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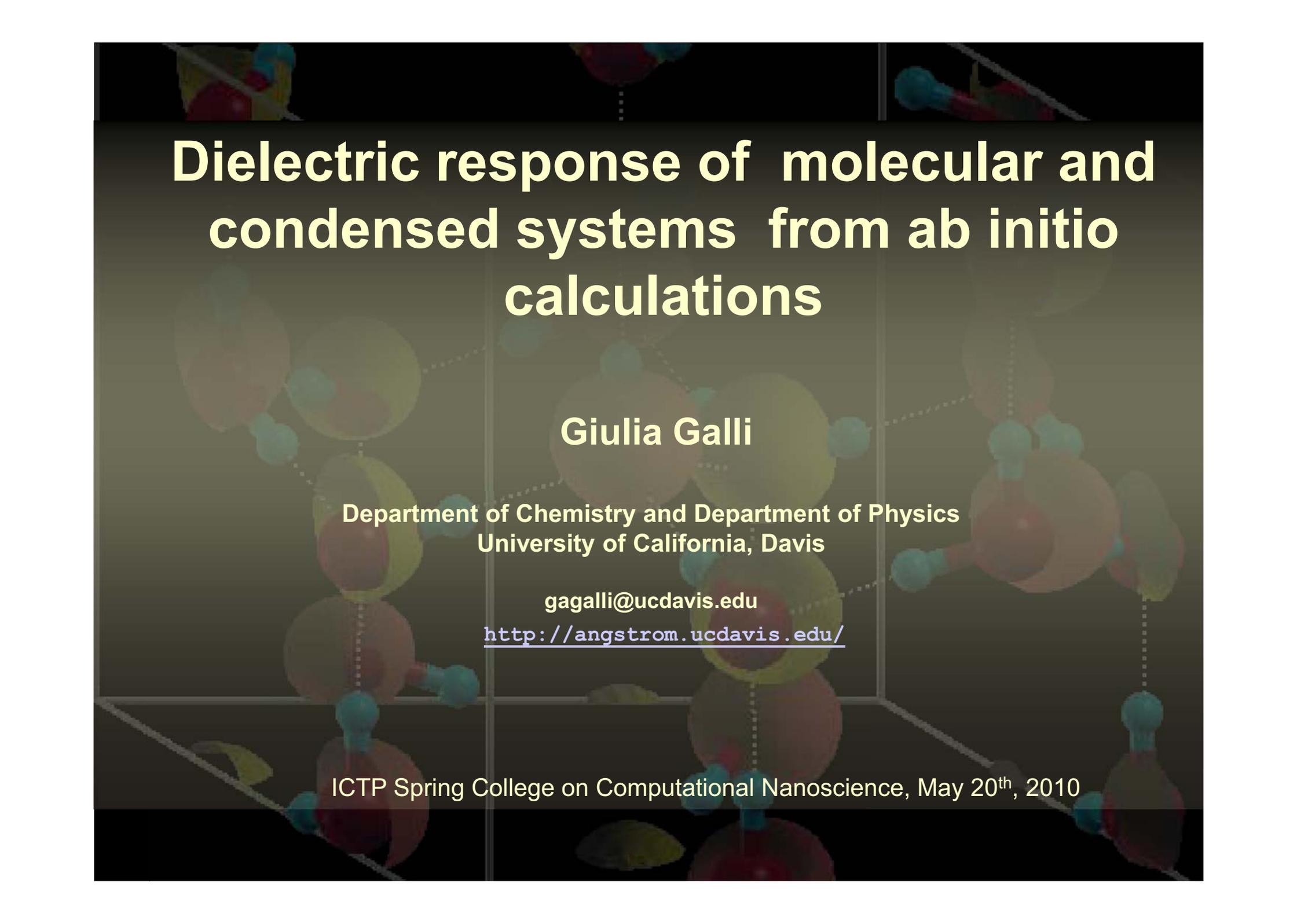
Spring College on Computational Nanoscience

17 - 28 May 2010

**Dielectric Response of Molecular and Condensed Systems From Ab Initio
Calculations**

Giulia GALLI

*Dept. of Chemistry and Dept. of Physics
University of California
Davis
U.S.A.*



Dielectric response of molecular and condensed systems from ab initio calculations

Giulia Galli

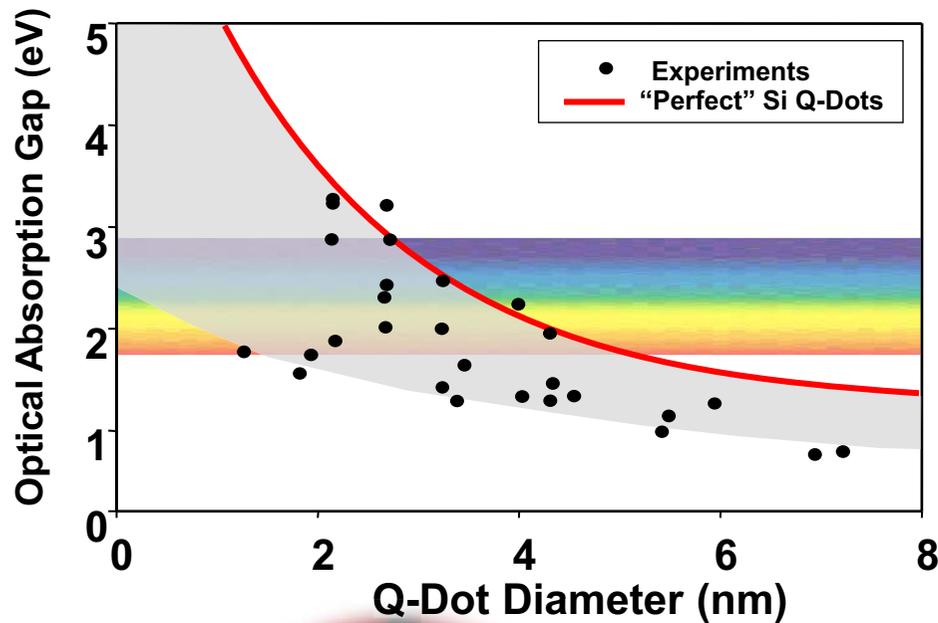
Department of Chemistry and Department of Physics
University of California, Davis

gagalli@ucdavis.edu

<http://angstrom.ucdavis.edu/>

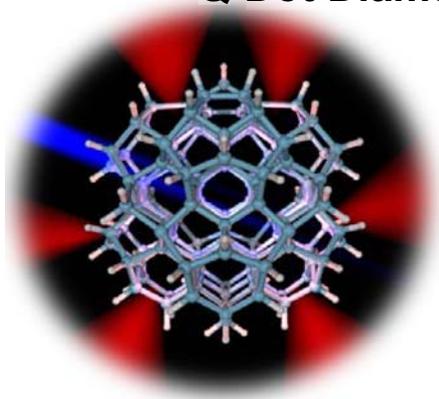
ICTP Spring College on Computational Nanoscience, May 20th, 2010

Optical properties of Si nanocrystals

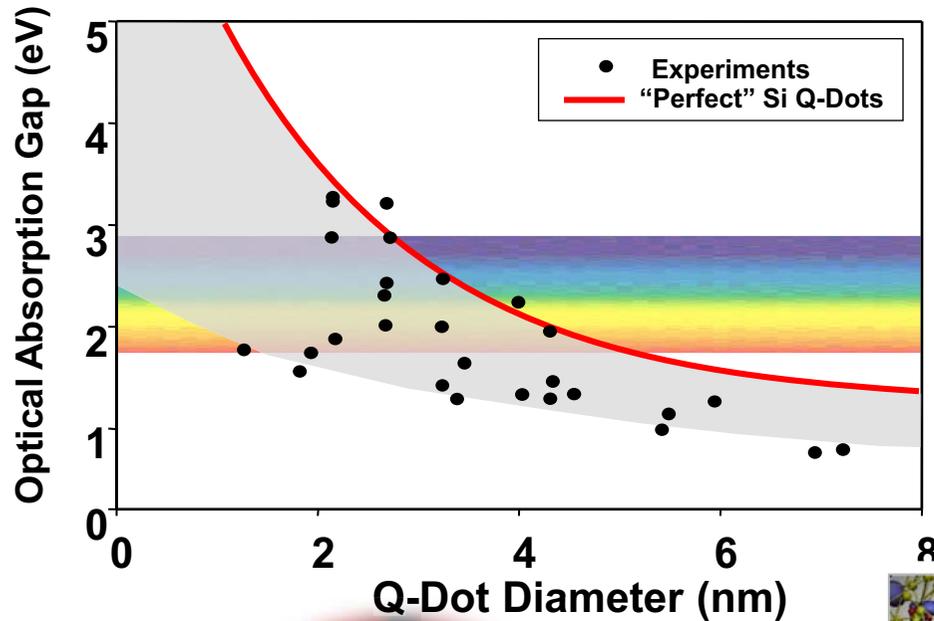


Key influence of surface structure and passivation on opto-electronic properties

Key role of first principles approach in elucidating interplay between structure and `function`

