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8 - 19 February 2010

Lighting and illumination engineering
I. Introduction

J. Koshel

The Univ. of Arizona & Photon Engineering, LLC
U.S.A.

Lighting and illumination engineering
I. Introduction

15 February 2010

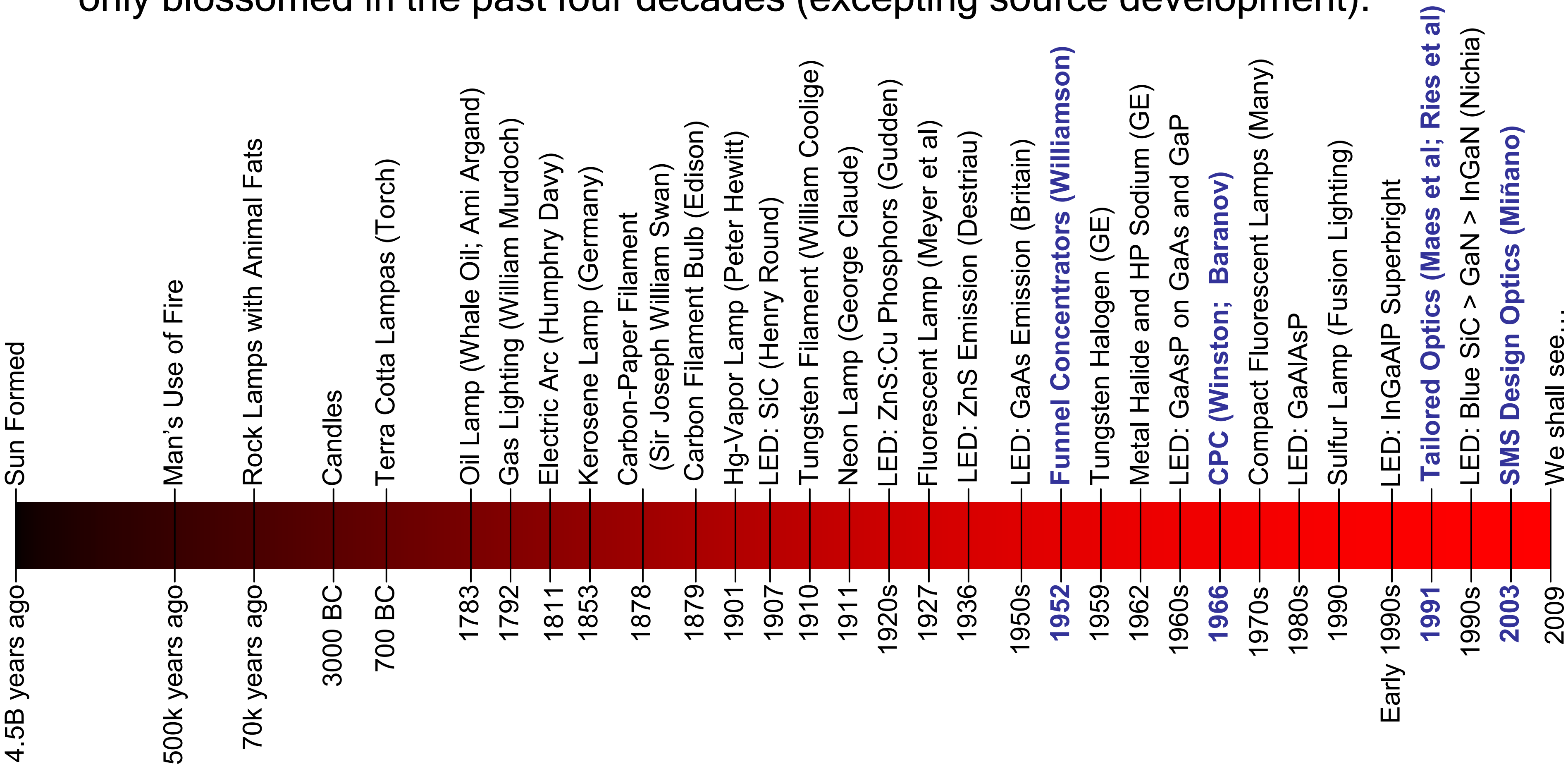
John Koshel
College of Optical Sciences – The Univ. of Arizona &
Photon Engineering, LLC

Where is Illumination Design?

- Academia:
  - Universidad Politécnica de Madrid (Spain)
  - Philips-University Marburg (Germany)
  - Ben-Gurion University of the Negev (Israel)
  - University of California/Merced (USA)
  - Lighting Research Center at RPI (USA)
- Industry:
  - Light Prescription Innovators (USA, Spain, PacRim)
  - Optics and Energy Concepts (Germany)
  - Software Companies (USA & Europe): Photon Engineering (FRED), ORA (LightTools), BRO (ASAP), LRC (TracePro), Optis (Solstis); ZDC (ZEMAX)
  - Large Companies (worldwide): 3M, SAIC, Visteon, Philips, LumiLeds
  - Small Companies (worldwide): Tailored Optics, Wavien
  - Consultants (worldwide)
- Government:
  - USA: Sandia, DARPA, NREL, LBL, DOE
  - World: Korea, Japan, Europe, Taiwan, Israel

When Illumination Design?

Illumination design has a rich history, but as a scientific / engineering discipline it has only blossomed in the past four decades (excepting source development).



Why Illumination Design?

- The Illumination Industry is Large: $100B+ worldwide.

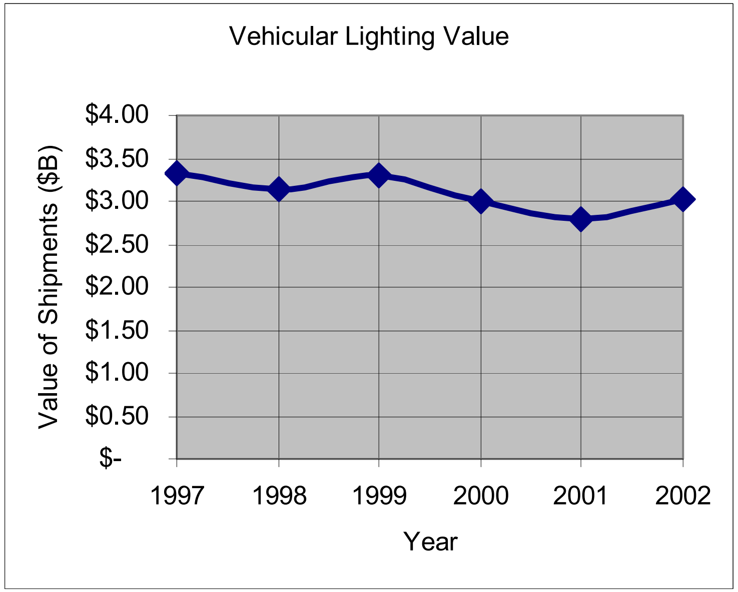
- Energy Costs are Increasing:
  - Electricity for all Lighting: 20% (1993; http://www.eia.doe.gov/emeu/recs/recs4a.html)
  - Electricity for Residential Lighting: 9.4% (1993; http://www.eia.doe.gov/emeu/recs/recs4a.html)
  - Wasted Lighting: ~25% unused
  - Light Pollution: $1B/yr lights sky (Batinsey, ANJEC Report 1994)

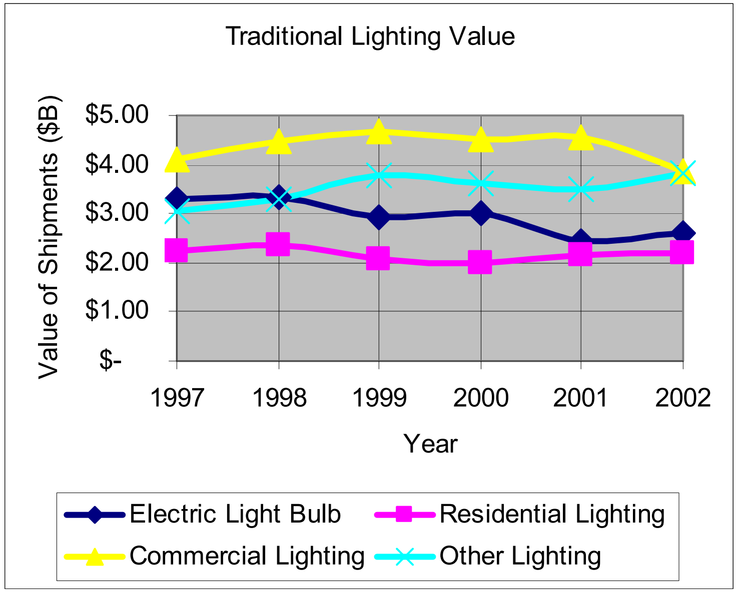
- There are many standards bodies dedicated to illumination requirements.

