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A quantum Lovász Local Lemma

The Lovász Local Lemma (LLL) is a powerful tool in probability theory to show the existence of combinatorial objects meeting a prescribed collection of “weakly dependent” criteria. We show that the LLL extends to a much more general geometric setting, where events are replaced with subspaces and probability is replaced with relative dimension, which allows to lower bound the dimension of the intersection of vector spaces under certain independence conditions. Our result immediately applies to the quantum k-SAT problem (where we ask whether a system discribed by a Hamiltonian that is a sum of projectors is frustration-free): For instance we show that any collection of rank 1 projectors with the property that each qubit appears in at most 2k/(e · k) of them, has a joint satisfiable state.

We then apply our results to the recently studied model of random k-QSAT. Recent works have shown that the satisfiable region extends up to a density of 1 in the large k limit, where the density is the ratio of projectors to qubits. Using a hybrid approach building on work by Laumann et al. we greatly extend the known satisfiable region for random k-QSAT to a density of Ω(2k/k2). Since our tool allows us to show the existence of joint satisfying states without the need to construct them, we are able to penetrate into regions where the satisfying states are conjectured to be entangled, avoiding the need to construct them, which has limited previous approaches to product states.

Joint work with Andris Ambainis and Or Sattath.