The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2144-7

Workshop on Localization Phenomena in Novel Phases of Condensed Matter

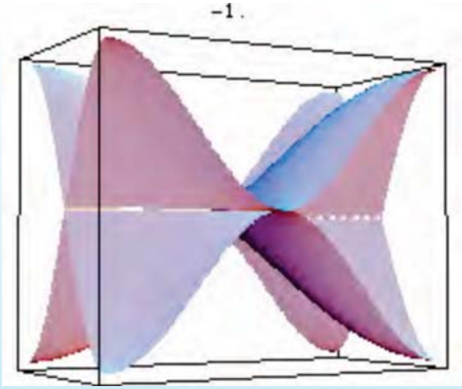
17 - 22 May 2010

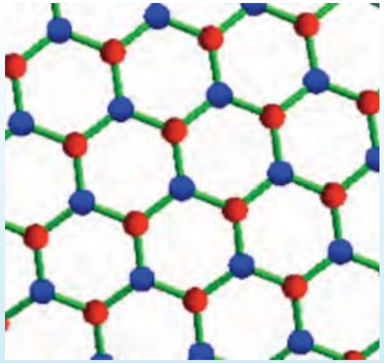
Dirac Fermions, Chern Numbers & Bulk-Edge Correspondence in Graphen with Randomness

Yasuhiro HATSUGAI
University of Tsukuba Institute of Physics, 1-1-1 Tennodai
Tsukuba, 305-8571
Ibaraki
JAPAN

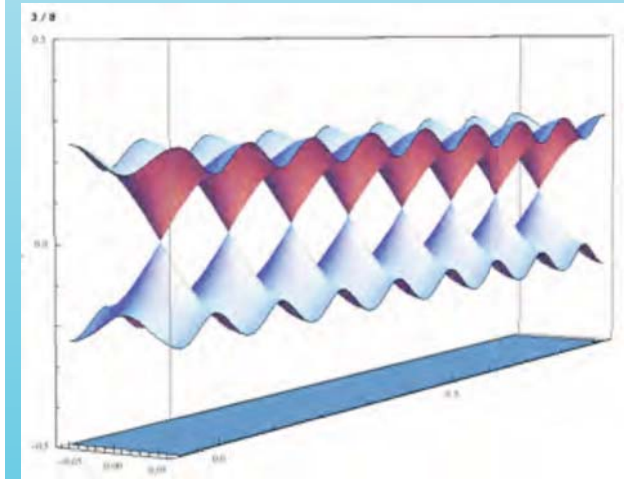
Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

ICTP May 20, 2010

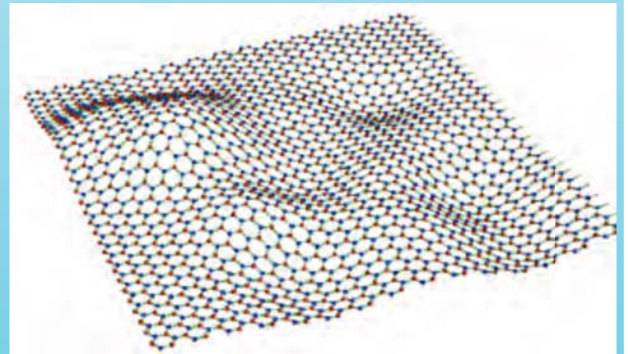




Dirac fermions, Chern numbers & bulk-edge correspondence in graphene with randomness



Institute of Physics
University of Tsukuba
JAPAN
Yasuhiro Hatsugai



University of Tsukuba

Talk today (Dirac fermions in condensed matter)

Emerging Dirac Fermions in condensed matter

- ☆ Bloch Electrons in a magnetic field (Hofstadter problem)
- ☆ Graphene & 2D gapless superconductors

Non Abelian gauge structures in condensed matter

- ☆ Quantum Hall effects, especially of Graphene
- ☆ Lattice gauge fields in a parameter space

Dirac Fermion & Zero modes

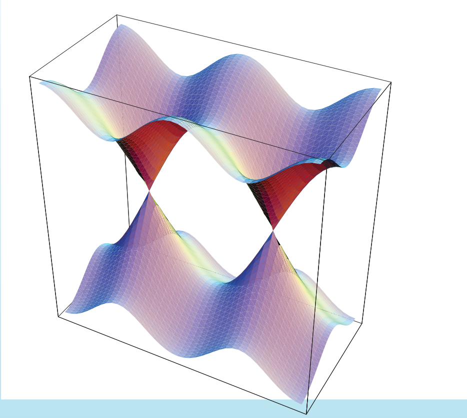
- ☆ Universality for the Zero modes of Dirac fermions
- ☆ Bulk-Edge correspondence of topological ordered states

Random Dirac ferimion & random Gauge field

- ☆ Ripples as Random gauge field in graphene
- ☆ Realization of exact fix point of random Dirac fermions

Massless Dirac Dispersions in Condensed Matter

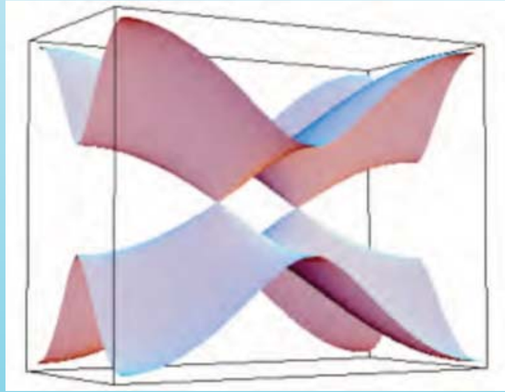
A



☆ π-flux phases in two-dimensions

Fisher and Fradkin (1985)
Affleck and Marston (1988)
Hasegawa-Lederer-Rice-Wiegmann (1989)

B

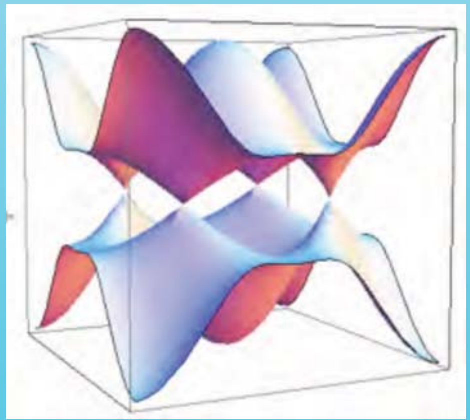


☆ Graphene as a 2D Carbon sheet

Wallace (1946)

2-Dirac cones

C

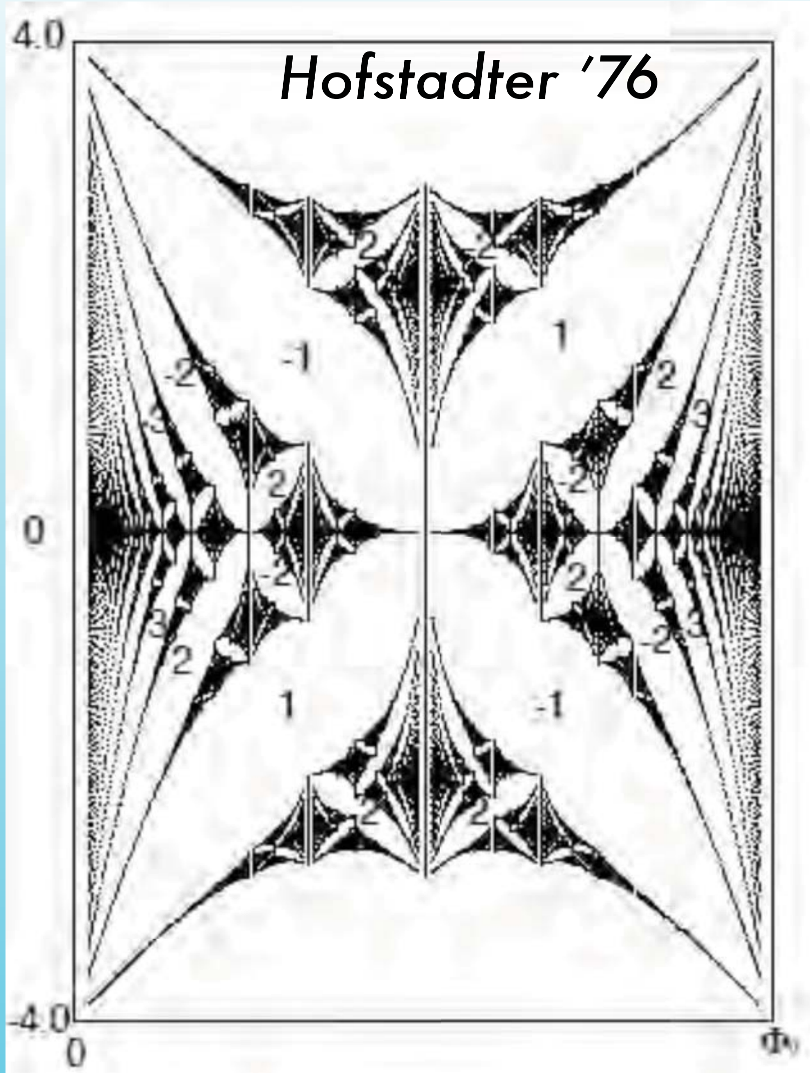


☆ Gapless Superconductor with point Nodes

d-wave superconductivity (high-T_C)

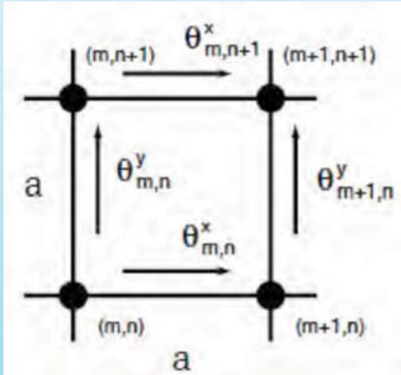
4-Dirac cones

Energy spectrum of Bloch Electrons in a magnetic field



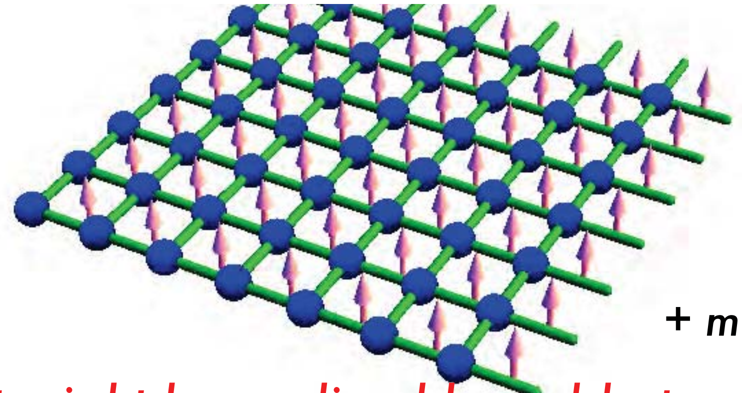
$$H = \sum_{\langle ij \rangle} c_i^\dagger e^{i\theta_{ij}} c_j + h.c.$$

c_i : electron annihilation operator



$$\sum_{j \in \langle ij \rangle} \theta_{ij} = 2\pi\phi_i = 2\pi\phi$$

Electrons in a 2D periodic potential

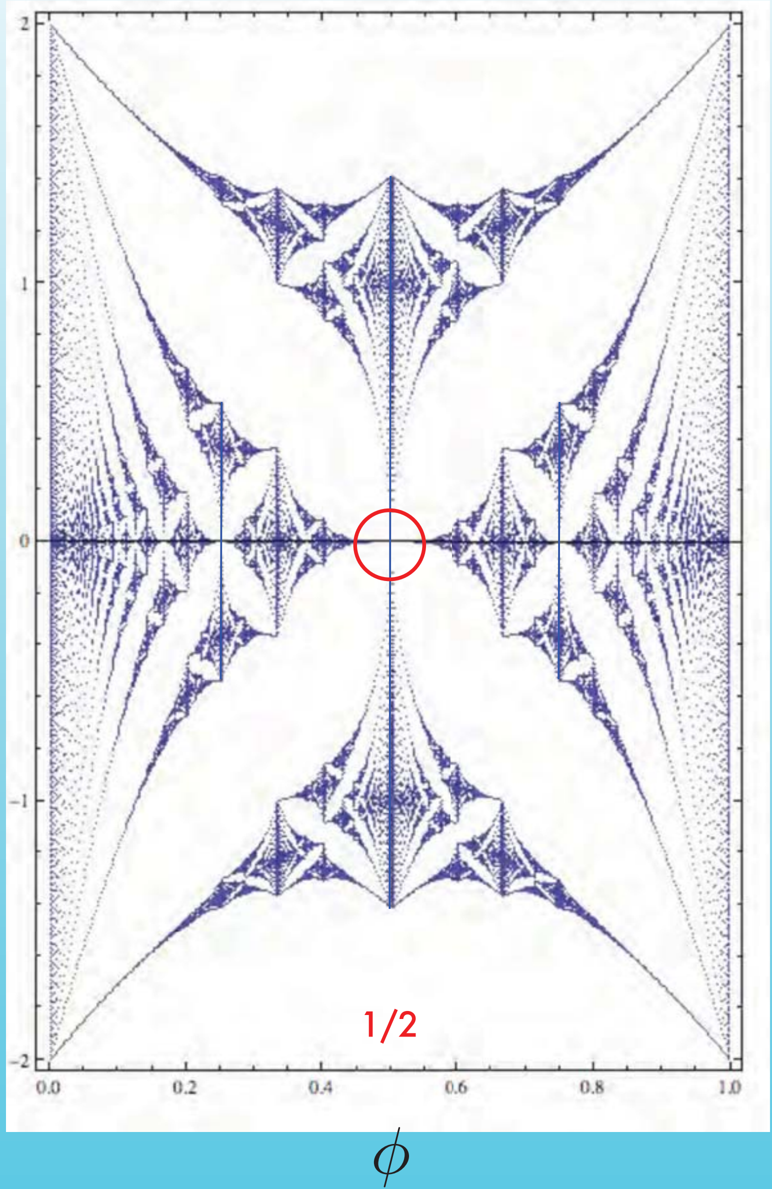


+ magnetic field

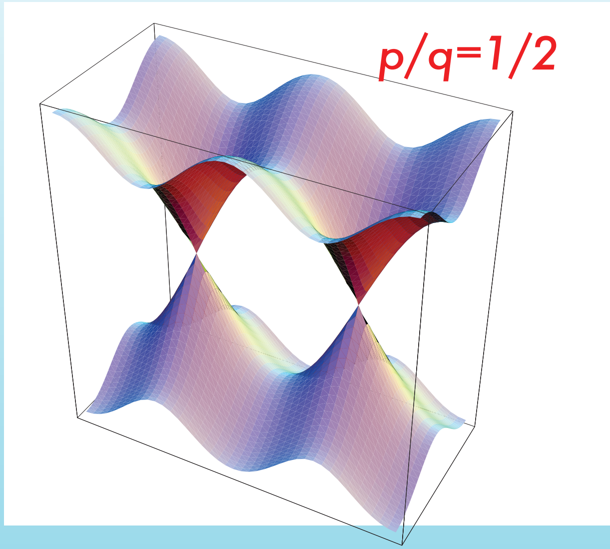
It might be realized by cold atoms?

The spectrum is fractal as a function of magnetic flux per plaquette

Are there massless Dirac dispersions ?

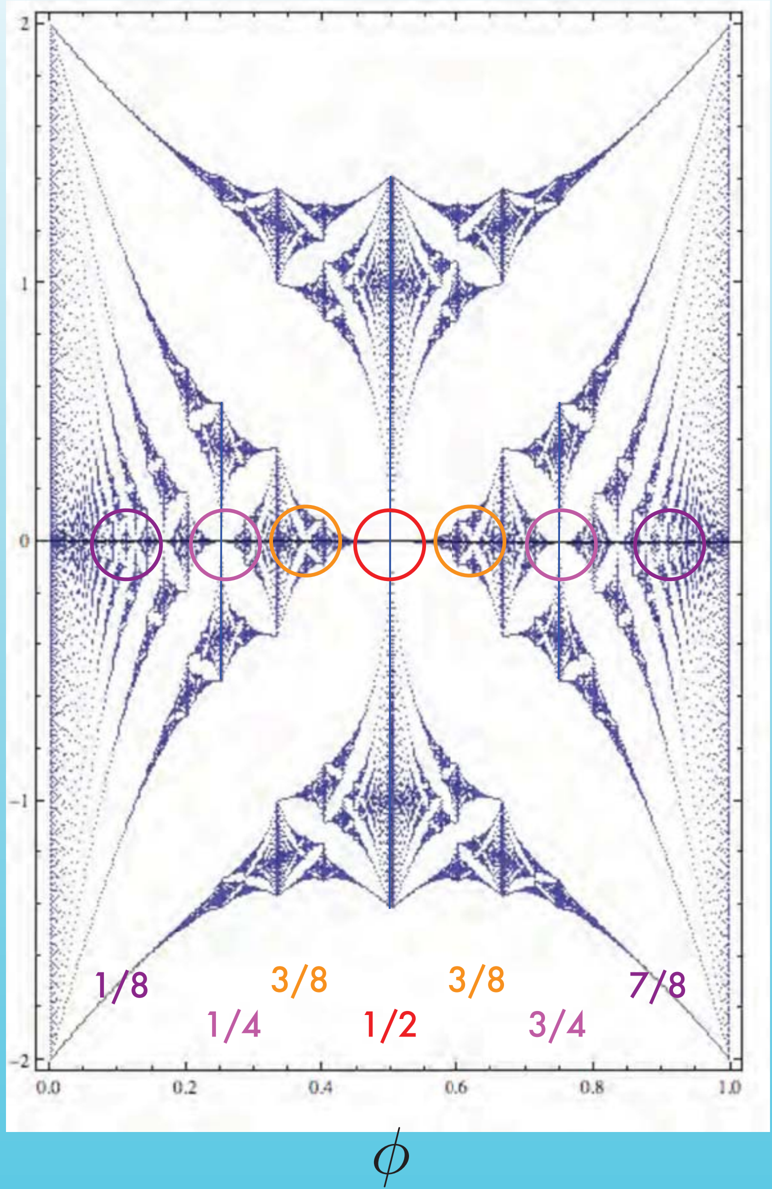


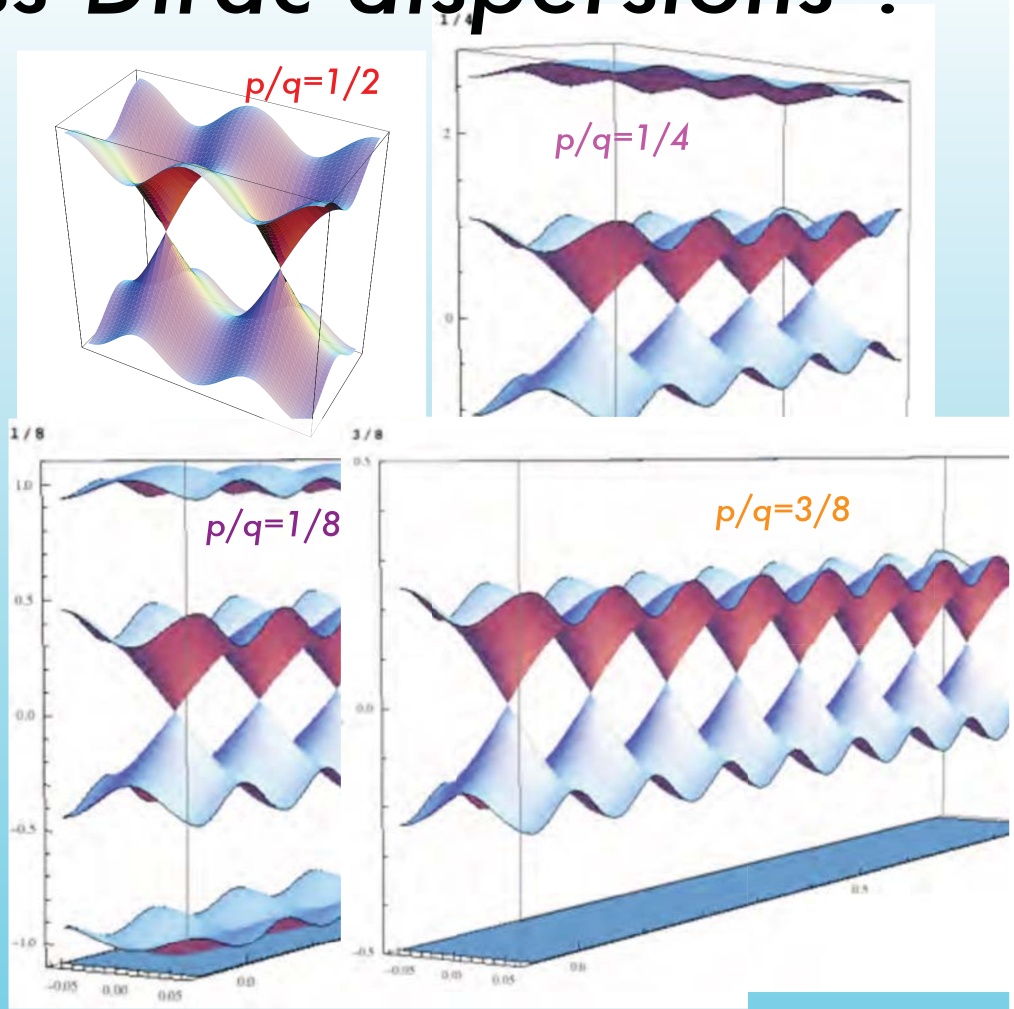
π-flux phases in two-dimensions



Fisher and Fradkin (1985)
Affleck and Marston (1988)
Hasegawa-Lederer-Rice-Wiegmann (1989)

Are there massless Dirac dispersions ?





