ASAP), LRC (TracePro), Optis (Solstis); ZDC (ZEMAX)
  - Large Companies (worldwide): 3M, SAIC, Visteon, Philips, LumiLeds
  - Small Companies (worldwide): Tailored Optics, Wavien
  - Consultants (worldwide)

- **Government:**
  - USA: Sandia, DARPA, NREL, LBL, DOE
  - World: Korea, Japan, Europe, Taiwan, Israel
Illumination design has a rich history, but as a scientific/engineering discipline it has only blossomed in the past four decades (excepting source development).
Why Illumination Design?

- The Illumination Industry is Large: $100B+ worldwide.

- Energy Costs are Increasing:
  - Electricity for all Lighting: 20% (1993; [http://www.eia.doe.gov/emeu/recs/recs4a.html](http://www.eia.doe.gov/emeu/recs/recs4a.html))
  - Electricity for Residential Lighting: 9.4% (1993; [http://www.eia.doe.gov/emeu/recs/recs4a.html](http://www.eia.doe.gov/emeu/recs/recs4a.html))
  - Wasted Lighting: ~25% unused
  - Light Pollution: $1B/yr lights sky (Batinsey, ANJEC Report 1994)

- There are many standards bodies dedicated to illumination requirements.
The Illumination Industry

- Displays (iSuppli, May 2005):
  - TFT-LCD: 59M units (’04) = $29.5B, 94.2M units (’05) = $47.1B
  - OLED: $408M (’05), $2.9B (’11)
- High-Brightness LEDs:
  - Strategies Unlimited (8/05): $3.7B (’04), $6.8B (’09)
  - iSuppli (8/05): $5.2B (’05); SSL: $144M (’05), $875M (’10)
- Data from US Census Bureau from 2005 Report:

There is also solar energy generation – we make the power that then we use for providing light

Vehicular Lighting Value

<table>
<thead>
<tr>
<th>Year</th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
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<td>Value of Shipments ($)</td>
<td>$4.00</td>
<td>$3.50</td>
<td>$3.00</td>
<td>$2.50</td>
<td>$2.00</td>
<td>$1.50</td>
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Traditional Lighting Value

<table>
<thead>
<tr>
<th>Year</th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Shipments ($)</td>
<td>$5.00</td>
<td>$4.00</td>
<td>$3.50</td>
<td>$3.00</td>
<td>$2.50</td>
<td>$2.00</td>
</tr>
</tbody>
</table>

- Electric Light Bulb
- Residential Lighting
- Commercial Lighting
- Other Lighting
How Illumination Design: Software

- **FRED - Photon Engineering**
- **TracePro - Lambda Research Corporation**
  - [http://www.lambdares.com/](http://www.lambdares.com/); also market OSLO
- **LightTools - Optical Research Associates**
  - [http://www.opticalres.com/](http://www.opticalres.com/); also market Code V
- **ASAP - Breault Research Organization, Inc.**
  - [http://www.breault.com](http://www.breault.com)
- There are others:
  - Optics programs: Solstis, OptiCAD, Zemax
  - Lighting programs: Photopia, LucidShape
  - Rendering programs: POVray
What is Illumination Engineering?

The Basics
Basis of Illumination Design

Take radiation from the source through an optical system to the target

**Optical design goals:**
- High transfer efficiency
- Distribution at target:
  - Spatial: Irradiance (W/m²) or Illuminance (lm/m²)
  - Angular: Radiant (W/sr) or Luminous (lm/sr) Intensity
  - Spatial-Angular: Radiance (W/m²/sr) or Luminance (lm/m²/sr)

**Other “important” demands:**
- Low system cost
- Low required volume
- Appearance, lit and unlit
- Color of light at target
- Electrical requirements
- Safety/Green
Illumination Optic Types
Reflectors in Illumination

Law of Reflection
\[ r'' = r - 2a \cos \theta \]
(Note - as law of refraction: \( n' = -n \))

How the software handles reflection
\[ L'' = L - 2a_L \cos \theta \]
\[ M'' = M - 2a_M \cos \theta, \text{ and} \]
\[ N'' = N - 2a_N \cos \theta. \]

Freeform Reflective Headlamp

Imaging shapes are conics – not optimal
Nonimaging shapes based on conics
Freeform, Tailored, Faceted can improve performance

From A. Gupta and K.J. Koshel, Handbook of Optics, Vol. II, 3rd Ed., M. Bass, Ed., 2009, Fig. 40.34.
Lenses in Illumination

