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19 - 23 April 2010

Mathematical modeling of systemic risk: addressing unintended consequences

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Outline

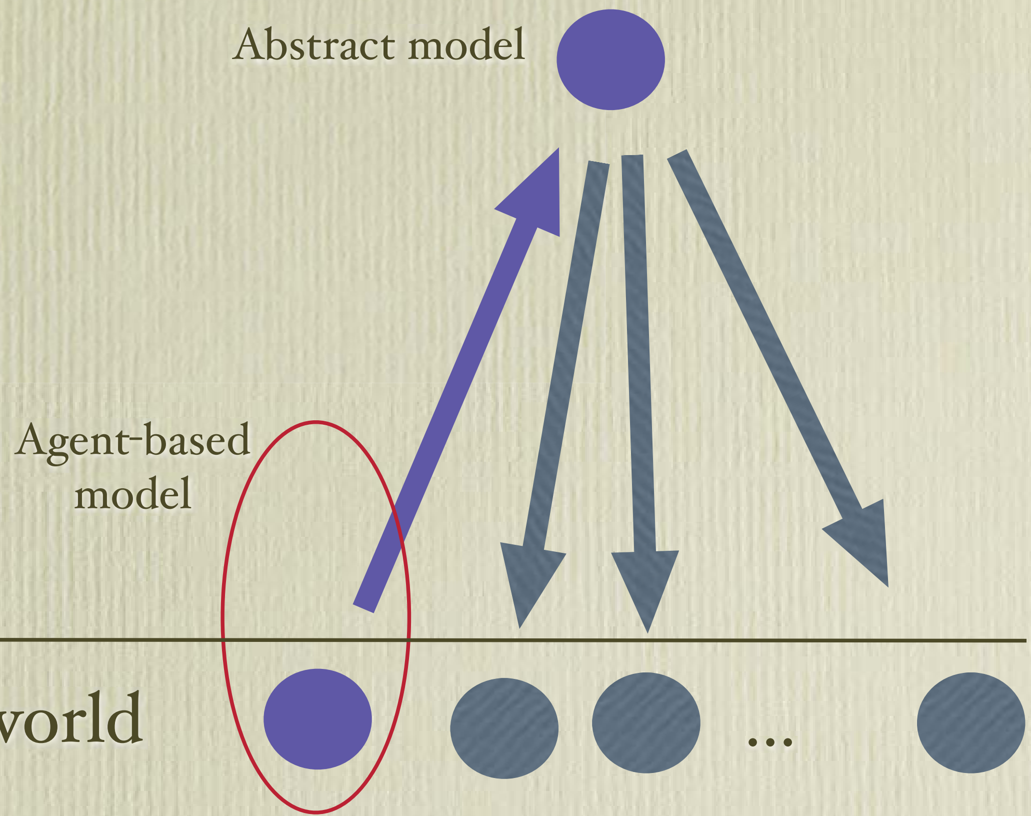
- General considerations:
  Risk in a complex world
- Examples:
  - Managing congestion in communication networks
  - Efficiency vs Stability in financial markets
  - The rise and fall of socio-economic networks
- Conclusions

Risk in a complex world

- Accounting for the human dimension:
  Economic incentives in Natural and technological systems
- Unintended consequences
- Extreme events: large shocks vs small perturbations in densely connected system?
- Efficiency - stability paradox (May '72)
- Time-scales: enough time to learn?

Modeling complex systems

- Agent-Based vs stylized/abstract models
  - Scenario generation prediction
  - Understanding, policy, regulation, design,

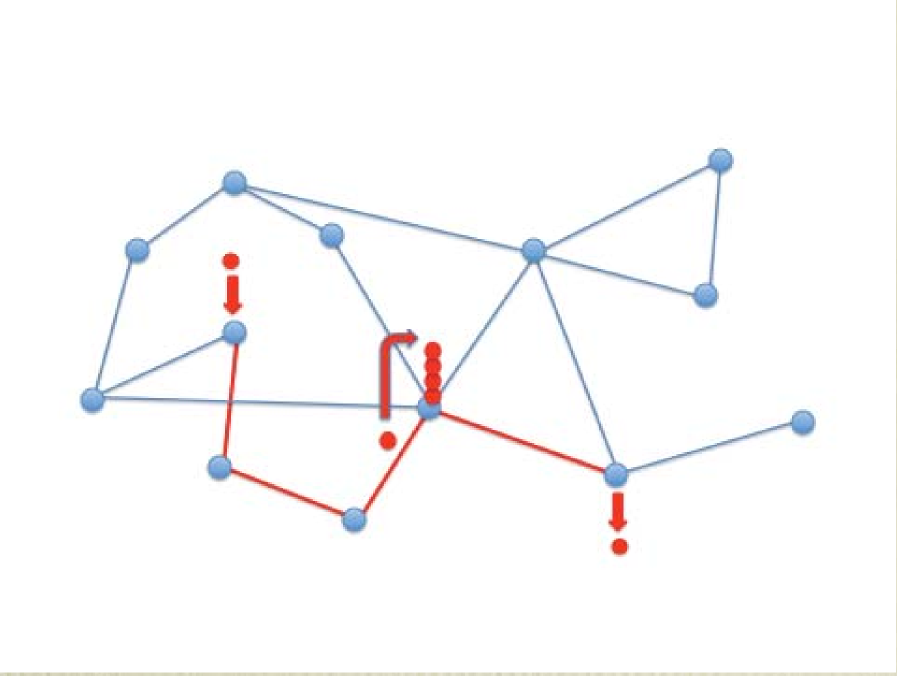


Congestion phenomena in complex networks

- with D. De Martino (SISSA), G. Bianconi, L. Dall'Asta (ICTP)
- Congestion phenomena:
  Internet traffic, urban traffic, power-grids, bureaucracy, ...
- Ingredients:
  finite capacity channels
  heterogeneous network
  source-destination mapping
  increasing traffic loads
  local congestion avoidance rules
- Congestion: what loads can the network support?
  Do congestion avoidance local protocols help?