Dirac fermions are everywhere for $\phi = p/q$

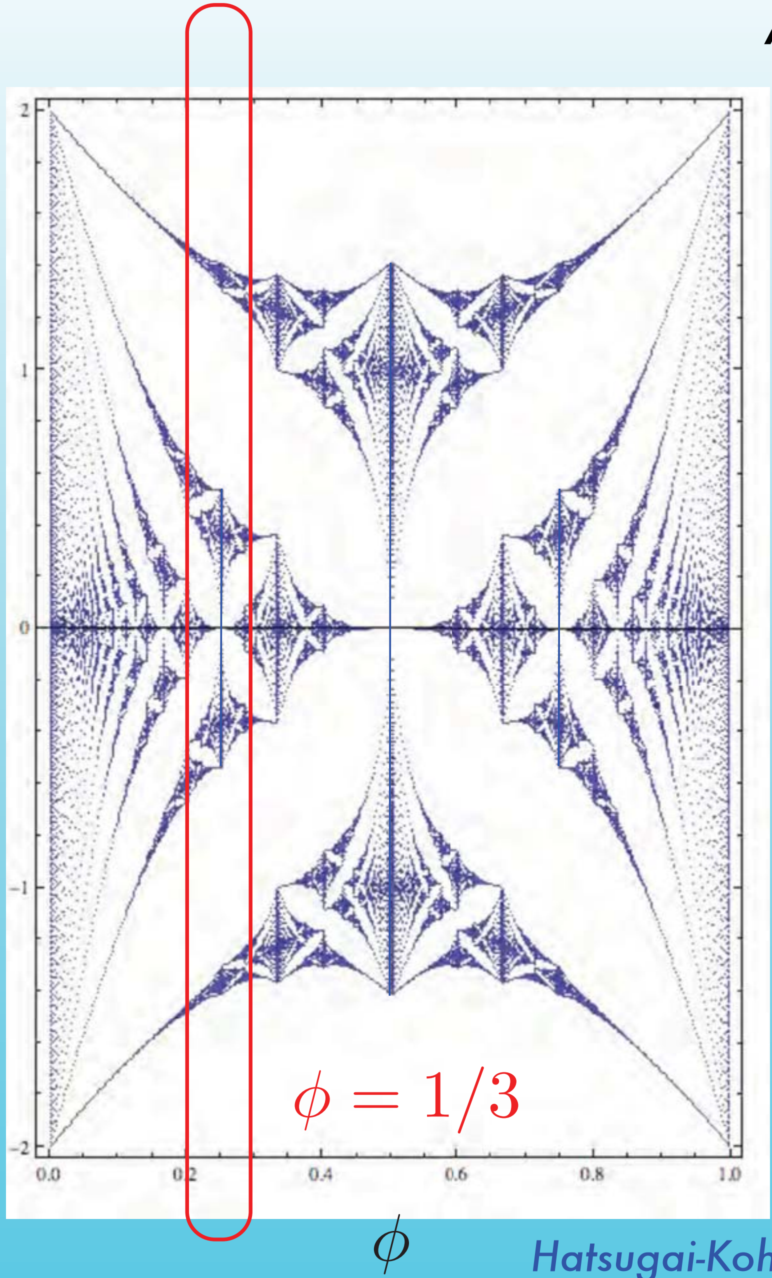
chiral symmetric q: even

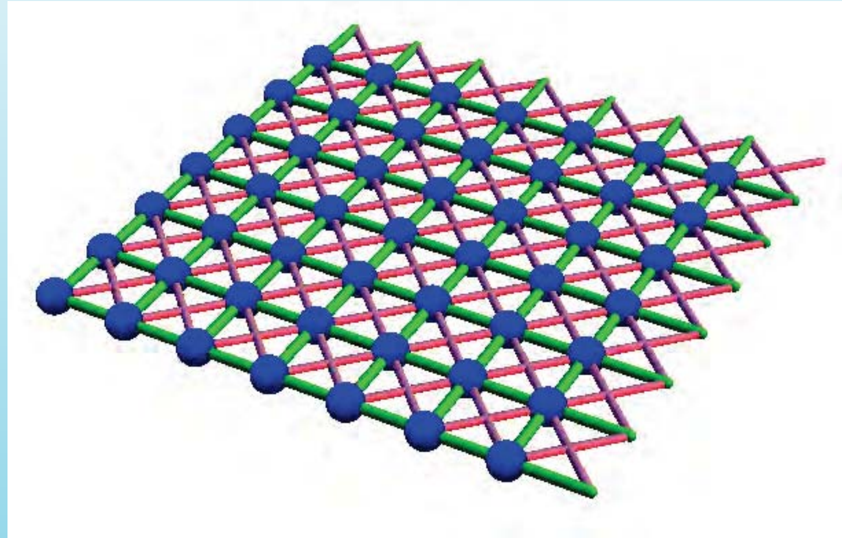
$$\{H, {}^{\exists}\gamma\} = 0, \quad \gamma^2 = 1$$

doubling Nielsen-Ninomiya '81

More Dirac dispersion !

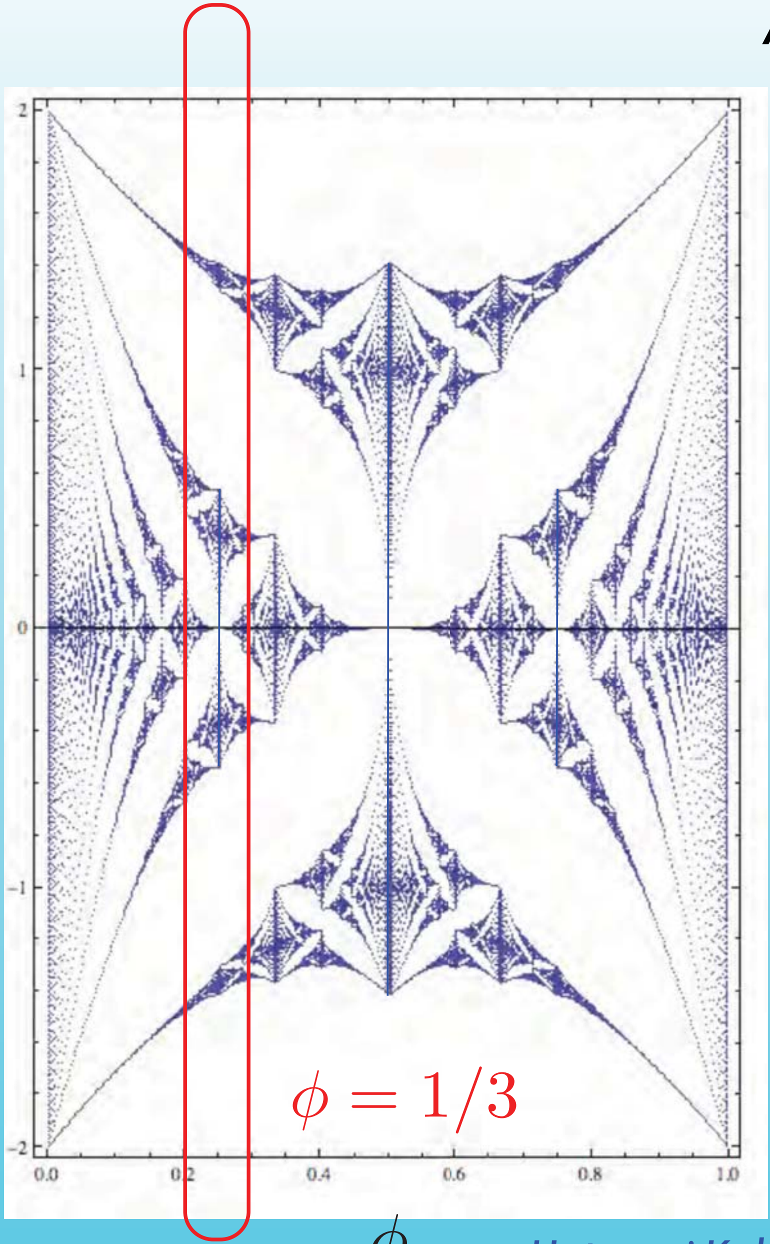
Adiabatic introduction of chiral symmetry breaking terms





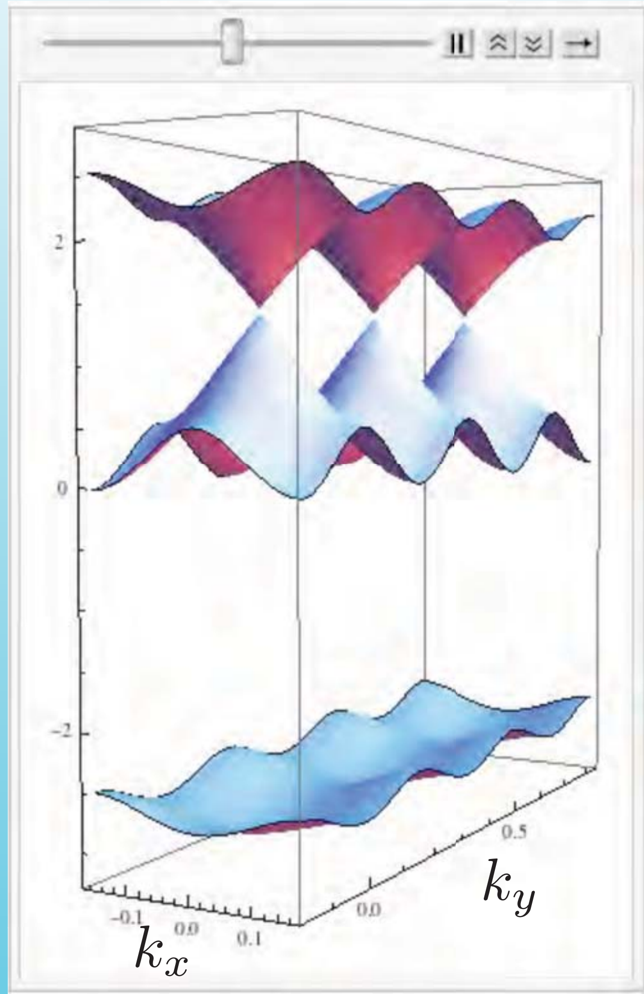
Hatsugai-Kohmoto'90

More Dirac dispersion !



Hatsugai-Kohmoto'90

Adiabatic introduction of chiral symmetry breaking terms



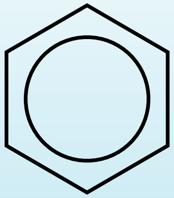
3 massless Dirac cones

in the momentum space

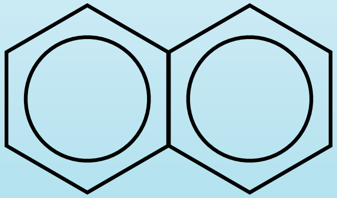
B

Graphene??

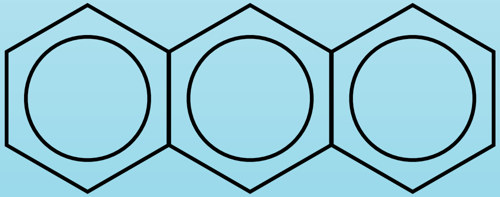
☆ π-electron systems



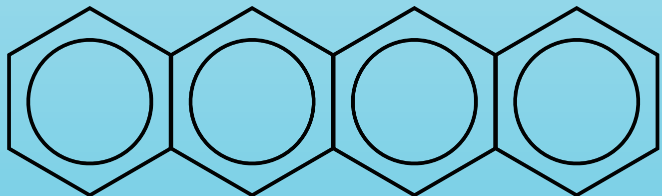
benzene



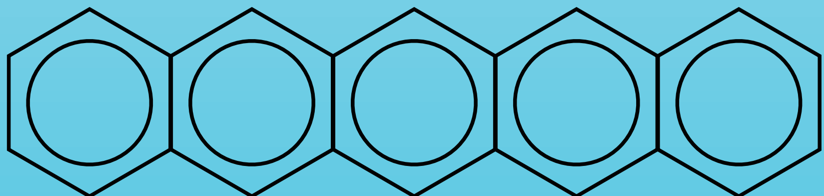
naphthalene



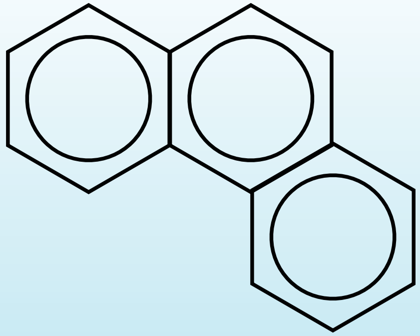
anthracene



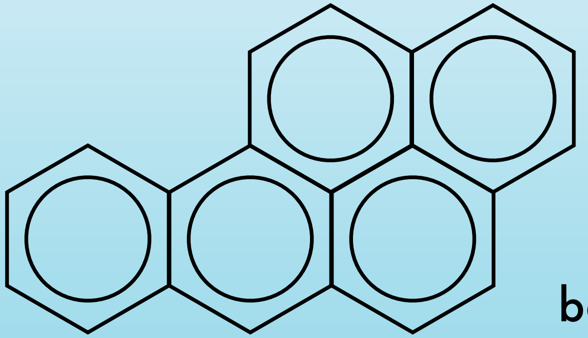
tetracene



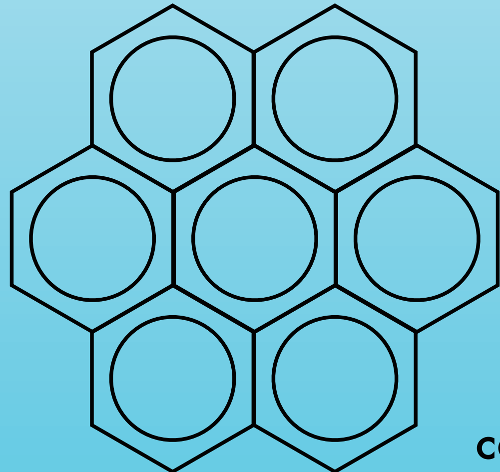
pentacene



phenanthrene



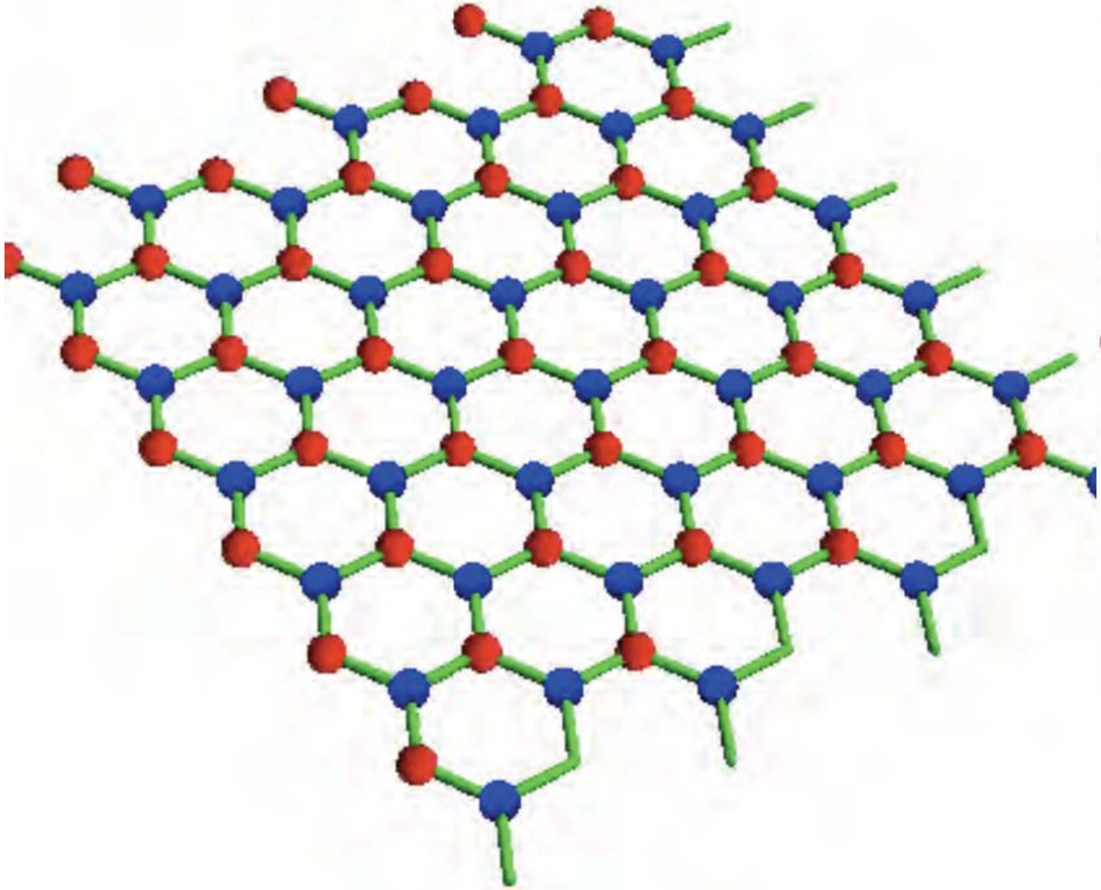
benzopyrene



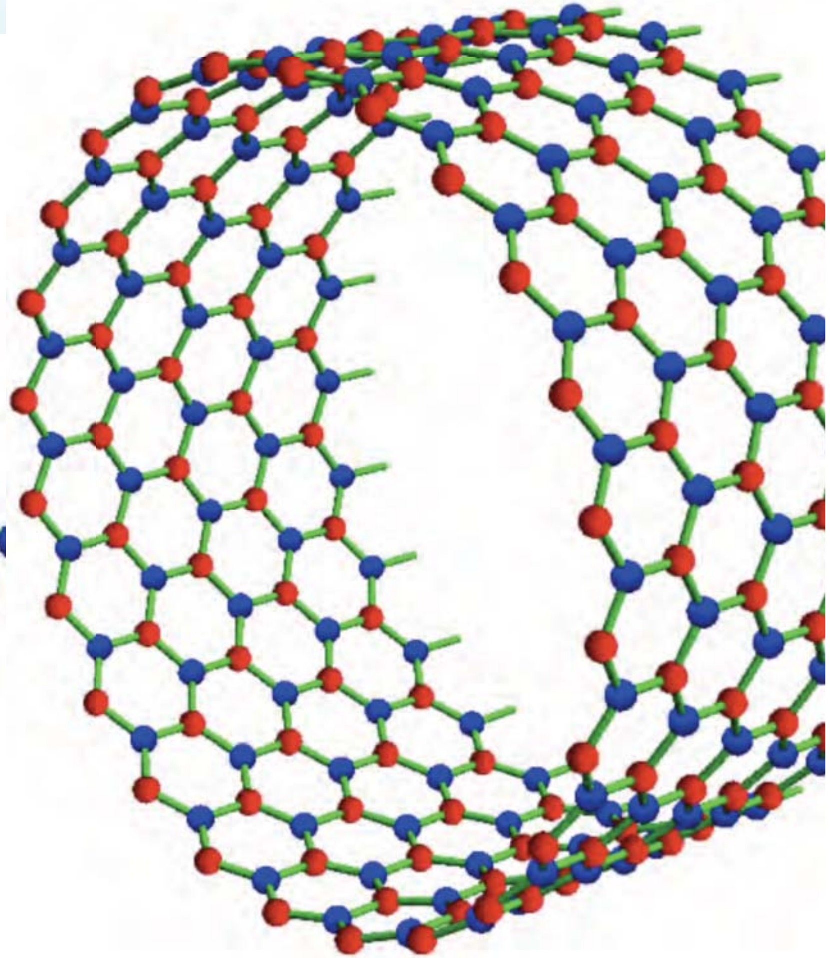
coronene

Graphene??

Graphene



Carbon nanotube

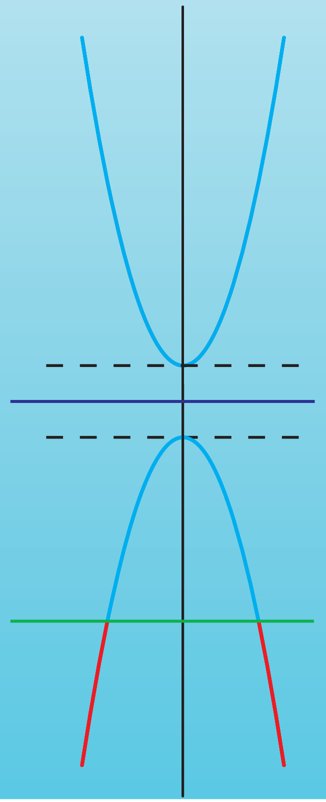


2D crystal: stable ??

What is special for the graphene

- 2Dimensional Semiconductors
- Energy gap is zero! Zero gap Semiconductors!