Law of Refraction
\[ n'r' = n'r + (n'\cos \theta' - n\cos \theta)a \]

Snell’s Law
\[ n'\sin \theta' = n\sin \theta \]

How the software handles refraction
\[ n'L' = nL + (n'\cos \theta' - n\cos \theta)a_L, \]
\[ n'M' = nM + (n'\cos \theta' - n\cos \theta)a_M, \]
\[ n'N' = nN + (n'\cos \theta' - n\cos \theta)a_N. \]

Freeform Lens as SOE

Refractors are common in the field of illumination
Standard lenses are not optimal – go to freeform
Best imaging: F/1, microscope, Luneberg lens

Lightpipes in Illumination

Critical Angle
\[ \theta_c = \arcsin\left(\frac{n'}{n}\right) \]

Law of Reflection
\[ r'' = r - 2a \cos \theta \]

How the software handles TIR
\[ L'' = L - 2a_L \cos \theta \]
\[ M'' = M - 2a_M \cos \theta, \text{ and} \]
\[ N'' = N - 2a_N \cos \theta. \]

Uses TIR to transport light from the source to the target
Number of freeform optics are being made with TIR
Hybrid optics typically incorporate TIR along with Refraction &/or Reflection
Limited design methods, but improving
Diffusers in Illumination

How the software handles Scatter

$$BSDF(\theta_i, \phi_i; \theta_{sc}, \phi_{sc}) = \frac{L(\theta_i, \phi_i; \theta_{sc}, \phi_{sc})}{E(\theta_i, \phi_i)} s r^{-1}$$

Total Integrated Scatter

$$TIS = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} BSDF(\theta, \phi) \cdot \sin \theta \cdot \cos \theta \cdot d\theta \cdot d\phi$$

Anisotropic Diffuser

Occurs in Reflection, Transmission, Volume

Tool often used to solve Hot or Cold spots
Hides the interior of the optic from inspection
Lossy method due to back scatter and out scatter out of region of interest

LabSphere Integrating Sphere

From R. Pfisterer, Stray light short course, Photon Engineering.
## Radiometric Units

<table>
<thead>
<tr>
<th>Term and Description</th>
<th>Symbol</th>
<th>Functional Form</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiant Energy</td>
<td>$Q_e$</td>
<td></td>
<td>J</td>
</tr>
<tr>
<td>Radiant Energy Density</td>
<td>$u_e$</td>
<td>$\frac{dQ_e}{dV}$</td>
<td>J/m³</td>
</tr>
<tr>
<td>Radiant Flux / Power</td>
<td>$\Phi_e$ or $P_e$</td>
<td>$\frac{dQ_e}{dt}$</td>
<td>J/s or W</td>
</tr>
<tr>
<td>Radiant Exitance</td>
<td>$M_e$</td>
<td>$\frac{d\Phi_e}{dA_{source}}$</td>
<td>W/m²</td>
</tr>
<tr>
<td>Irradiance</td>
<td>$E_e$</td>
<td>$\frac{d\Phi_e}{dA_{target}}$</td>
<td>W/m²</td>
</tr>
<tr>
<td>Radiant Intensity</td>
<td>$I_e$</td>
<td>$\frac{d\Phi_e}{d\Omega}$</td>
<td>W/sr</td>
</tr>
<tr>
<td>Radiance</td>
<td>$L_e$</td>
<td>$\frac{d^2\Phi_e}{dA_{proj}d\Omega}$</td>
<td>W/m²/sr</td>
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### Photometric Units

<table>
<thead>
<tr>
<th>Term and Description</th>
<th>Symbol</th>
<th>Functional Form</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminous Energy</td>
<td>( Q_\nu )</td>
<td></td>
<td>( \text{T} )</td>
</tr>
<tr>
<td>Luminous Energy Density</td>
<td>( u_\nu )</td>
<td>( \frac{dQ_\nu}{dV} )</td>
<td>( \text{T/m}^3 )</td>
</tr>
<tr>
<td>Luminous Flux / Power</td>
<td>( \Phi_\nu ) or ( P_\nu )</td>
<td>( \frac{dQ_\nu}{dt} )</td>
<td>( \text{lm} )</td>
</tr>
<tr>
<td>Luminous Exitance</td>
<td>( M_\nu )</td>
<td>( \frac{d\Phi_\nu}{dA_{\text{source}}} )</td>
<td>( \text{lx} )</td>
</tr>
<tr>
<td>Illuminance</td>
<td>( E_\nu )</td>
<td>( \frac{d\Phi_\nu}{dA_{\text{target}}} )</td>
<td>( \text{lx} )</td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>( I_\nu )</td>
<td>( \frac{d\Phi_\nu}{d\Omega} )</td>
<td>( \text{cd} )</td>
</tr>
<tr>
<td>Luminance</td>
<td>( L_\nu )</td>
<td>( \frac{d^2\Phi_\nu}{dA_{\text{source}} d\Omega} )</td>
<td>( \text{nt} )</td>
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</table>
Other Photometric Units