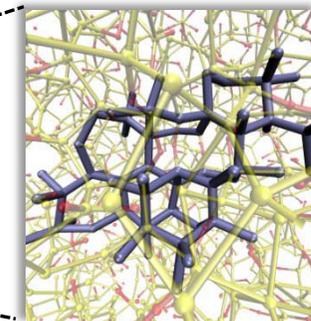
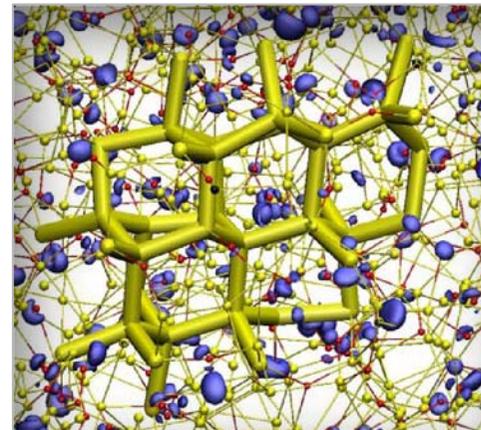
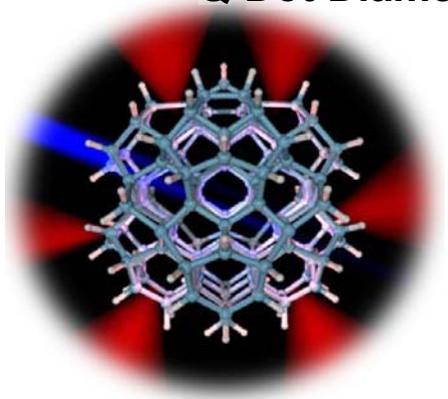


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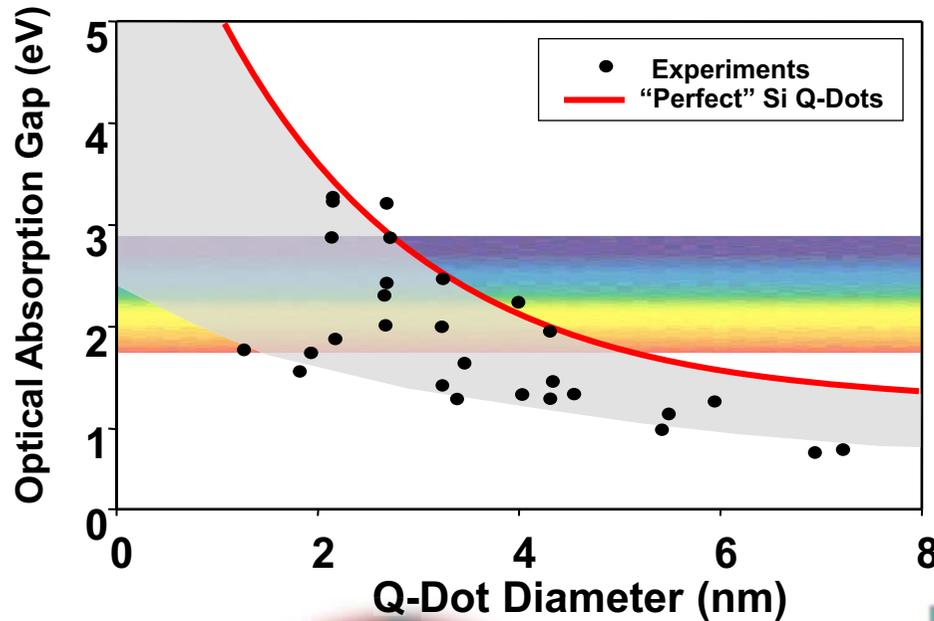


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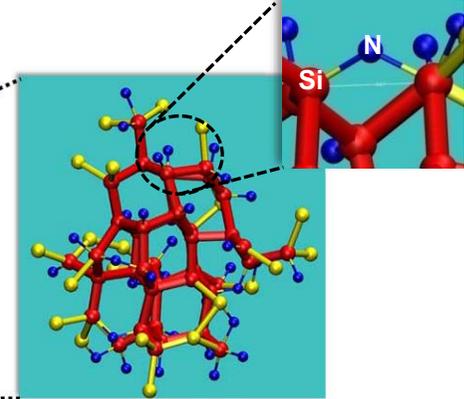
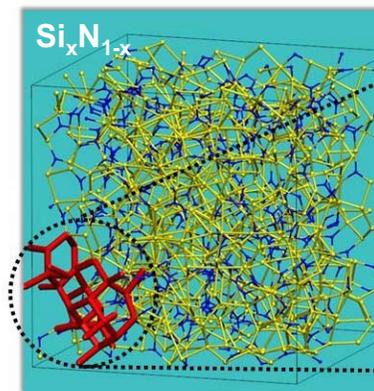
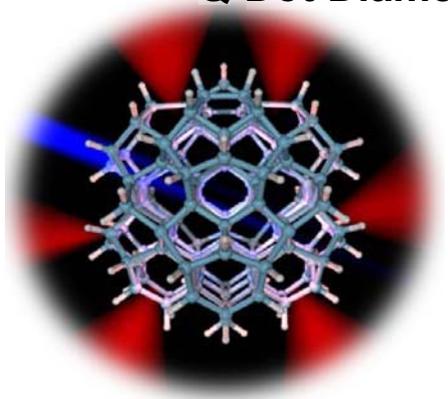


Optical properties of Si nanocrystals

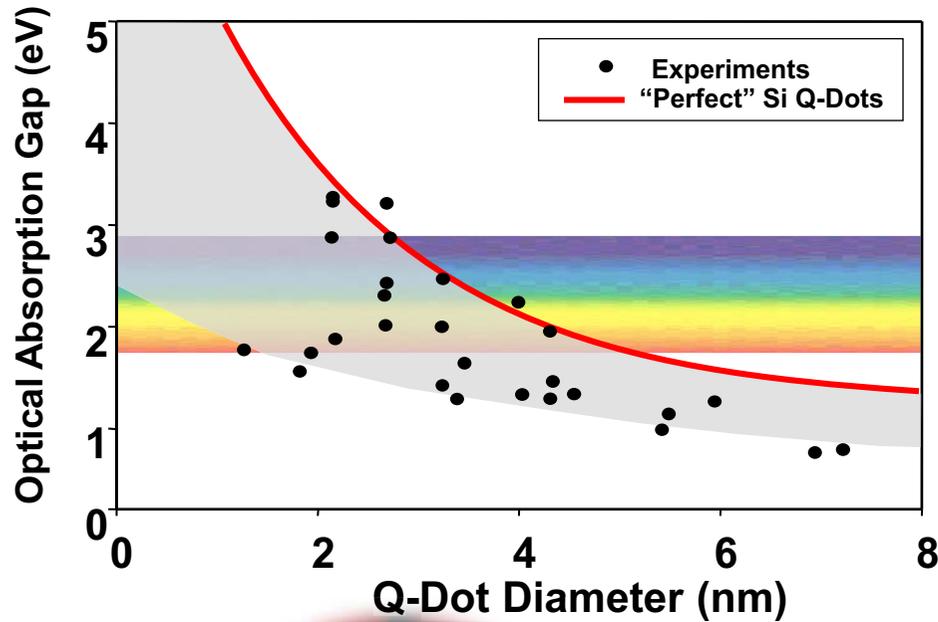


Key influence of **surface structure** and **passivation** on opto-electronic properties

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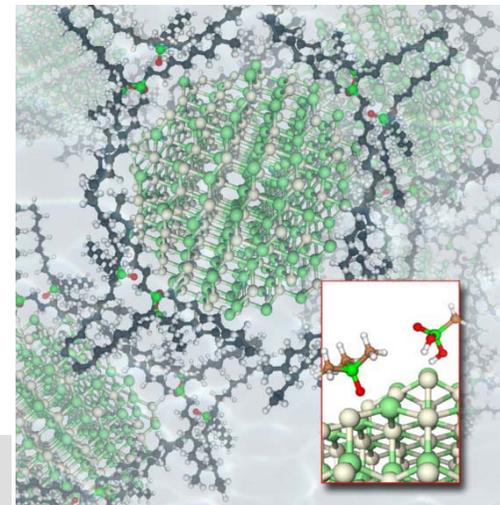
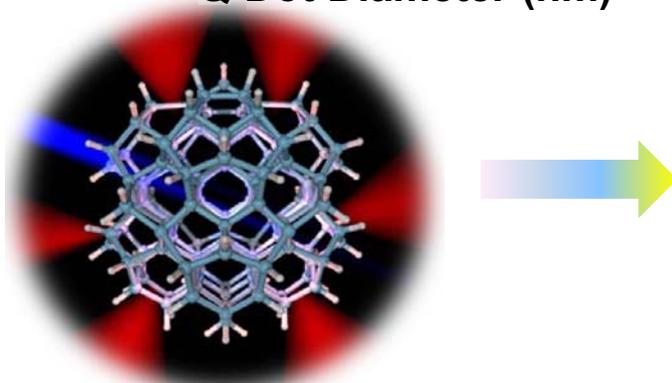