The Illumination Industry

- Displays (iSuppli, May 2005):
  - TFT-LCD: 59M units ('04) = $29.5B, 94.2M units ('05) = $47.1B
  - OLED: $408M ('05), $2.9B ('11)
- High-Brightness LEDs:
  - Strategies Unlimited (8/05): $3.7B ('04), $6.8B ('09)
  - iSuppli (8/05): $5.2B ('05); SSL: $144M ('05), $875M ('10)
- Data from US Census Bureau from 2005 Report:

There is also solar energy generation – we make the power that then we use for providing light



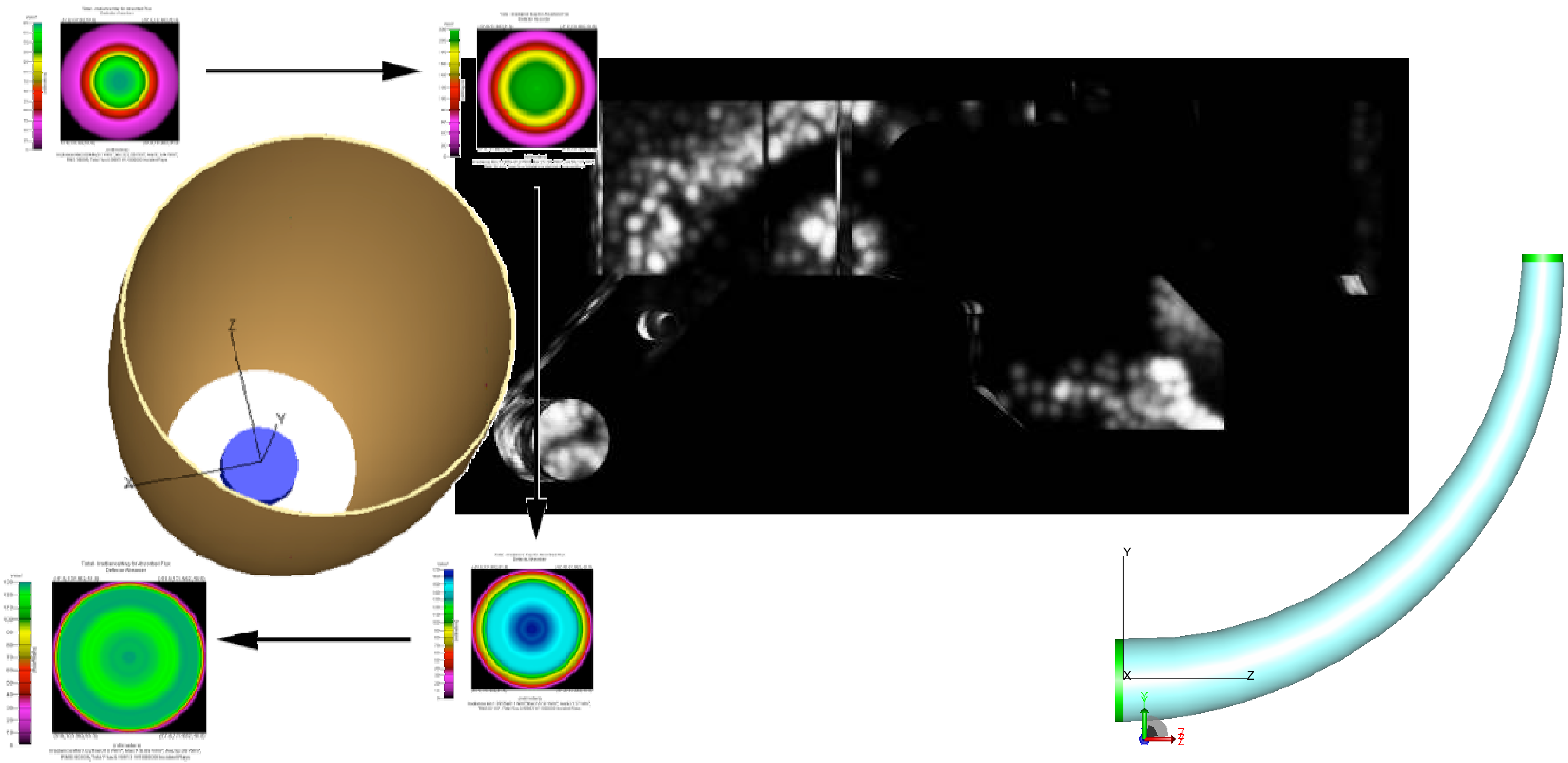


From http://www.census.gov/econ/census02/

How Illumination Design: Software

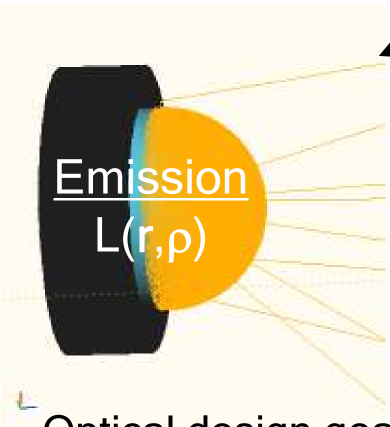
- FRED – Photon Engineering
  - http://www.photonengr.com/
- TracePro – Lambda Research Corporation
  - http://www.lambdares.com/; also market OSLO
- LightTools – Optical Research Associates
  - http://www.opticalres.com/; also market Code V
- ASAP – Breault Research Organization, Inc.
  - http://www.breault.com
- There are others:
  - Optics programs: Solstis, OptiCAD, Zemax
  - Lighting programs: Photopia, LucidShape
  - Rendering programs: POVray

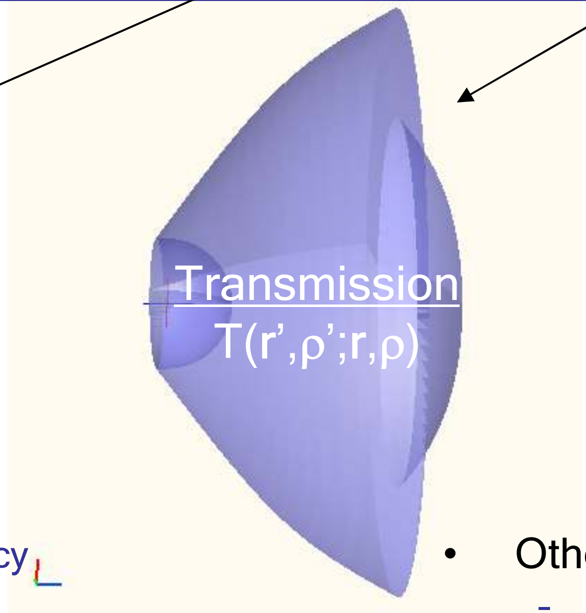
What is Illumination Engineering?
The Basics

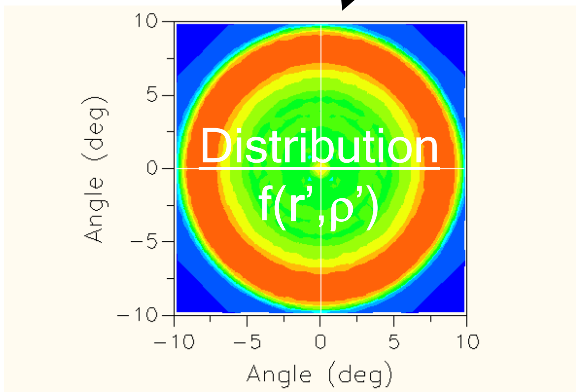


Basis of Illumination Design

Take radiation from the source through an optical system to the target



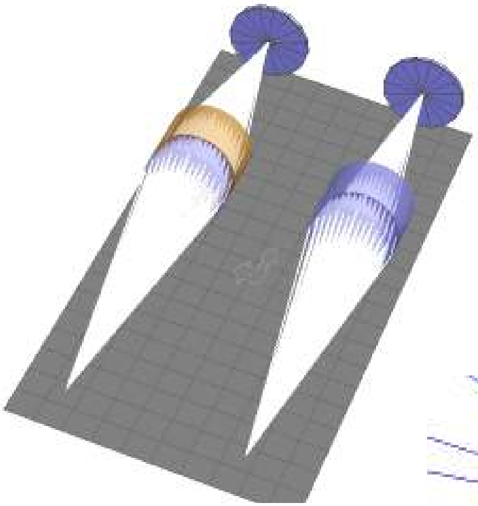


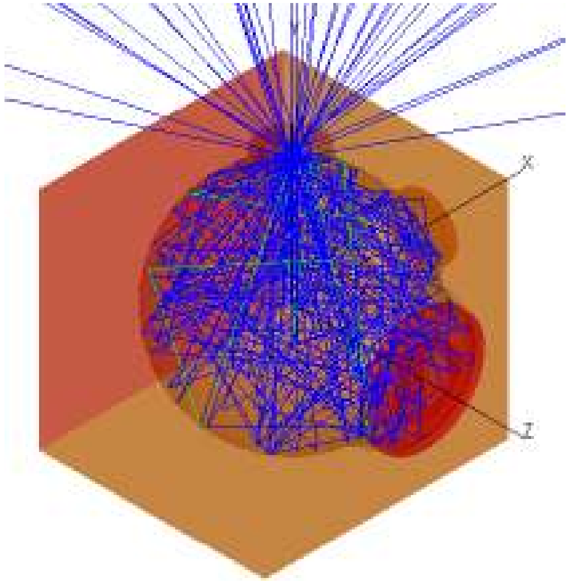


- Optical design goals:
  - High transfer efficiency
  - Distribution at target:
    - Spatial: Irradiance(W/m^2) or Illuminance (lm/m^2)
    - Angular: Radiant (W/sr) or Luminous (lm/sr) Intensity
    - Spatial-Angular: Radiance ($W/m^2/sr$) or Luminance ($lm/m^2/sr$)

- Other "important" demands:
  - Low system cost
  - Low required volume
  - Appearance, lit and unlit
  - Color of light at target
  - Electrical requirements
  - Safety/Green

Illumination Optic Types





Reflectors in Illumination

<u>Law of Reflection</u>

$$\mathbf{r}'' = \mathbf{r} - 2\mathbf{a}\cos\theta$$

(Note - as law of refraction: *n*’ = -*n*)

<u>How the software handles reflection</u>

$$L'' = L - 2a_L, \cos\theta$$

$$M'' = M - 2a_M, \cos\theta, \text{and}$$

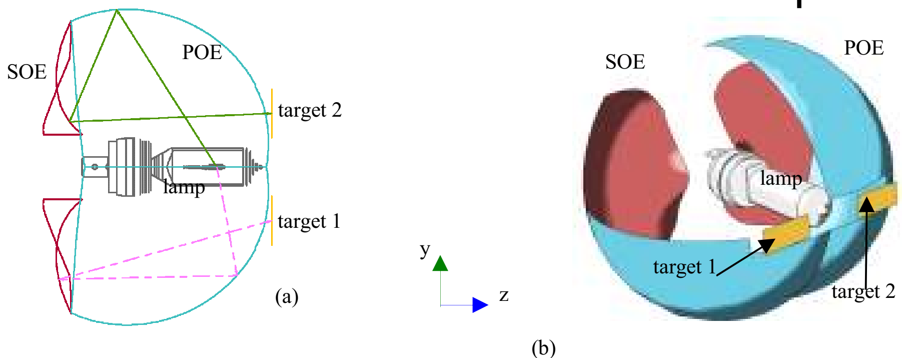
$$N'' = N - 2a_N, \cos\theta.$$

From A. Gupta and R. J. Koshel, *Handbook of Optics*, Vol. II, 3rd Ed., M. Bass, Ed., 2009, Fig. 40.34.