Minimal ingredients of a model for Internet traffic

- Packets generated at rate p
- FIFO queues, finite bandwidth
- Packets hopping
- Absorption (reaching destination)
- Traffic avoidance protocols: if the number of packets on a node exceeds a threshold then incoming packets are rejected with probability η
- Congestion indicator: n. of undelivered packets per unit time

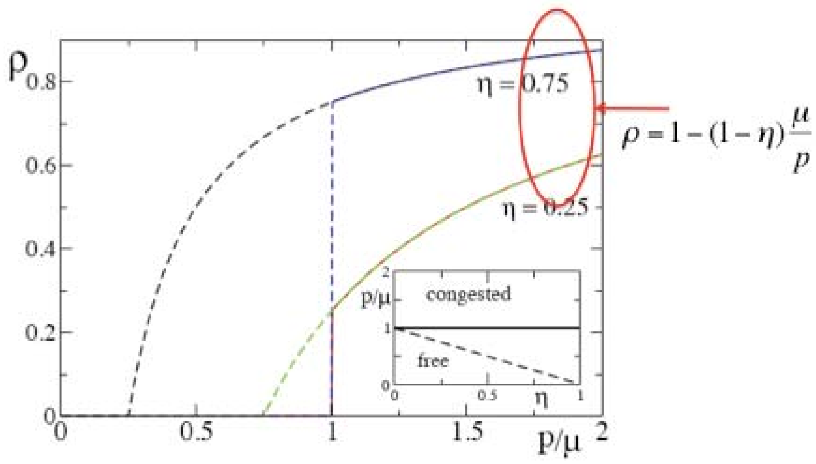


$$\rho = \lim_{t \to \infty} \frac{M(t+\tau) - M(t)}{p \tau N}$$

Managing congestion

Homogeneous Networks

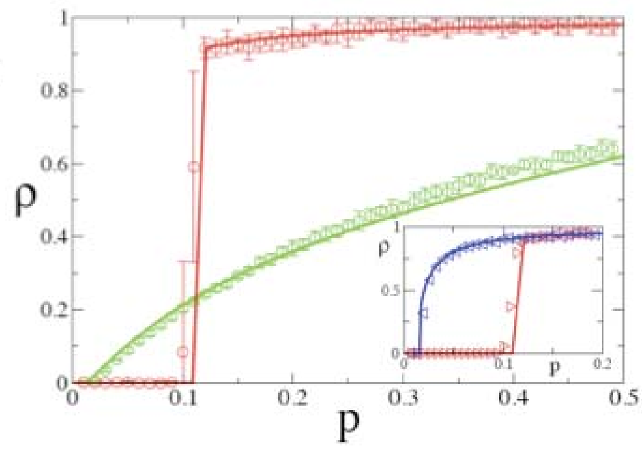
Transition and phase diagram on regular graphs (all free / all congested)



Heterogeneous Networks

Phase diagram of scale-free network with $P(k) \propto k^{-3}$
N = 3000, k_{max}=110, μ=0.2, n^* = 10

red η=0.95
green η = 0.05



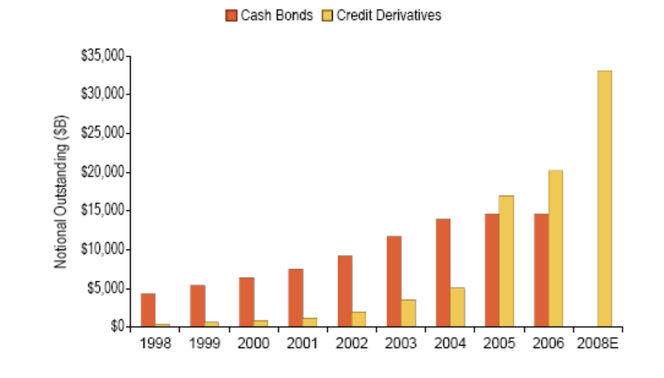
Traffic avoidance protocols
1- help only in heterogeneous networks
2- change the nature of the transition
3- introduce hysteresis and coexistence

Financial crisis:

3. Crash!!!
Trading in ABS froze
Interbank market froze

2. … bad news …

1a. growth of complex credit derivative products



Sources: British Bankers' Association Credit Derivatives Report 2006, Bank for International Settlements and ISDA.
Note: Cash bonds through June 2006.

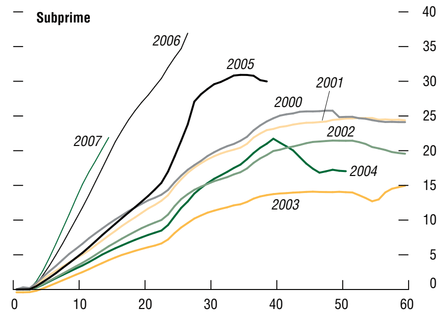


Figure 1.8. U.S. Mortgage Delinquencies by Vintage Year
(60+ day delinquencies, in percent of original balance)

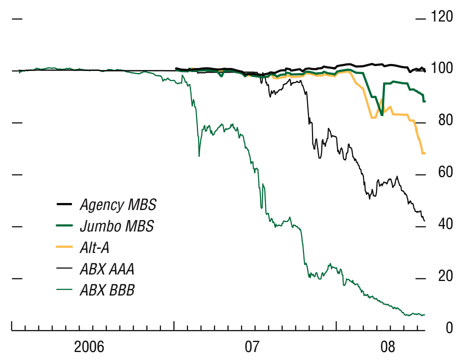
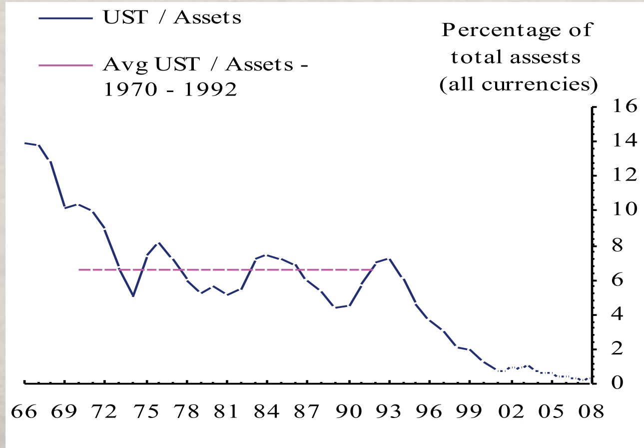
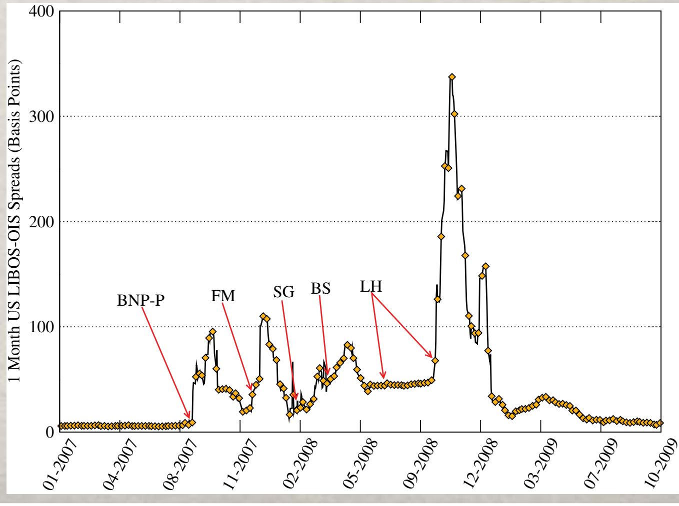