<table>
<thead>
<tr>
<th>Unit</th>
<th>Abbreviation</th>
<th>Form</th>
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<tbody>
<tr>
<td>Illuminance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>foot-candle</td>
<td>fc</td>
<td>lm/ft²</td>
</tr>
<tr>
<td>phot</td>
<td>ph</td>
<td>lm/cm²</td>
</tr>
<tr>
<td>Luminance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>apostilb</td>
<td>asb</td>
<td>cd/π/m²</td>
</tr>
<tr>
<td>foot-lambert</td>
<td>fl</td>
<td>cd/π/ft²</td>
</tr>
<tr>
<td>lambert</td>
<td>L</td>
<td>cd/π/cm²</td>
</tr>
<tr>
<td>stilb</td>
<td>sb</td>
<td>cd/cm²</td>
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</table>
Other Terminology in Radiometry

**Solid Angle**

\[ d\Omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi \]

\[ \Omega = 4\pi \sin^2 \frac{\theta_0}{2} \]

**Lambertian**

\[ L(\mathbf{r}, \theta, \phi) = L(\mathbf{r}) = L_s \]

**Isotropic**

\[ L(\mathbf{r}, \theta, \phi) = \frac{L(\mathbf{r})}{\cos \theta} = \frac{L_s}{\cos \theta} \]

**Projected Area**

\[ dA_{s, \text{proj}} = dA_s \cos \theta \]

**Projected Solid Angle**

Analogous to Projected Area, one can project the solid angle

\[ d\Omega_{\text{proj}} = d\Omega \cos \theta \]
Intensity

\[ \int \frac{d^2 \Phi}{d\Omega} = \int dI = \int_{\Omega} L_s \cos \theta dA \]

- Integrating provides:
  \[ I = \cos \theta \int_{\Omega} L_s dA = I_s \cos \theta, \]
  where \( D \) is the surface of the source and \( I_s = \int_{\Omega} L_s dA \)

- For a Lambertian, uniform emitter: \( I = A_s L_s \cos \theta \)

- For an Isotropic, uniform emitter: \( I_{iso} = A_s L_s \)
Irradiance and Illuminance

- A detector area subtends an elemental solid angle:
  \[ d\Omega = \frac{dA_{\text{proj}}}{r^2} = \frac{dA \cos \theta}{r^2} \]

- Using the Expression for Intensity:
  \[ E = \frac{d\Phi}{dA} = \frac{d\Phi \cos \theta}{r^2 d\Omega} = \frac{I \cos \theta}{r^2} \]

- Lambertian Exitance:
  \[ M = L_s \int_0^{\pi/2} \int_0^{2\pi} \cos \theta \sin \theta d\theta d\phi \]

  \[ = M_{\text{lam}} = \pi L_s, \]

- Isotropic Exitance:
  \[ M_{\text{iso}} = 2\pi L_s \]
Connection of Photometric and Radiometric

- Radiometric to photometric:

\[ f_v = h(\lambda)K(\lambda)f_e = Ch(\lambda)V(\lambda)f_e \]

  - \( f_v \) = Photometric quantity (e.g., illuminance)
  - \( f_e \) = Radiometric quantity (e.g., irradiance)
  - \( K(\lambda) \) = Luminous efficacy
  - \( V(\lambda) \) = Luminous Efficiency
  - \( h(\lambda) \) = filter function

- If you have a spectral bandwidth:

\[ f_v = \int_{0}^{\infty} h(\lambda)CV(\lambda)f_e(\lambda)d\lambda \]

  - \( f_e(\lambda) \) is in units per wavelength

Day peak (photopic) = 555 nm with C = 683 lm/W
Night peak (scotopic) = 507 nm with C = 1700 lm/W
Note: actually it makes no sense to discuss lumens for scotopic vision
Combined vision (mesopic) = contributions from both photopic and scotopic dependent on light level
Étendue