Headlamp from 2004 Honda Civic

Array / Facet

Freeform Reflective Headlamp



From J. C. Minano, *Advanced Nonimaging/Illumination Optics*, J. Koshel, Ed., to be published 2010.

Imaging shapes are conics – not optimal

Nonimaging shapes based on conics

Freeform, Tailored, Faceted can improve performance

Lenses in Illumination

From J. C. Minano, *Advanced Nonimaging/Illumination Optics*, J. Koshel, Ed., to be published 2010.

Freeform Lens as SOE

Law of Refraction

$$n'\mathbf{r}' = n\mathbf{r} + \left(n'\cos\theta' - n\cos\theta\right)\mathbf{a}$$

Snell's Law

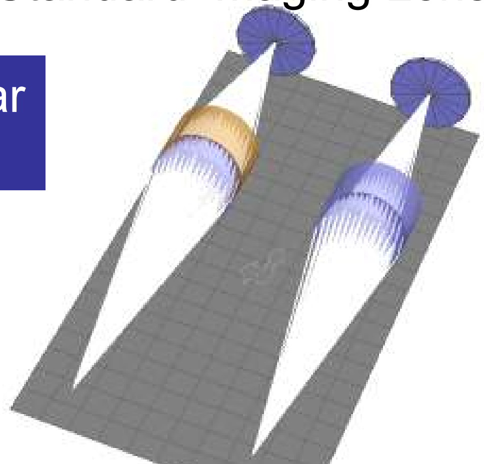
$$n'\sin\theta' = n\sin\theta$$

How the software handles refraction

$$n'L' = nL + \left(n'\cos\theta' - n\cos\theta\right)a_L,$$
$$n'M' = nM + \left(n'\cos\theta' - n\cos\theta\right)a_M, \text{and}$$
$$n'N' = nN + \left(n'\cos\theta' - n\cos\theta\right)a_N.$$

Array: Pillow or Lenticular
Facet: Fresnel

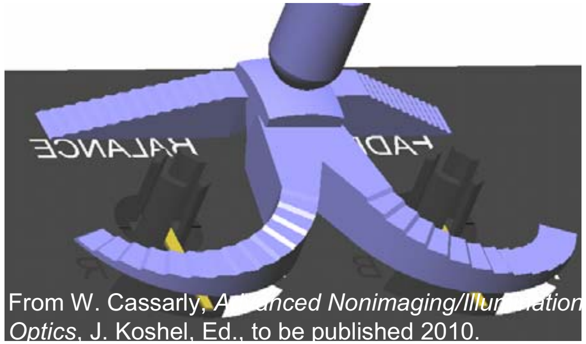
Standard Imaging Lens



Edmund Doublet: sample file from FRED

- Refractors are common in the field of illumination
- Standard lenses are not optimal – go to freeform
- Best imaging: F/1, microscope, Luneberg lens

Lightpipes in Illumination



From W. Cassarly, *Advanced Nonimaging/Illumination Optics*, J. Koshel, Ed., to be published 2010.

Lightpipe used in dashboard

Critical Angle

$$\theta_c = \arcsin\left(\frac{n'}{n}\right)$$

Law of Reflection

$$\mathbf{r}'' = \mathbf{r} - 2\mathbf{a}\cos\theta$$

How the software handles TIR

$$L'' = L - 2a_L, \cos\theta$$
$$M'' = M - 2a_M, \cos\theta, \text{and}$$
$$N'' = N - 2a_N, \cos\theta.$$

- Uses TIR to transport light from the source to the target
- Number of freeform optics are being made with TIR
- Hybrid optics typically incorporate TIR along with Refraction &/or Reflection
- Limited design methods, but improving

Optical Sciences/Univ. of Arizona: Desert Bloom by Christopher Ries, Donated by Schott

Glass Sculpture

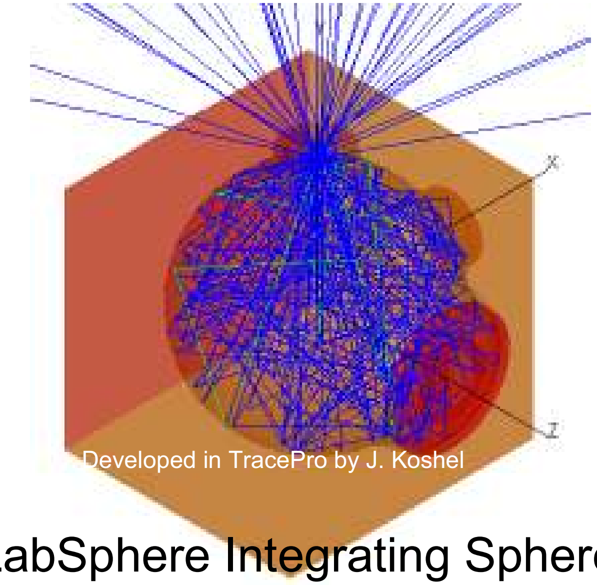
Diffusers in Illumination

How the software handles Scatter

$$BSDF(\theta_i,\phi_i;\theta_{sc},\phi_{sc}) \equiv \frac{L(\theta_i,\phi_i;\theta_{sc},\phi_{sc})}{E(\theta_i,\phi_i)} sr^{-1}$$

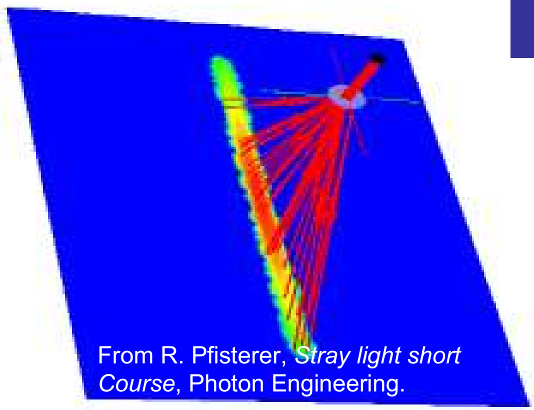
Total Integrated Scatter

$$TIS = \int_0^{2\pi} \int_0^{\pi/2} BSDF(\theta,\phi)\cdot\sin\theta\cdot\cos\theta\cdot d\theta\, d\phi$$



LabSphere Integrating Sphere

Anisotropic Diffuser



Occurs in Reflection, Transmission, Volume

Tool often used to solve Hot or Cold spots

Hides the interior of the optic from inspection

Lossy method due to back scatter and out scatter out of region of interest

Radiometric Units

Term and Description	Symbol	Functional Form	SI Units
Radiant Energy	Q_e		J
Radiant Energy Density *Radiant energy per unit volume*	u_e	$\dfrac{dQ_e}{dV}$	J/m^3
Radiant Flux / Power *Radiant energy per unit time*	Φ_e or P_e	$\dfrac{dQ_e}{dt}$	J/s or W
Radiant Exitance *Radiant flux per unit source area*	M_e	$\dfrac{d\Phi_e}{dA_{source}}$	W/m^2
Irradiance *Radiant flux per unit target area*	E_e	$\dfrac{d\Phi_e}{dA_{target}}$	W/m^2
Radiant Intensity *Radiant flux per unit solid angle*	I_e	$\dfrac{d\Phi_e}{d\Omega}$	W/sr
Radiance *Radiant flux per unit projected area per unit solid angle*	L_e	$\dfrac{d^2\Phi_e}{dA_{s,proj}d\Omega}$	W/m^2/sr

Photometric Units

Term and Description	Symbol	Functional Form	Units
Luminous Energy	Q_ν		T
Luminous Energy Density *Luminous energy per unit volume*	u_ν	$\frac{dQ_\nu}{dV}$	T/m^3
Luminous Flux / Power *Luminous energy per unit time*	Φ_ν or P_ν	$\frac{dQ_\nu}{dt}$	lm
Luminous Exitance *Luminous flux per unit source area*	M_ν	$\frac{d\Phi_\nu}{dA_{source}}$	lx
Illuminance *Luminous flux per unit target area*	E_ν	$\frac{d\Phi_\nu}{dA_{target}}$	lx
Luminous Intensity *Luminous flux per unit solid angle*	I_ν	$\frac{d\Phi_\nu}{d\Omega}$	cd
Luminance *Luminous flux per unit projected area per unit solid angle*	L_ν	$\frac{d^2\Phi_\nu}{dA_{s,proj}d\Omega}$	nt

Other Photometric Units

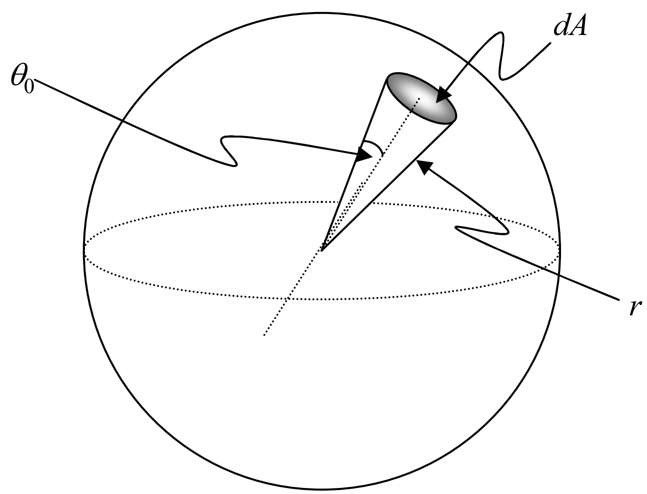
Unit	Abbreviation	Form
Illuminance		
foot-candle	fc	lm/ft^2
phot	ph	lm/cm^2
Luminance		
apostilb	asb	$cd/\ \pi/m^2$
foot-lambert	fL	$cd/\ \pi/ft^2$
lambert	L	$cd/\pi/cm^2$
stilb	sb	cd/cm^2