Figure 1.9. Prices of U.S. Mortgage-Related Securities
(In U.S. dollars)

Sources: JPMorgan Chase & Co.; and Lehman Brothers.
Note: ABX = an index of credit default swaps on mortgage-related asset-backed security; MBS = mortgage-backed security.

1b. decreasing liquidity in banks





Figures from Global Financial Stability Report Oct. 2008

Issues

- Efficiency vs stability
- Modeling the collapse of trade networks
- On market impact

Increasing complexity in financial markets

- The financial innovation spiral. Expansion in the repertoire of trading instruments (e.g. credit derivatives)
- Speculators' arm-race. Expansion in traders' types and trading strategies (e.g. proliferation of hedge funds)
- Efficiency:
  - Approaching the limit of *complete markets*:
    more financial instruments enables hedging risks more efficiently (R. Merton & Z. Bodie '05, R. Shiller '08)
  - Approaching the limit of *informationally efficient markets*:
    arm race of speculators provides liquidity and aggregates efficiently information into prices (E. Fama '65)

Increasing complexity in a simple economy

The market (returns)

demand

Investors

$$\Leftarrow \begin{matrix} z_1 \\ \vdots \\ z_k \\ \vdots \\ z_N \end{matrix} \Leftarrow \begin{matrix} r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\ \vdots & \ddots & \vdots & & \vdots \\ r_k^1 & \cdots & r_k^\omega & \cdots & r_k^\Omega \\ \vdots & & \vdots & \ddots & \vdots \\ r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega \end{matrix} \Leftarrow$$

Financial industry (banks)

$$\Downarrow$$

$$\max_{\vec{z} \geq 0} E\left[u\left(c(\vec{z})\right)\right]$$

$$r_{N+1}^1 \quad \cdots \quad r_{N+1}^\omega \quad \cdots \quad r_{N+1}^\Omega$$

financial innovation

Increasing complexity in a simple economy

The market (returns)

demand

Investors $\Leftarrow$

$$\begin{matrix} z_1 & & r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ z_k & \Leftarrow & r_k^1 & \cdots & r_k^\omega & \cdots & r_k^\Omega \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ z_N & & r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega \end{matrix}$$

$\Leftarrow$ Financial industry (banks)

$$\max_{\vec{z}\geq 0} E\left[u\left(c(\vec{z})\right)\right]$$

$$N, \Omega \to \infty, \quad n = \frac{N}{\Omega}$$

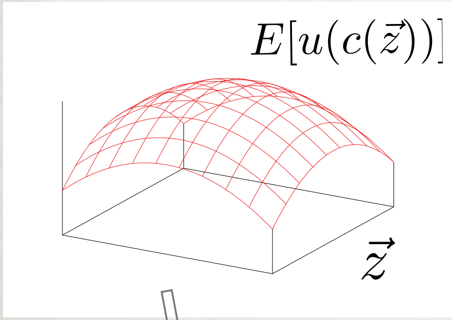
A creative financial sector

- Financial instruments are drawn at random from a probability distribution with

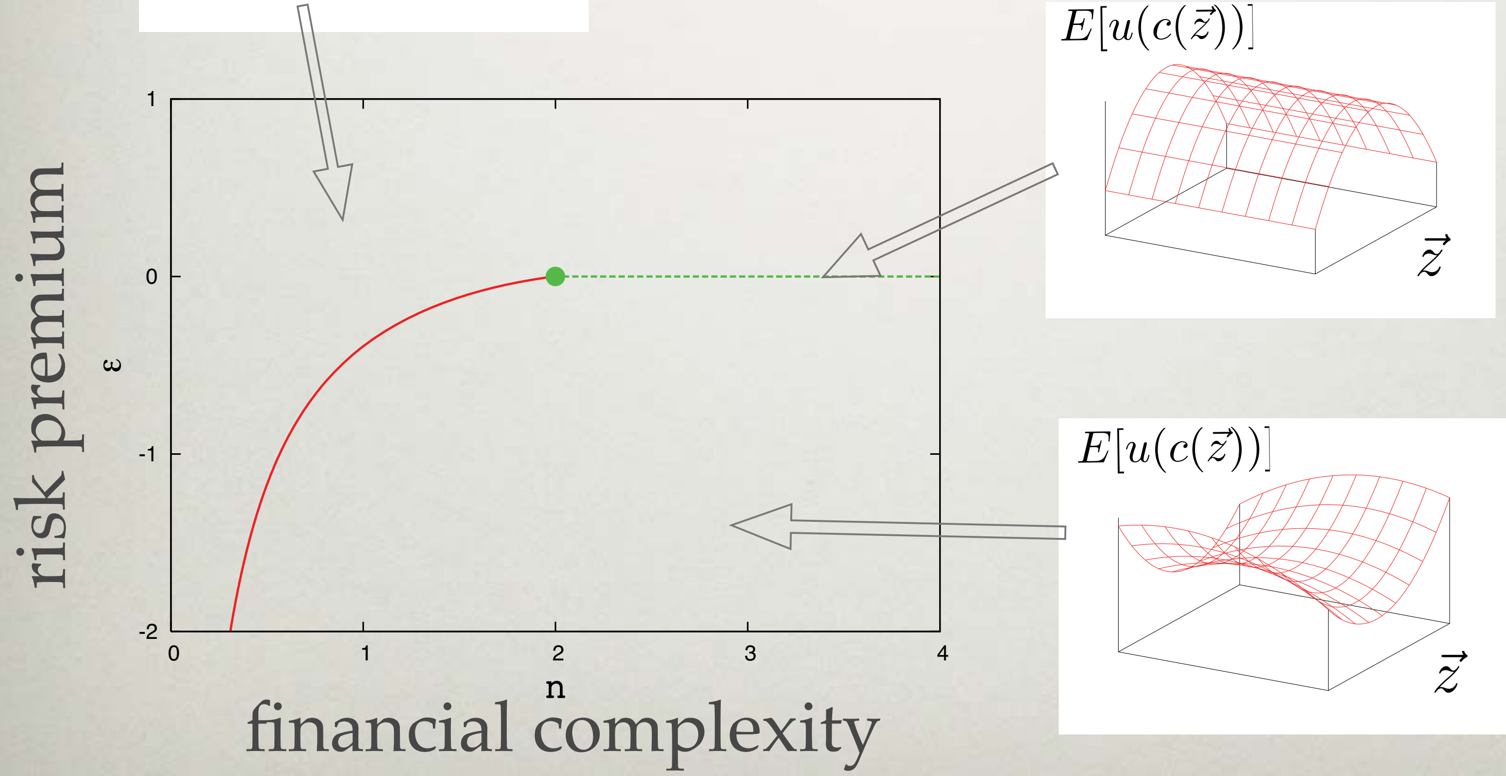
$$E_\pi[r_i] = \sum_\omega \pi^\omega r_i^\omega = -\frac{\epsilon}{\Omega}, \qquad \mathrm{Var}[r_i] = \frac{1}{\Omega}, \quad i = 1, \ldots, N$$