Normal Semiconductors



parabolic dispersion

$$E(k) = \frac{\hbar^2 k^2}{2m^*}$$

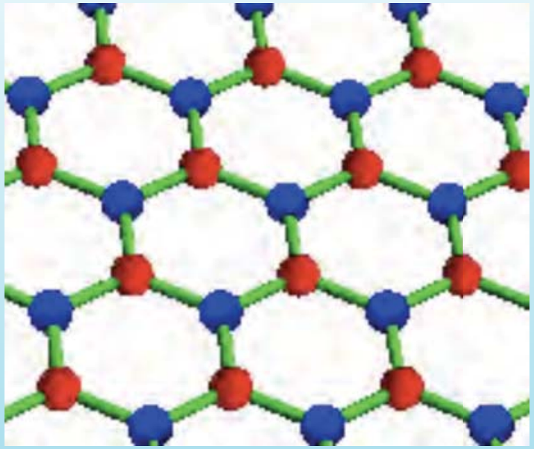
Effective Theory:

Schrodinger Equation
with effective mass

$$\begin{aligned} H\psi &= E\psi \\ H &= \frac{\boldsymbol{p}^2}{2m^*} \end{aligned}$$

$$\begin{aligned} \boldsymbol{p} &= -i\hbar\nabla \\ &= -i\hbar\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) \end{aligned}$$

Realization of massless Dirac fermions



☆ On a honeycomb lattice

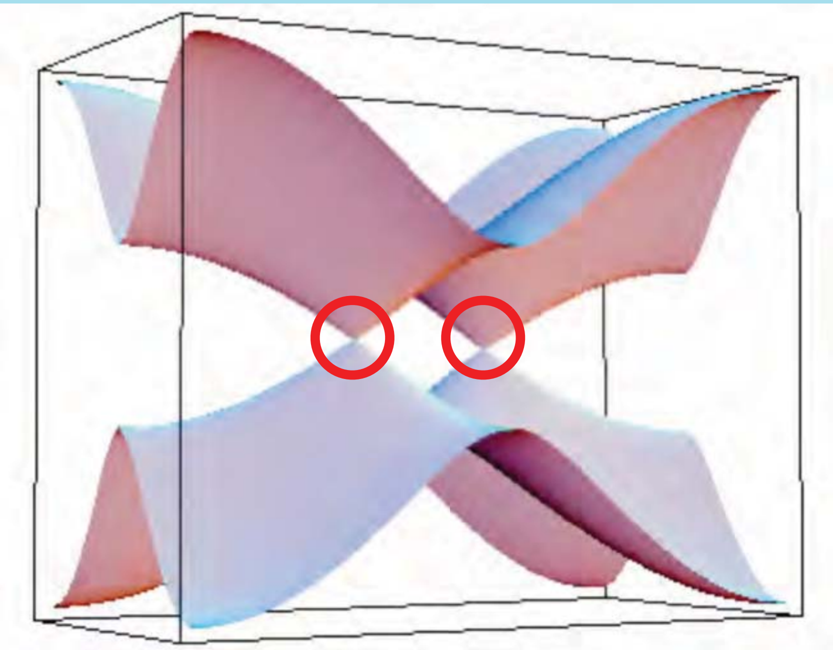
without any regularization

Wallace '47
Semenoff '85
Haldane '88

chiral symmetric　　doubling

$$\{H, {}^{\exists}\gamma\} = 0, \quad \gamma^2 = 1$$

Nielsen-Ninomiya '81



linear dispersion

$$E(k) = \pm c|k|, \ |k| = \sqrt{k_x^2 + k_y^2}$$

Effective Theory:

massless Dirac Equation

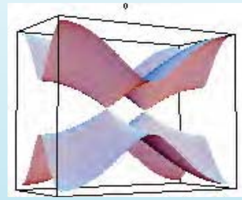
$$H\psi = E\psi$$

$$H = c\boldsymbol{\sigma}\cdot\boldsymbol{p} = c(\sigma_x p_x + \sigma_y p_y)$$

$$= c\begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

Is this anomalous behavior specific to the honeycomb lattice ?

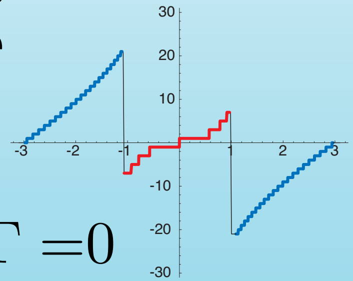
No! : It has topological Stability



- ☆ Vanishing DOS of the Dirac Fermions'

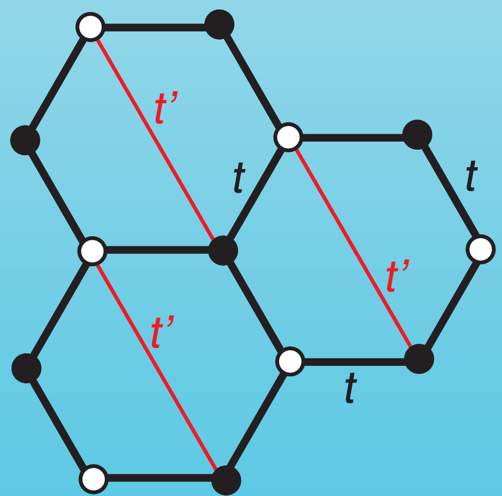
Hatsugai, Fukui, Aoki, '06

- ☆ Anomalous behavior of the Hall conductance



Chiral Symmetry (Bipartite Structure)

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0$$



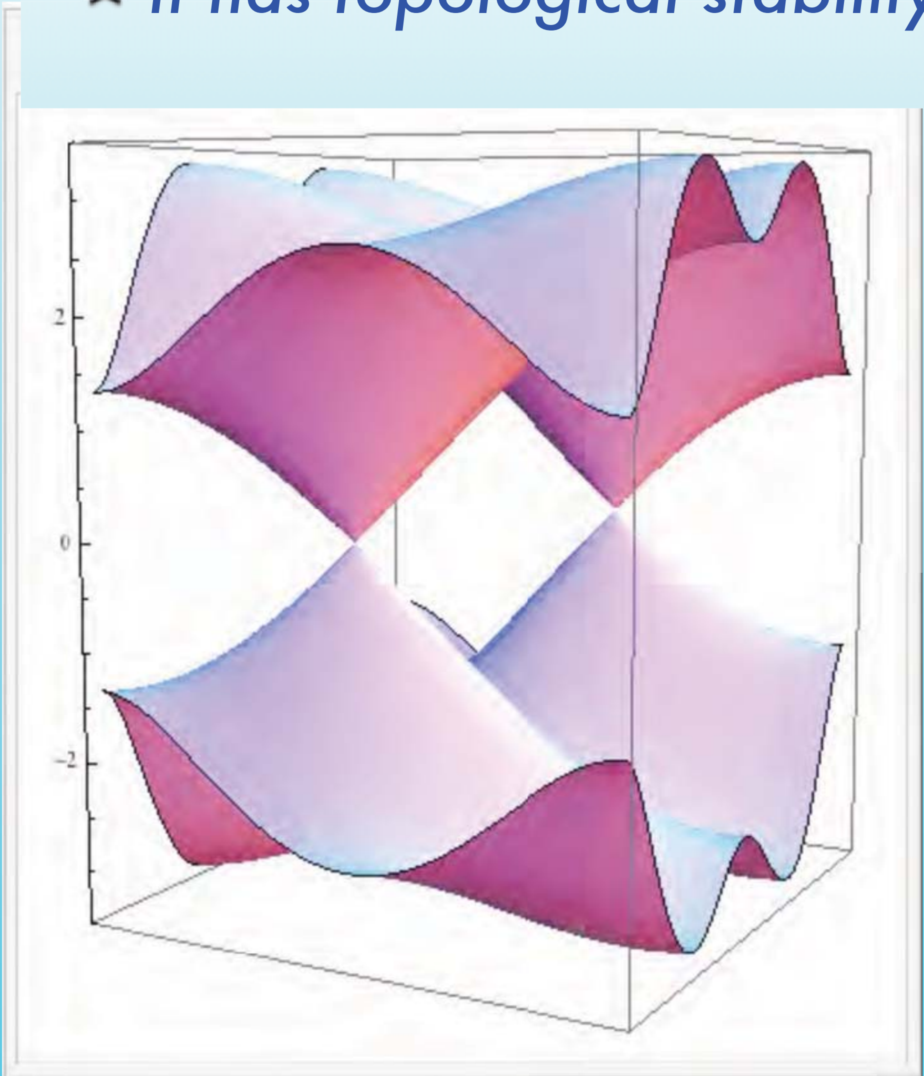
To demonstrate

introduce 2nd nearest neighbor hopping

Dirac Cones are Stable!

- The Dirac Cornes are not accidental
- It has topological stability

$t'/t = 1$: Square Lattice
$t'/t = 0$:Honeycomb Lattice
$t'/t = -1$: π Flux State



$$-3 < \frac{t'}{t} < 1 \rightarrow$$

Doubled Dirac Cones

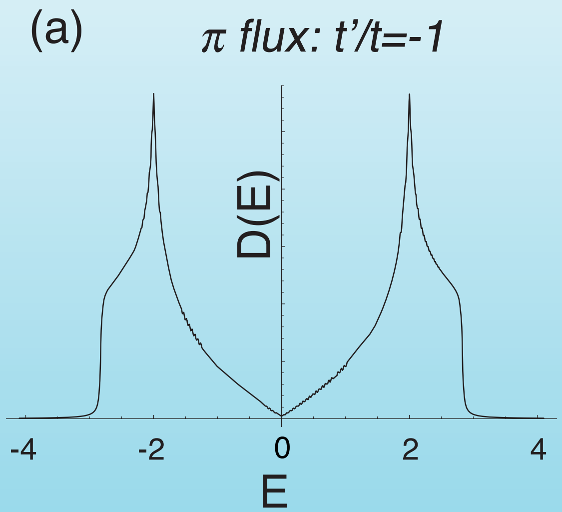
$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0$$

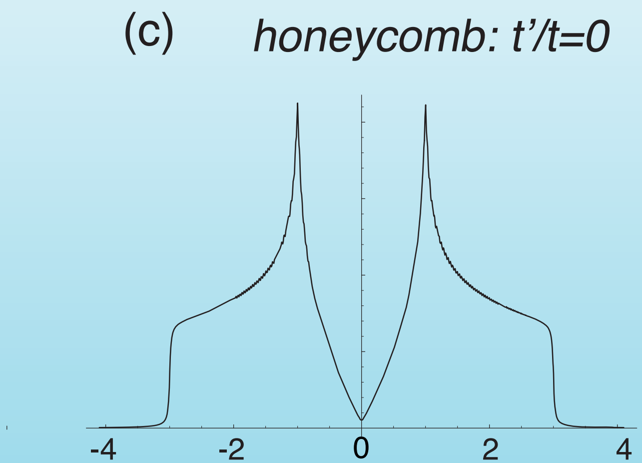
- Dirac Cones are stable for small but finite perturbation
- It can be gapped, if it's large.

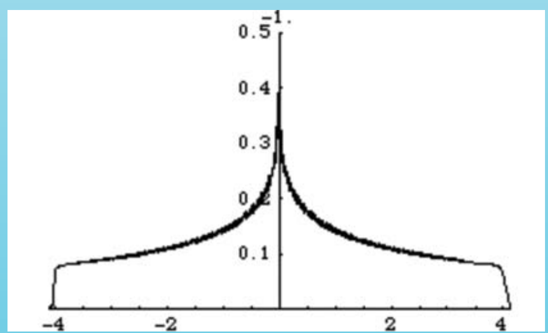
Hatsugai, Fukui, Aoki, '06

Density of States

- Vanishing DOS near the zero energy







$t'/t = 1$: Square Lattice

$t'/t = 0$:Honeycomb Lattice

$t'/t = -1$: π Flux State

Stability of the Dirac Cornes!

Topological Stability of the Dirac Cornes

$$H(k_x, k_y) = \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix} \qquad \{H, \Gamma\} = 0 \qquad \Gamma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

chiral symmetric

$$\Delta = -t(1 + e^{ik_y} + e^{ik_x}(1 + re^{-ik_y}), \qquad r = t'/t$$

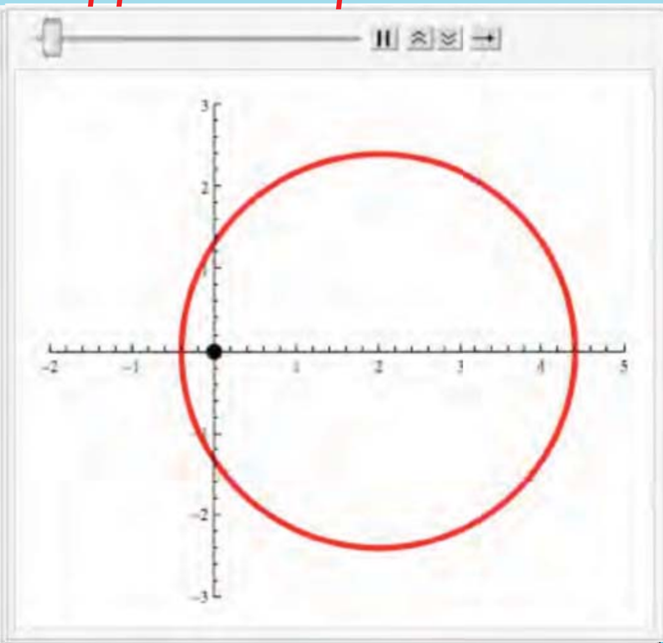
$$E(k_x, k_y) = \pm|\Delta|$$

$$= \pm\sqrt{(1 + \cos k_x + \cos k_y)^2 + (\sin k_x + \sin k_y)^2}$$

☆ General zeros of $\Delta(k_x, k_y)$ ⟶ Dirac Cones

Gapless

Gapped if the perturbation is too large



$\Delta(k_x, k_y)$, $k_x : 0 \to 2\pi$: loop $C(k_y)$ in $\mathbb{C}$

loop $C(k_y)$ moves : $k_y : 0 \to 2\pi$

The loop cut the origin → Dirac Cones

Topological Stability

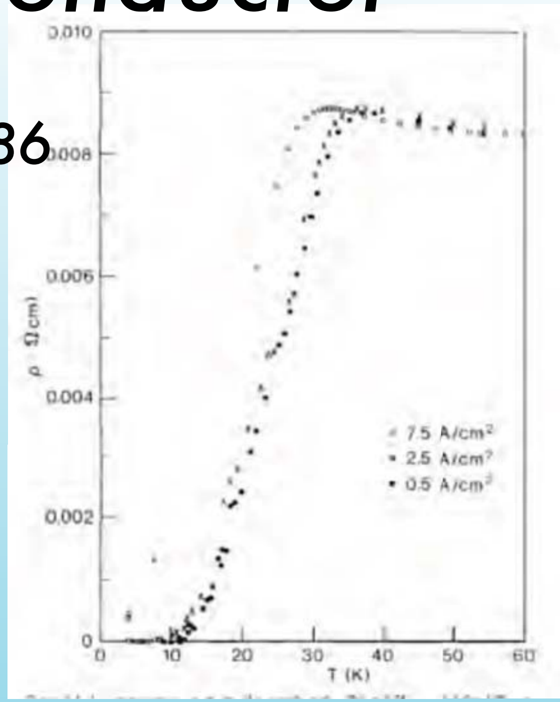
of

the doubled Dirac Cones

2D d-wave superconductor

High-T_C superconductivity

Bednorz-Muller '86



PEROVSKITE-TYPE OXIDES -
THE NEW APPROACH TO HIGH-T_c
SUPERCONDUCTIVITY

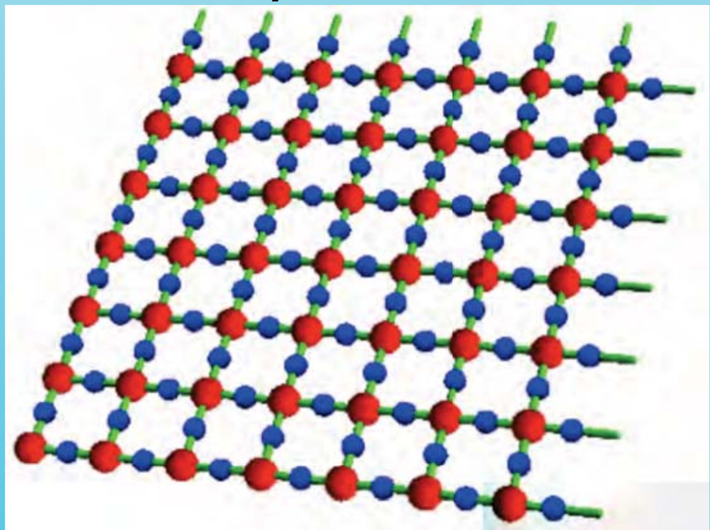
Nobel lecture, December 8, 1987

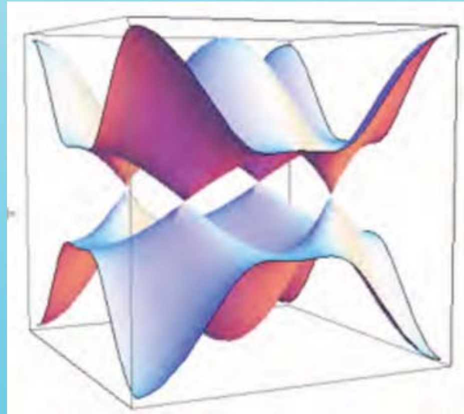
Nobelprize.org

by

J. GEORG BEDNORZ and K. ALEX MÜLLER

2D CuO_2 plane





Energy gap

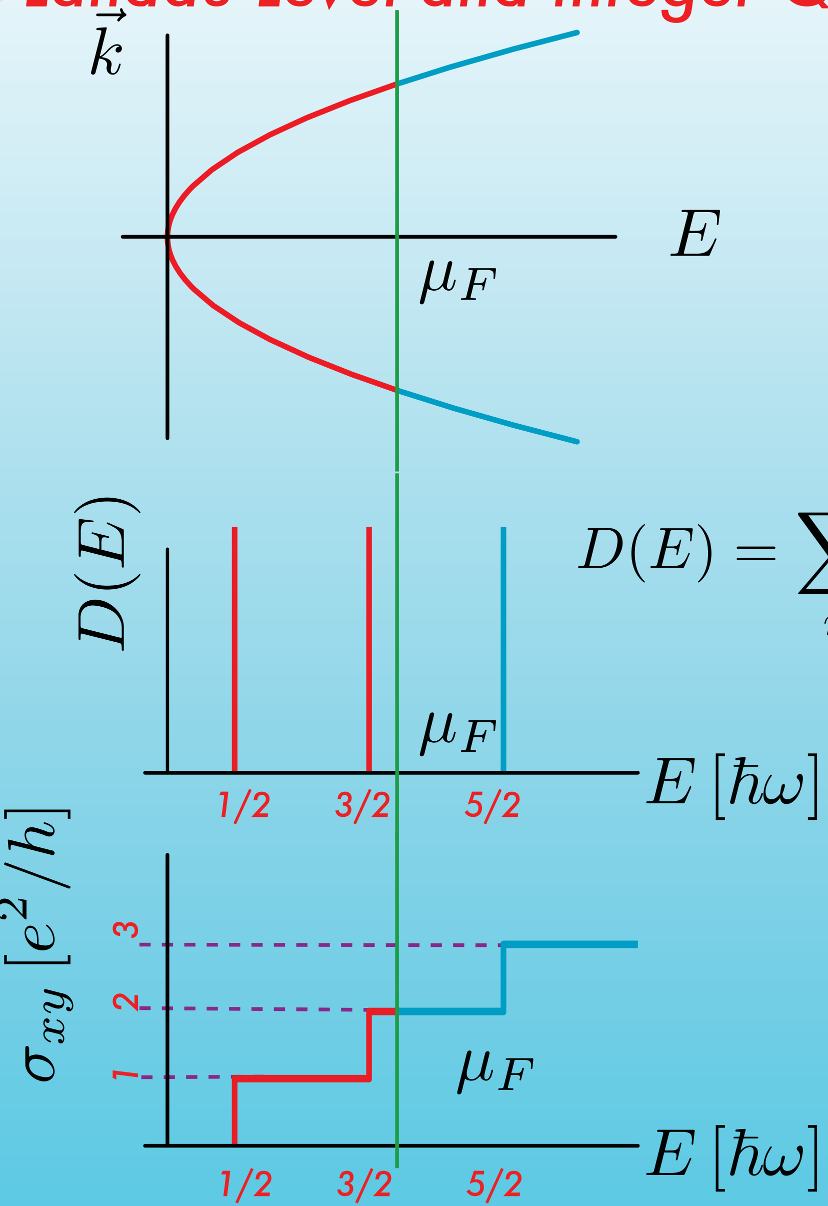
4 Dirac cones

2D gapless d-wave superconductor

P.A. Lee '93, ...