Other Terminology in Radiometry

Solid Angle

$$d\Omega = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

$$\Omega = 4\pi \sin^2 \frac{\theta_0}{2}$$



Lambertian

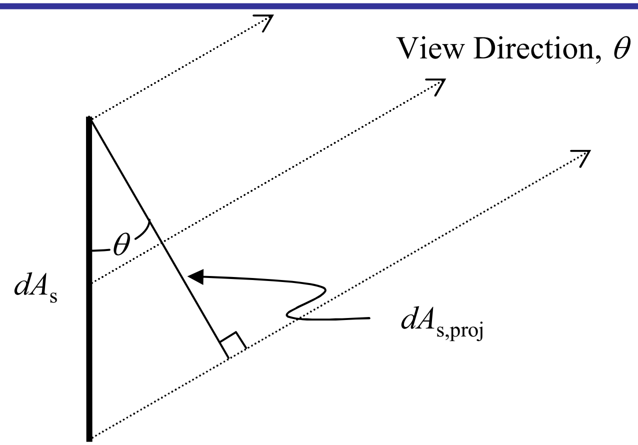
$$L(\mathbf{r},\theta,\phi) = L(\mathbf{r}) = L_s$$

Isotropic

$$L(\mathbf{r},\theta,\phi) = \frac{L(\mathbf{r})}{\cos\theta} = \frac{L_s}{\cos\theta}$$

Projected Area

$$dA_{s,proj} = dA_s \cos\theta$$



Projected Solid Angle

Analogous to Projected Area, one can project the solid angle

$$d\Omega_{proj} = d\Omega \cos\theta$$

Intensity

$$\int \frac{d^2\Phi}{d\Omega} = \int dI = \int_{\mathrm{D}} L_s \cos\theta dA$$

- Integrating provides:

$$\boxed{I = \cos\theta \int_D L_s dA = I_s \cos\theta,}$$

where D is the surface of the source and $I_s = \int_D L_s dA$

- For a Lambertian, uniform emitter: $I = A_s L_s \cos\theta$

- For an Isotropic, uniform emitter: $I_{iso} = A_s L_s$

Irradiance and Illuminance

- A detector area subtends an elemental solid angle:

$$d\Omega = \frac{dA_{proj}}{r^2} = \frac{dA\cos\theta}{r^2}$$

- Using the Expression for Intensity:

$$E = \frac{d\Phi}{dA} = \frac{d\Phi\cos\theta}{r^2 d\Omega} = \frac{I\cos\theta}{r^2}$$

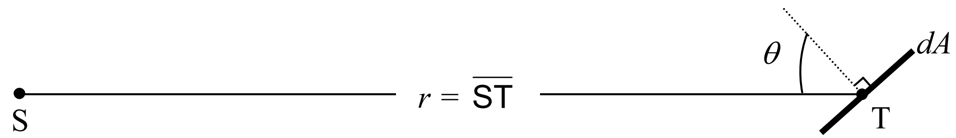
- Lambertian Exitance:

$$M = L_s \int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi$$

$$= M_{lam} = \pi L_s,$$

- Isotropic Exitance:

$$M_{iso} = 2\pi L_s$$



Connection of Photometric and Radiometric

- Radiometric to photometric:

$$\boxed{\begin{aligned} f_v &= h(\lambda)K(\lambda)f_e \\ &= Ch(\lambda)V(\lambda)f_e \end{aligned}}$$

  - f_v = Photometric quantity (e.g., illuminance)
  - f_e = Radiometric quantity (e.g., irradiance)
  - $K(\lambda)$ = Luminous efficacy
  - $V(\lambda)$ = Luminous Efficiency
  - $h(\lambda)$ = filter function
- If you have a spectral bandwidth:

$$\boxed{f_v = \int_0^\infty h(\lambda)CV(\lambda)f_e(\lambda)d\lambda}$$

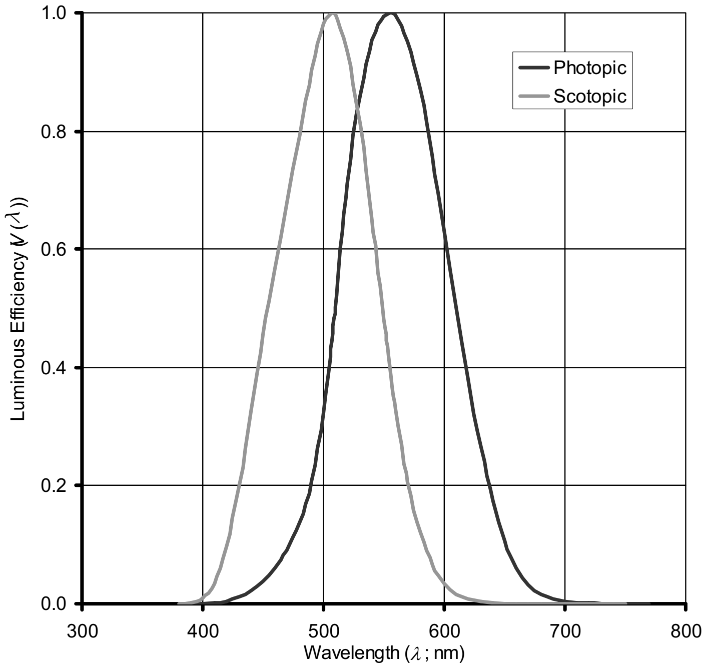
  - $f_e(\lambda)$ is in units per wavelength

Day peak (photopic) = 555 nm with C = 683 lm/W

Night peak (scotopic) = 507 nm with C = 1700 lm/W

Note: actually it makes no sense to discuss lumens for scotopic vision

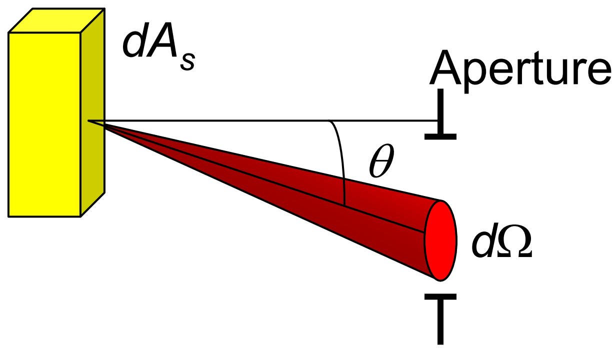
Combined vision (mesopic) = contributions from both photopic and scotopic dependent on light level



Étendue

- French Word:
  - Verb: extended
  - Noun: reach
- Étendue is a geometric factor:

$$\boxed{\mathcal{E} = n^2 \iint_{\text{aperture}} \cos\theta \, dA_s \, d\Omega}$$



- It describes the flux propagation characteristics of an optical system:

<u>Arbitrary Source Radiance</u>

$$\Phi = \iint_{\text{aperture}} L(\mathbf{r}, \hat{\mathbf{a}}) \cos\theta \, dA_s \, d\Omega,$$

<u>Lambertian Source Radiance</u>

$$\Phi = L_s \iint_{\text{aperture}} \cos\theta \, dA_s \, d\Omega$$

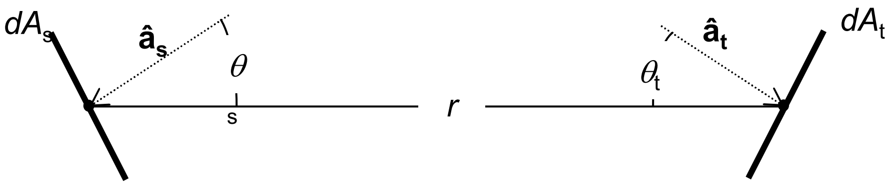
$$= \frac{L_s \mathcal{E}}{n^2}$$

Conservation of Étendue

- Étendue is conserved in a lossless optical system.
- By “giving up” étendue one reduces the flux transmission characteristics of the system.

- Number of proofs:
  - Laws of thermodynamics: Boyd, Radiometry…, Wiley pp. 77-79 (1983).
  - Liouville’s Theorem in statistical mechanics: Winston et al, Nonimaging Optics, Elsevier Academic Press, Appendix A.3 (2005).
  - Stokes’ Theorem applied to Hamiltonian systems: Miñano and Benítez, SPIE Proc. 5529, pp. 87-95 (2004).
  - Conservation of Radiance: shown here.
  - Conservation of Generalized Étendue: shown here.

Proof: Conservation of Radiance



- Homogeneous Medium:

$$\frac{L_s}{L_t} = \frac{d^2\Phi_s \big/ \cos\theta_s dA_s d\Omega_s}{d^2\Phi_t \big/ \cos\theta_t dA_t d\Omega_t}$$

- Lossless - $\Phi_s = \Phi_t$:

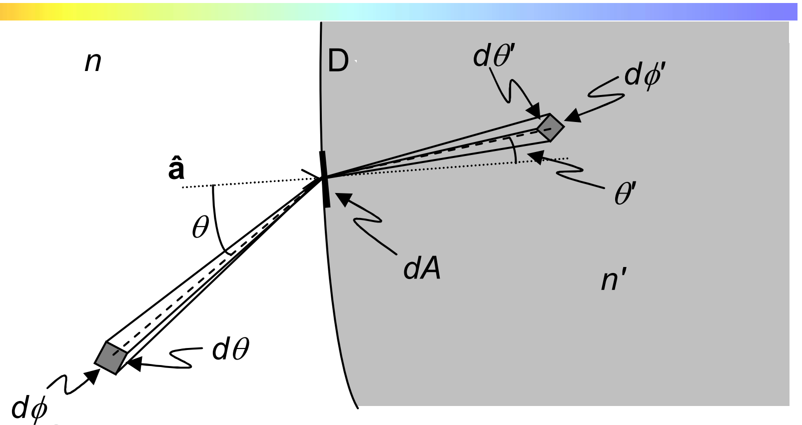
$$\frac{L_s}{L_t} = \frac{\cos\theta_t dA_t \left(dA_s \cos\theta_s \big/ r^2\right)}{\cos\theta_s dA_s \left(dA_t \cos\theta_t \big/ r^2\right)} = 1$$