- Key variables:
  - financial complexity: $n=N/\Omega$
  - risk premium: ε
- Note: Successful innovations ($z_i>0$) are not independent draws

Instability with increasing financial complexity



- ϕ Ω free variables (z_i>0), Ω constraints
  ε<0 ⇒ unstable directions can appear (arbitrages)

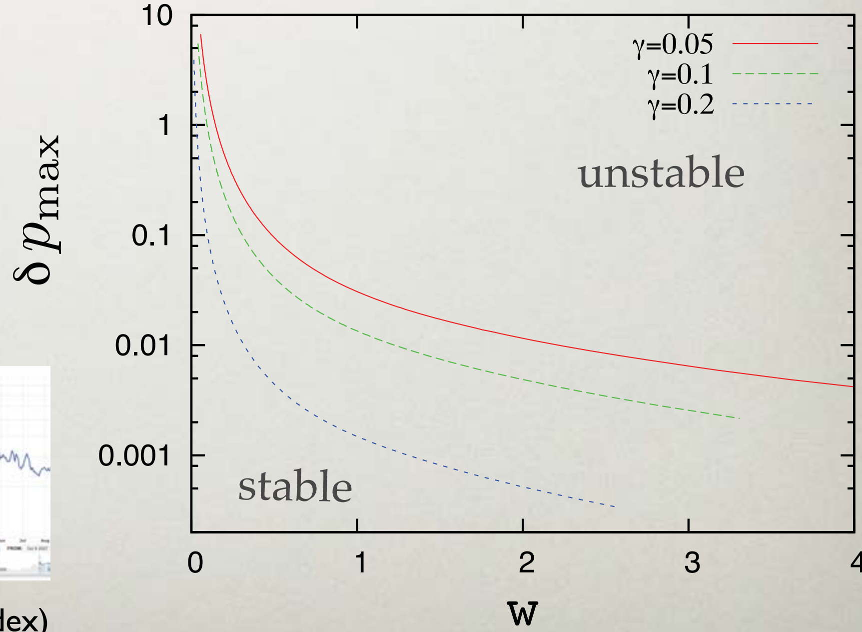


Stability and the size of financial markets

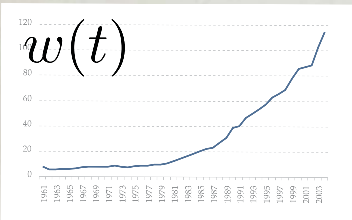
- Relative size of financial markets ≈ volume of trading for hedging one unit of a new asset $w = \sqrt{\sum_i v_i^2}$
- Financial stability: → price uncertainty $$\frac{\delta z}{z} = \frac{1}{z}\frac{\delta z}{\delta p}\delta p = \frac{\chi}{z}\delta p \ll 1$$

$$\delta p_{\max} = \frac{z}{\chi}$$

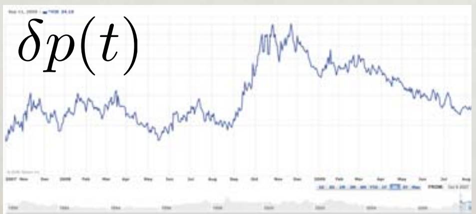
Stability diagram



- E.g. Iceland:



Size of financial markets/GDP



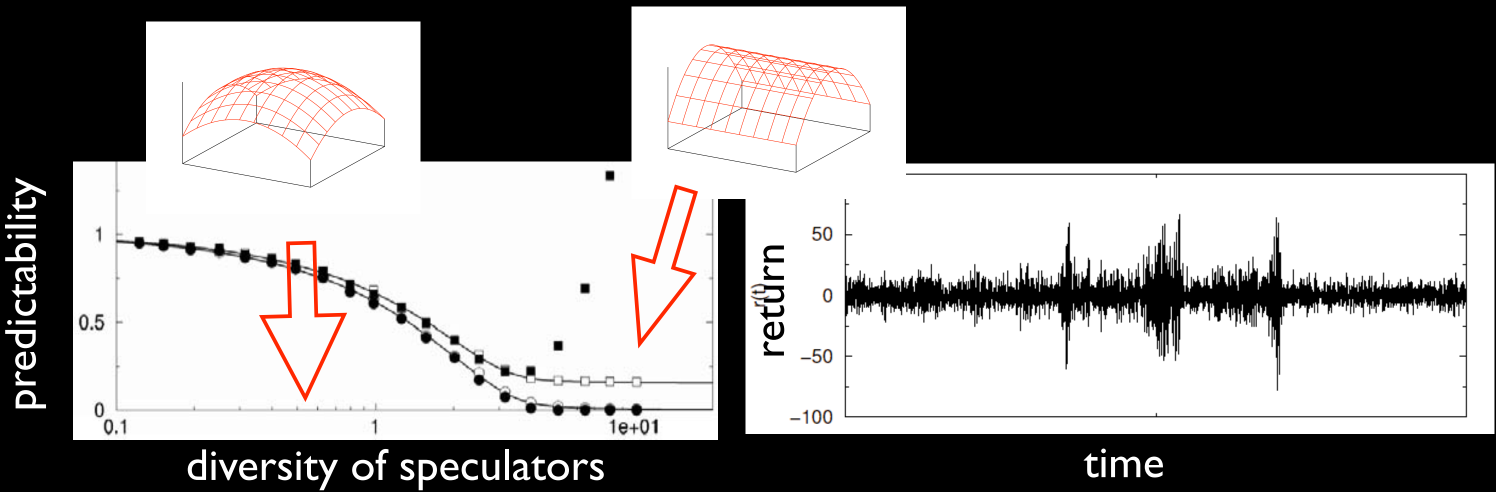
price volatility (CBOE-VIX index)