Conventional QHE

☆ **Landau Level and Integer QHE**



$$E(B=0) = \frac{\hbar^2}{2m}k^2$$

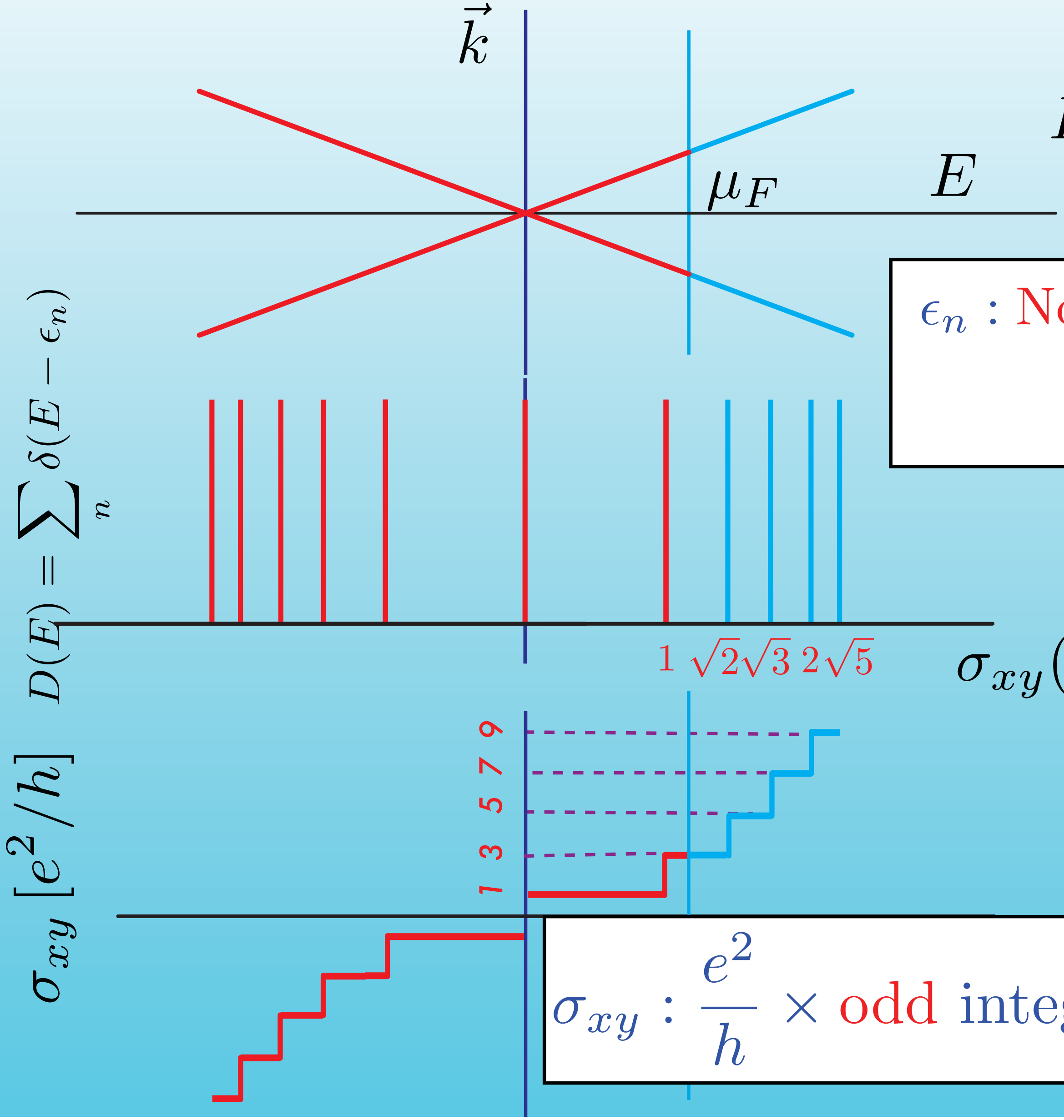
$$D(E) = \sum_n \delta(E - \epsilon_n) \qquad \epsilon_n = \hbar\omega(n + \frac{1}{2})$$

$$\sigma_{xy}(\mu_F) = \frac{e^2}{\hbar}n, \quad n = 1, 2, 3, \cdots$$

$$\epsilon_{n-1} < \mu_F < \epsilon_n$$

QHE of Graphene (Gapless Semiconductor)

☆ Landau Level of Doubled Dirac Fermions



$$E(B=0) = \pm \hbar c |\vec{k}|$$

ϵ_n : No zero point energy shift

$$\epsilon_n = \pm C\sqrt{Bn}$$

McClure, 1956

$$\sigma_{xy}(\mu_F) = \frac{e^2}{\hbar}(2n+1)$$

$$n = 0, 1, 2, 3, \cdots$$

$$\epsilon_{n-1} < \mu_F < \epsilon_n$$

$\sigma_{xy} : \dfrac{e^2}{h} \times$ odd integer

Zheng-Ando 2002

Gusynin-Sharapov, 2005

Peres-Guinea-Neto 2006

...

Theoretical Background

Why the QHE of graphene is interesting ?

generic Topological Insulators

gapped & topologically nontrivial

Topological Insulators

Gapped Quantum Liquids

Featureless !!

Use Geometrical Phases of the Quantum states

To characterize the topological insulators

- ✓ Berry phases
- ✓ Chern numbers

  1st, 2nd, ...

multi-component : non Abelian

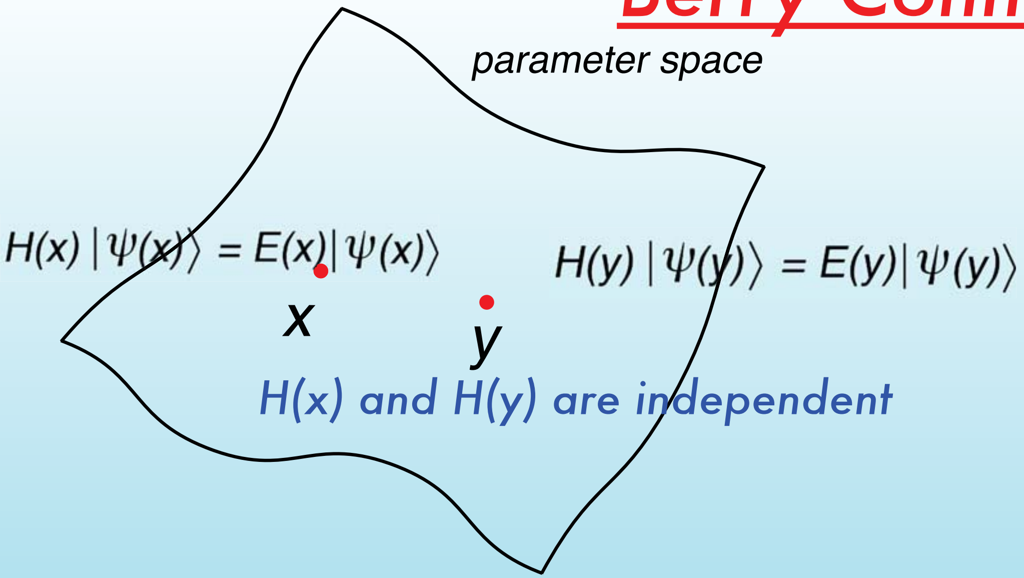
✓ Berry connections

$$A = \Psi^\dagger d\Psi = \Psi^\dagger \partial_\mu \Psi \, dx^\mu$$

$$\Psi = (|\psi_1\rangle, \cdots, |\psi_M\rangle)$$

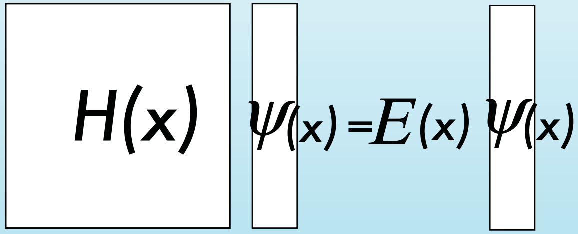
Berry Connection?

Berry '84



Eigenvectors (space)

with Parameters



(Abelian)

Information between nearby states

Fiber Bundle

Berry connection : $A_\psi = \langle \psi | d\psi \rangle = \langle \psi | \frac{d}{dx} \psi \rangle dx.$

Gauge Transformation

$$|\psi(x)\rangle = |\psi'(x)\rangle e^{i\Omega(x)}$$

gauge potential

$$A_\psi = A'_\psi + id\Omega = A'_\psi + i\frac{d\Omega}{dx}dx$$

Geometrical quantities

$$i\gamma_C(A_\psi) = \int_C A_\psi$$

: Berry phase :gauge dependent $\mod 2\pi$

$$C_1(A_\psi) = \frac{1}{2\pi i}\int_S dA_\psi$$

: 1st Chern # :gauge invariant

Quantized Hall Conductance as the Chern numbers

☆ TKNN formula: σ_{xy} as a topological invariant

Kubo formula

Thouless-Kohmoto-Nightingale-den Nijs '82

$$\sigma_{xy} = \frac{e^2}{h} \sum_{\ell:\epsilon_\ell(k)<E_F} C_\ell$$

Need to sum over bands below E_F

$$C_\ell = \frac{1}{2\pi i}\int_{T^2:\mathrm{BZ}} F_\ell$$

:First Chern number of the ℓ-th Band

$$F_\ell = dA_\ell = \langle d\psi_\ell | d\psi_\ell \rangle$$

$$A_\ell = \langle \psi_\ell | d\psi_\ell \rangle$$

Parameter space : Momentum

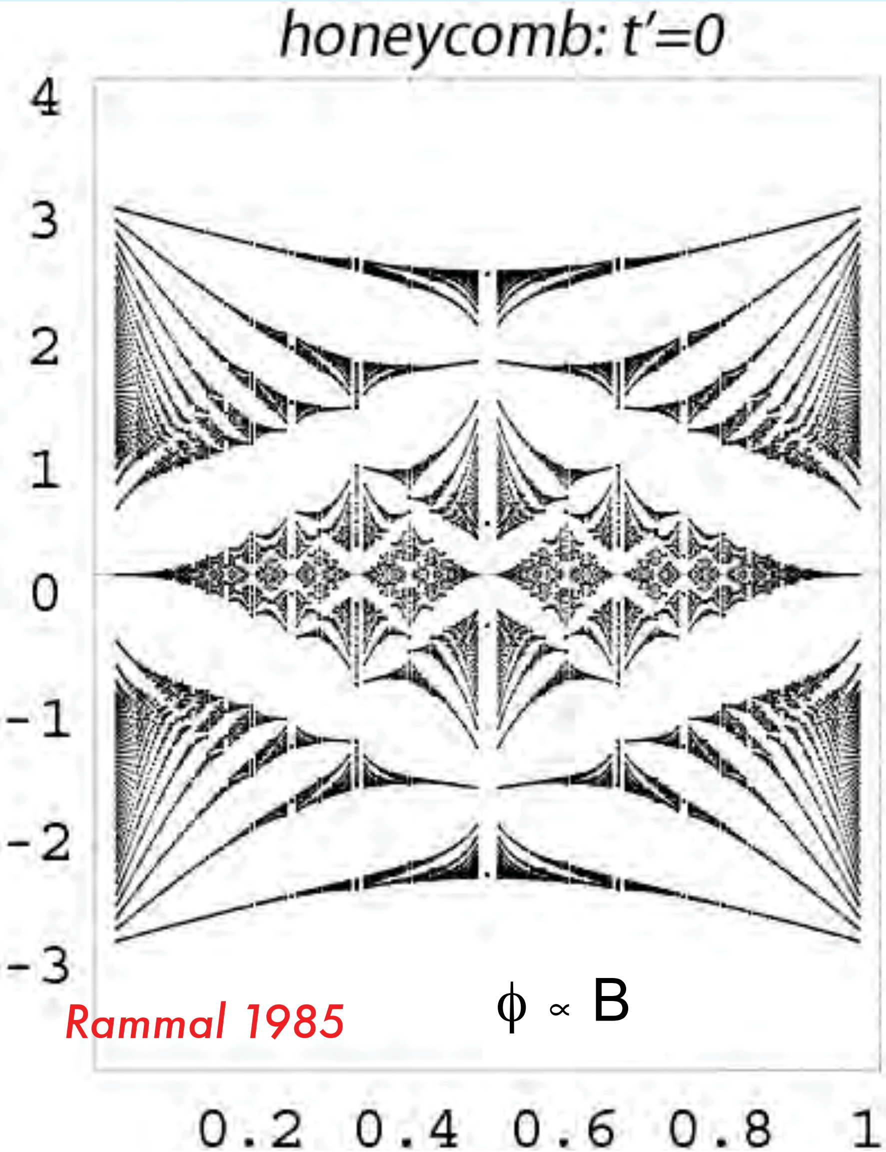
$$H(k)|\psi_\ell(k)\rangle = \epsilon(k)|\psi_\ell(k)\rangle$$

$$k \in T^2_{\mathrm{BZ}} = \{k = (k_x, k_y)|\ 0 \le k_x, k_y \le 2\pi\}$$

$$d = dk_\mu \frac{\partial}{\partial k_\mu}$$

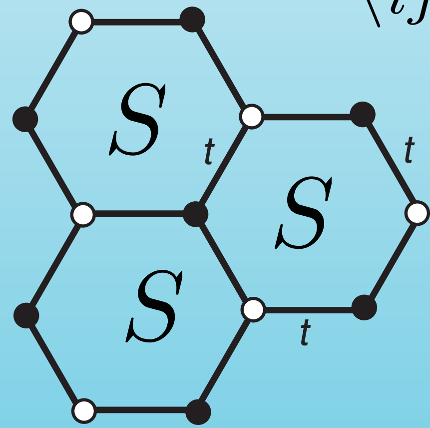
Graphene in a magnetic field

☆ *Tight-binding model on a honeycomb lattice*



$$H = \sum_{\langle ij \rangle} t_{ij} e^{i\theta_{ij}}$$

$$2\pi\phi_P = \sum_{\langle ij \rangle \in P} \theta_{ij}$$



$$\phi_P = \phi = \frac{p}{q}$$

$$(p, q) = 1$$

$$\phi = \frac{BS}{\Phi_0}$$

Φ_0 : flux quantum

E=0 Landau level :
outside Onsager's semiclassical quantisation scheme

Bulk Hall conductance of graphene

☆ Hall conductance by Chern number

Counting vortices in the band

$$\sigma_{xy}^{j} = \frac{e^2}{h}\sum_{\ell=1}^{j} C_\ell, \quad C_\ell = \frac{1}{2\pi i}\int_{BZ} dA_\ell, \quad A_\ell = \langle \psi_\ell | d\psi_\ell \rangle$$

$\epsilon_\ell(k) < \mu_F, \ \ell = 1, \cdots, j$

Thouless-Kohmoto-Nightingale-den Nijs 1982
with randomness Aoki-Ando 1986

graphene

Calculating each of them separately
is unstable for the weak field !

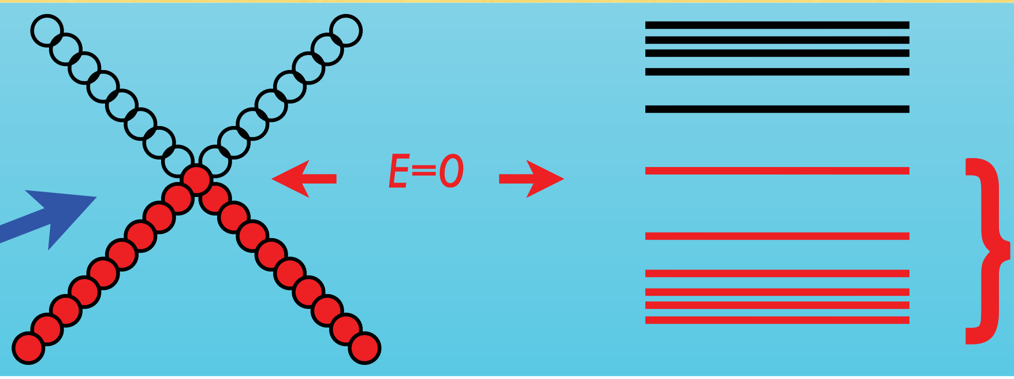
E=0

Need to sum over them

Sum over the filled bands

Need to sum for many bands until E=0

Numerical difficulty for the weak field
Level crossing below E_F (with randomness)
Need to fill negative energy Dirac sea



Graphene: Bulk σ_{xy} of the Filled Dirac Sea

Hatsugai, Fukui, Aoki, '06

- Integration of the NonAbelian Berry Connection of the filled "Fermi Sea" & "Dirac Sea"

$$H_j(k)|\psi_j(k)\rangle = \epsilon_j(k)|\psi_j(k)\rangle$$

$$\Psi = (|\psi_1\rangle, \cdots, |\psi_M\rangle)$$ Collect M states below the Fermi level

$$A_{FS} \equiv \Psi^\dagger d\Psi = \begin{pmatrix} \langle\psi_1^\dagger|d\psi_1\rangle & \cdots & \langle\psi_1^\dagger|d\psi_M\rangle \\ \vdots & \ddots & \vdots \\ \langle\psi_M^\dagger|d\psi_1\rangle & \cdots & \langle\psi_M^\dagger|d\psi_M\rangle \end{pmatrix}$$

Wilczek & Zee '84

Matrix vector potential of the Fermi (Dirac) Sea
Non Abelian extension for the Chern numbers

Technology #1

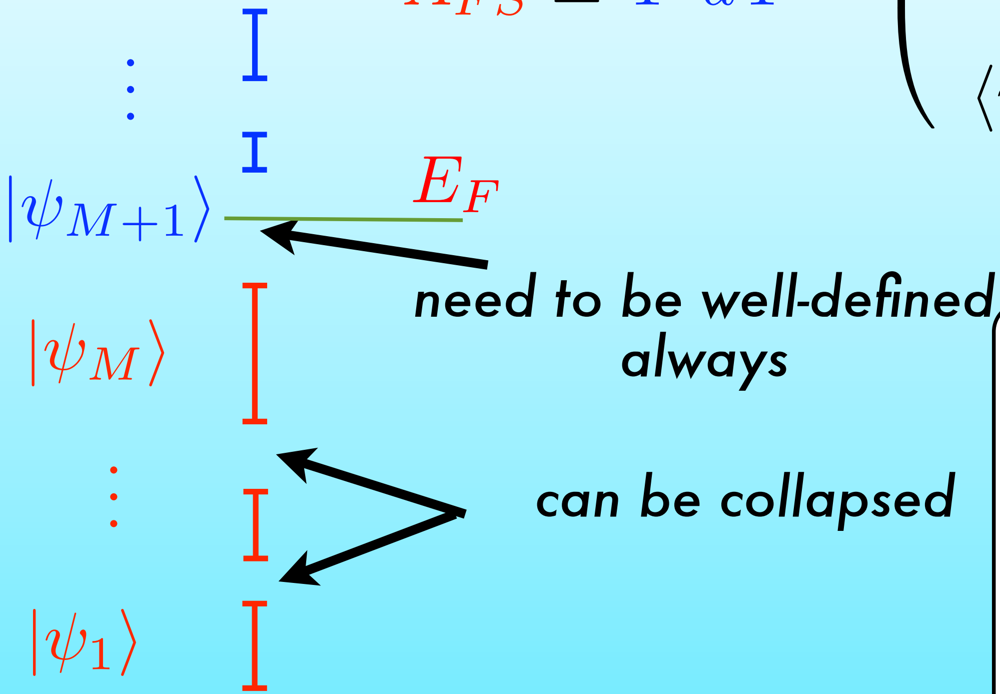
$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\int_{T^2} \mathrm{Tr}_M dA_{FS}$$

Y.H., J. Phys. Soc.Jpn. 73, 2604 (2004), ibid 74,1374(2005)

Stability of the Berry connection

$$\Psi = (\,|\psi_1\rangle, \cdots, |\psi_M\rangle)$$

$$A_{FS} \equiv \Psi^\dagger d\Psi = \begin{pmatrix} \langle\psi_1^\dagger|d\psi_1\rangle & \cdots & \langle\psi_1^\dagger|d\psi_M\rangle \\ \vdots & \ddots & \vdots \\ \langle\psi_M^\dagger|d\psi_1\rangle & \cdots & \langle\psi_M^\dagger|d\psi_M\rangle \end{pmatrix}$$



Gauge transformation

$$\Psi = \Psi\,'\omega, \quad \omega \in U(M)$$

mix among the filled states

Physical quantities like the Hall conductance should be stable if the many-body gap remains open

Y.H., J. Phys. Soc.Jpn. 73, 2604 (2004),ibid 74,1374(2005)

Non abelian Berry Connection and Gauge Transformation

- Berry's Connection $\boldsymbol{\Psi} = (|\psi_1\rangle, |\psi_2\rangle, \cdots, |\psi_M\rangle)$

$$\mathcal{A} = \boldsymbol{\Psi}^\dagger d\boldsymbol{\Psi} = \begin{pmatrix} \langle\psi_1| \\ \vdots \\ \langle\psi_M| \end{pmatrix} (|d\psi_1\rangle, \cdots, |d\psi_M\rangle)$$

- Gauge Transformation (Base Change)

$$\boldsymbol{\Psi}' = \boldsymbol{\Psi}\boldsymbol{\omega} = (|\psi_1\rangle, |\psi_2\rangle, \cdots, |\psi_M\rangle) \begin{pmatrix} \omega_{11} & \cdots & \omega_{1M} \\ \vdots & \ddots & \vdots \\ \omega_{M1} & \cdots & \omega_{MM} \end{pmatrix}$$

$$\boldsymbol{\mathcal{A}}' = \boldsymbol{\Psi}' d\boldsymbol{\Psi}' = \boldsymbol{\omega}^\dagger \boldsymbol{\Psi}^\dagger (d\boldsymbol{\Psi}\boldsymbol{\omega} + \boldsymbol{\Psi} d\boldsymbol{\omega})$$

$$= \boldsymbol{\omega}^\dagger \boldsymbol{\mathcal{A}} \boldsymbol{\omega} + \boldsymbol{\omega}^\dagger d\boldsymbol{\omega}$$

- Field Strength $\boldsymbol{\mathcal{F}} = d\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{A}}$

$$\boldsymbol{\mathcal{F}}' = \boldsymbol{\omega}^{-1} \boldsymbol{\mathcal{F}} \boldsymbol{\omega}$$

$$\mathrm{Tr}\ \boldsymbol{\mathcal{F}} = \mathrm{Tr}\ \boldsymbol{\mathcal{F}}' = \mathrm{Tr}\ d\boldsymbol{\mathcal{A}}$$

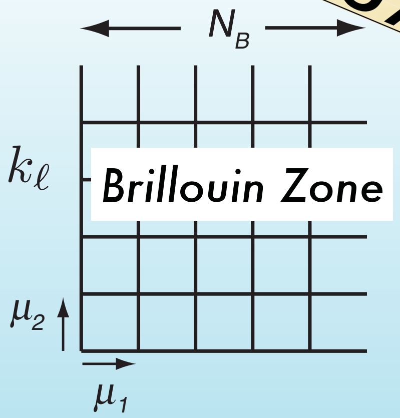
Wilczek & Zee '84

Numerical Technique from the Lattice gauge theory

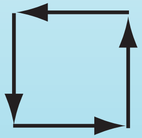
☆ Topological Invariant on Discretized Lattice

Lattice in k space (discretization for the integral)

Technical Advantage for large Chern Numbers



$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\sum F_{1234}$$ gauge invariant & integer



$$F_{1234} = \mathrm{Im}\,\log U_{12}U_{23}U_{34}U_{41}$$

$$\longrightarrow \quad U_{mn} = \det_j \Psi_m^\dagger \Psi_n, \quad \Psi_n = (\psi_1(k_n), \cdots, \psi_j(k_n))$$