- Conservation of Radiance:

$$\boxed{L_s = L_t}$$

- Upon Refraction:

$$L' = L\frac{\cos\theta d\Omega}{\cos\theta' d\Omega'}$$



- Solid Angle Ratio:

$$\frac{d\Omega}{d\Omega'} = \frac{\sin\theta d\theta d\phi}{\sin\theta' d\theta' d\phi'} = \frac{n'^2 \cos\theta'}{n^2 \cos\theta}$$

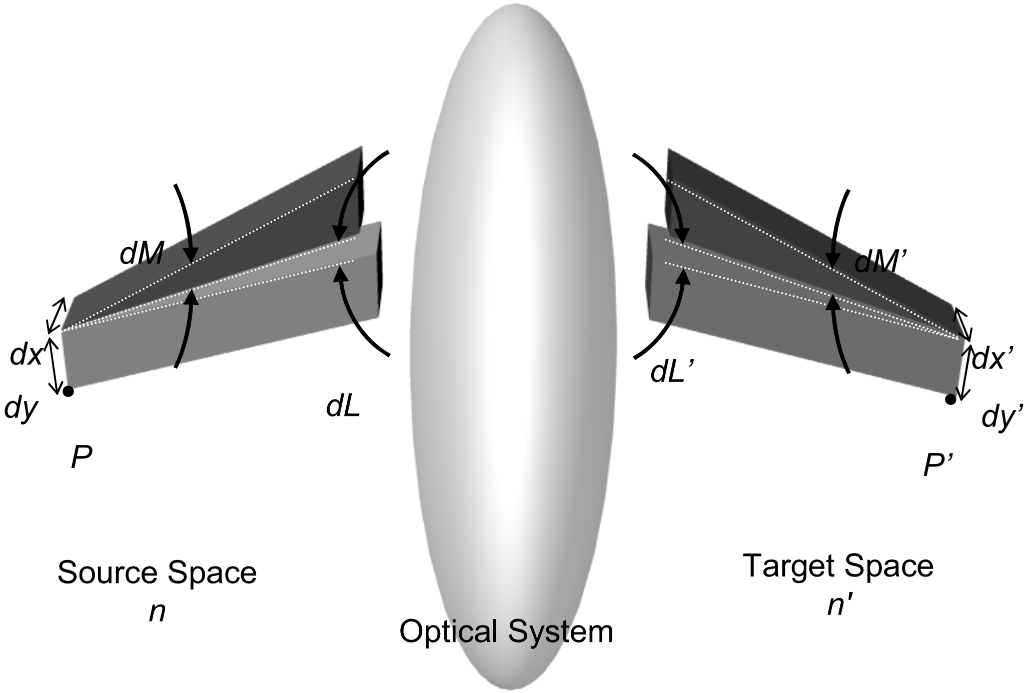
- Radiance Theorem:

$$\boxed{\frac{L}{n^2} = \frac{L'}{n'^2}}$$

- Conservation of Étendue:

$$\boxed{d^2\Phi = \frac{L_s}{n^2} d^2\mathcal{E}} \qquad \boxed{\mathcal{E} = \iint_{\text{aperture}} d^2\mathcal{E}}$$

Proof: Generalized Étendue



- Generalized Étendue:

$$\boxed{dxdydpdq = dx'dy'dp'dq'}$$

- Point Characteristic:

$$V(\mathbf{r};\mathbf{r}') = \int_P^{P'} nds = V(P') - V(P)$$

- At P and P':

$$\nabla V(\mathbf{r}) = -n\mathbf{s} \quad \text{and}$$

$$\nabla V(\mathbf{r}') = n'\mathbf{s}',$$

- Ray Components:

$$\begin{pmatrix} V_{xx} & V_{xy} & 1 & 0 \\ V_{yx} & V_{yy} & 0 & 1 \\ V_{x'x} & V_{x'y} & 0 & 0 \\ V_{y'x} & V_{y'y} & 0 & 0 \end{pmatrix} \begin{pmatrix} dx \\ dy \\ dp \\ dq \end{pmatrix} = \begin{pmatrix} -V_{xx'} & -V_{xy'} & 0 & 0 \\ -V_{yx'} & -V_{yy'} & 0 & 0 \\ -V_{x'x'} & -V_{x'y'} & 1 & 0 \\ -V_{y'x'} & -V_{y'y'} & 0 & 1 \end{pmatrix} \begin{pmatrix} dx' \\ dy' \\ dp' \\ dq' \end{pmatrix}$$

- Matrix Form:

$$\mathbf{Aw} = \mathbf{Bw}' \rightarrow \mathbf{w}' = \mathbf{Jw} \rightarrow \mathbf{J} = \mathbf{B}^{-1}\mathbf{A}$$

- Jacobian:

$$dx'dy'dp'dq' = \frac{|\mathbf{A}|}{|\mathbf{B}|} dxdydpdq$$

- Determinants:

$$|\mathbf{A}| = V_{x'x}V_{y'y} - V_{x'y}V_{y'x}$$

$$|\mathbf{B}| = V_{xx'}V_{yy'} - V_{xy'}V_{yx'}$$

Analogies for Étendue

- Radiance: associated with source emission.
- Brightness: Luminance but based on actual observer.
- Throughput: flux transmission capabilities of an imaging system.
- Extent: Direct analogy for étendue.
- Lagrange Invariant: $$H = nhu = n'h'u'$$ $$H = n(\bar{y}u - \bar{u}y) = n'(\bar{y}u' - \bar{u}'y)$$
- Abbe Sine Condition: $nh\sin\theta = n'h'\sin\theta'$

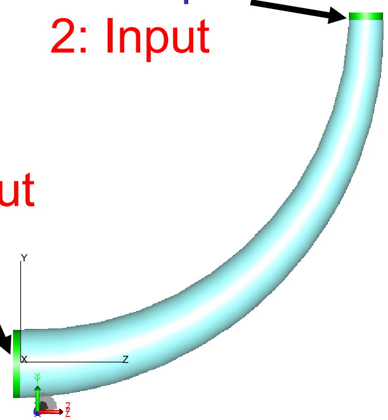
What Does Étendue Mean?

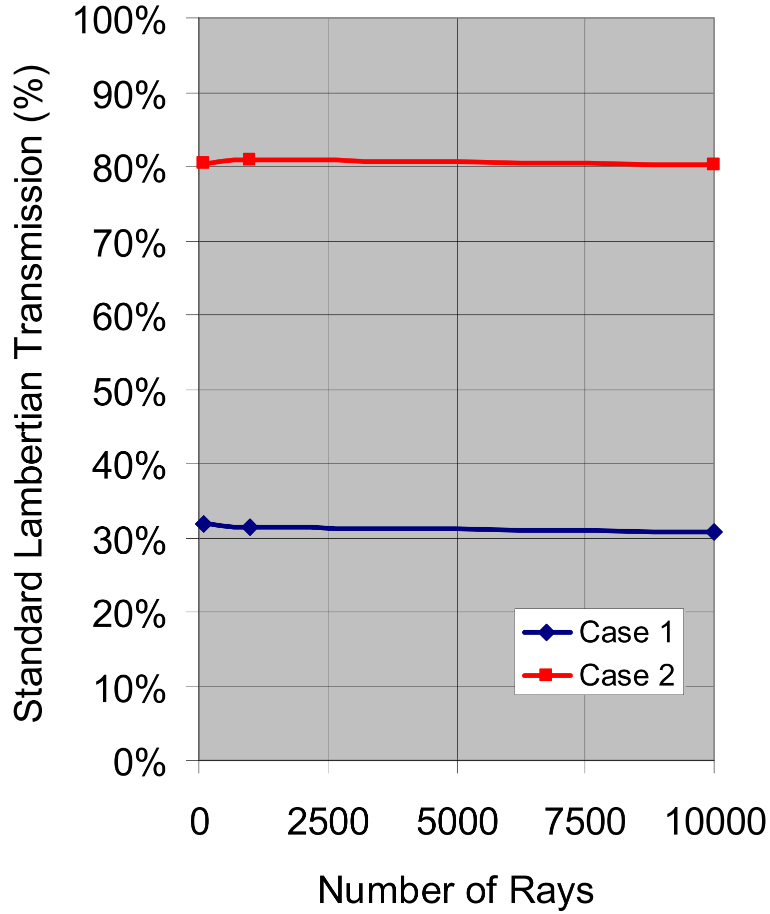
- Simply :
  - Large Area - Small Angle

    ↕
  - Small Area - Large Angle
- Example - consider a flared lightpipe:
  - Case 1: Flares at input end
  - Case 2: Flares at output end