Optical properties of Si nanocrystals



Key influence of surface structure and passivation on opto-electronic properties

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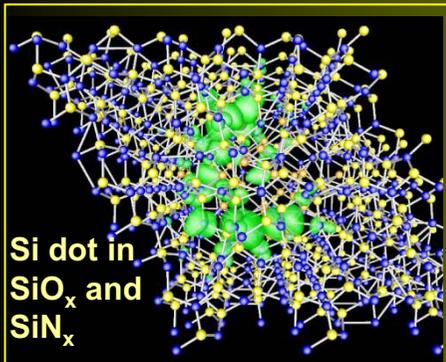
A. Puzder, A. Williamson, F.Gygi *G. G.* PRL. 04; A. Puzder, A. Williamson, N.Zaitseva, *G. G.*, L.Manna and A.P.Alivisatos, Nanoletters 04



Nanostructures in realistic environments

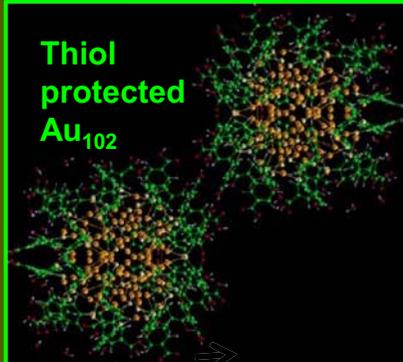
- Nanoparticles **embedded in solid matrices**
- Nanoparticles in **solutions**
- **Arrays, assemblies** of nanostructures