Fermi Sea of j filled bands

c.f. polarization (Berry phase) King-Smith & Vanderbilt '93

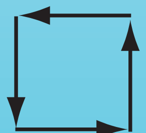
Abelian

$\xrightarrow{U_\mu(k_\ell)}$

$$U_\mu(k_\ell) \equiv \langle n(k_\ell)|n(k_\ell+\hat\mu)\rangle/\mathcal{N}_\mu(k_\ell)$$

$$\mathcal{N}_\mu(k_\ell) = |\langle n(k_\ell)|n(k_\ell+\hat\mu)\rangle|$$

$\tilde F_{12}(k_\ell)$



$$\tilde F_{12}(k_\ell) \equiv \ln U_1(k_\ell)U_2(k_\ell+\hat 1)U_1(k_\ell+\hat 2)^{-1}U_2(k_\ell)^{-1}$$

$$-\pi < \tilde F_{12}(k_\ell)/i \le \pi$$ (principal value)

Luscher '82, '99
Phillips & Stone
Fujiwara & Suzuki

Without Gauge fixing

Fukui-Hatsugai-Suzuki 2005

Admissibility and Critical Mesh Size

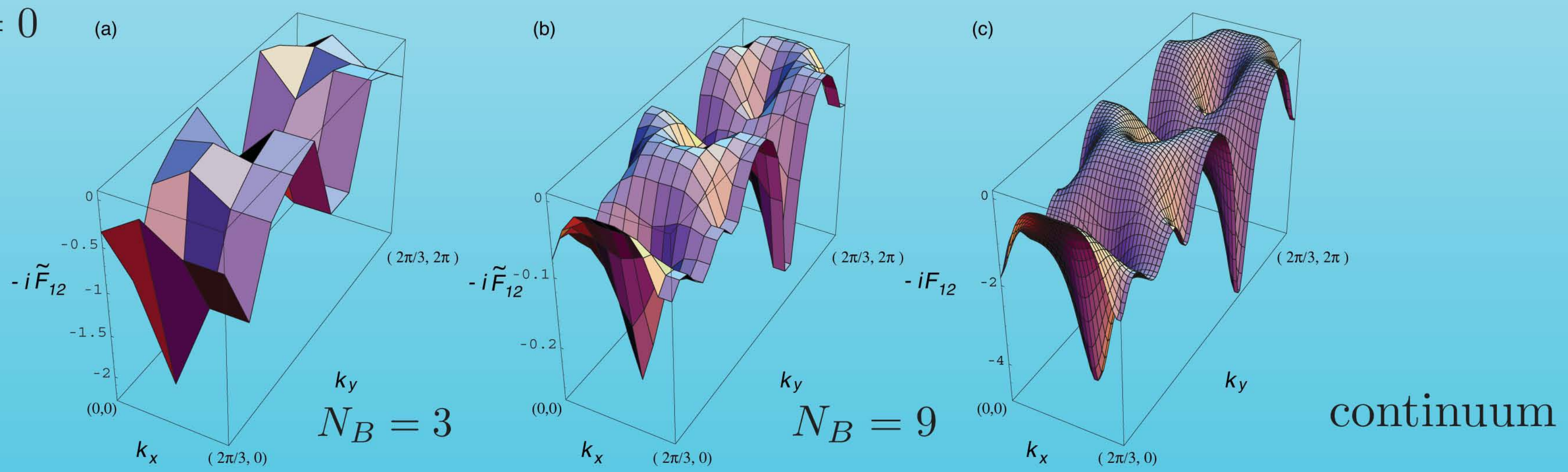
Fukui-Hatsugai-Suzuki 2005

- Admissibility Condition: $|\tilde{F}_{12}| < \pi$ Luscher '82, '
  - Need this for consistency
- Critical Mesh Size : $N_B^C = \mathcal{O}(\tilde{c})$ $\tilde{c} = c \quad \text{if } N_B > N_B^C$
  - To reproduce the one in the continuum
- Coarse discretization work quite well !!
  - Advantage for Practical Numerical Calculation $\tilde{c} = 2 \quad \text{for both (a) and (b)}$

$$\tilde{c} = \frac{1}{2\pi i}\sum_\ell \tilde{F}_{12}(k_\ell) \xrightarrow{N_B\to\infty} c = \frac{1}{2\pi i}\int \mathrm{Tr}\, dA$$

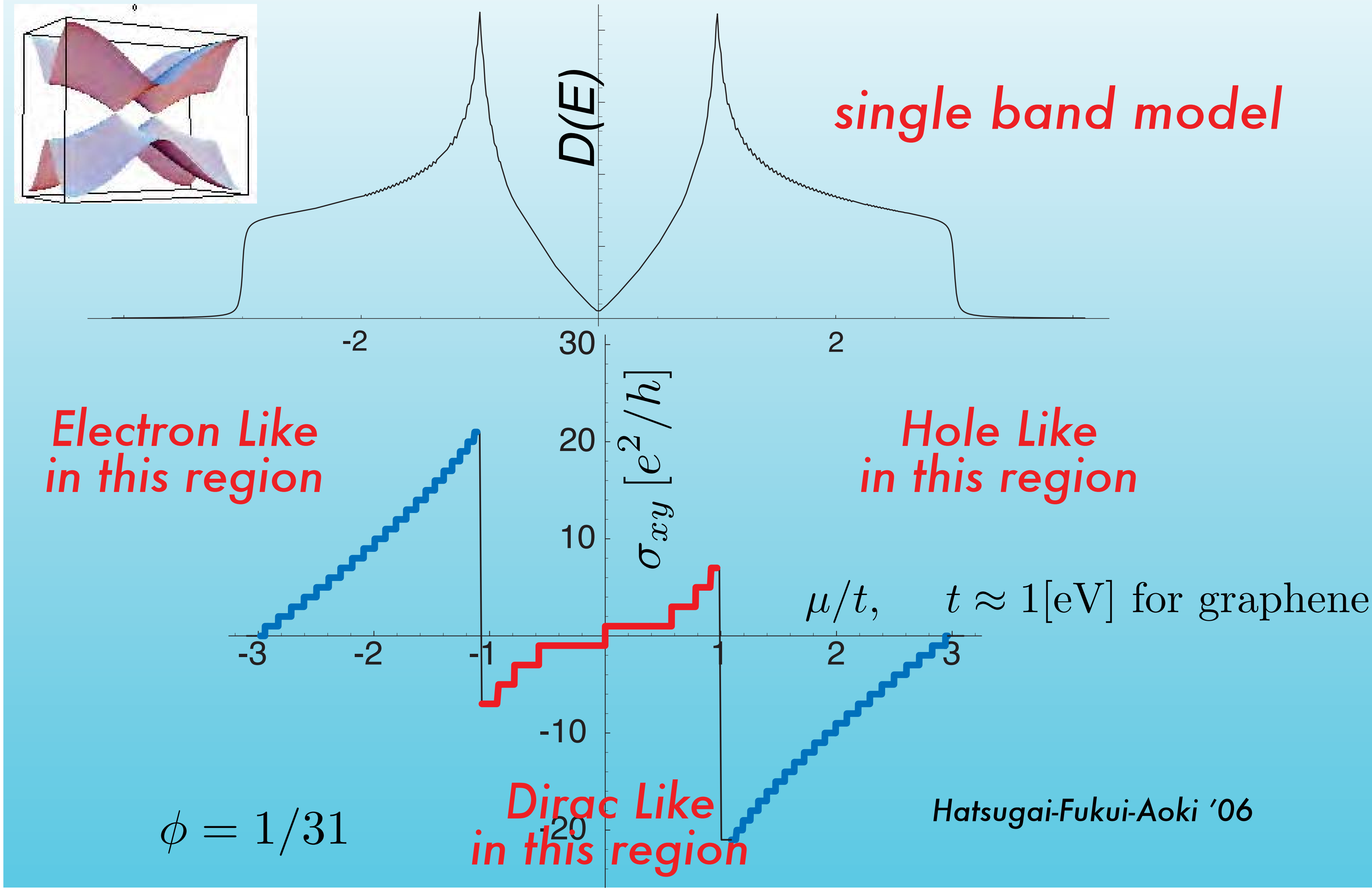
Lattice Field Strength F_{12} and Continuum Field Strength F_{12}

$t' = 0$



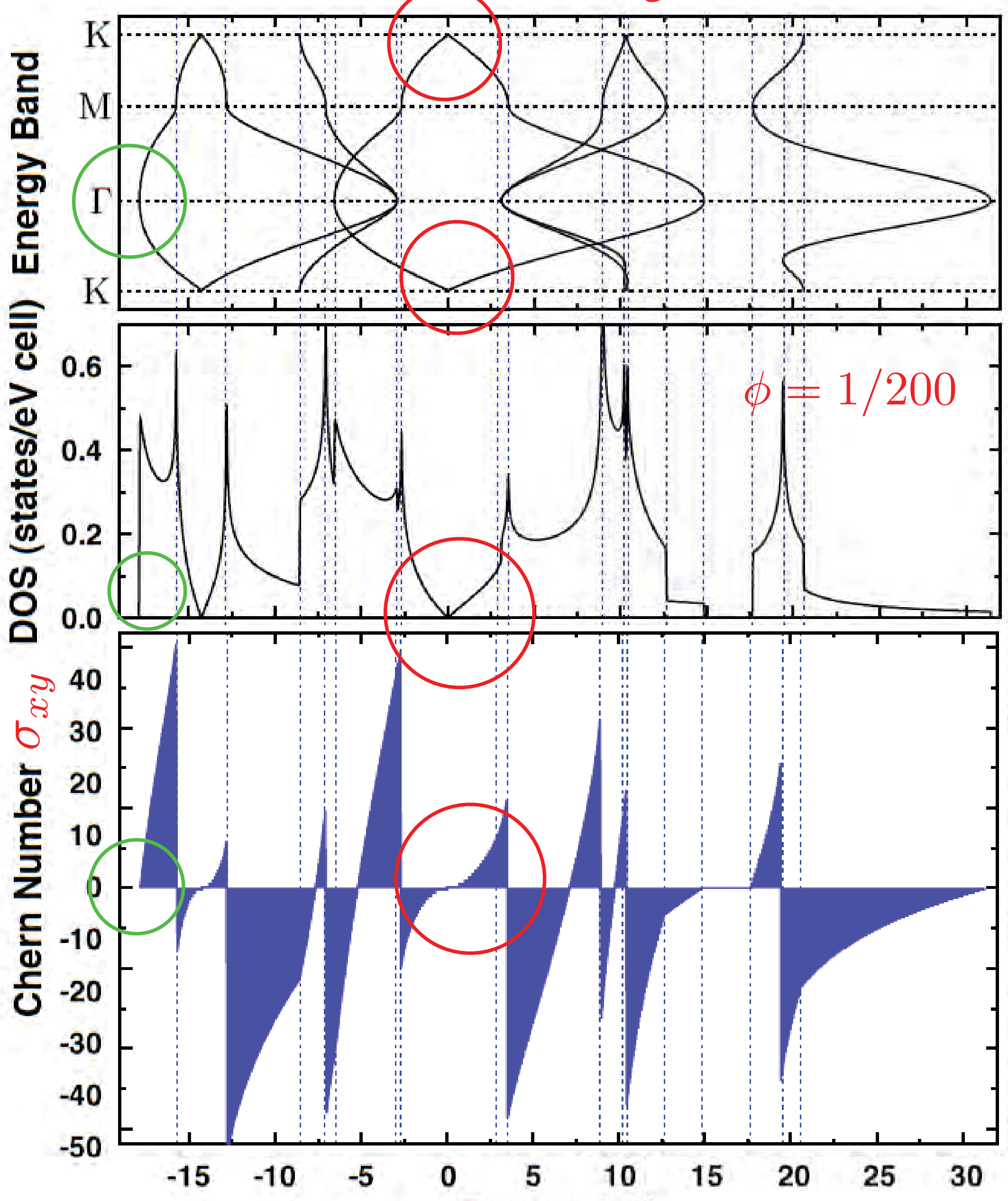
Hall Conductace vs chemical potential

- ☆ Accurate Hall conductance over whole spectrum

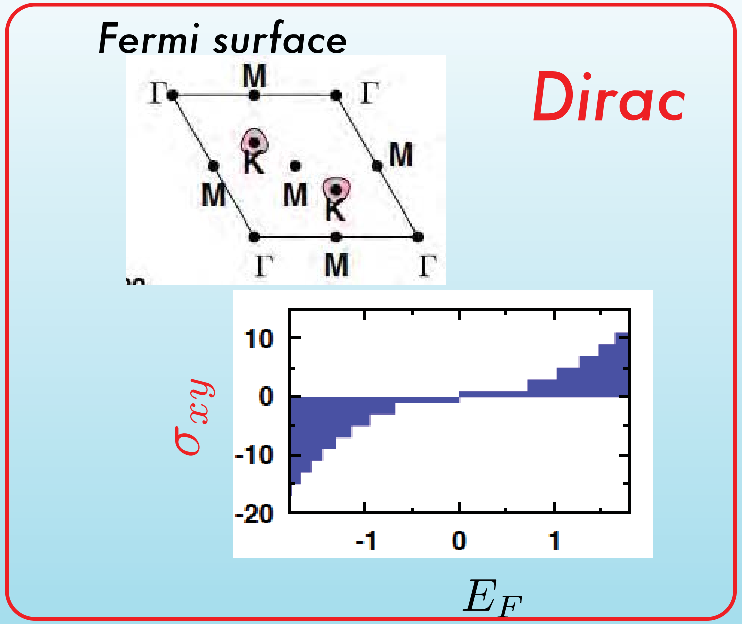


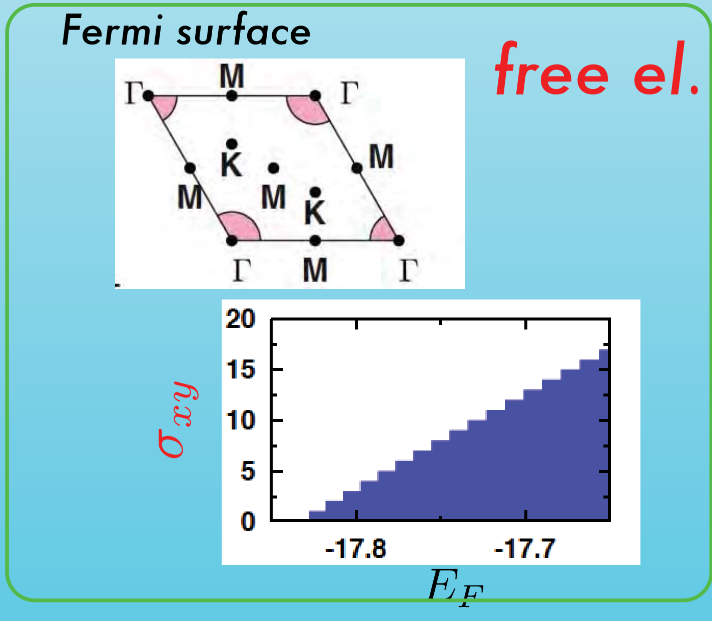
semi-classical quantization: $\frac{S(\varepsilon)}{\Omega_{BZ}} = (n+\gamma)\phi;\ \gamma = \frac{1}{2}$ or 0

Chern numbers (σ_{xy}) based on Realistic Band Calc.



quantized everywhere





M.Arai and Y.Hatsugai, Phys.Rev. B79, 075429 (2009)

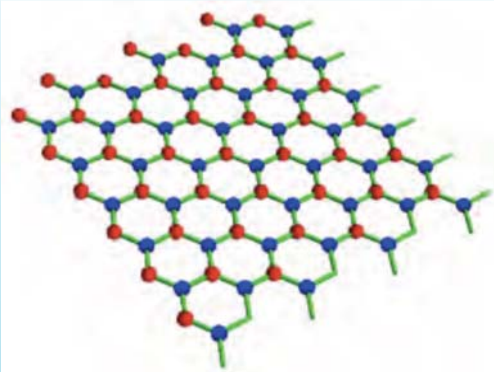
Universality in the zero modes of Dirac Fermions

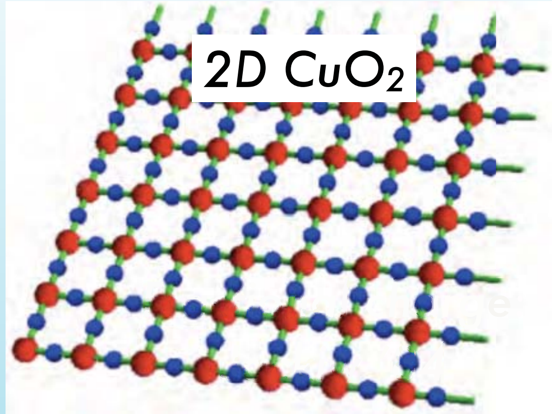
1D Dirac fermions :

Zero mode localized states

Su-Schriefer-Heeger '79
Witten '81

Universality in the zero modes of Dirac Fermions





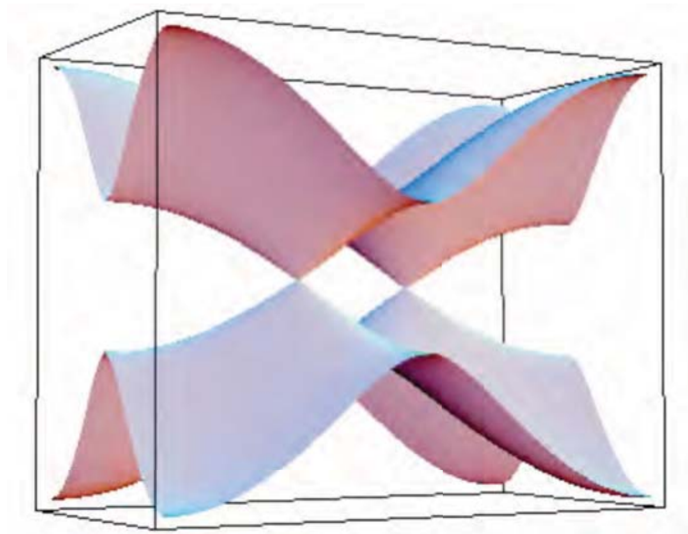
2D Dirac fermions :

Edge States

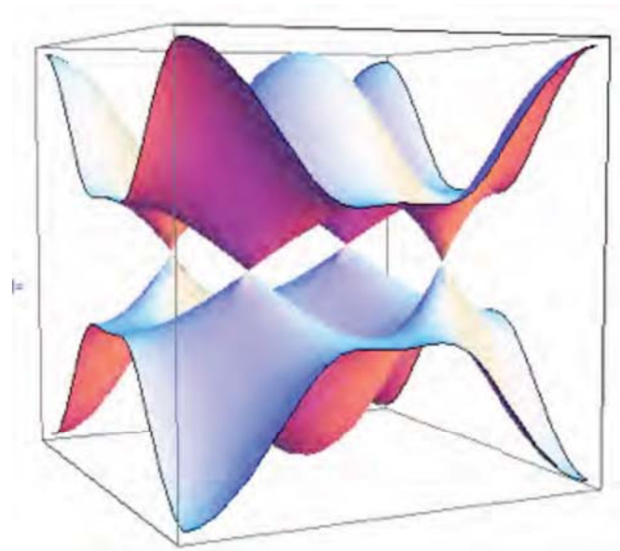
Zero mode localized states ??