- French Word:
  - Verb: extended
  - Noun: reach
- Étendue is a geometric factor:
  \[ \mathcal{E} = n^2 \int \! \int \cos \theta dA_s d\Omega \]
- It describes the flux propagation characteristics of an optical system:

  **Arbitrary Source Radiance**
  \[ \Phi = \int \! \int L(r, \hat{a}) \cos \theta dA_s d\Omega, \]

  **Lambertian Source Radiance**
  \[ \Phi = L_s \int \! \int \cos \theta dA_s d\Omega \]
  \[ = \frac{L_s \mathcal{E}}{n^2} \]
Conservation of Étendue

• Étendue is conserved in a lossless optical system.
• By “giving up” étendue one reduces the flux transmission characteristics of the system.

• Number of proofs:
  - Conservation of Radiance: shown here.
  - Conservation of Generalized Étendue: shown here.
Proof: Conservation of Radiance

- Homogeneous Medium:
  \[
  L_s = \frac{d^2 \Phi_s}{\cos \theta_s dA_s d\Omega_s} \\
  L_t = \frac{d^2 \Phi_t}{\cos \theta_t dA_t d\Omega_t}
  \]
- Lossless - \( \Phi_s = \Phi_t \):
  \[
  \frac{L_s}{L_t} = \frac{\cos \theta_t dA_t (dA_s \cos \theta_s / r^2)}{\cos \theta_s dA_s (dA_t \cos \theta_t / r^2)} = 1
  \]
- Conservation of Radiance:
  \[
  L_s = L_t
  \]
- Upon Refraction:
  \[
  L' = L \frac{\cos \theta \partial \Omega}{\cos \theta' \partial \Omega'}
  \]
- Solid Angle Ratio:
  \[
  \frac{d\Omega}{d\Omega'} = \frac{\sin \theta \partial \theta \partial \phi}{\sin \theta' \partial \theta' \partial \phi'} = \frac{n'^2 \cos \theta'}{n^2 \cos \theta}
  \]
- Radiance Theorem:
  \[
  \frac{L}{n^2} = \frac{L'}{n'^2}
  \]
- Conservation of Étendue:
  \[
  d^2 \Phi = \frac{L_s}{n^2} d^2 \xi
  \]
  \[
  \xi = \iint d^2 \xi
  \]
Proof: Generalized Étendue

- At $P$ and $P'$:
  \[ \nabla V(r) = -ns \quad \text{and} \quad \nabla V(r') = n's', \]
- Ray Components:
  \[
  \begin{pmatrix}
    V_{xx} & V_{xy} & 1 & 0 & dx' \\
    V_{yx} & V_{yy} & 0 & 1 & dy' \\
    V_{xx}' & V_{xy}' & 0 & 0 & dp' \\
    V_{yx}' & V_{yy}' & 0 & 1 & dq'
  \end{pmatrix} = \begin{pmatrix}
    -V_{xx} & -V_{xy} & 0 & 0 & dx' \\
    -V_{yx}' & -V_{yy}' & 0 & 0 & dy' \\
    -V_{xx}' & -V_{xy}' & 1 & 0 & dp' \\
    -V_{yx}' & -V_{yy}' & 0 & 1 & dq'
  \end{pmatrix}
  \]
- Matrix Form:
  \[ Aw = Bw' \rightarrow w' = Jw \rightarrow J = B^{-1}A \]
- Jacobian:
  \[ dx'dy'dp'dq' = \begin{vmatrix} A \\ B \end{vmatrix} dx dy dp dq \]
- Determinants:
  \[
  \begin{vmatrix} A \\ B \end{vmatrix} = V_{x'x}V_{y'y} - V_{x'y}V_{y'x},
  \begin{vmatrix} A \\ B \end{vmatrix} = V_{xx}'V_{yy'} - V_{xy}'V_{yx'}
  \]
Analogies for Étendue

- Radiance: associated with source emission.
- Brightness: Luminance but based on actual observer.
- Throughput: flux transmission capabilities of an imaging system.
- Extent: Direct analogy for étendue.

- Lagrange Invariant:
  \[ H = nhu = n'h'u' \]
  \[ H = n(\overline{yu} - \overline{uy}) = n'(\overline{yu}' - \overline{u'y}) \]

- Abbe Sine Condition:
  \[ nh \sin \theta = n'h' \sin \theta' \]
What Does Étendue Mean?