Inorganic and inorganic/organic interfaces

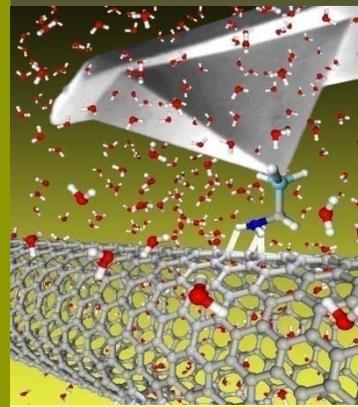


T.Li, D.Donadio, F.Gygi and G.G. (2010); Y.Li, F.Gygi and G.G. ACS Nano 2008

Thiol protected Au_{102}

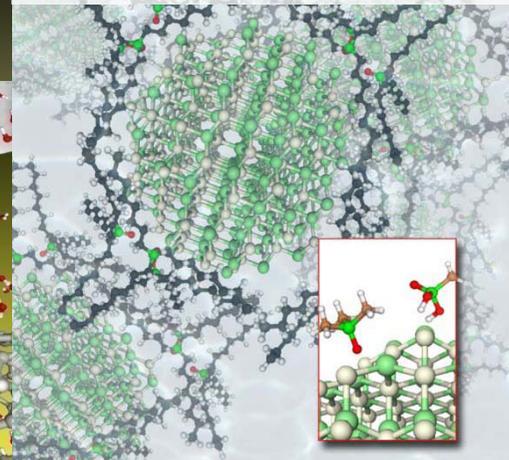


CNT/AFM tip in solvent



LeMieux et al. Nature Nanotech. 2007

CdSe in solution

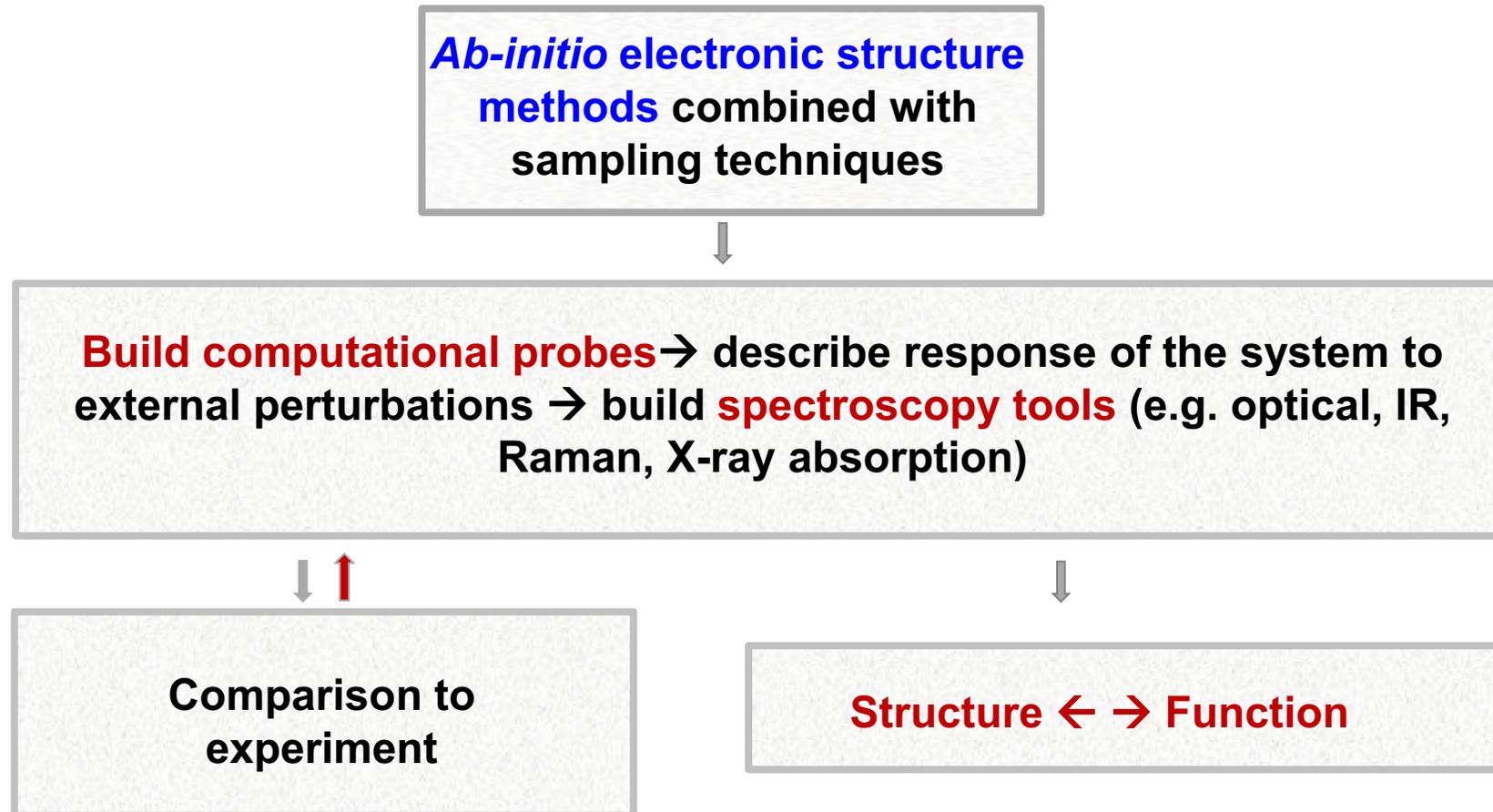


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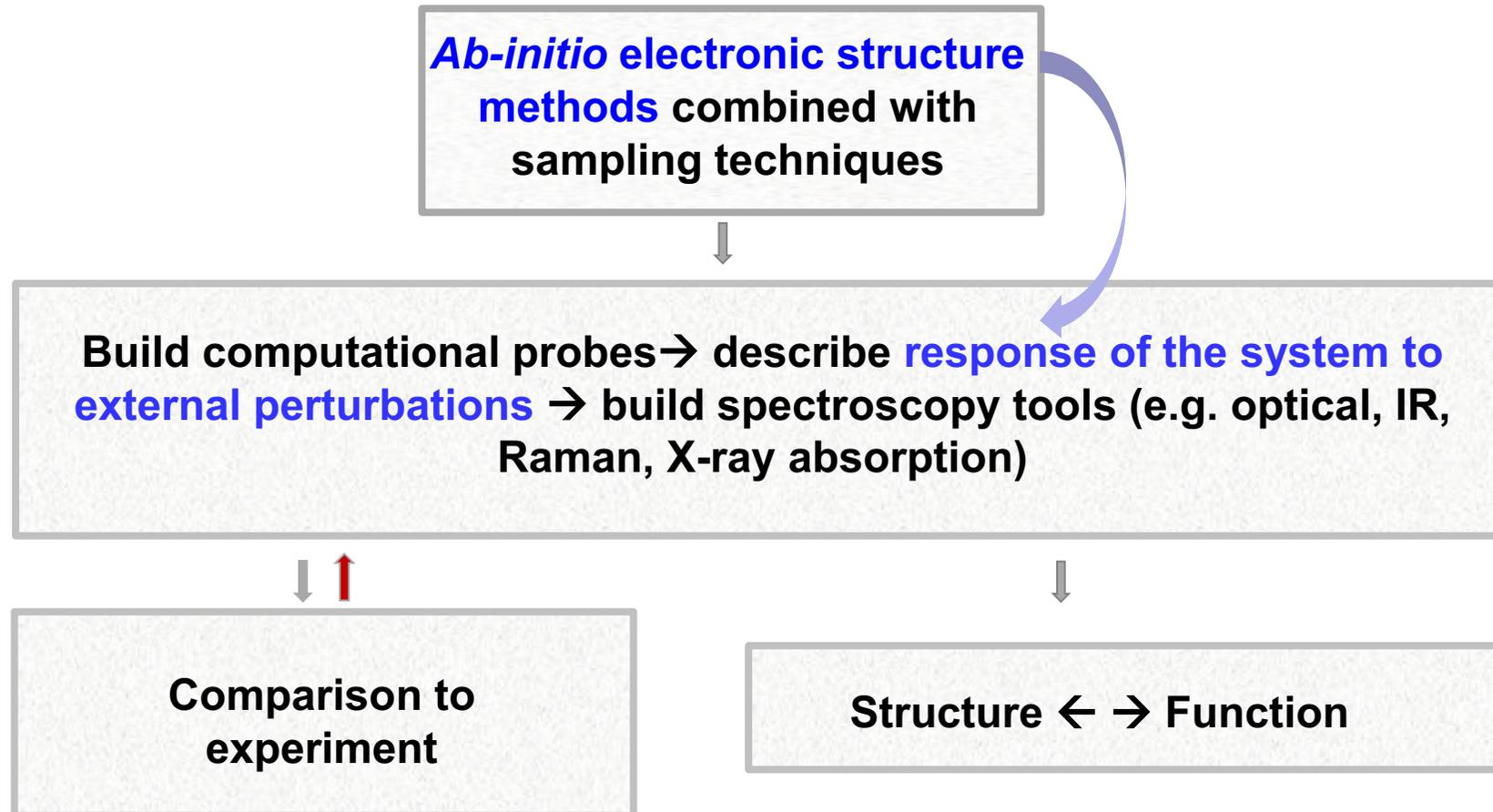
**Nanostructured materials:
Complex, large condensed systems**



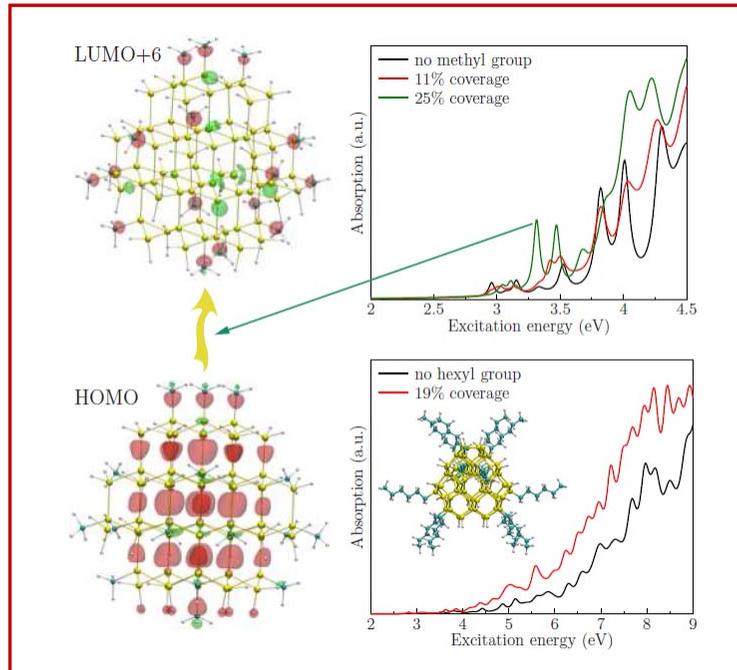
Predictive modeling of multiple properties with controllable accuracy



Predictive modeling of multiple properties with controllable accuracy



High energy excitations of Si nanoparticles



A.Gali, M.Voros, D.Rocca, G.Zimanyi and G.G.,
Nanolett. 2009, and MRS Bulletin 2010

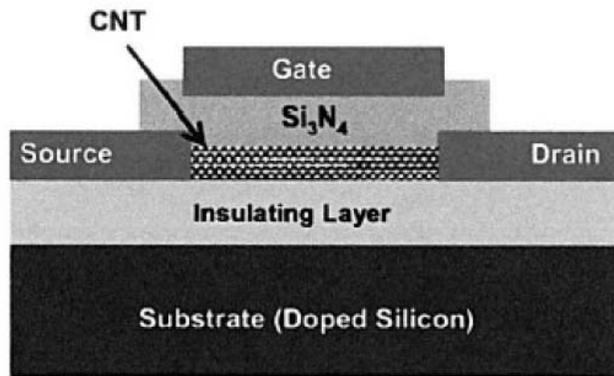
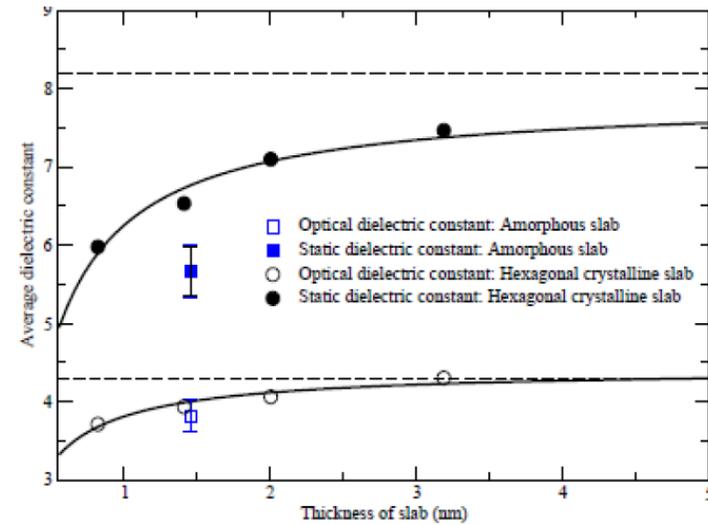
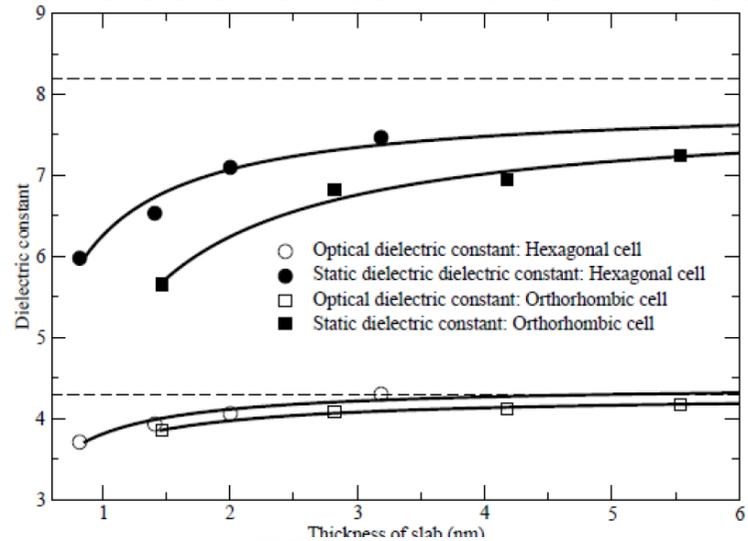
Time-dependent density functional (TDDFT) calculations of excitation spectra:

- **Surface reconstruction** may change excitation spectra both at low and high energies above the gap
- Absorption may be enhanced non-linearly by the presence of **alkyl groups**, and by the **interaction between NPs**.
- Insight into interpretation of experiments on **Multi-Electron-Generation** in colloidal NPs.