YH, '09 (review)

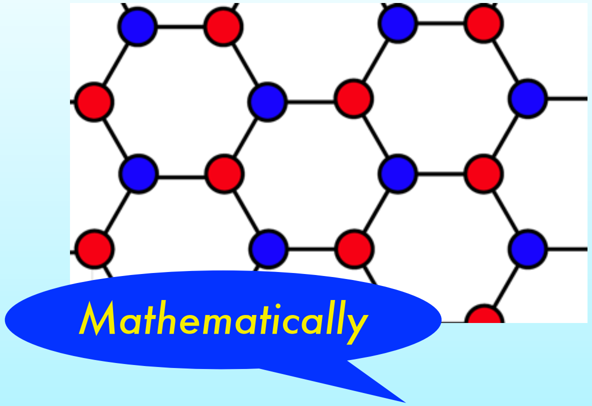
Graphene

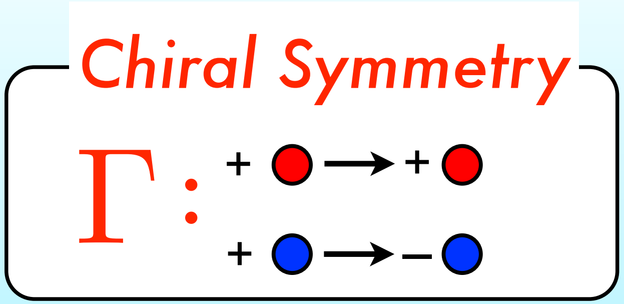


d-wave superconductor



Chiral Symmetry of Graphene





Hamiltonian anti-commutes with $^{\exists}\Gamma$

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0 \ , \ \Gamma^2 = 1$$

Hopping between ● ↔ ●

$$H = \begin{pmatrix} \boldsymbol{O} & \boldsymbol{D} \\ \boldsymbol{D}^\dagger & \boldsymbol{O} \end{pmatrix} \qquad \Gamma = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{O} \\ \boldsymbol{O}^\dagger & -\boldsymbol{I} \end{pmatrix}$$

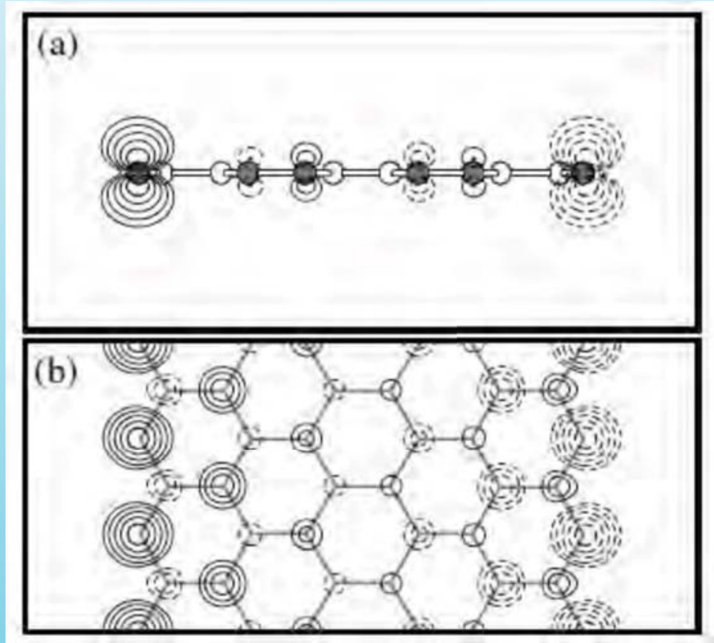
If the zero mode exists,

$$\Gamma \psi_{E=0} = \pm \psi_{E=0}$$

➡ **Finite amplitude only on ● or ●**

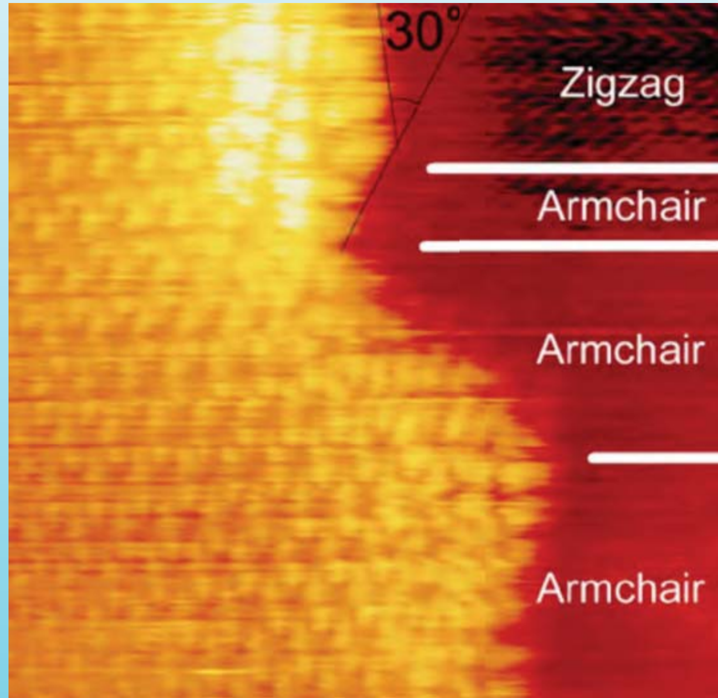
It's real !

First principle calculation



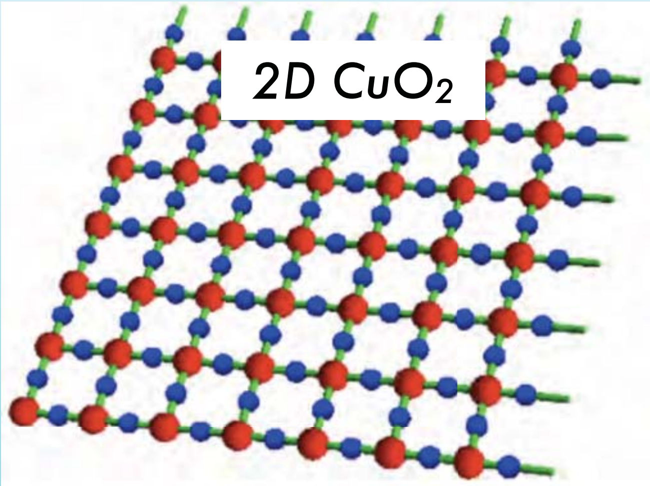
Okada and Oshiyama,
Phys. Rev. Lett. 87, 146803 (2001)

STM image

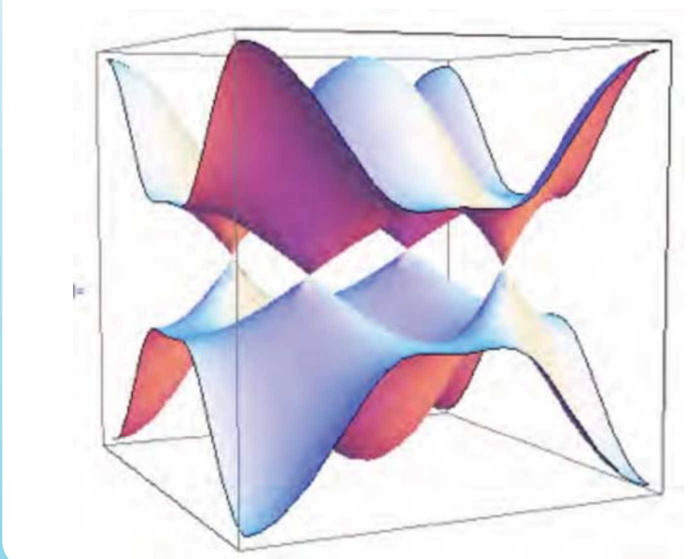


Kobayashi et al,
Phys. Rev. B71, 193406 (2005)

Zero mode localized states ??

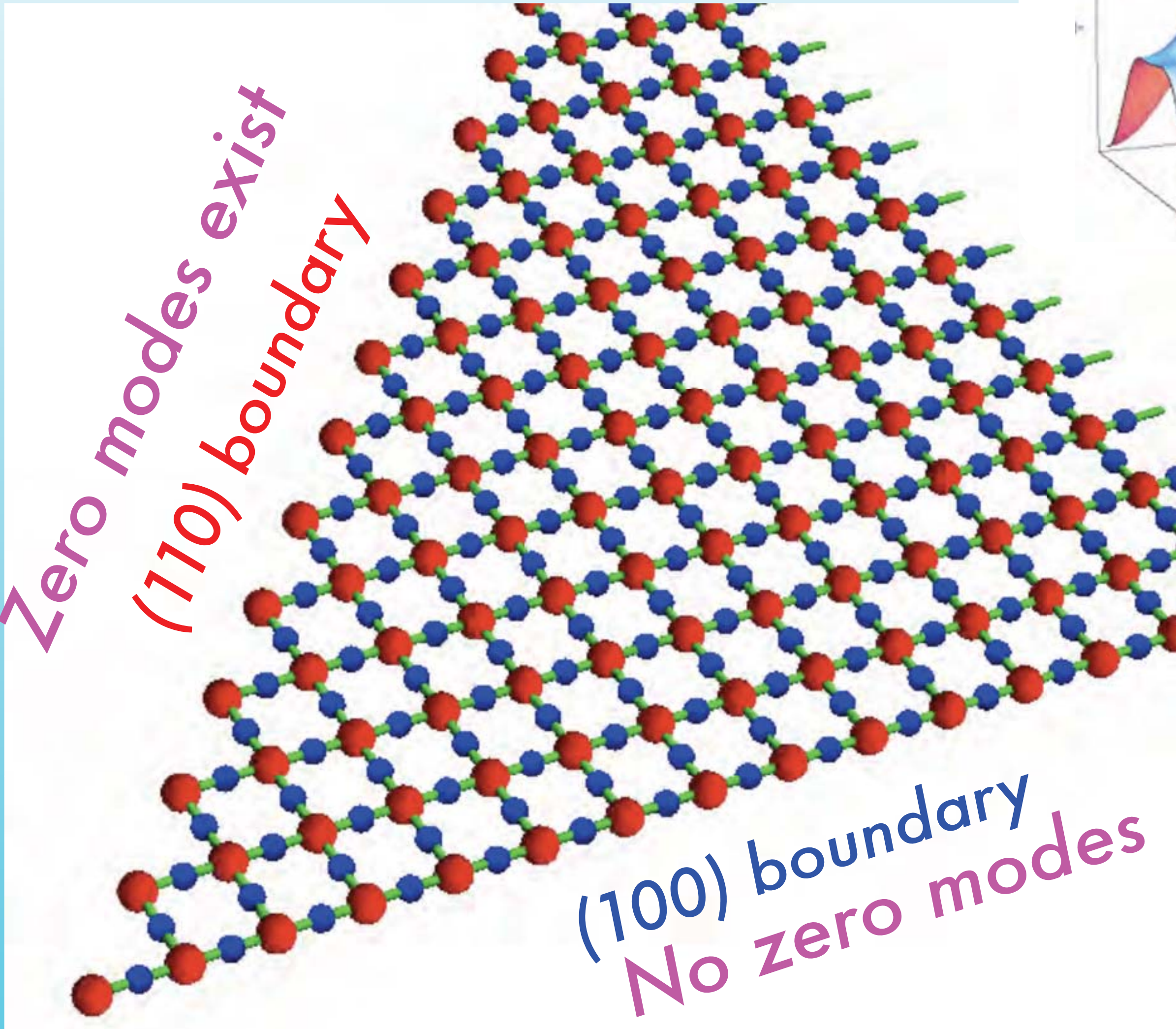


d-wave superconductor

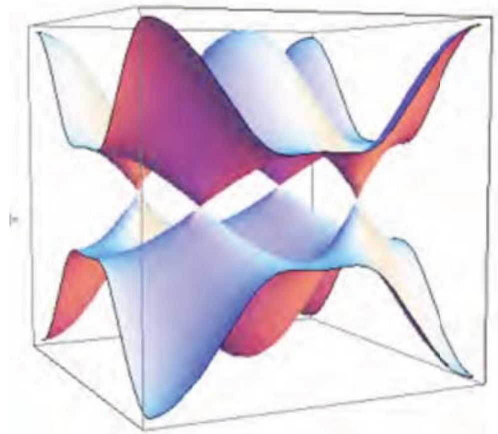


Zero mode localized states ??

d-wave superconductor



d-wave superconductor



Andreev
bound states

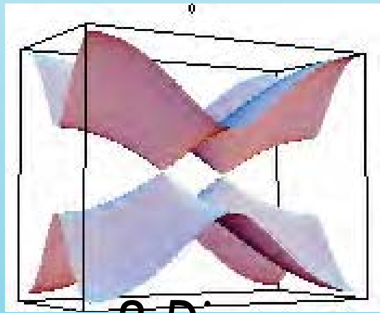
Hu, '94

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

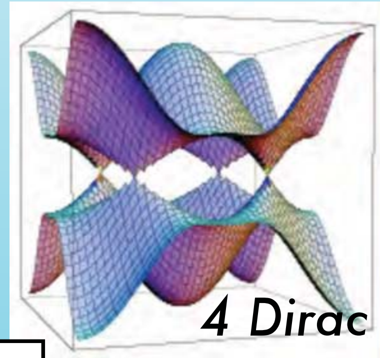
1. Zero energy edge states of graphene
   - Boundary Magnetic moments of graphene
2. Andreev bound states of d-wave superconductors
   - Zero bias conductance peak

graphene



2 Dirac cones

d-wave superconductor



4 Dirac cones

These 2 systems are topologically equivalent

Symmetry protected
Zero modes of Dirac fermions
:1D Flat Band of edge states

Γ :Bipartite

(A-B sublattice symmetry)

Γ :Time Reversal

(Real Order parameter)

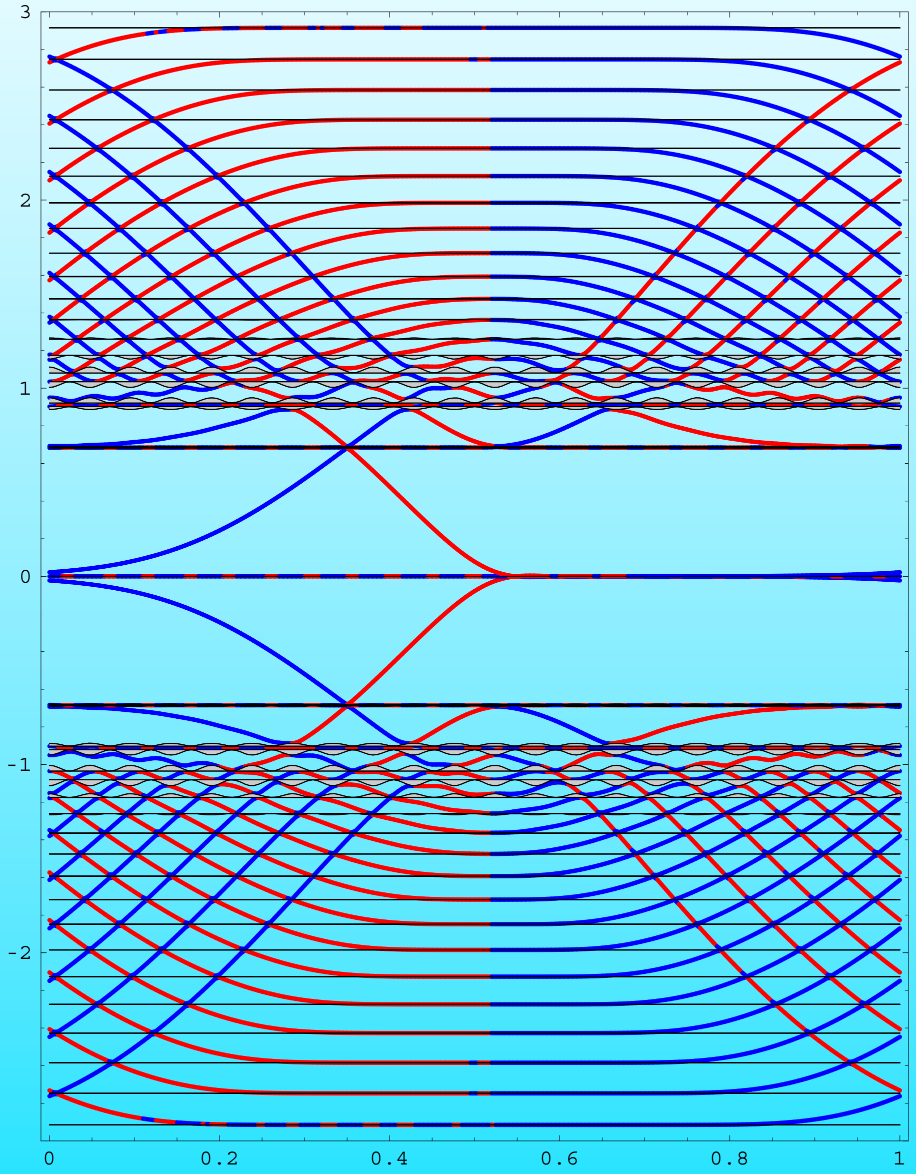
$^{\exists}\Gamma$ chiral symmetry

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0 \,, \quad \Gamma^2 = 1$$

With magnetic field

M. Arikawa, H. Aoki & YH

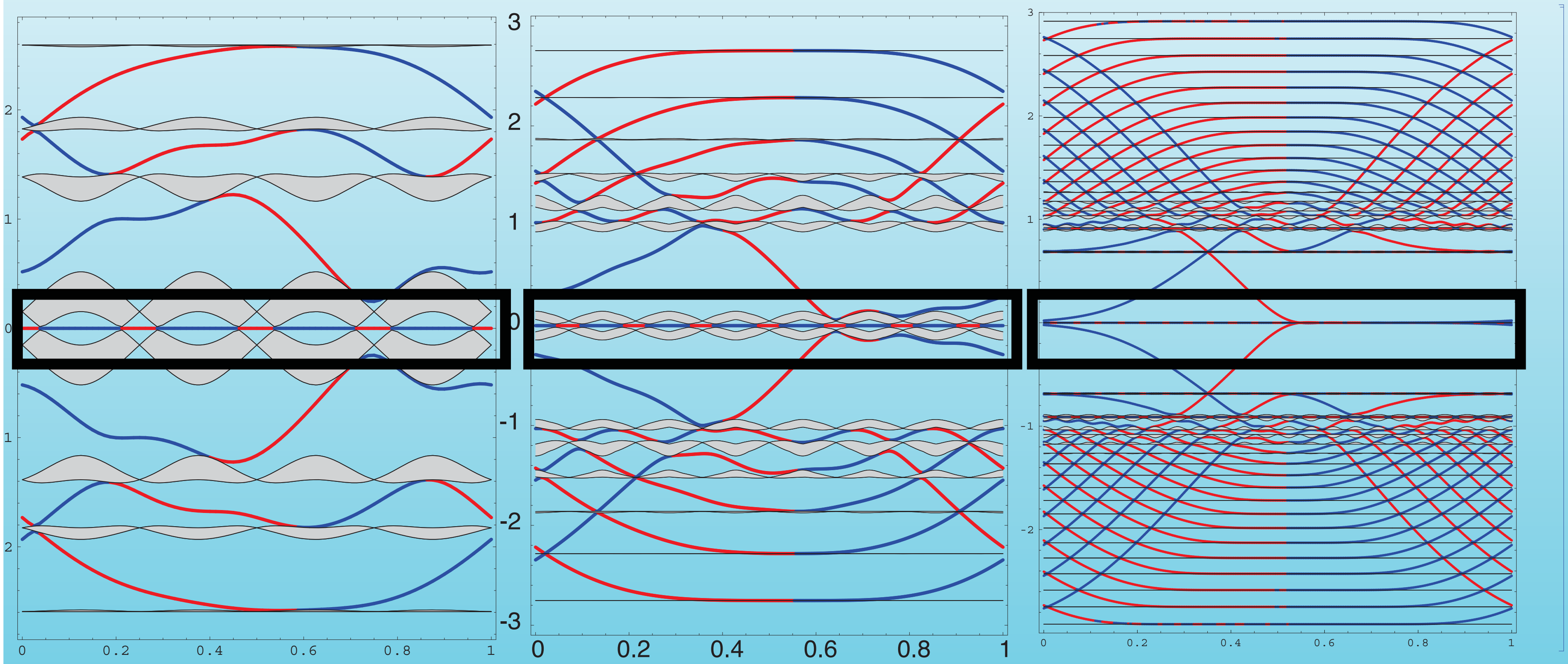
Phys. Rev. B79, 075429 (2
arXiv: & 0806.2429



Landau Levels of graphene with edges

Close look at E=0

☆ *n=0 Landau Level*



$\phi = 1/4$ $\phi = 1/7$ $\phi = 1/21$

strong ⟷ weak

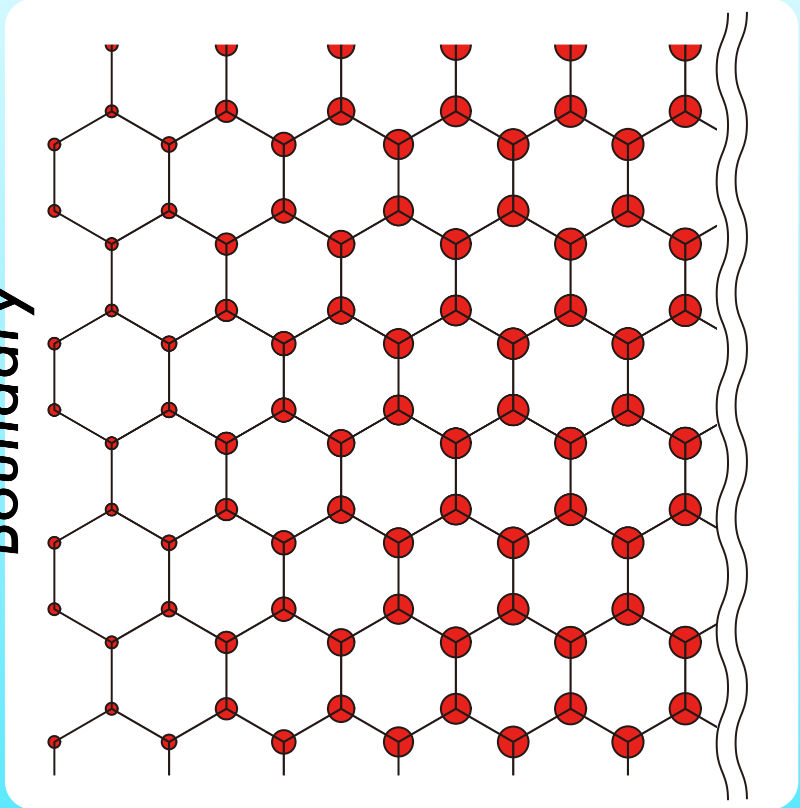
LDOS around E=0 with Landau Level

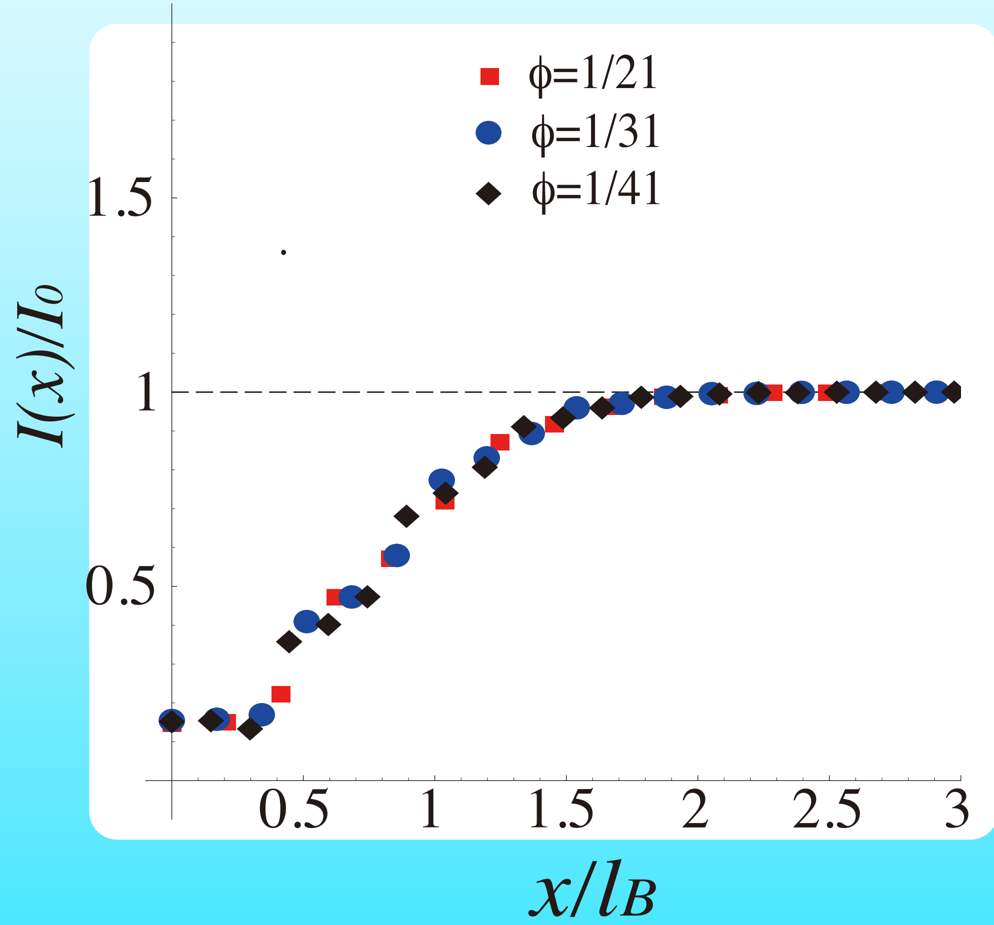
$$I(x) = \frac{1}{2\pi}\int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y \, |\Psi(x, k_y, E)|^2$$