Conclusions:

- The proliferation of financial instruments, even in an ideal world (perfect competition and full information), leads to systemic instability
  - Complete markets lie on a critical line with infinite susceptibility
  - A competitive financial sector is expected to converge to this singularity
  - The volume generated by banks to hedge financial instruments they sell diverges as markets approaches completeness
  - Learning to invest optimally is hard (Brock, Hommes, Wagener 2006)
- The larger (and more complex) the financial market is, the more price indeterminacy is problematic
  - Institution should grow in size with financial complexity
  - Quantitative measure of financial stability based on price indeterminacy and relative size of financial sector?

Financial complexity and market information efficiency

- Markets as information “food chain” (e.g. Minority Games)



- Excess volatility as signature of market information efficiency (Challet, MM, Zhang ’05)

  Non-informed traders dominate in efficient markets (Caccioli, MM, Economics, ’10)

  Market impact matters and it regularizes instability in portfolio selection (Caccioli, Still, MM, Kondor 2010)

Systemic stability in financial markets

- Stability requires new math and new observables:
Susceptibility and response functions

- Stability is eroded by increased complexity
  - excess volatility as market become informationally efficient (Minority Game)
  - systemic instability and divergent volumes required by hedging, as markets approach completeness, even in an ideal setting (Wegener et al. ’06, MM ’09)

  → stability and efficiency are incompatible (K. Iwai, ’08)

- Stability as a common good:
measures for its efficient provision are needed!

- Competitive equilibria ≠ Nash equilibria even for $N \to \infty$
Market impact (liquidity) matters!

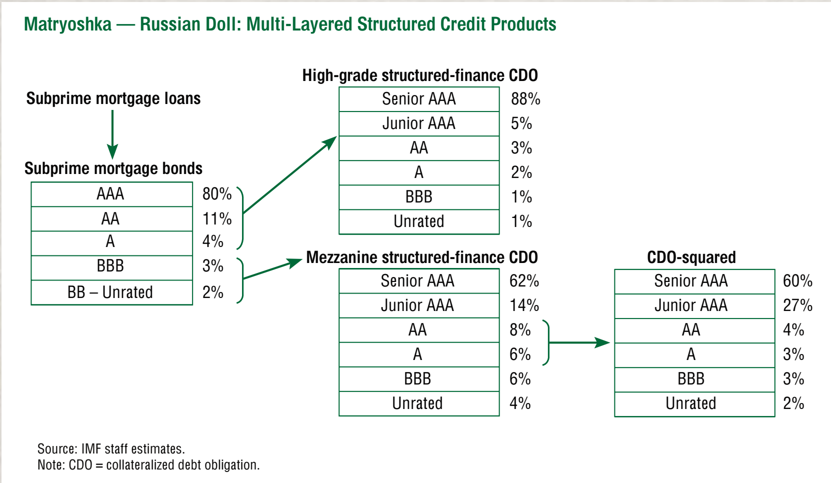
- Similarity with May’s bio-diversity paradox (R. May ’72)
and instability of risk measures (I. Kondor et al. ’07)

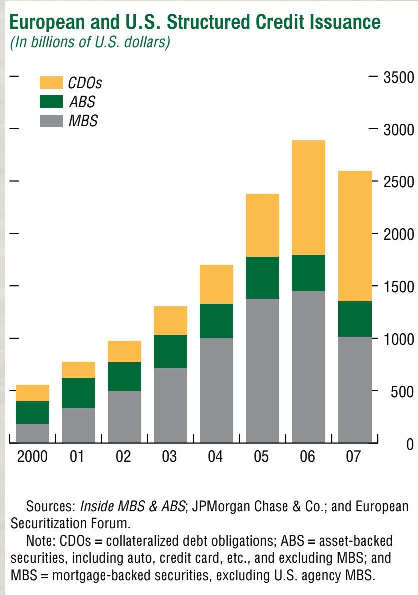
Liquidity crisis and the evaporation of trust

Matteo Marsili, Kartik Anand (ICTP),
Alan Kirman (Marseille) and Prasanna Gai (Camberra)

The collapse of credit derivative markets

- Securitization: originate and distribute





- Pros: diversification, control on risk, sure return for financial institutions, apparent liquidity

ABS = Asset Backed Security
MBS = Mortgage " " "
CDO = Collateralized Debt Obligation
CDS = Credit Default Swap

- Cons: lack of transparency, complexity, moral hazard, risk concentration in balance sheets of banks, ...

To check or not to check?

- Over-reliance on credit rating:
  "[...] some institutional investors have relied too heavily on ratings in their investment guidelines and choices, in some cases fully substituting ratings for independent risk assessment and due diligence".
  (report of the Financial Stability Forum 2008)

- Market for ABS perceived as liquid:
  "The high volume of outstanding mortgage securities, combined with the large number of investors who hold these securities, creates a sizable and active secondary market". (pamphlet of The Bond Market Association, 2002)

- Lending to unreliable borrowers (sub-primes):
  "Securitization increases the distance between the originator of the loan and the party that bears the default risk inherent in the loan. Since soft information about borrowers is unverifiable to a third party, the increase in distance results in lenders choosing to not collect soft information about borrowers".
  (Rajan, Seru, Vig 2008).