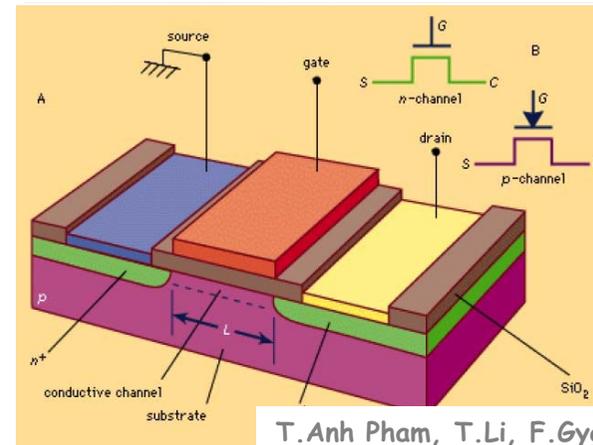


Nanoscale interfaces in electric fields: high- κ dielectrics

Dielectric properties of ordered and disordered silicon nitride films as a function of thickness



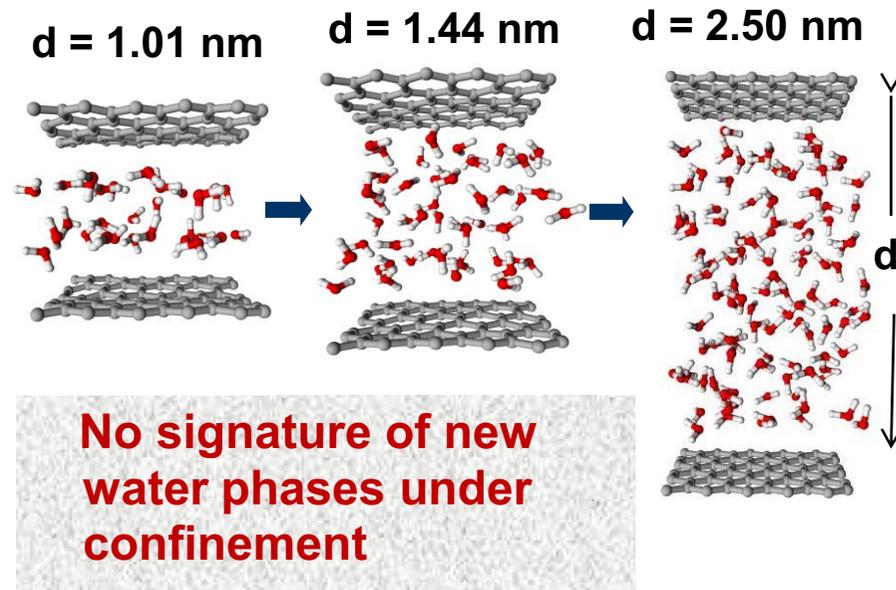
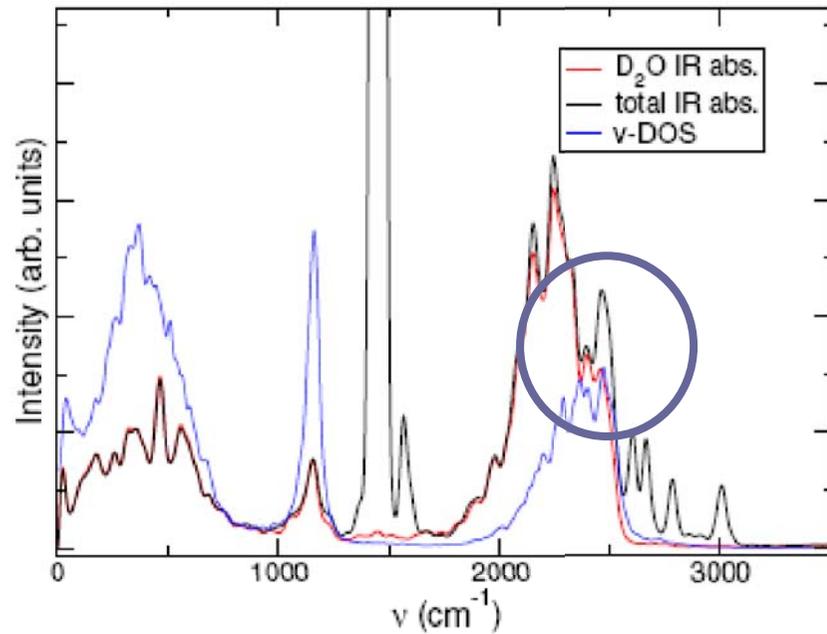
Schematic cross-section view of top-gate nanotube transistor.



T.Anh Pham, T.Li, F.Gygi and G.G., APL 2010 and in preparation



Water confined at the nanoscale: Infrared spectra



IR spectra and power spectra show remarkable differences highlighting the importance of electronic effects

Subtle electronic interactions present also in the case of highly hydrophobic substrates

Important implications for the interpretation of "hydrophobic effect(s)"



Dielectric response of molecules and condensed systems

- In general, *ab-initio* calculations of **excited state properties** of condensed systems and nanostructures involve evaluation of dielectric matrices

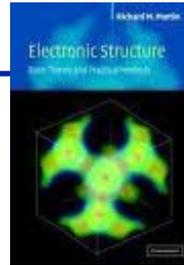
$$\varepsilon^{-1}(r_1, r_2; \omega) \longrightarrow \varepsilon_{G, G'}^{-1}(q, \omega)$$

For example to obtain: photoemission spectra ($\Sigma = iGW; W = \varepsilon^{-1}v$)
absorption spectra ($\text{Im}[1/\varepsilon_{0,0}^{-1}(q=0, \omega)]$); electron energy loss spectra ($-\text{Im}[\varepsilon_{0,0}^{-1}(q, \omega)]$).....

- Likewise, calculations of **long range correlation effects** (e.g. RPA correlation energies) involve knowledge of dielectric response functions.



Outline



- Basic Concepts: Response functions
R.M.Martin: [Electronic Structure](#)
- Approximate yet accurate dielectric matrices from eigenvalue decomposition, using Density Functional Perturbation Theory
[Hugh Wilson, Deyu Lu and Francois Gygi](#)
- Interpretation of dielectric eigenmodes: water and ice
[Deyu Lu and Francois Gygi](#)
- RPA correlation energies: binding of organic crystals (benzene, methane) and SAM on surfaces
[Yan Li , Deyu Lu, Viet Nguyen and Dario Rocca](#)
- Solution of Bethe-Salpeter Equations (BSE): Si clusters and charge transfer excitations
[Dario Rocca and Deyu Lu](#)



Maxwell equations in terms of **total field E**

- Maxwell equations: $Q = -e$; $n = \text{density}$

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 4\pi Qn, & \nabla \times \mathbf{E}(t) &= -\frac{1}{c} \frac{d\mathbf{B}}{dt}, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{B}(t) &= \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{d\mathbf{E}}{dt},\end{aligned}$$

$$\nabla \cdot \mathbf{j} = -Q \frac{dn}{dt}.$$

- **Internal and External** charges and currents:

$$— n = n_{\text{int}} + n_{\text{ext}}; \mathbf{j} = \mathbf{j}_{\text{int}} + \mathbf{j}_{\text{ext}}$$

- **Polarization** : defined to *within an additive constant* (one computes *polarization differences*)

$$\mathbf{P}(\mathbf{r}, t) = \int^t dt' \mathbf{j}_{\text{int}}(\mathbf{r}, t'),$$

$$\nabla \cdot \mathbf{P}(\mathbf{r}, t) = -Qn_{\text{int}}(\mathbf{r}, t).$$



Maxwell equations in terms of external field \mathbf{D}

- $\mathbf{D} = \mathbf{E} + 4\pi \mathbf{P}$ \mathbf{D} = external field, independent of the material

$$\begin{aligned} \nabla \cdot \mathbf{D} &= 4\pi Q_{\text{ext}}, & \nabla \times \mathbf{E}(t) &= -\frac{1}{c} \frac{d\mathbf{B}}{dt}, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{B}(t) &= \frac{4\pi}{c} \mathbf{j}_{\text{ext}} + \frac{1}{c} \frac{d\mathbf{D}}{dt}. \end{aligned}$$

- Relation between current and total field and density and total field

$$\mathbf{j}_{\text{int}}(\mathbf{r}, t) = \int d\mathbf{r}' \int dt' \sigma(\mathbf{r}, \mathbf{r}', t - t') \mathbf{E}(\mathbf{r}', t'),$$

$$\mathbf{j}_{\text{int}}(\mathbf{r}, \omega) = \int d\mathbf{r}' \sigma(\mathbf{r}, \mathbf{r}', \omega) \mathbf{E}(\mathbf{r}', \omega),$$

$$\mathbf{D}(\mathbf{r}, \omega) = \int d\mathbf{r}' \epsilon(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{E}(\mathbf{r}', \omega) \quad \text{or} \quad \mathbf{E}(\mathbf{r}, \omega) = \int d\mathbf{r}' \epsilon^{-1}(\mathbf{r}, \mathbf{r}', \omega) \mathbf{D}(\mathbf{r}', \omega),$$