1: Output
2: Input

1: Input
2: Output

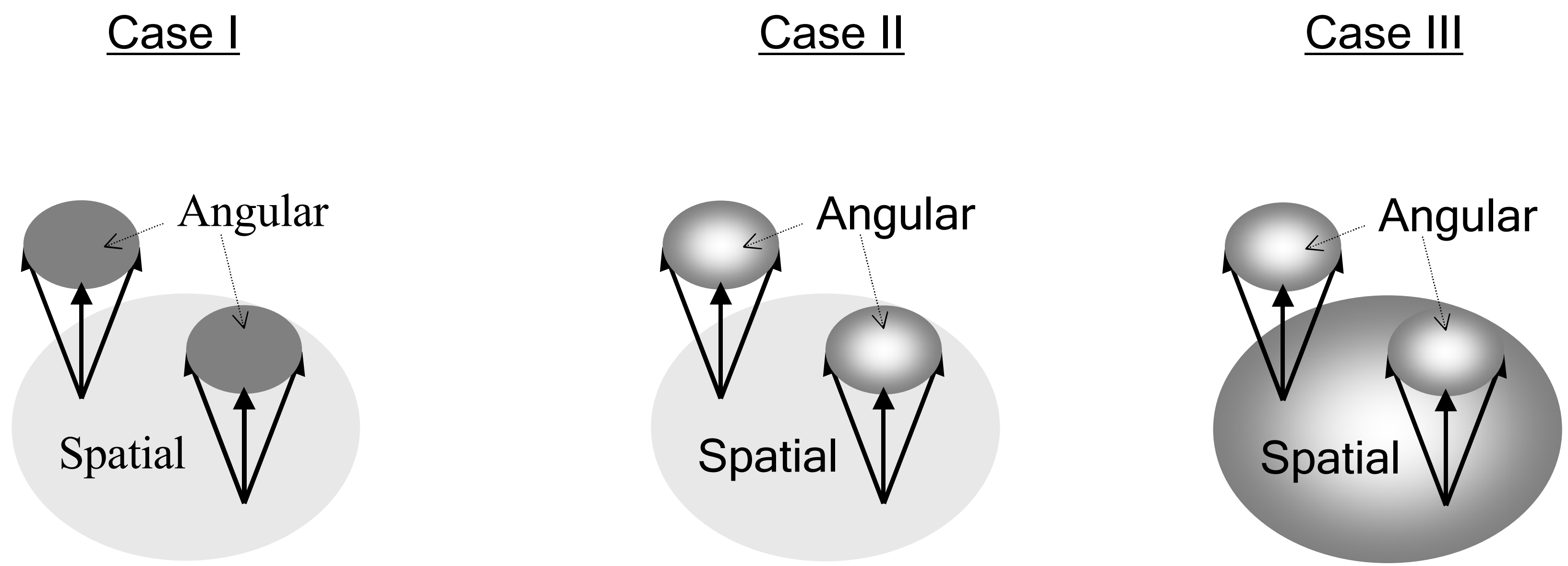




Examples

Illustrate the use of Etendue

Three Examples to Consider



Spatial: Uniform
Angular: Lambertian

Spatial: Uniform
Angular: Non Lambertian

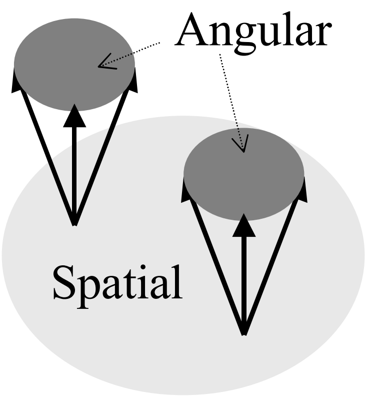
Spatial: Non uniform
Angular: Non Lambertian

Case I: Uniform, Lambertian Disk

- Integrate the Etendue equation:

$$\xi_s(\theta_a) = n^2 A_s \int_0^{2\pi} \int_0^{\theta_a} \cos\theta \sin\theta \, d\theta \, d\phi = \pi n^2 A_s \sin^2\theta_a$$

  - Full area of the source: A_s and
  - Angular range of $\theta \in [-\theta_a, \theta_a]$



- We can find the flux expressing with the radiance L_s and source radius r_s:

$$\Phi = \pi L_s A_s \sin^2\theta_a = \pi^2 L_s r_s^2 \sin^2\theta_a$$

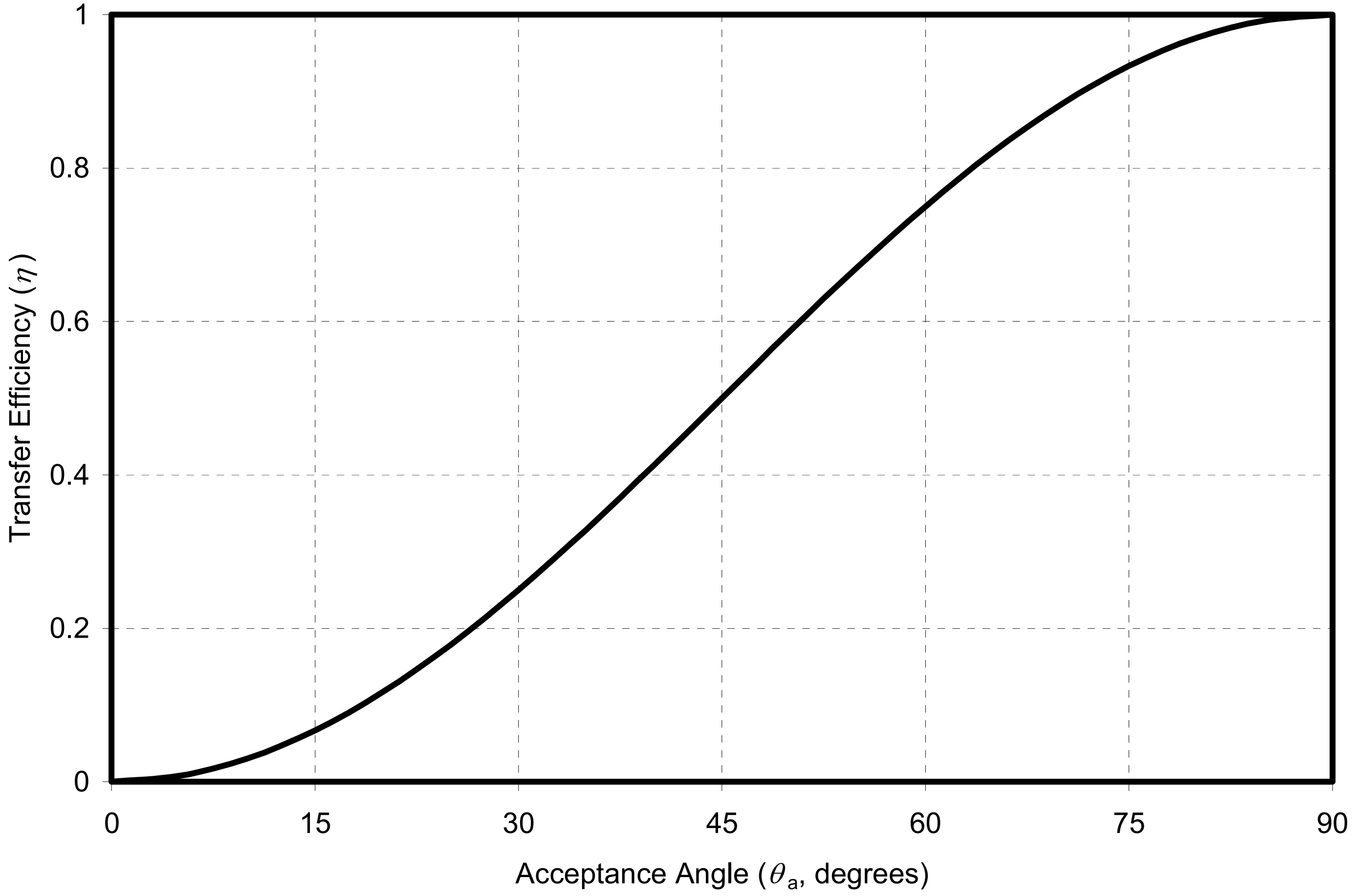
- The total flux is found when $\theta_a = \pi/2$:

$$\Phi(\theta_a = \pi/2) = \Phi_{opt} = \pi L_s A_s = \pi^2 L_s r_s^2$$

- Suppose we have a system with a FOV of θ_a and can capture the full spatial extent:

$$\eta(\theta_a) = \frac{\Phi(\theta_a)}{\Phi_{opt}} = \frac{\xi_s(\theta_a)}{\xi_{opt}} = \sin^2\theta_a$$

Case I: Acceptance Angle Plot



Aside: Better Way to Plot

- We have assumed that the full spatial extent of the source perfectly matches the field of view of the optical system:

$$\boxed{\eta(r,\theta_a)=\frac{\Phi(r,\theta_a)}{\Phi_{opt}}=\frac{\xi(r,\theta_a)}{\xi_{opt}}=\bar{\xi}(r,\theta_a)=\frac{r^2\sin^2\theta_a}{r_s^2}=\bar{r}^2\sin^2\theta_a}$$

- We have used a Lambertian angular distribution and uniform spatial distribution, which do not happen readily in real systems
  - Cases II and III show the limitation of this assumption
  - Homework problem #1 also tackles this issue

- There are hereunto undeveloped limitations in the transfer efficiency of systems whose cross-sectional shape evolves with propagation
  - Skew invariance – next lecture
  - Homework problem #2 tackles this issue