A simple game

- The rule:
  buy an ABS without checking whether it is "toxic" or not

- Strategy: $z_i=1 \Leftrightarrow$ follow the rule ($i=1,\ldots,N$ labels agents)
  $z_i=0 \Leftrightarrow$ don't, i.e. check before buying, this costs χ_i
  Idea: checking is costly, if majority follows the rule, then I better follow it too

- Prob{ABS is toxic when checked} = p (bad news: p larger than expected)

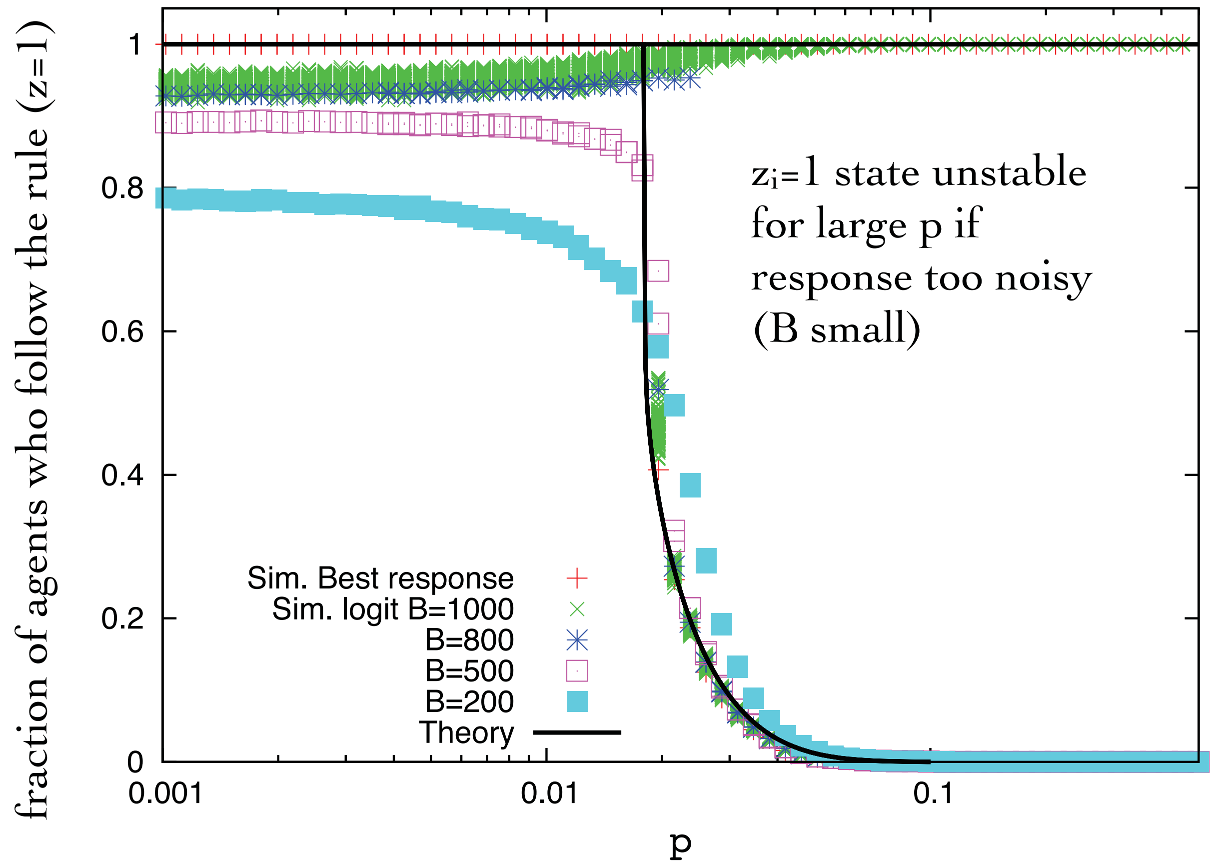
- Agents connected in a network (OTC market):
  i trades with j drawn at random among his neighbors

- Payoffs: pay a price p_0 to seller
  resell at $p_2 < p_0$ if buyer checks & ABS toxic
  resell at $p_1 > p_0$ else
  checking costs $-\chi_i$ (drawn from pdf $\Phi(\chi)$)

  (reduce # params. by rescaling: $p_1-p_2=1$, $c=p_0-p_2$)

	check & toxic	no check
$z_i=0$	$-\chi_i$	$1-c-\chi_i$
$z_i=1$	$-c$	$1-c$

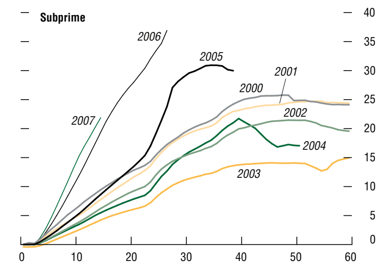
Rule epidemics



$$P\{z_i = 1\} \propto e^{B[u_i(1)-u_i(0)]}$$

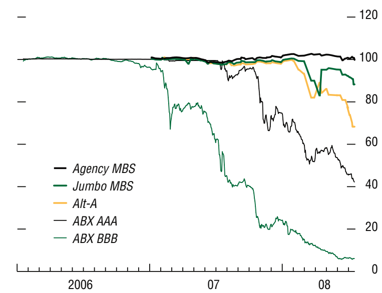
Back to ABS story:

Figure 1.8. U.S. Mortgage Delinquencies by Vintage Year
(60+ day delinquencies, in percent of original balance)



increasing p ...