- Response to the total field $\mathbf{E} \rightarrow$

$$\epsilon(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{1} \delta(\mathbf{r} - \mathbf{r}') + \frac{4\pi i}{\omega} \sigma(\mathbf{r}, \mathbf{r}', \omega).$$

- Response to the external field $\mathbf{D} \rightarrow$

$$\epsilon^{-1}(\mathbf{r}, \mathbf{r}', \omega)$$



Response in terms of scalar potentials

- $\mathbf{E} = -\nabla V(\mathbf{r})$: field derived from potential \rightarrow
in Fourier space: $\mathbf{E}(\mathbf{q}) = i \mathbf{q} V(\mathbf{q})$ is longitudinal ($//$ to \mathbf{q})

$$\epsilon^{-1}(\mathbf{q}, \mathbf{q}', \omega) = \frac{\delta V_{\text{total}}^C(\mathbf{q}, \omega)}{\delta V_{\text{ext}}(\mathbf{q}', \omega)}$$

External potential

$$\epsilon(\mathbf{q}, \mathbf{q}', \omega) = \frac{\delta V_{\text{ext}}(\mathbf{q}, \omega)}{\delta V_{\text{total}}^C(\mathbf{q}', \omega)}$$

Coulomb potential (V_{xc} excluded)

- How do we use electronic structure, theory and calculations to obtain the dielectric response?
 - Derive an expression of the **direct and inverse dielectric response in terms** of single particle (Kohn-Sham) **electronic states ψ_i from density response functions**



Static density response function

- Response of the electrons to a **variation of the total potential** at $\mathbf{r}=\mathbf{r}'$

$$\chi_n^0(\mathbf{r}, \mathbf{r}') = \frac{\delta n(\mathbf{r})}{\delta V_{\text{eff}}(\mathbf{r}')} = 2 \sum_{i=1}^{\text{occ}} \sum_j^{\text{empty}} \frac{\psi_i^*(\mathbf{r})\psi_j(\mathbf{r})\psi_j^*(\mathbf{r}')\psi_i(\mathbf{r}')}{\epsilon_i - \epsilon_j}$$

Definition

Perturbation theory

$$\chi_n^0(\mathbf{r}, \mathbf{r}') = \sum_{i=1}^{\text{occ}} \psi_i^*(\mathbf{r})G_0^i(\mathbf{r}, \mathbf{r}')\psi_i(\mathbf{r}'), \quad G_0^i(\mathbf{r}, \mathbf{r}') = \sum_{j \neq i}^{\infty} \frac{\psi_j(\mathbf{r})\psi_j^*(\mathbf{r}')}{\epsilon_i - \epsilon_j}$$

Independent particle Green function



Density response function in Fourier space

- Response to **specific Fourier components**:

$$\Delta V_{\text{eff}}(\mathbf{r}) = \Delta V_{\text{eff}} e^{i\mathbf{q}\cdot\mathbf{r}} \text{ and } n(\mathbf{q}') = \int d\mathbf{r} n(\mathbf{r}) e^{i\mathbf{q}'\cdot\mathbf{r}}$$



$$\chi_n^0(\mathbf{q}, \mathbf{q}') = \frac{\delta n(\mathbf{q}')}{\delta V_{\text{eff}}(\mathbf{q})} = 2 \sum_{i=1}^{\text{occ}} \sum_j^{\text{empty}} \frac{M_{ij}^*(\mathbf{q}) M_{ij}(\mathbf{q}')}{\epsilon_i - \epsilon_j}$$

$\langle \psi_i | e^{i\mathbf{q}\cdot\mathbf{r}} | \psi_j \rangle$

- **Homogeneous system**: χ^0 differs from 0 only for $\mathbf{q} = \mathbf{q}'$
- **Non interacting electrons**: the variation of the total potential equals the variation of the external potential $\rightarrow \chi^0$ is the response to an external perturbation
- **Mean field theory (Kohn-Sham)**: internal fields vary



Density response function within Kohn-Sham theory

- Electrons are independent particles in the potential V_{eff} :

$$\chi_n^0(\mathbf{q}, \mathbf{q}') = \frac{\delta n(\mathbf{q}')}{\delta V_{\text{eff}}(\mathbf{q})} = 2 \sum_{i=1}^{\text{occ}} \sum_j^{\text{empty}} \frac{M_{ij}^*(\mathbf{q}) M_{ij}(\mathbf{q}')}{\epsilon_i - \epsilon_j} \leftarrow \langle \psi_i | e^{i\mathbf{q}\cdot\mathbf{r}} | \psi_j \rangle$$

- Response to the **external field**:

$$\chi(\mathbf{r}, \mathbf{r}') = \frac{\delta n(\mathbf{r})}{\delta V_{\text{ext}}(\mathbf{r}')} \quad \text{or} \quad \chi(\mathbf{q}, \mathbf{q}') = \frac{\delta n(\mathbf{q})}{\delta V_{\text{ext}}(\mathbf{q}')}$$

- Relationship between **response to the total** (χ^0) **and the external field** (χ):

$$\chi = \chi^0 [1 - \chi^0 K]^{-1} \quad \text{or} \quad \chi^{-1} = [\chi^0]^{-1} - K$$

Random Phase Approximation: $f_{\text{xc}} = 0$

$$K(\mathbf{q}, \mathbf{q}') = \frac{\delta V_{\text{int}}(\mathbf{q})}{\delta n(\mathbf{q}')} = \frac{4\pi}{q^2} \delta_{\mathbf{q}, \mathbf{q}'} + \frac{\delta^2 E_{\text{xc}}[n]}{\delta n(\mathbf{q}) \delta n(\mathbf{q}')} \equiv V_C(q) \delta_{\mathbf{q}, \mathbf{q}'} + f_{\text{xc}}(\mathbf{q}, \mathbf{q}')$$



Dynamical response

- Time dependence of Kernel:

$$K(\mathbf{q}, \mathbf{q}', t - t') = \frac{\delta V_{\text{int}}(\mathbf{q}, t)}{\delta n(\mathbf{q}', t')} = \frac{4\pi}{q^2} \delta_{\mathbf{q}, \mathbf{q}'} \delta(t - t') + \frac{\delta^2 E_{\text{xc}}[n]}{\delta n(\mathbf{q}, t) \delta n(\mathbf{q}', t')},$$

↓ FT

$$K(\mathbf{q}, \mathbf{q}', \omega) = V_C(q) \delta_{\mathbf{q}, \mathbf{q}'} + f_{\text{xc}}(\mathbf{q}, \mathbf{q}', \omega).$$

- Generalization of the relationship between response to the total (χ^0) and the external field (χ):

$$\chi(\omega) = \chi^0(\omega) [1 - \chi^0(\omega) K(\omega)]^{-1}.$$

- Kramers-Kronig relations: →

$$\begin{aligned} \text{Re}\chi(\omega) &= -\frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\text{Im}\chi(\omega')}{\omega - \omega'}, \\ \text{Im}\chi(\omega) &= \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\text{Re}\chi(\omega')}{\omega - \omega'}. \end{aligned}$$



Response in terms of scalar potentials

- $\mathbf{E} = -\nabla V(\mathbf{r})$: field derived from potential \rightarrow
in Fourier space: $\mathbf{E}(\mathbf{q}) = i \mathbf{q} V(\mathbf{q})$ is longitudinal (// to \mathbf{q})

$$\epsilon^{-1}(\mathbf{q}, \mathbf{q}', \omega) = \frac{\delta V_{\text{total}}^C(\mathbf{q}, \omega)}{\delta V_{\text{ext}}(\mathbf{q}', \omega)}$$

External potential

$$\epsilon(\mathbf{q}, \mathbf{q}', \omega) = \frac{\delta V_{\text{ext}}(\mathbf{q}, \omega)}{\delta V_{\text{total}}^C(\mathbf{q}', \omega)}$$

Coulomb potential (V_{xc} excluded)

- **How do we use electronic structure, theory and calculations to obtain the dielectric response?**
 - Derive an expression of the **direct and inverse dielectric response in terms** of single particle (Kohn-Sham) **electronic states ψ_i from density response functions**

$$\epsilon^{-1}(\mathbf{q}, \mathbf{q}', \omega) = \delta(\mathbf{q} - \mathbf{q}') + V_C(q)\chi(\mathbf{q}, \mathbf{q}', \omega),$$

$$\epsilon^{-1} = 1 + \frac{V_C \chi^0}{1 - (V_C + f_{\text{xc}})\chi^0} = \frac{1 - f_{\text{xc}}\chi^0}{1 - (V_C + f_{\text{xc}})\chi^0}.$$



