The statistical mechanics of random quantum satisfiability

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We present an account of our research programme to transfer insights and methods from quantum statistical mechanics to the study of quantum complexity theory. We study random quantum k-QSAT, Bravyi's generalisation of classical k-SAT studied extensively in classical statistical mechanics. In our approach, the ratio of clauses to qubits, M/N, acts as a control parameter with which to tune from low density, easily satisfied to high density, highly frustrated instances of QSAT.

We characterize the phase diagram of random k-QSAT using several complementary approaches. We examine unentangled (product) satisfying states and discover a geometric criterion for deciding when a QSAT interaction graph is generically product satisfiable. Applied to the random graph ensemble, this criterion provides improved lower bounds on the location of the SAT--UNSAT transition. Coupled with recent work on the quantum Lovasz local lemma, this shows the existence of an entanglement transition in the satisfying space of the random ensemble at large k. For k=3 and k=4, we present numerical results which provide mild evidence for a similar transition at smaller k. Finally, we examine the UNSAT regime in detail, where it has recently been shown that the threshold for random quantum k-SAT is strictly less than the conjectured classical k-SAT threshold.

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