Armchair

x

Boundary





Suppression near the edge

Standard behavior due to edge potential

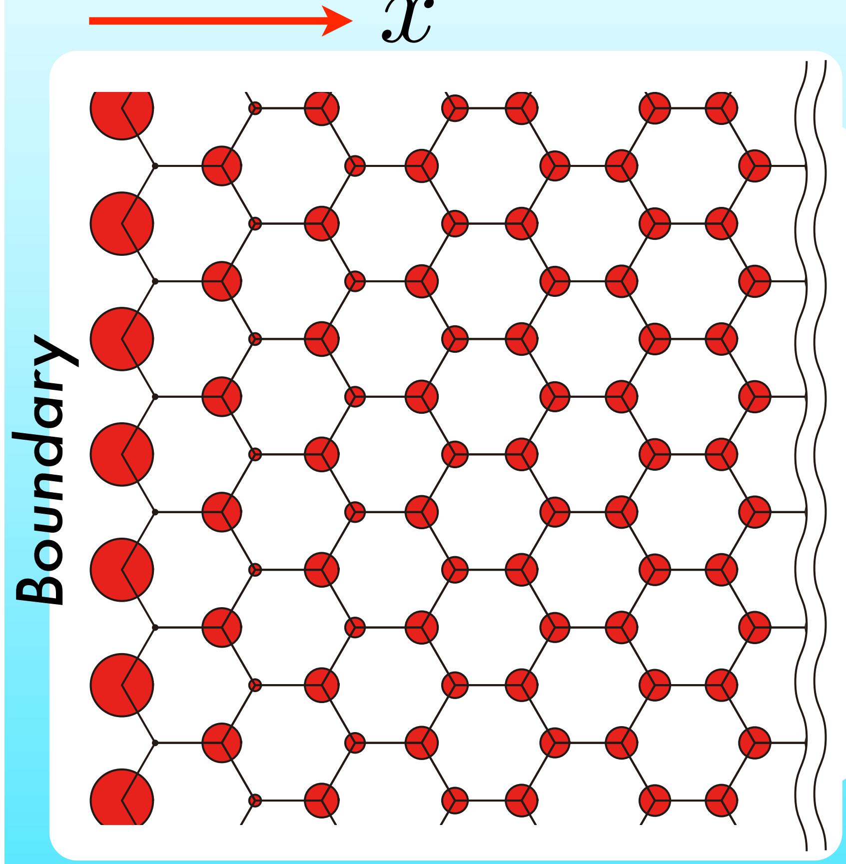
LDOS around E=0 with Landau Level

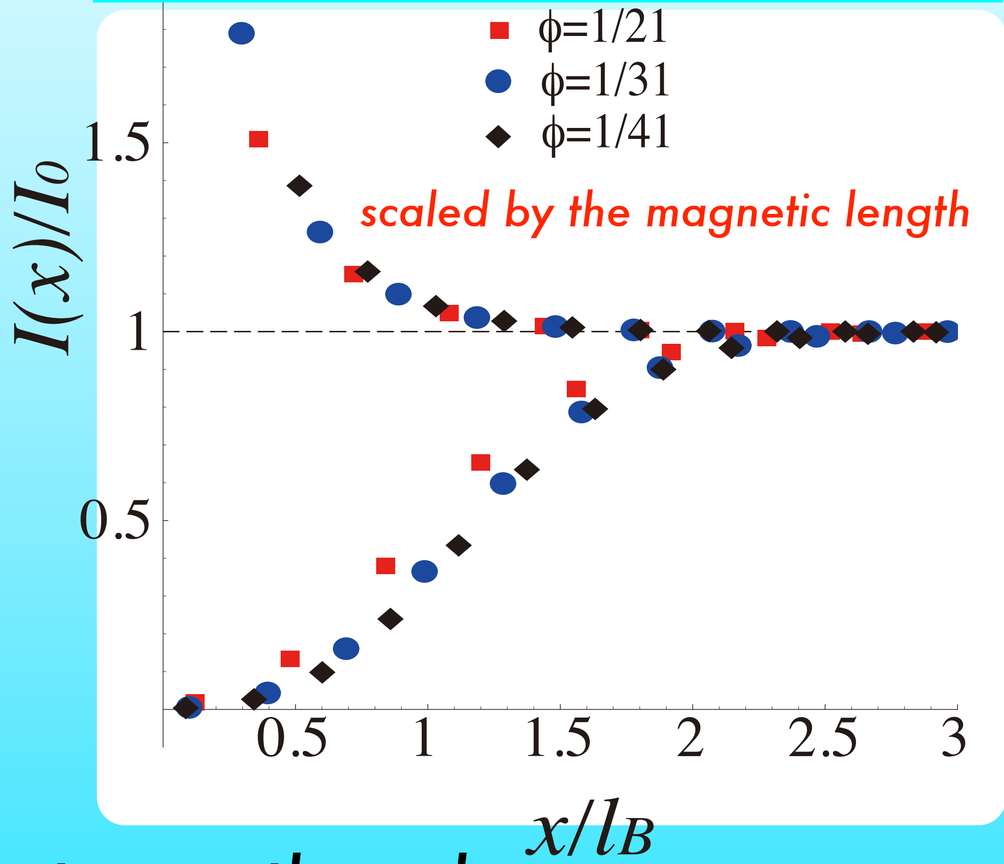
$$I(x) = \frac{1}{2\pi}\int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y\, |\Psi(x, k_y, E)|^2$$

M. Arikawa, H. Aoki & YH

Phys. Rev. B79, 075429 (2009),arXiv: & 0806.2429

Zigzag





STM observable

Strong enhancement near the edge

Characteristic feature of the Graphene Zigzag edges!

Why the Edge States are there??

Accidental ?

NO !

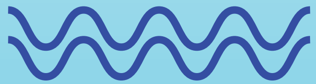
Inevitable reasons

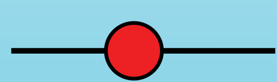
Universal Structures behind:

Bulk determines the edges

<u>Bulk-Edge correspondence</u>

Universality

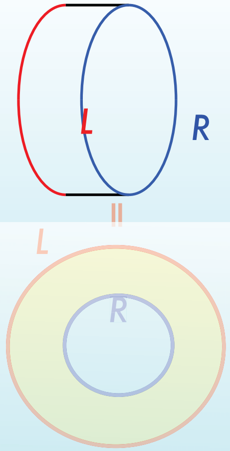




Bulk state (scattering state) Bulk Gap Non trivial Vacuum	Control with each other ↔	Edge state (Bound state) Particles in the gap

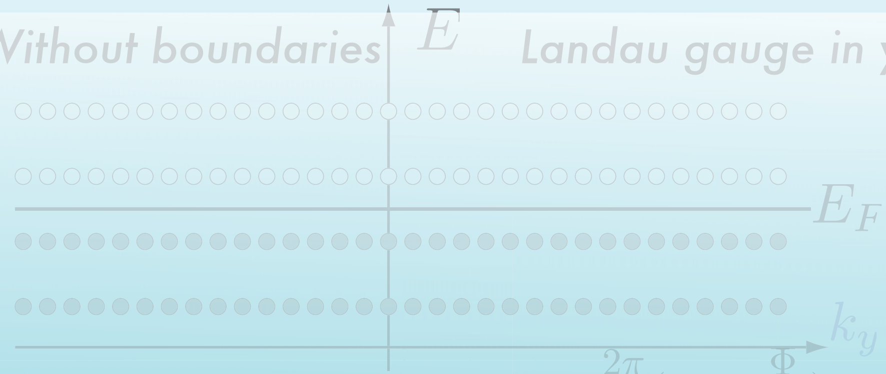
Quantum Hall Effects by edge states

☆ Edge states and Hall conductance σ_{xy} Halperin '82



Without boundaries

Landau gauge in y

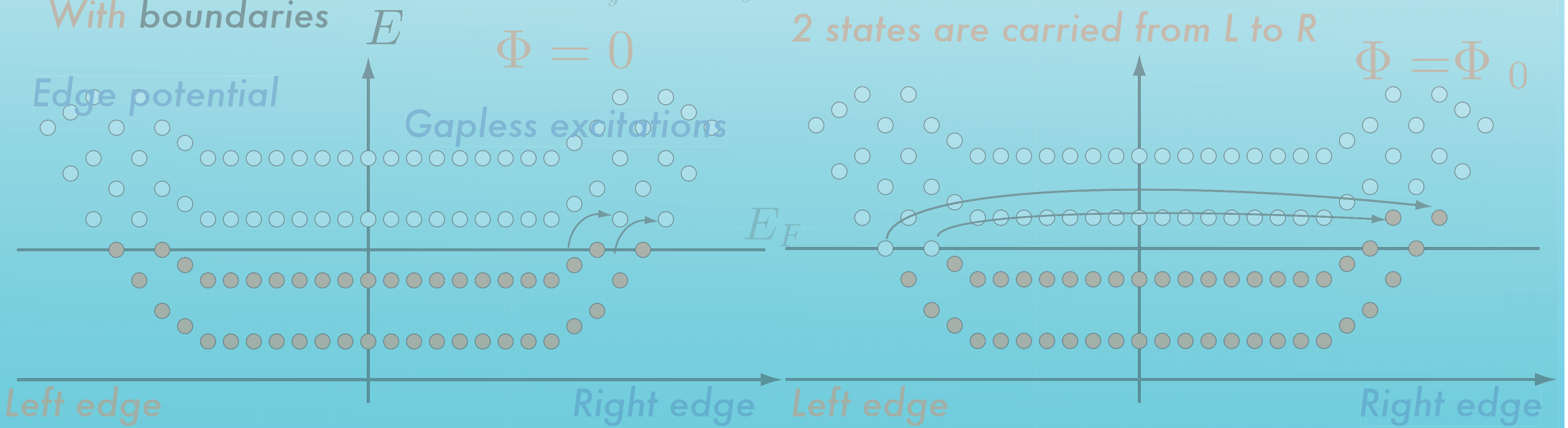


$$H_{2D} = \sum_{k_y} H_{1D}(k_y)$$

$H_{1D}(k_y)$: harmonic osc. centered at $\langle x \rangle \sim \ell_B^2 k_y$

$k_y = \frac{2\pi}{L_y}(n_y + \frac{\Phi}{\Phi_0})$ $n_y = 0, \pm 1, \pm 2, \cdots$

With boundaries



Laughlin's undetermined n : # of Landau Levels below E_F

Edge states are essential in the QHE !

Hall Conductance has a Topological meaning

- Discussion by the Bloch electrons (Peierls substitution)
  - preserve U(1) gauge symmetry
  - without cutoff ambiguity
  - recover continuum theory by scaling limit (weak field limit)

$$H = \sum_{\langle ij \rangle} c_i^\dagger e^{i\theta_{ij}} c_j \qquad 2\pi\phi = \sum_{\langle ij \rangle \in P} \theta_{ij} \qquad \phi = \frac{Ba^2}{\Phi_0}$$

P : plaquette

When E_F is in the j-th gap   Two topological quantities

$$\sigma_{xy}^{\text{bulk}} = \frac{e^2}{h} \sum_{\ell:\epsilon_\ell(k)<E_F} C_\ell$$

Sum of the First Chern numbers below E_F
Thouless-Kohmoto-Nightingale-den Nijs '82

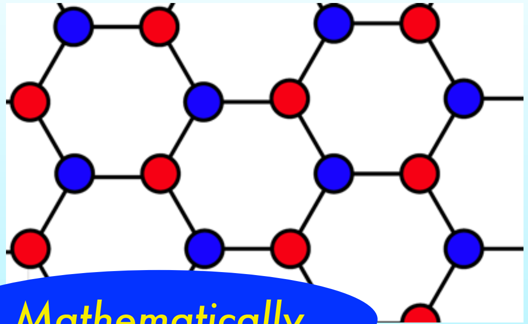
$$\sigma_{xy}^{\text{edge}} = \frac{e^2}{h} I(\alpha_j, C^j)$$

Winding number of the edge state in the complex energy surface   Hatsugai '93a

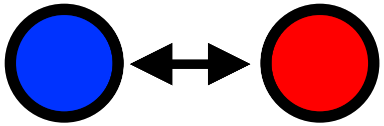
Bulk — Edge Correspondence   Hatsugai '93b

$$\sigma_{xy}^{\text{bulk}} = \sigma_{xy}^{\text{edge}}$$

Chiral Symmetry of Graphene



Chiral Symmetry

Equivalence of Hopping between 

Mathematically

Hamiltonian anti-commutes with $^{\exists}\Gamma$

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \ \Gamma^2 = 1$$

$$H = \begin{pmatrix} \boldsymbol{O} & \boldsymbol{D} \\ \boldsymbol{D}^{\dagger} & \boldsymbol{O} \end{pmatrix} \qquad \Gamma = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{O} \\ \boldsymbol{O}^{\dagger} & -\boldsymbol{I} \end{pmatrix}$$

Perturbations

- Uniform site energy (○)
- Staggered site energies (×)
- Antiferromagnetic order (×)
- Bond order (○)
- Random potential (×)
- Random hopping (○)

Ripples ?

Quantum Hall plateau transition

- Quantum phase transition driven by disorder among states with different topological numbers

Focus: Delocalized states within Landau Levels

Khmelnitskii '84
Laughlin '84
Kivelson-Lee-Zhang '92
Ludwig-Fisher-Shankar-Grinstein '94
Sheng-Wen '97
Hatsugai-Ishibashi-Morita '99

- Hall plateau transition of graphene & effects of randomness in graphene

Many...

Suzuura-Ando '02
McCann et al. '06
Aleiner-Efetov '06
Altland '06
Ostrovsky-Gornyi-Mirlin '06
Sheng-Sheng-Wen '06
Schweitzer-Markos '08
Nomura-Ryu-Koshino-Mudry-Furusaki '08

Our focus

Quantum Hall plateau transition
Chiral symmetry & n=0 L.L. of graphene

Ripples of graphene

☆ Ripples as a random gauge field

Neto-Guinea-Peres-Novoselov-Geim '09

(Meyer, Geim et al, Nature 2007)

locally gauge out by gauge transformation

↓

hopping amplitude is modified (can be phases)

Random hopping model on a honeycomb lattice
with spatial correlation

Zero mode:Index theorem !?
Atiyah-Singer

Aharonov-Casher '79

Ludwig-Fisher-Shankar-Grinstein'94

Model

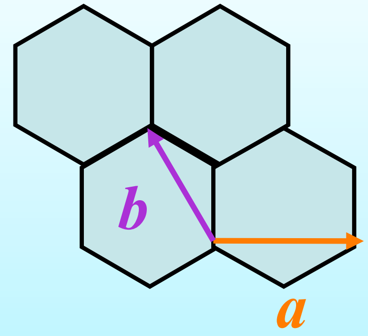
c.f. Dirac fermions on square lattice

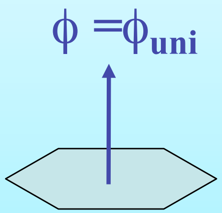
Y.Hatsugai, X.G.Wen, M.Kohmoto, Phys.Rev. B56, 1061 (1997)

2D Honeycomb Lattice in disordered hopping amplitude

$$H = \sum_{<r,r'>} t_{r,r'} e^{i\theta_{rr'}} c_r^\dagger c_{r'}$$

$$t_{r,r'} = t + \delta t_{r,r'}$$





$$\sum_{\hexagon} \theta_{r,r'} = -2\pi\phi_{uni}/\phi_0$$

Disordered components δt

$$P(\delta t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp(-\delta t^2 / 2\sigma_t^2)$$

$$W_{\delta t} = \sigma_t \sqrt{12}$$ **effective width**

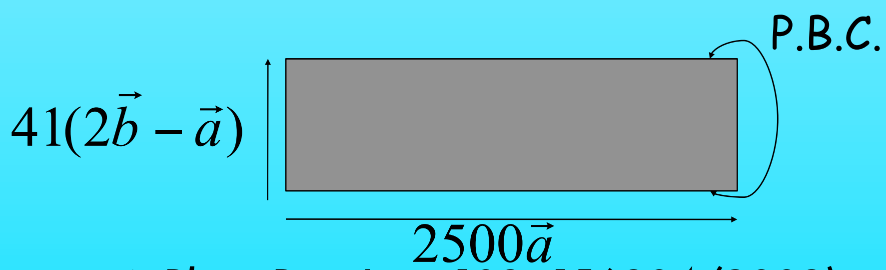
$$\langle \delta t_{r_i} \delta t_{r_j} \rangle = \sigma_t^2 \exp(-|r_i - r_j|^2 / 4\eta_t^2)$$

η_t : correlation of random hopping

Density of states

The Green function method
Schweitzer, Kramer, MacKinnon (1984)

$$\rho(E) = -\frac{1}{\pi} \langle \mathrm{Im} G_{r,r}(E + i\gamma) \rangle_r$$



Correlated Random Hopping

T. Kawarabayashi, H.Aoki and Y.Hatsugai, Phys. Rev. Lett. 103, 156804 (2009)

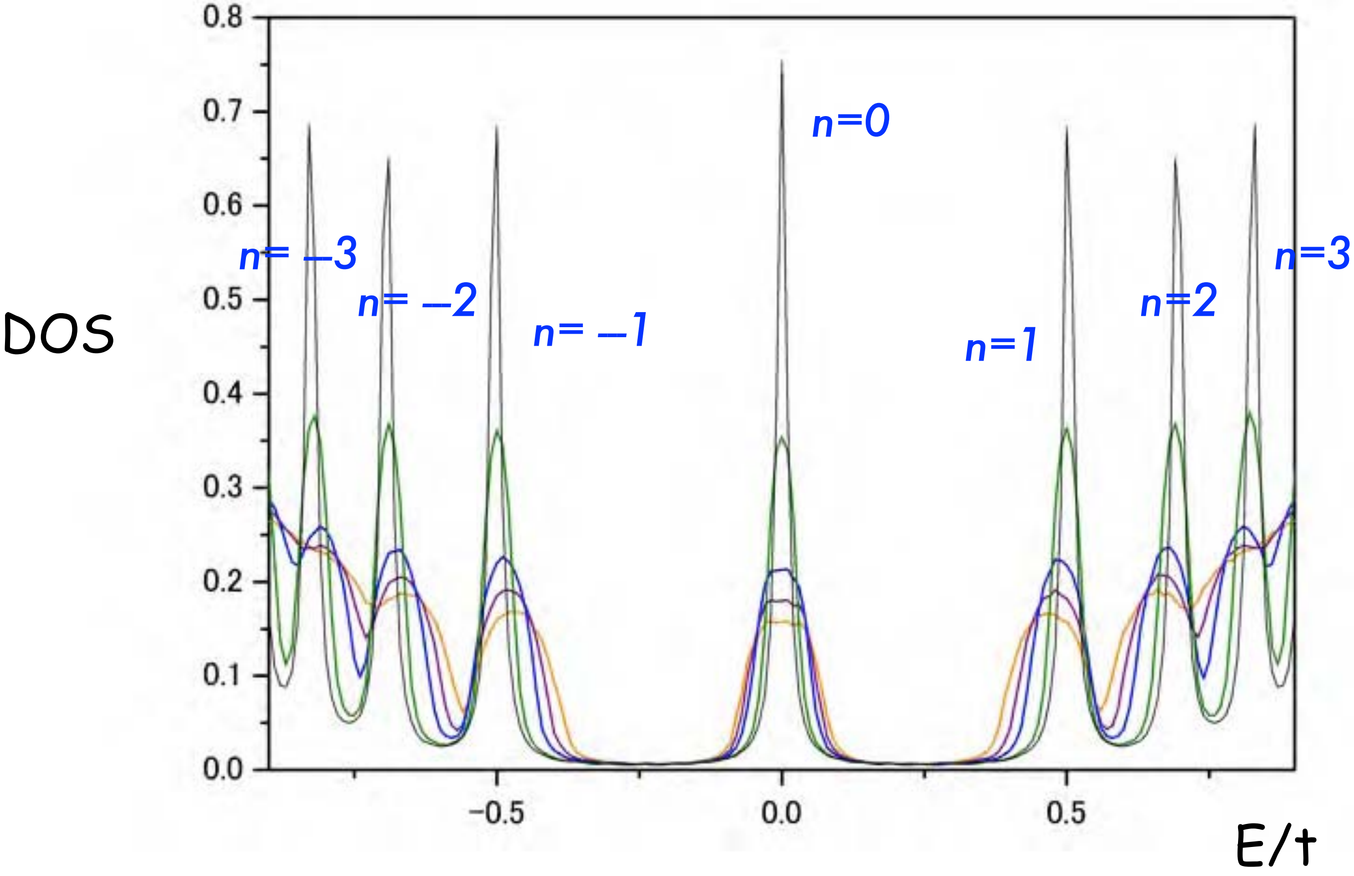
Broadening of Landau levels

$\phi_{uni}/\phi_0 = 1/41$ ($l = 2.4|a|$)