Dielectric matrices, dielectric function, dielectric tensors

- In a crystal $\mathbf{q} = \mathbf{k} + \mathbf{G}$ where \mathbf{k} is in the first Brillouin zone and \mathbf{G} is a reciprocal lattice vector

$$\epsilon^{-1}(\mathbf{q}, \mathbf{q}', \omega) = \delta(\mathbf{q} - \mathbf{q}') + V_C(q)\chi(\mathbf{q}, \mathbf{q}', \omega),$$



Matrices

$$\epsilon_{\mathbf{G}\mathbf{G}'}(\mathbf{k}, \omega) \quad \epsilon_{\mathbf{G}\mathbf{G}'}^{-1}(\mathbf{k}, \omega).$$

- Optical excitations \rightarrow long wavelengths $\rightarrow \mathbf{G}, \mathbf{G}' = \mathbf{0} \rightarrow$ macroscopic dielectric function :

$$\epsilon(\mathbf{k}, \omega) = \frac{\delta V_{\text{ext}}(\mathbf{k}, \omega)}{\delta V_{\text{total}}^C(\mathbf{k}, \omega)} = \frac{1}{\epsilon_{00}^{-1}(\mathbf{k}, \omega)}.$$

- Dielectric tensors:

$$\epsilon(\mathbf{k}, \omega) = \lim_{|\mathbf{k}| \rightarrow 0} \hat{\mathbf{k}}_\alpha \epsilon_{\alpha\beta}(\mathbf{k}, \omega) \hat{\mathbf{k}}_\beta.$$

- In a cubic crystal:

$$\epsilon_{\alpha\beta} = \epsilon \delta_{\alpha\beta}.$$



Calculation of dielectric matrices

Within the RPA approximation ($f_{xc} = 0$)

$$\epsilon_{G,G'}(\mathbf{q}) = \delta_{G,G'} - \frac{4\pi e^2}{|\mathbf{q} + \mathbf{G}|^2} \frac{4}{N_k \Omega} \sum_{\mathbf{k}, v, c} \frac{\langle v, \mathbf{k} | e^{-i(\mathbf{q} + \mathbf{G}) \cdot \mathbf{r}} | c, \mathbf{k} + \mathbf{q} \rangle \langle c, \mathbf{k} + \mathbf{q} | e^{i(\mathbf{q} + \mathbf{G}') \cdot \mathbf{r}'} | v, \mathbf{k} \rangle}{E_{v, \mathbf{k}} - E_{c, \mathbf{k} + \mathbf{q}}}$$

Similarity transformation to an Hermitian matrix:

$$\tilde{\epsilon}_{G,G'}(\mathbf{q}) = \frac{|\mathbf{q} + \mathbf{G}|}{|\mathbf{q} + \mathbf{G}'|} \epsilon_{G,G'}(\mathbf{q})$$

Eigenvalues of the dielectric matrix are real and greater than or equal to 1

Direct, straightforward calculation of dielectric matrices is prohibitive for large systems



Iterative calculations of dielectric matrices

- Represent dielectric matrix by its **eigenvalue decomposition** and **truncate sum** over eigenvalues to an *appropriate, small number*

$$\tilde{\epsilon} = \sum_{i=1}^N \tilde{\mathbf{v}}_i \lambda_i \tilde{\mathbf{v}}_i^H = \sum_{i=1}^N \tilde{\mathbf{v}}_i (\lambda_i - 1) \tilde{\mathbf{v}}_i^H + I$$

- **Compute eigenvalues and eigenvectors using Density Functional Perturbation Theory*** (DFPT) → avoid costly calculation of empty single particle states

— Compute **charge density variation** from solution of Sternheimer equation (1st order perturbation theory)

$$\Delta n(\mathbf{r}) = 4 \operatorname{Re} \sum_v \psi_v^*(\mathbf{r}) \Delta \psi_v(\mathbf{r}).$$

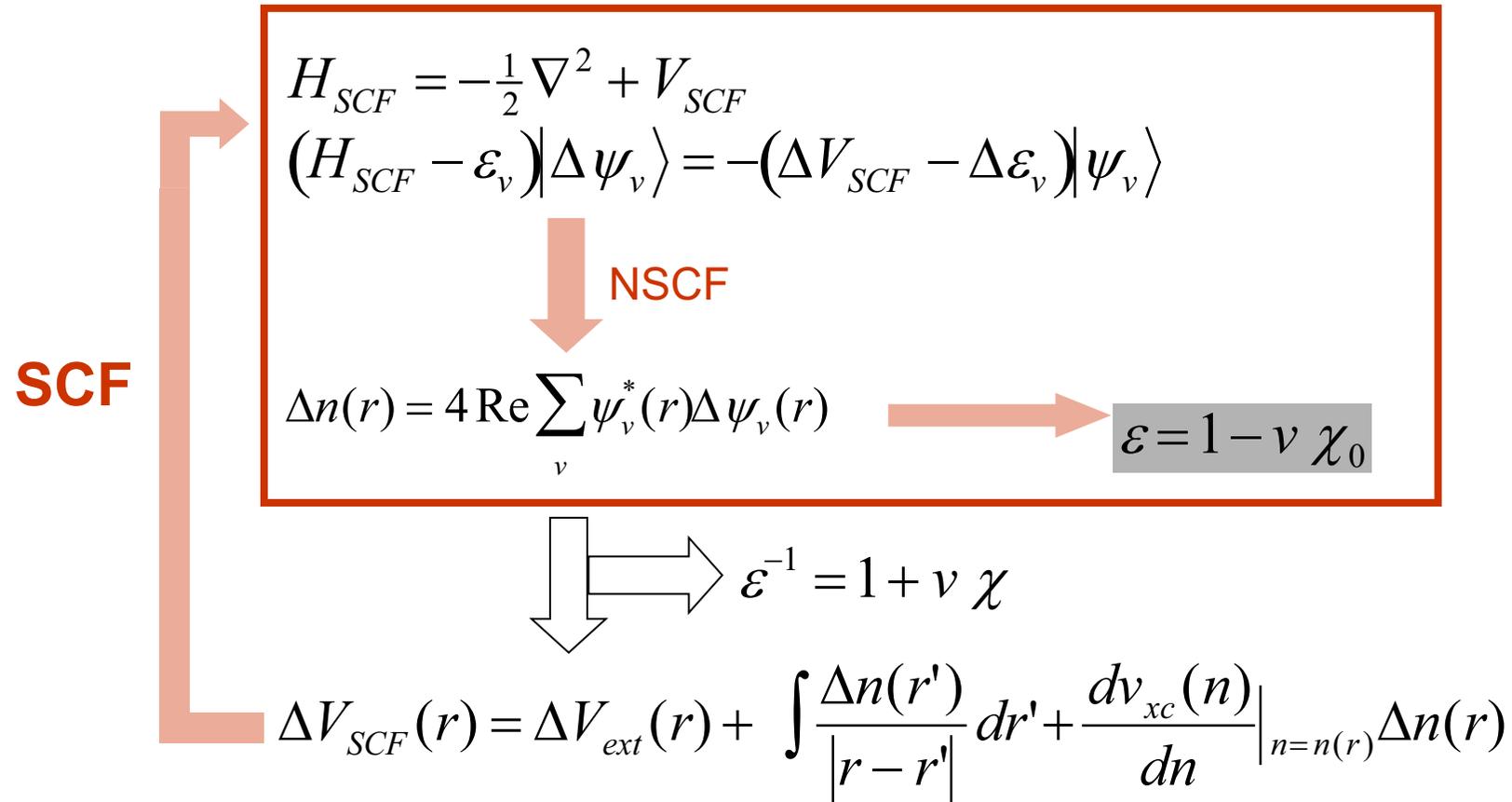
— The action of $(\epsilon - I)$ on any trial potential is then obtained from:

$$(\epsilon - I) \Delta V_{\text{SCF}} = -v_c \chi_0 \Delta V_{\text{SCF}} = -v_c \Delta n.$$

* S. Baroni, et al., Rev. Mod. Phys., 73:515, 2001.



DFPT and Sternheimer equation



The Sternheimer equation is solved non-self-consistently.