Figure 1.9. Prices of U.S. Mortgage-Related Securities
(In U.S. dollars)



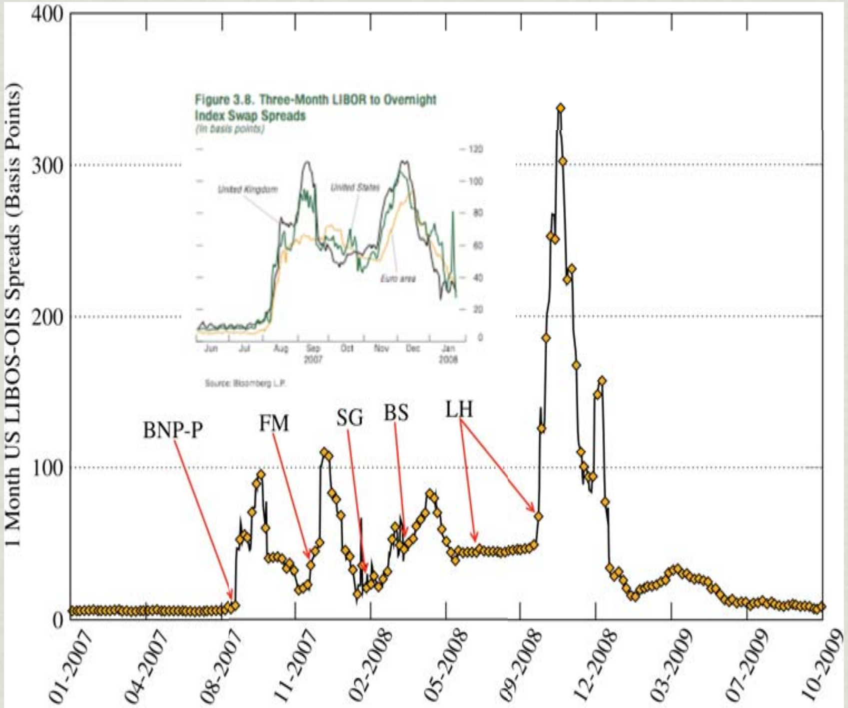
Sources: JPMorgan Chase & Co.; and Lehman Brothers.
Note: ABX = an index of credit default swaps on mortgage-related asset-backed security; MBS = mortgage-backed security.

... sharp transition!

The evaporation of Trust

Credit markets - Investors lend monies to each other with the promise of repayment.

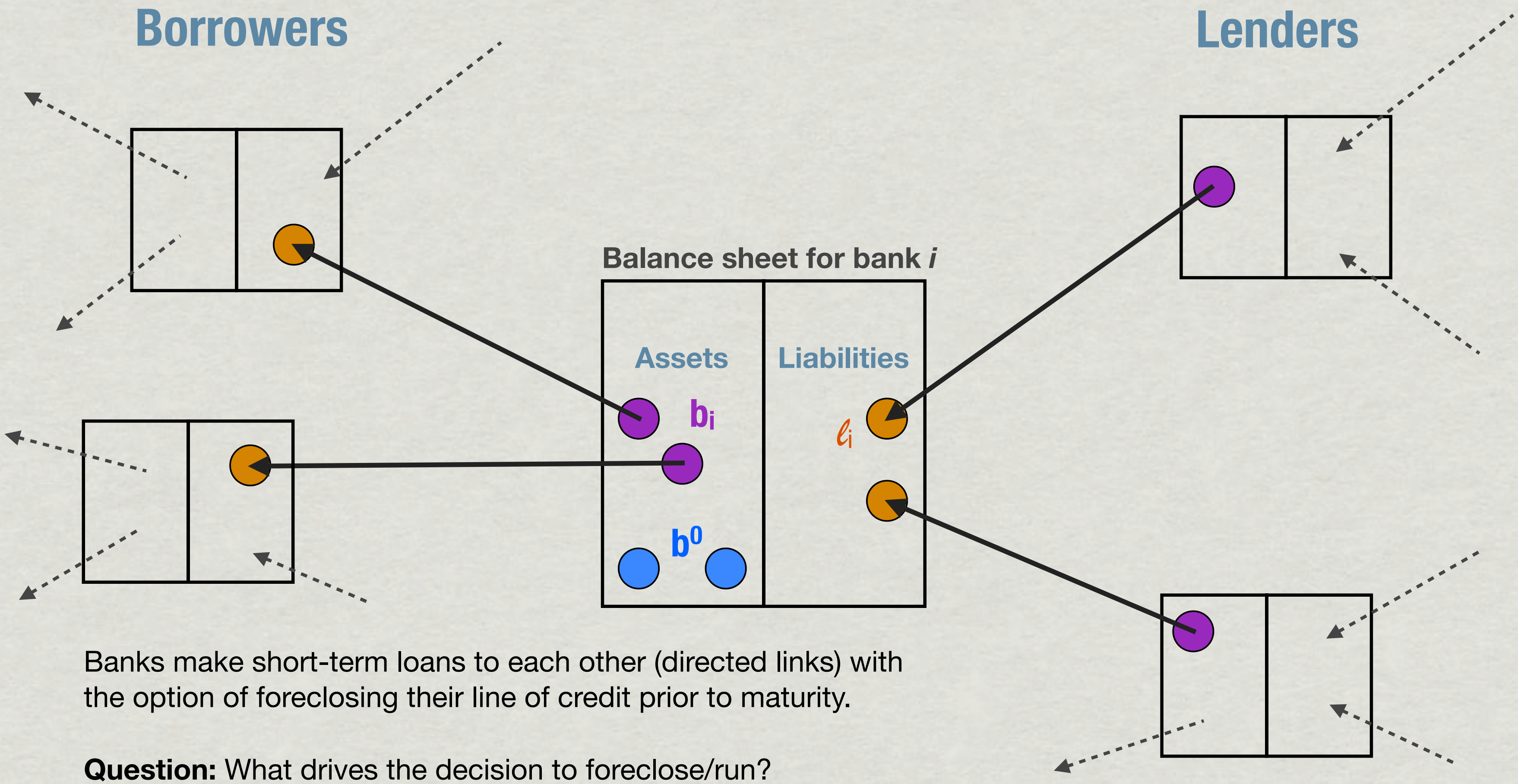
> “A ***credit crunch*** is a breakdown in trust. [...] That loss of trust has been the root cause of the devastating impact felt globally since the credit crunch began. Events of the past two years can be re-told as a story of the progressive breakdown in trust.” (Haldane, 2009)



Good and bad equilibria: being solvent is easy when credit is easily accessible, but when people do not trust each other, it is difficult to be trustworthy

When and why does an economy falls from the good to the bad equilibrium?

Credit networks



Banks make short-term loans to each other (directed links) with the option of foreclosing their line of credit prior to maturity.

Question: What drives the decision to foreclose/run?

Challenge: Multiple *foreclose games* simultaneously being played across the network.

Strategic uncertainty: Larry Summer's game

Everyone invests $10 with me.

Expectation ~ earn $11, assuming I stay solvent.