- <u>Transfer Efficiency versus Fractional Etendue Plots!</u>

Case I: Fractional Etendue Plot – Constant *r*

Spatial: *r* is constant
Angular: θ_a varies

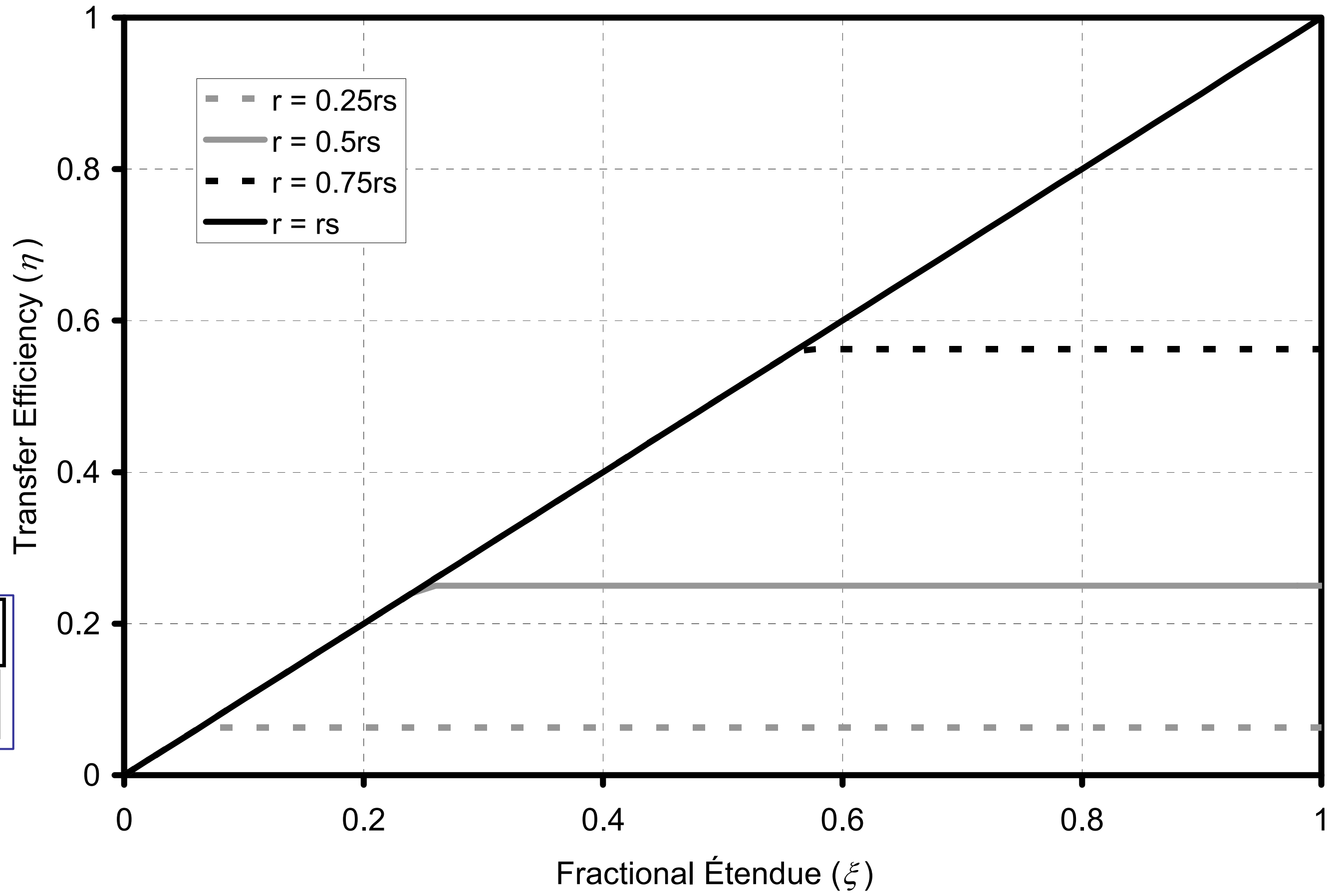
θ_a varies to maintain conservation of etendue

Maximum value of θ_a is $\pi/2$

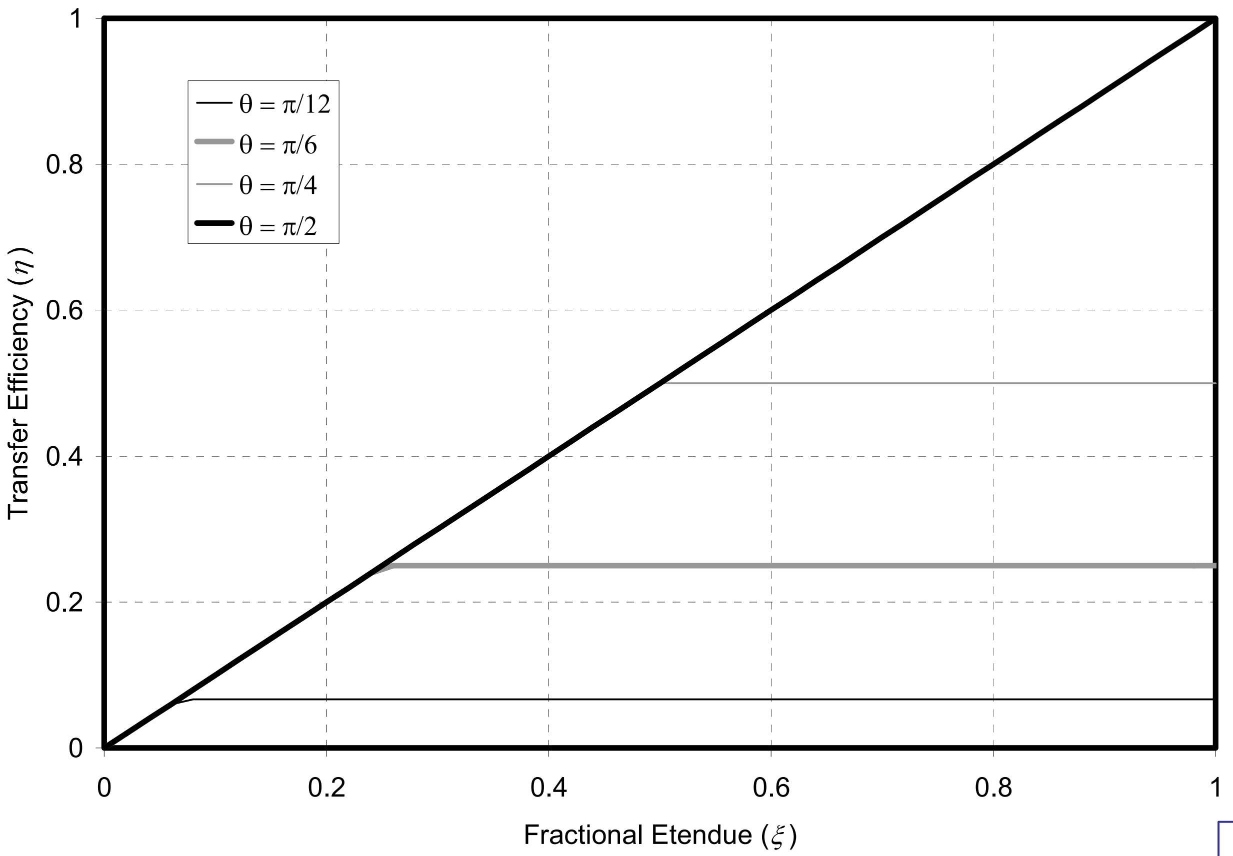
$$\eta\left(\bar{\xi};\theta_a\right)=\begin{cases}\bar{\xi}, & \bar{\xi}\in\left[0,\bar{r}^2\right]\\ \bar{r}^2, & \bar{\xi}\in\left(\bar{r}^2,1\right]\end{cases}$$

where

$$\bar{r}=r/r_s$$



Case I: Fractional Etendue Plot – Constant θ_a



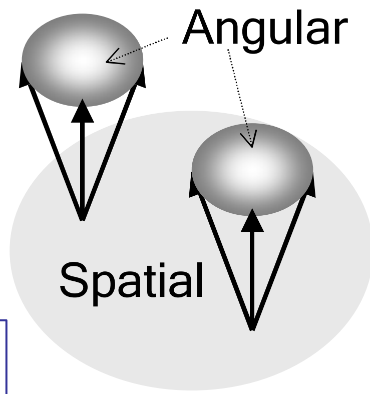
Spatial: r varies
Angular: θ_a constant

r varies to maintain conservation of etendue

Maximum value of r is r_s

$$\eta(\bar{\xi};r)=\begin{cases}\bar{\xi}, & \bar{\xi}\in[0,\sin^2\theta_a]\\ \sin^2\theta_a, & \bar{\xi}\in(\sin^2\theta_a,1]\end{cases}$$

Case II: Uniform, Isotropic Disk



- The radiance is given by:

$$\boxed{L(\mathbf{r},\theta_a,\phi)=\frac{L(\mathbf{r})}{\cos\theta_a}=\frac{L_s}{\cos\theta_a}}$$

- We find the transfer efficiency:

$$\boxed{\eta(R,\theta_a)=\frac{\Phi(R,\theta_a)}{\Phi_{opt}}=\frac{r^2}{r_s^2}(1-\cos\theta_a)=\bar{r}^2(1-\cos\theta_a)=\bar{\xi}\,\frac{(1-\cos\theta_a)}{\sin^2\theta_a}}$$

  - Where we used conservation of etendue in the last equality:

$$\boxed{\frac{\xi(r,\theta_a)}{\xi_{opt}}=\bar{\xi}(r,\theta_a)=\frac{r^2\sin^2\theta_a}{r_s^2}=\bar{r}^2\sin^2\theta_a}$$

- Constant acceptance angle:

$$\boxed{\eta(\bar{\xi};r)=\begin{cases}\bar{\xi}\,\dfrac{(1-\cos\theta_a)}{\sin^2\theta_a}, & \bar{\xi}\in\left[0,\sin^2\theta_a\right]\\ 1-\cos\theta_a, & \bar{\xi}\in\left(\sin^2\theta_a,1\right]\end{cases}}$$

Constant radius:

$$\boxed{\eta(\bar{\xi};\theta_a)=\begin{cases}\bar{\xi}\,\dfrac{(1-\cos\theta_a)}{\sin^2\theta_a}, & \bar{\xi}\in\left[0,\bar{r}^2\right]\\ \bar{r}^2, & \bar{\xi}\in\left(\bar{r}^2,1\right]\end{cases}}$$