- Simply:
  - Large Area - Small Angle
  - Small Area - Large Angle
- Example - consider a flared lightpipe:
  - Case 1: Flares at input end
  - Case 2: Flares at output end

1: Output
2: Input

1: Input
2: Output
Examples

Illustrate the use of Etendue
Three Examples to Consider

Case I

Spatial: Uniform
Angular: Lambertian

Case II

Spatial: Uniform
Angular: Non Lambertian

Case III

Spatial: Non uniform
Angular: Non Lambertian
Case I: Uniform, Lambertian Disk

- Integrate the Etendue equation:
  \[ \xi_s(\theta_a) = n^2 A_s \int_0^{2\pi} \int_0^{\theta_a} \cos \theta \sin \theta d\theta d\phi = \pi n^2 A_s \sin^2 \theta_a \]
  - Full area of the source: \( A_s \) and
  - Angular range of \( \theta \in [-\theta_a, \theta_a] \)
- We can find the flux expressing with the radiance \( L_s \) and source radius \( r_s \):
  \[ \Phi = \pi L_s A_s \sin^2 \theta_a = \pi^2 L_s r_s^2 \sin^2 \theta_a \]
- The total flux is found when \( \theta_a = \pi/2 \):
  \[ \Phi(\theta_a = \pi/2) = \Phi_{opt} = \pi L_s A_s = \pi^2 L_s r_s^2 \]
- Suppose we have a system with a FOV of \( \theta_a \) and can capture the full spatial extent:
  \[ \eta(\theta_a) = \frac{\Phi(\theta_a)}{\Phi_{opt}} = \frac{\xi_s(\theta_a)}{\xi_{opt}} = \sin^2 \theta_a \]
Case I: Acceptance Angle Plot
Aside: Better Way to Plot

- We have assumed that the full spatial extent of the source perfectly matches the field of view of the optical system:

\[ \eta(r, \theta_a) = \frac{\Phi(r, \theta_a)}{\Phi_{opt}} = \frac{\xi(r, \theta_a)}{\xi_{opt}} = \frac{\bar{\xi}(r, \theta_a) = \frac{r^2 \sin^2 \theta_a}{r^2_s}}{= \bar{r}^2 \sin^2 \theta_a} \]

- We have used a Lambertian angular distribution and uniform spatial distribution, which do not happen readily in real systems
  - Cases II and III show the limitation of this assumption
  - Homework problem #1 also tackles this issue

- There are heretofore undeveloped limitations in the transfer efficiency of systems whose cross-sectional shape evolves with propagation
  - Skew invariance - next lecture
  - Homework problem #2 tackles this issue

- **Transfer Efficiency versus Fractional Etendue Plots!**
Case I: Fractional Etendue Plot – Constant \( r \)

Spatial: \( r \) is constant
Angular: \( \theta_a \) varies

\( \theta_a \) varies to maintain conservation of etendue

Maximum value of \( \theta_a \) is \( \pi/2 \)

\[
\eta(\xi; \theta_a) = \begin{cases} 
\frac{\xi}{\bar{r}^2}, & \xi \in [0, \bar{r}^2] \\
\frac{\bar{r}^2}{\bar{r}^2}, & \xi \in [\bar{r}^2, 1]
\end{cases}
\]

where

\[
\bar{r} = r/r_s
\]
Case I: Fractional Etendue Plot – Constant $\theta_a$

Spatial: $r$ varies
Angular: $\theta_a$ constant

$r$ varies to maintain conservation of etendue

Maximum value of $r$ is $r_s$

$$\eta(\xi; r) = \begin{cases} \xi, & 0 \leq \xi \leq \sin^2 \theta_a \\ \sin^2 \theta_a, & \sin^2 \theta_a \leq \xi \leq 1 \end{cases}$$
Case II: Uniform, Isotropic Disk

- The radiance is given by:
  \[
  L(r, \theta_a, \phi) = \frac{L(r)}{\cos \theta_a} = \frac{L_s}{\cos \theta_a}
  \]

- We find the transfer efficiency:
  \[
  \eta(R, \theta_a) = \frac{\Phi(R, \theta_a)}{\Phi_{opt}} = \frac{r^2}{r_s^2} (1 - \cos \theta_a) = \frac{\bar{R}^2 (1 - \cos \theta_a)}{\xi \sin^2 \theta_a}
  \]
  - Where we used conservation of etendue in the last equality:
  \[
  \frac{\xi(r, \theta_a)}{\xi_{opt}} = \frac{\bar{\xi}(r, \theta_a)}{r_s^2} = \bar{r}^2 \sin^2 \theta_a
  \]