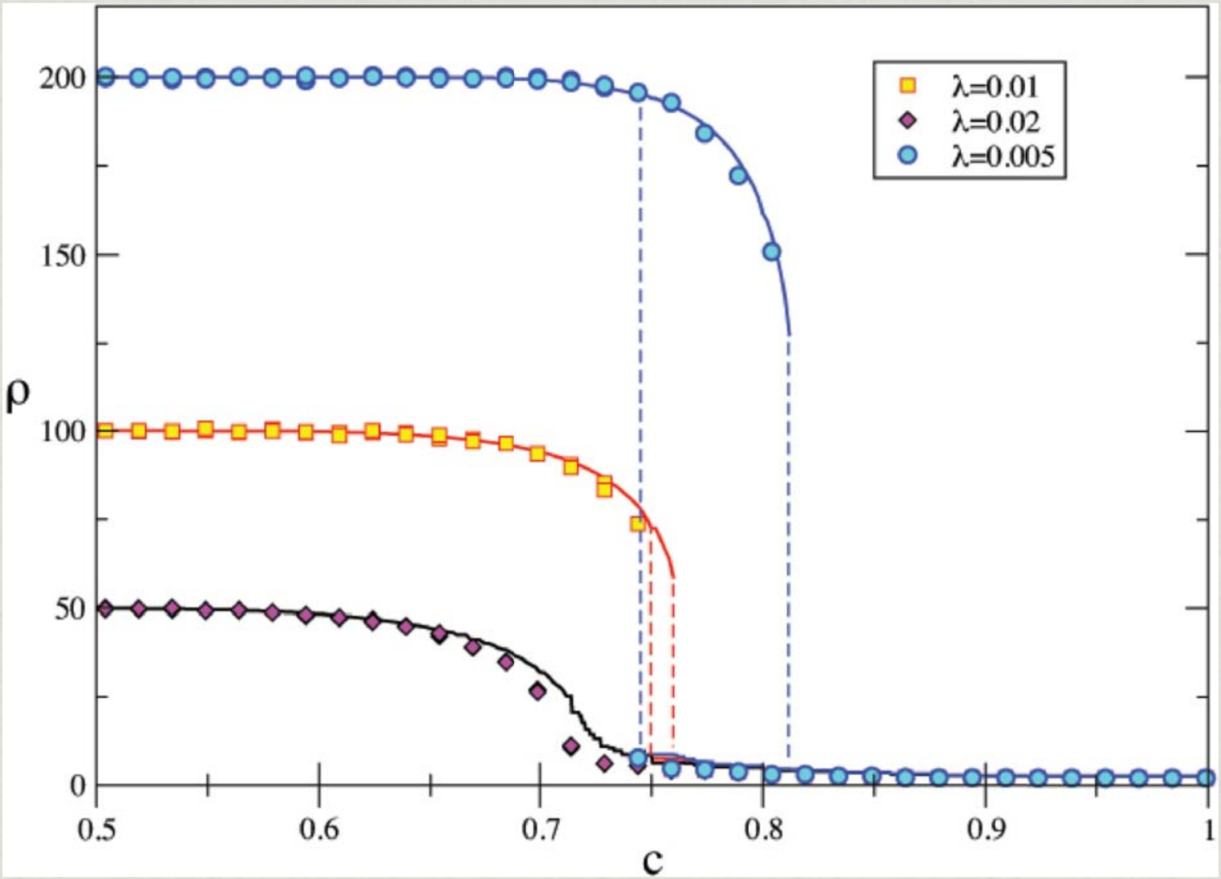
If I go bankrupt, you loose the $10 investment.

Proposition ~ I *won't* go bankrupt if at most only one-third of you choose to withdraw.

- **FORMAL GAMES PROPOSED BY SHIN & MORRIS (2004)**
- **INTERACTING SIMULTANEOUS "GAMES" ON THE NODES A CREDIT NETWORK**

The evaporation of trust

circles ~ simulations
lines ~ numerical solution of master equation



ρ = CREDIT NETWORK DENSITY

COST OF MIS-COORDINATION

Determinants and policy

- Maturity mistmatch:
  Sharp transition only for small λ/ν
- Transparency: ν
  many unstable banks for small n, but fewer defaults
- Interest rates and c
- Bailouts and b_0
- Capital/liquidity requirements:
  $b_0 = \beta + \alpha\, b \rightarrow \quad c' = c - \alpha$
  $b_0 = \beta + \alpha\, l \rightarrow \quad c' = c/(1 + \alpha)$

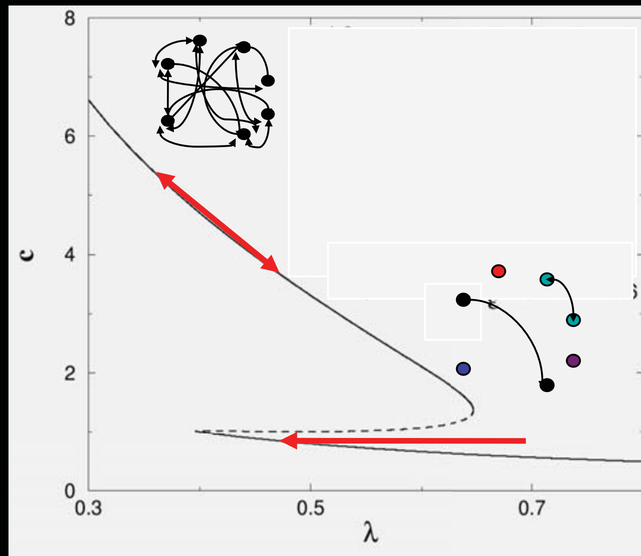
The rise and fall of networked societies

E.g. R&D networks, scientific collaboration, web communities, etc.

Networks = chance & necessity

Link form depending on:
information diffusion,
search-ability,
coordination,
proximity,
similarity,
social ranking,
technological levels,
reputation/trust, ...

Network density



Volatility

(Ehrhardt, Marsili, Vega-Redondo '06)

Summary

- From individual behavior to collective dynamics: statistical mechanics of systemic stability
- Unintended consequences of enhancing efficiency
  - sharp transition in congestion phenomena
  - instability from financial innovation (May's paradox)
- Systemic failure in networks:
  - epidemics of rules and strategic uncertainty
  - positive feedback: homogeneity and network density

  $\Rightarrow$ sharp transition, hysteresis and resilience
- Some insights on measures, policy and regulation
- ... work in progress...