Case II: Fractional Etendue Plot – Constant *r*

Spatial: *r* is constant
Angular: θ_a varies

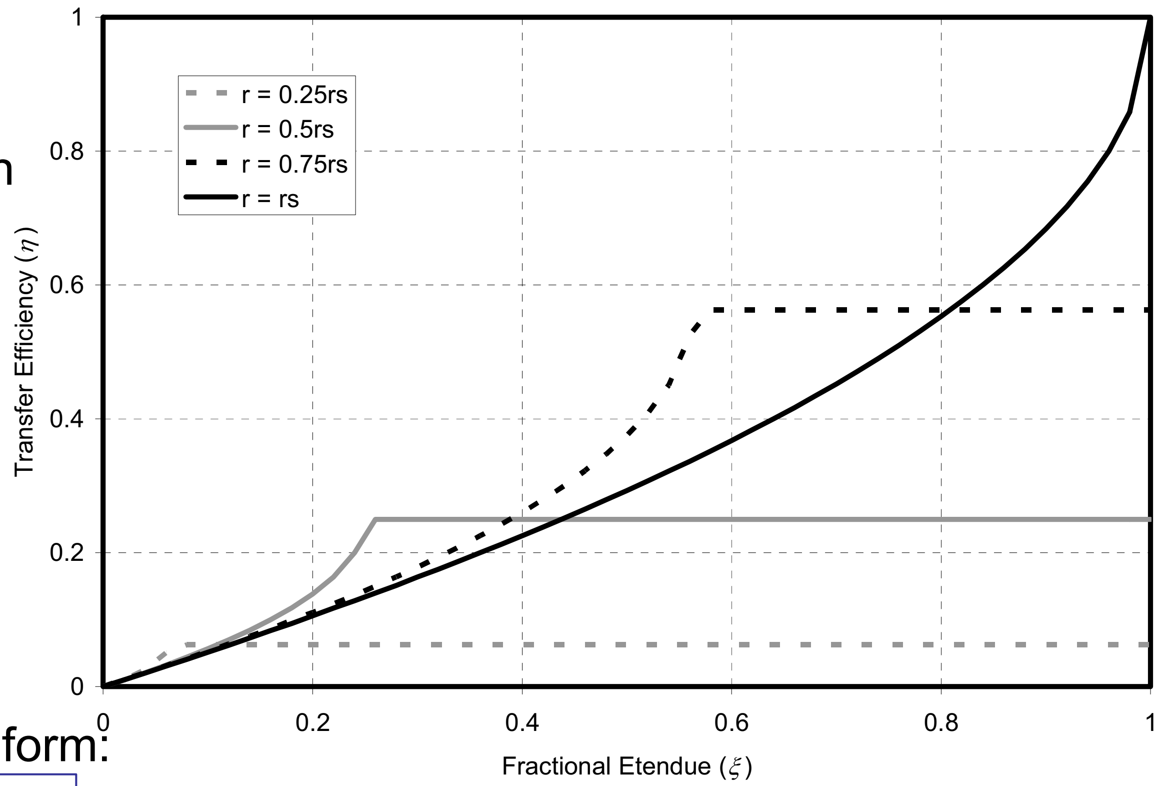
θ_a varies to maintain conservation of etendue

Maximum value of θ_a is $\pi/2$:

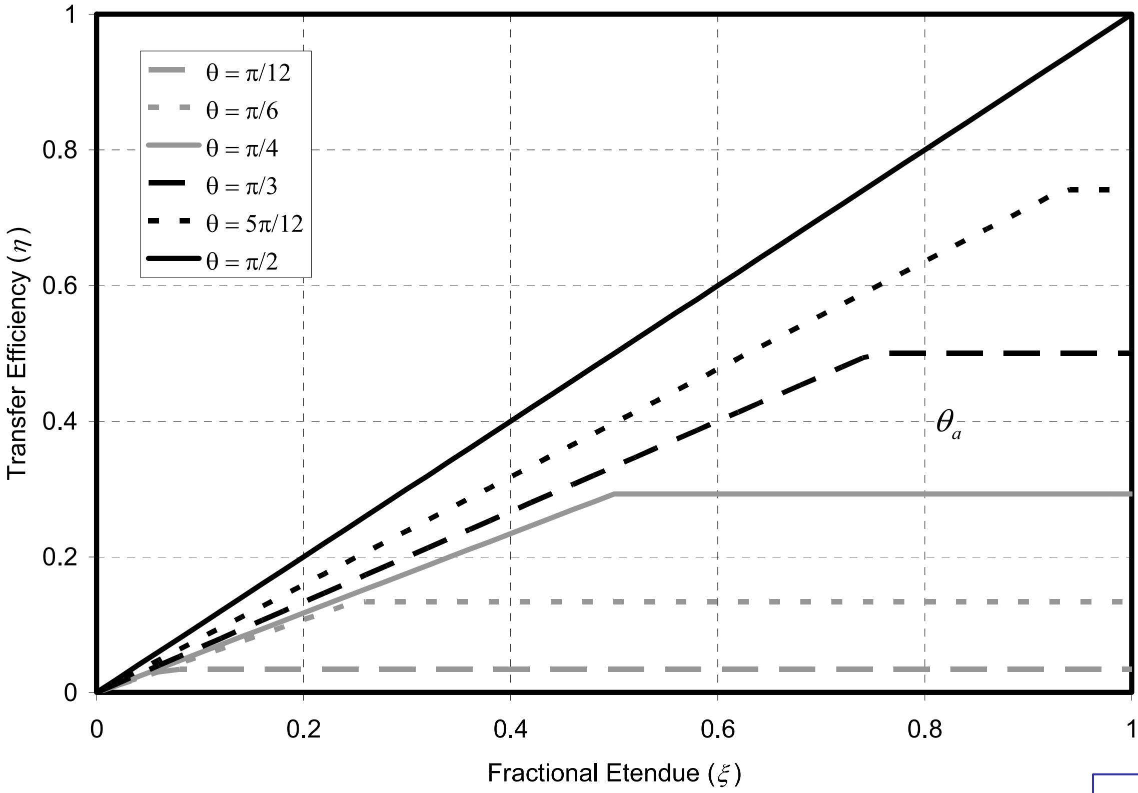
$$\eta(\bar{\xi};\theta_a)=\begin{cases}\bar{\xi}\dfrac{(1-\cos\theta_a)}{\sin^2\theta_a}, & \bar{\xi}\in[0,\bar{r}^2] \\ \bar{r}^2, & \bar{\xi}\in(\bar{r}^2,1]\end{cases}$$

Use conservation to get in better form:

$$\eta(\bar{\xi};\theta_a)=\begin{cases}\bar{r}^2\left(1-\sqrt{1-\bar{\xi}/\bar{r}^2}\right), & \bar{\xi}\in[0,\bar{r}^2] \\ \bar{r}^2, & \bar{\xi}\in(\bar{r}^2,1]\end{cases}$$



Case II: Fractional Etendue Plot – Constant θ_a



Spatial: *r* varies
Angular: θ_a constant

r varies to maintain conservation of etendue

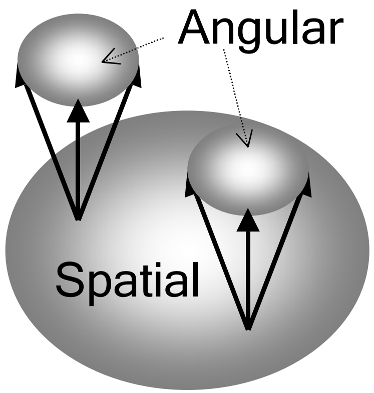
Maximum value of *r* is r_s

$$\eta(\bar{\xi};r)=\begin{cases}\bar{\xi}\dfrac{(1-\cos\theta_a)}{\sin^2\theta_a}, & \bar{\xi}\in\left[0,\sin^2\theta_a\right]\\ 1-\cos\theta_a, & \bar{\xi}\in\left(\sin^2\theta_a,1\right]\end{cases}$$

Case III: Non Uniform, Isotropic Disk

- The radiance is given by:

$$\boxed{L(r,\theta_a) = L_s \frac{(1 - r/r_s)}{\cos\theta_a} = L_s \frac{(1 - \bar{r})}{\cos\theta_a}}$$



- This is homework problem #1!
- Steps to solve:
  - Find the transfer efficiency as a function of fractional r and θ_a
  - Use conservation of etendue to remove the radius or acceptance angle to keep only constant terms in the equation
    - Radius constant: remove the acceptance angle terms
    - Acceptance angle constant: remove the radius terms
  - Now you will have the transfer efficiency as a function of fractional etendue and the one constant term!
  - Plot

Discussion

- Adding source complexity (aka realism):
  - Calculations are harder
  - Give better approximations to transfer efficiency / flux at target
- Disk source geometry:
  - Simple “enough” and “illustrative”
  - Not realistic for an emitter by itself
  - May be used to approximate the output aperture of an emitter coupled to an optic (however, you should do etendue calculation through the optic!)
- Disk source distribution:
  - Angular: isotropic is not typical, Lambertian is better
  - Spatial: non uniform is typical
- More realistic source?
  - Geometry: how about a tube to approximate arc or filament
  - Distribution: uniform and Lambertian

The Skew Invariant

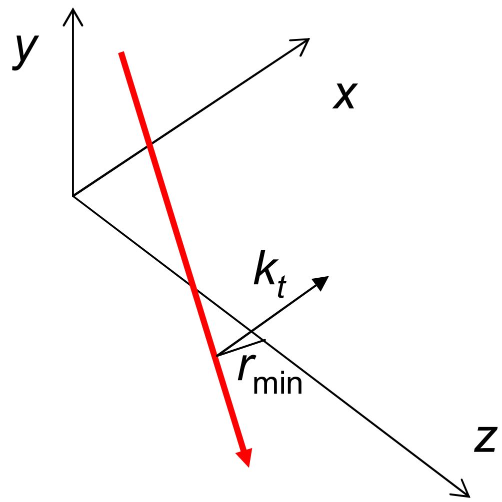
- On a Piecewise Basis Étendue Plays Crucial Role in Determination of Output Distribution.
- Rotationally Symmetric Systems: Rotational Skew Invariant of Each Ray is Conserved.
  - Skewness Distribution Dependent on Étendue:

$$f_{\text{skew}}(s) = \frac{d\mathcal{E}(s)}{ds} \quad \text{where } s = r_{\min} k_t$$

$r_{\min}$: ray's closest approach to optical axis

k_t: tangential component of ray's propagation direction

with $|\vec{k}| = n$



- System Symmetry Must be Broken to Overcome the Skew Invariant.

Design Methods

- Number of Current Design Methods:
  - Edge Ray Design: Compound Parabolic Concentrator (Hinterberger and Winston, Rev. Sci. Instrum. 37, 1094-1095, 1966).
  - Tailored Edge Ray Design: Cosine-Corrected Freeform Illuminator (Ries and Muschaweck, JOSA A 19, 590-595, 1996).
  - Simultaneous Multiple Surfaces: Hybrid Optic (Benítez et al, Opt. Eng. 43, 1489-1502, 2004).
  - Non-Edge Ray Design: Multiple Source Concentrator (Koshel and Walmsley, Opt. Eng. 43, 1511-1521, 2004).
  - Flow-Line Method: LED Backlight (Miñano et al, published in SPIE Proc. Vol. 5942, 2005).
- New Algorithms (and Improvements) All the Time!
- Caveat: Poor Source Model Leads to Poor Design!