- Constant acceptance angle:
  \[
  \eta(\bar{\xi}; \theta_a) = \begin{cases}
  \frac{\bar{\xi} (1 - \cos \theta_a)}{\sin^2 \theta_a}, & \bar{\xi} \in [0, \sin^2 \theta_a] \\
  1 - \cos \theta_a, & \bar{\xi} \in (\sin^2 \theta_a, 1]
  \end{cases}
  \]

- Constant radius:
  \[
  \eta(\bar{\xi}; \theta_a) = \begin{cases}
  \frac{\bar{\xi} (1 - \cos \theta_a)}{\sin^2 \theta_a}, & \bar{\xi} \in [0, \bar{R}^2] \\
  \bar{R}^2, & \bar{\xi} \in (\bar{R}^2, 1]
  \end{cases}
  \]
Case II: Fractional Etendue Plot – Constant \( r \)

Spatial: \( r \) is constant
Angular: \( \theta_a \) varies

\( \theta_a \) varies to maintain conservation of etendue

Maximum value of \( \theta_a \) is \( \pi/2 \):

\[
\eta(\bar{\xi}; \theta_a) = \begin{cases} 
\frac{(1 - \cos \theta_a)}{\sin^2 \theta_a}, & \bar{\xi} \in [0, \bar{r}^2] \\
\frac{\bar{r}^2}{\bar{r}^2}, & \bar{\xi} \in (\bar{r}^2, 1]
\end{cases}
\]

Use conservation to get in better form:

\[
\eta(\bar{\xi}; \theta_a) = \begin{cases} 
\bar{r}^2 \left(1 - \sqrt{1 - \frac{\bar{\xi}}{\bar{r}^2}}\right), & \bar{\xi} \in [0, \bar{r}^2] \\
\bar{r}^2, & \bar{\xi} \in (\bar{r}^2, 1]
\end{cases}
\]
Case II: Fractional Etendue Plot – Constant $\theta_a$

Spatial: $r$ varies
Angular: $\theta_a$ constant

$r$ varies to maintain conservation of etendue

Maximum value of $r$ is $r_s$

$$\eta(\bar{\xi};r) = \begin{cases} \bar{\xi} \frac{1 - \cos \theta_a}{\sin^2 \theta_a}, & \bar{\xi} \in \left[0, \sin^2 \theta_a\right] \\ 1 - \cos \theta_a, & \bar{\xi} \in \left(\sin^2 \theta_a, 1\right) \end{cases}$$
Case III: Non Uniform, Isotropic Disk

- The radiance is given by:

\[
L(r, \theta_a) = L_s \frac{(1 - r/r_s)}{\cos \theta_a} = L_s \frac{(1 - \bar{r})}{\cos \theta_a}
\]

- This is homework problem #1!
- Steps to solve:
  - Find the transfer efficiency as a function of fractional \( r \) and \( \theta_a \)
  - Use conservation of etendue to remove the radius or acceptance angle to keep only constant terms in the equation
    - Radius constant: remove the acceptance angle terms
    - Acceptance angle constant: remove the radius terms
  - Now you will have the transfer efficiency as a function of fractional etendue and the one constant term!
  - Plot
Discussion

- Adding source complexity (aka realism):
  - Calculations are harder
  - Give better approximations to transfer efficiency / flux at target
- Disk source geometry:
  - Simple “enough” and “illustrative”
  - Not realistic for an emitter by itself
  - May be used to approximate the output aperture of an emitter coupled to an optic (however, you should do etendue calculation through the optic!)
- Disk source distribution:
  - Angular: isotropic is not typical, Lambertian is better
  - Spatial: non uniform is typical
- More realistic source?
  - Geometry: how about a tube to approximate arc or filament
  - Distribution: uniform and Lambertian
The Skew Invariant

- On a Piecewise Basis Étendue Plays Crucial Role in Determination of Output Distribution.
  - Skewness Distribution Dependent on Étendue:

\[ f_{\text{skew}}(s) = \frac{d\varepsilon(s)}{ds} \text{ where } s = r_{\text{min}}k_t \]

- Skewness Distribution Dependent on Étendue:

\[ f_{\text{skew}}(s) = \frac{d\varepsilon(s)}{ds} \text{ where } s = r_{\text{min}}k_t \]

- System Symmetry Must be Broken to Overcome the Skew Invariant.
Design Methods

- **Number of Current Design Methods:**

- New Algorithms (and Improvements) All the Time!
- **Caveat:** Poor Source Model Leads to Poor Design!