uncorrelated disordered hopping

$W_{\delta t}/t = 0, 0.2, 0.4, 0.5, 0.6$

finite width for any n

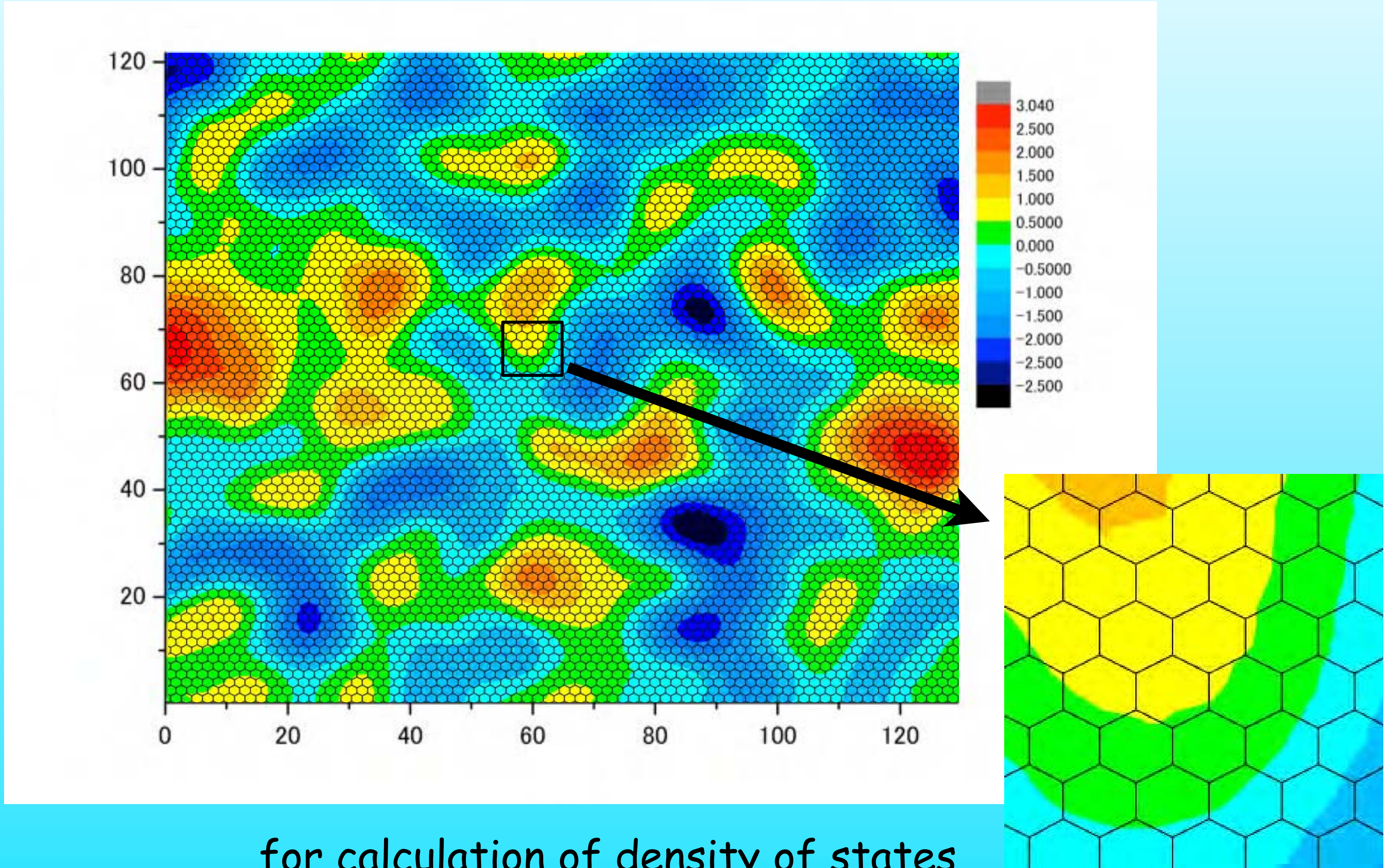


T. Kawarabayashi, H.Aoki and Y.Hatsugai, Phys. Rev. Lett. 103, 156804 (2009)

Correlated Random Hopping

(distribution of gauge field) $\sqrt{3}\eta_t / |\vec{a}| = 5.0$

Landscape of hopping amplitude $W_{\delta t}/t = 2.0$

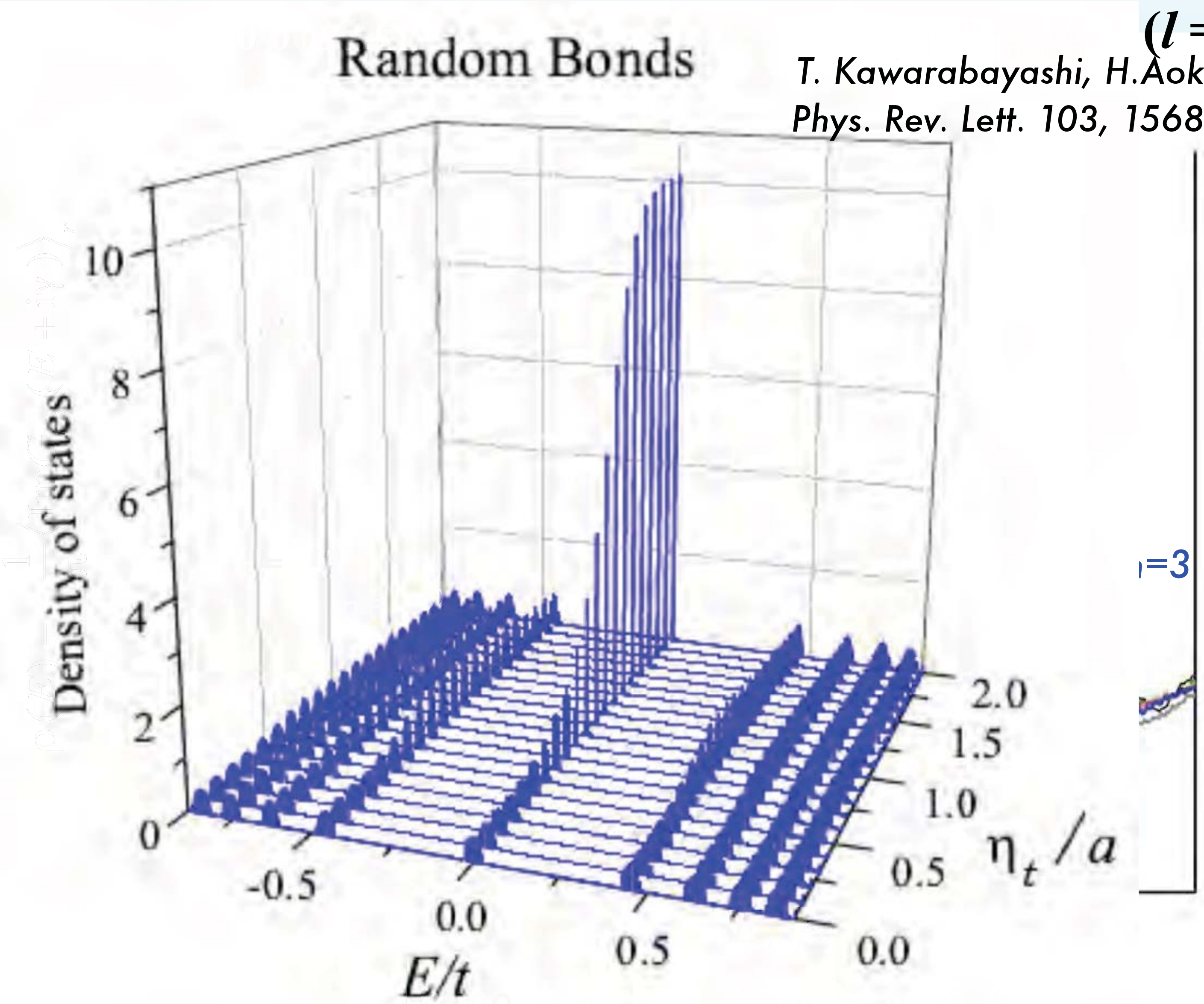


for calculation of density of states

Effect of spatial correlation

$(l = 2.4|a|)$

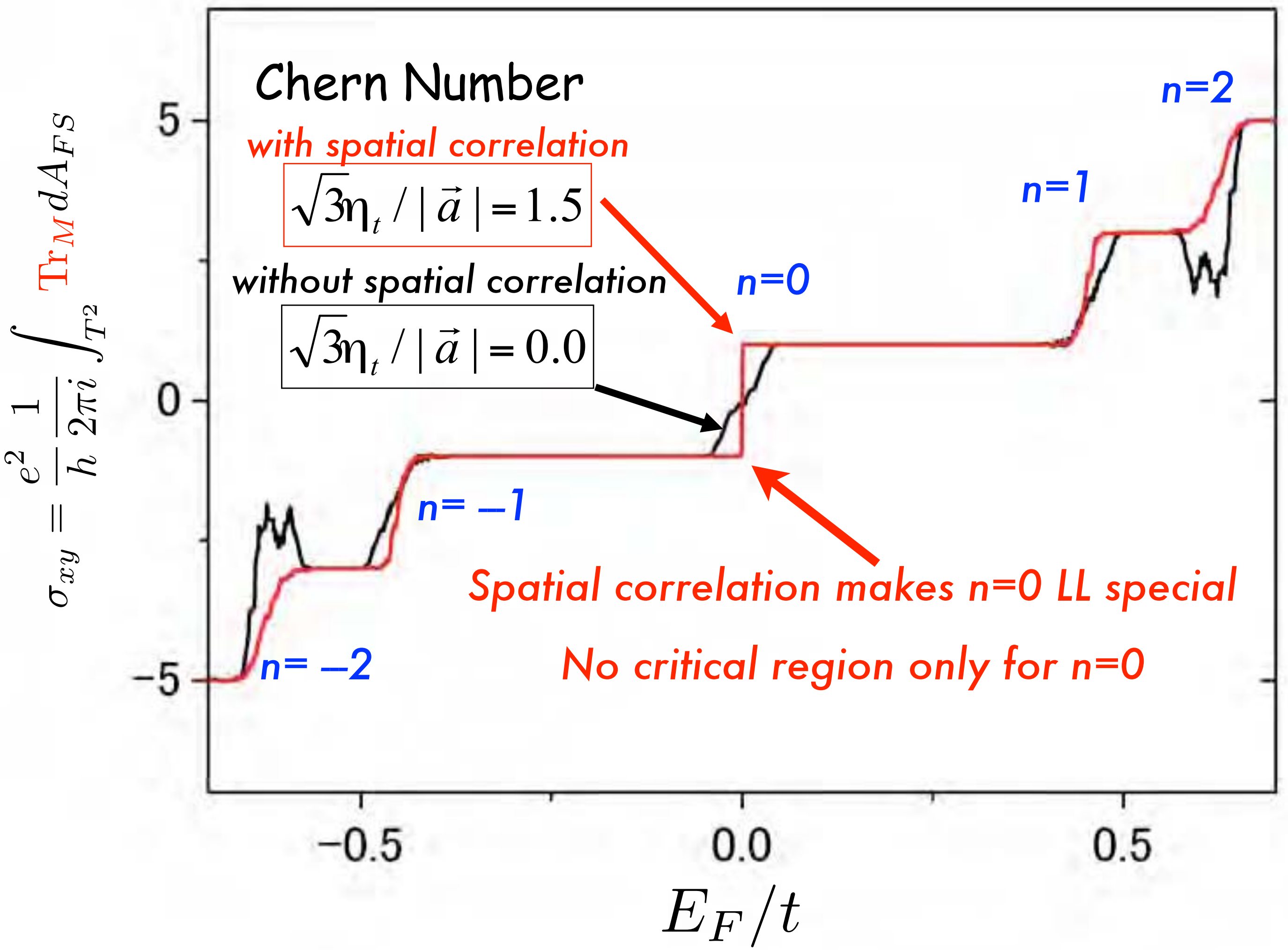
T. Kawarabayashi, H.Aoki and Y.Hatsugai
Phys. Rev. Lett. 103, 156804 (2009)



almost no broadening when the correlation exceeds lattice constant

T. Kawarabayashi, H.Aoki and Y.Hatsugai
arXiv 0904.1927, 0908.4134

$W_{\delta t}/t = 0.4$ $\phi_{uni}/\phi_0 = 1/50$ System size 20×20



100 samples

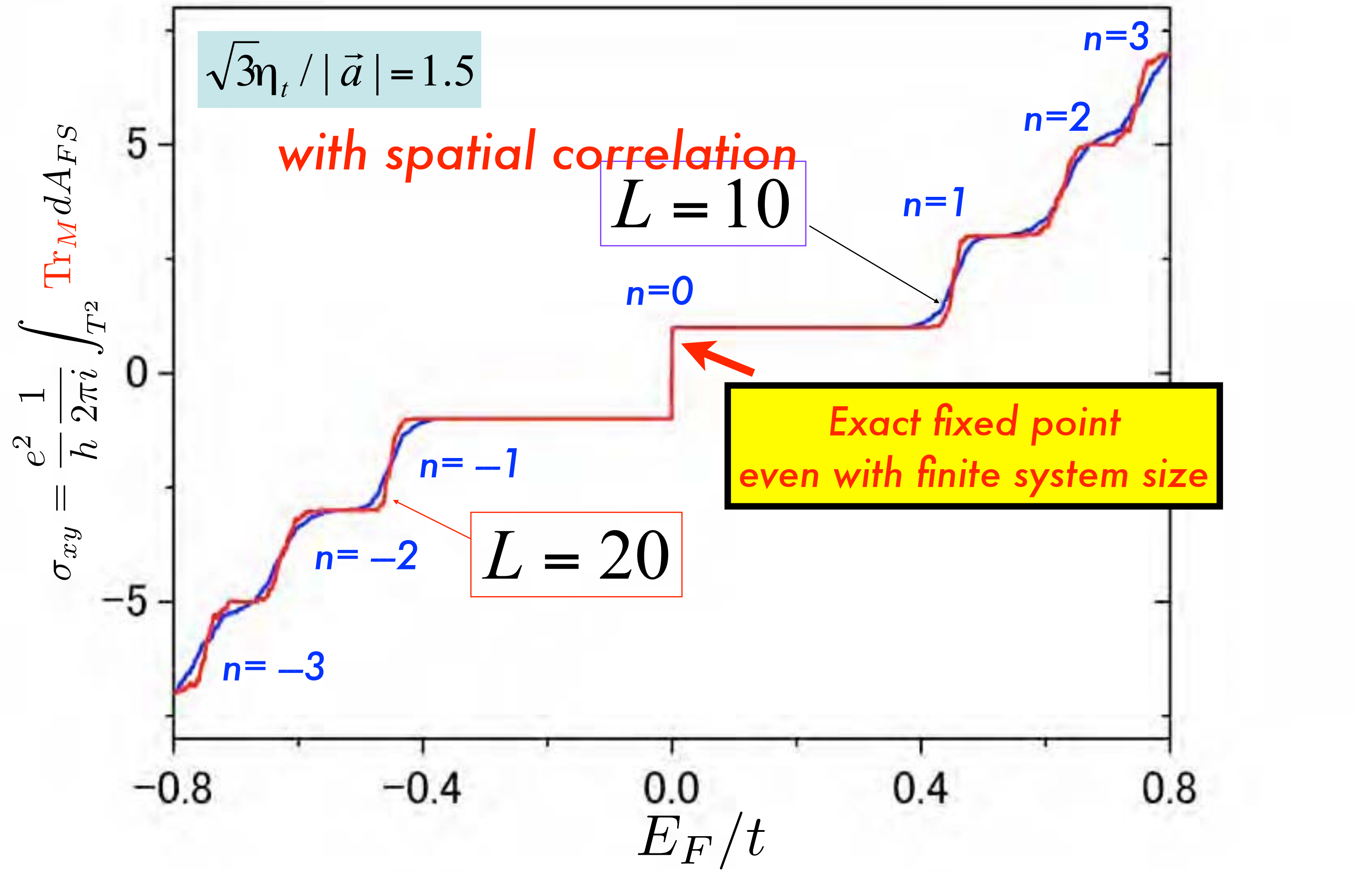
T. Kawarabayashi, H.Aoki and Y.Hatsugai
Phys. Rev. Lett. 103, 156804 (2009)

T. Kawarabayashi, H.Aoki and Y.Hatsugai, Phys. Rev. Lett. 103, 156804 (2009)

Size-dependence

$L \times L$

$W_{\delta t}/t = 0.4 \quad \phi_{uni}/\phi_0 = 1/50$



100 samples

c.f. Critical zero mode of Random Dirac fermion:
Y.Morita & Y.Hatsugai, Phys.Rev. Lett. 79, 3728,(1997)

AC Hall Effect

We have learned chiral symmetry brings about anomalous n=0 LL in dc response.

What about AC response or optical Hall conductivity? $\sigma_{xy}(\omega)$

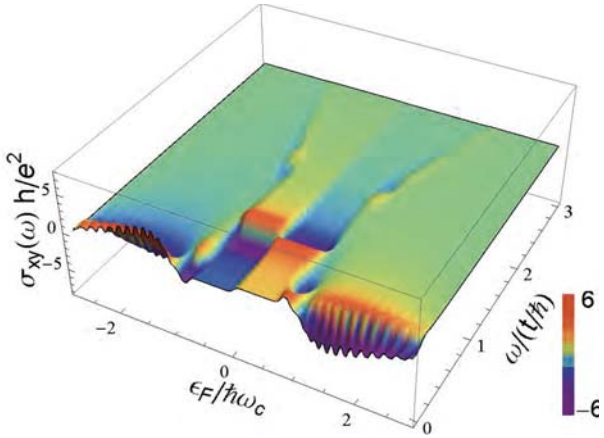
T. Morimoto, H. Aoki, Y. Hatsugai, Phys. Rev. Lett. 103, 116803 (2009)

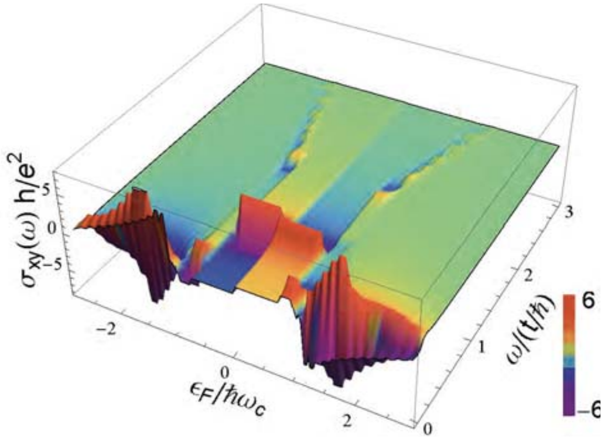
Site disorder $H = \sum t c_i^\dagger c_j + \delta\varepsilon(r) c_i^\dagger c_i$

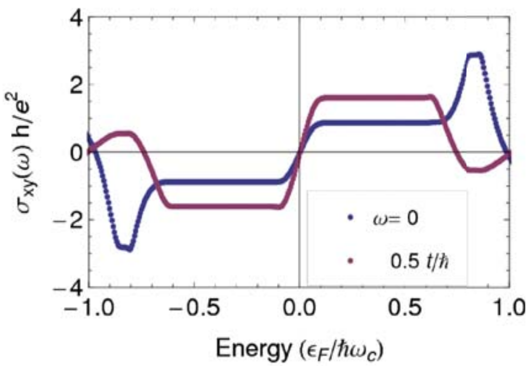
Not chiral symmetric

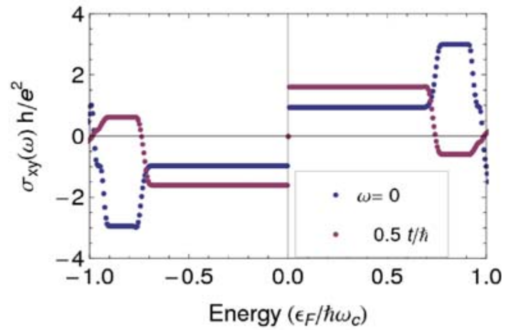
Random bonds $H = \sum (t + \delta t(r)) c_i^\dagger c_j$

Chiral symmetric









n=0 LL step is blurred

Chiral symmetry ensures sharp
n=0 LL step in ac response!

T. Kawarabaysshi, T. Morimoto, Y. Hatsugai, H. Aoki '10

Recent works have been done with

H. Aoki, U. Tokyo

T. Fukui, Ibaraki U.

M. Arai, NIMS

T. Kawarabayashi, Toho U.

T. Morimoto, U. Tokyo

M. Arikawa, U. Tsukuba

Thank you