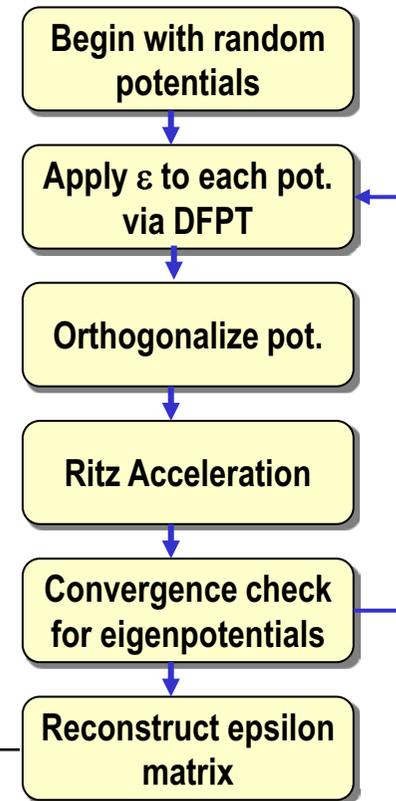


Iterative procedure based on Density Functional Perturbation Theory

$$\tilde{\epsilon} = \sum_{i=1}^N \tilde{\mathbf{v}}_i \lambda_i \tilde{\mathbf{v}}_i^H = \sum_{i=1}^N \tilde{\mathbf{v}}_i (\lambda_i - 1) \tilde{\mathbf{v}}_i^H + I$$

- Calculation of **empty electronic states**, calculation and storage of full dielectric matrix and inversion of ϵ are **avoided**
- **Scaling:** $N_{\text{eig}} N_{\text{pw}} N_v^2$ (instead of $N_{\text{pw}}^2 N_v N_c$)
- Efficient evaluation of ϵ^{-1} at different \mathbf{q} points and (eventually) at different MD steps is possible
- Incorporation of XC kernel is straightforward

$$\tilde{\epsilon} = I + \sum_i \tilde{\mathbf{v}}_i^* (\lambda_i - 1) \tilde{\mathbf{v}}_i$$



Power method

$A \in \mathbb{R}^{n \times n}$ symmetric

$X = [x_1, \dots, x_n] \in \mathbb{R}^{n \times n}$
eigenvectors of A

$$A x_i = \lambda_i x_i$$

assume $|\lambda_1| < |\lambda_2| < \dots < |\lambda_n|$

start from a normalized random vector $q^{(0)}$

$$1) z^{(k)} = A q^{(k-1)}$$

$$2) q^{(k)} = \frac{z^{(k)}}{\|z^{(k)}\|}$$

$$3) \lambda^{(k)} = (q^{(k)})^T A q^{(k)}$$

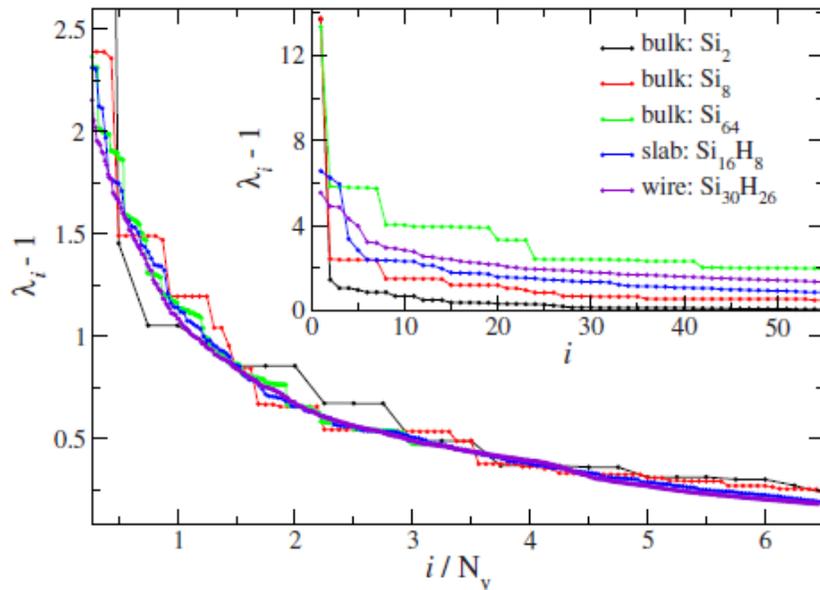
repeat 1), 2), 3), until converged

- The sequence $q^{(k)}$ converges to the eigenvector x_n
- The sequence $\lambda^{(k)}$ converges to λ_n
- The power method finds the largest eigenvalue of a matrix and its associated eigenvector, if λ_n is a dominant eigenvalue

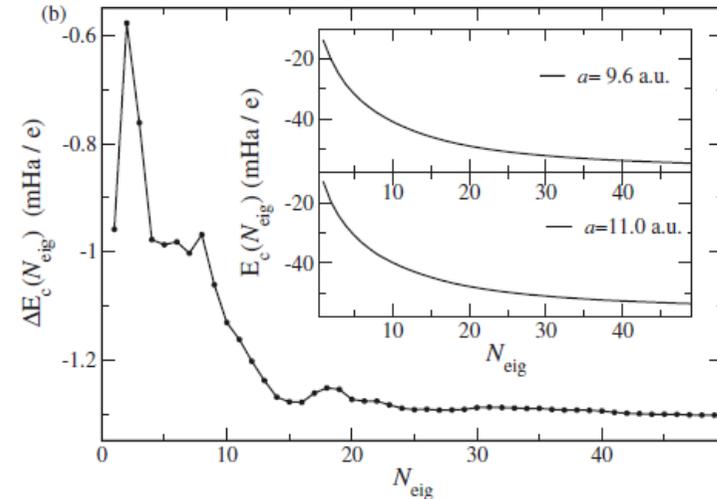


Well behaved decay of dielectric matrix eigenvalues for a broad class of systems

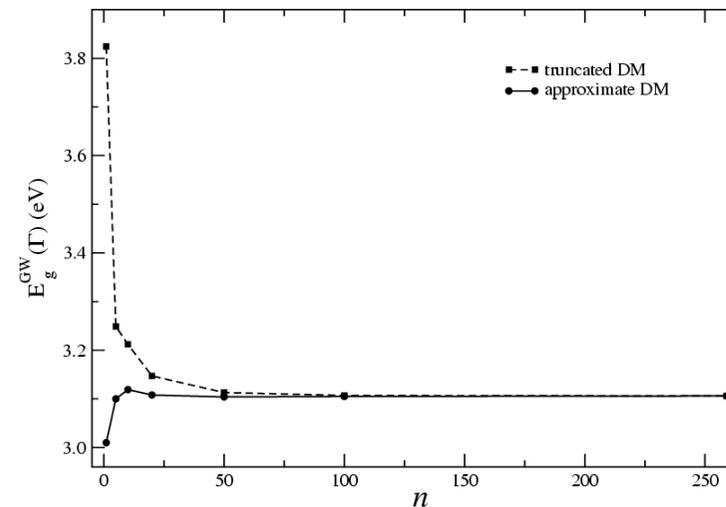
$$\tilde{\epsilon} = I + \sum_i \tilde{\mathbf{v}}_i^* (\lambda_i - 1) \tilde{\mathbf{v}}_i$$



Bulk Si RPA Correlation energy

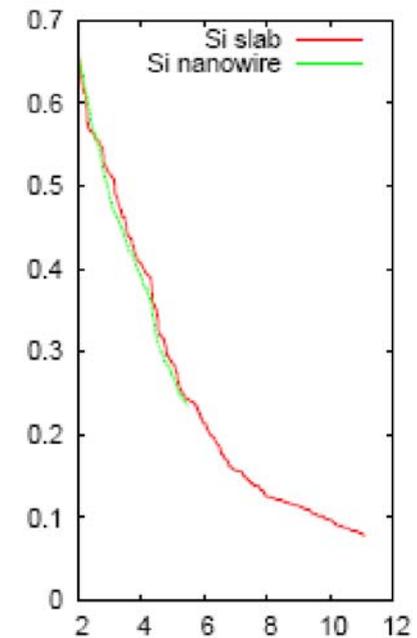
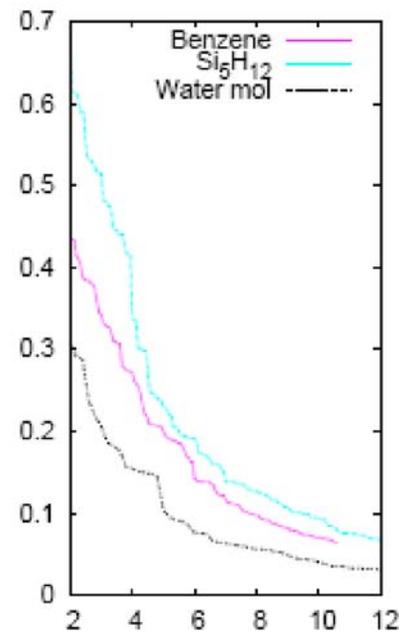
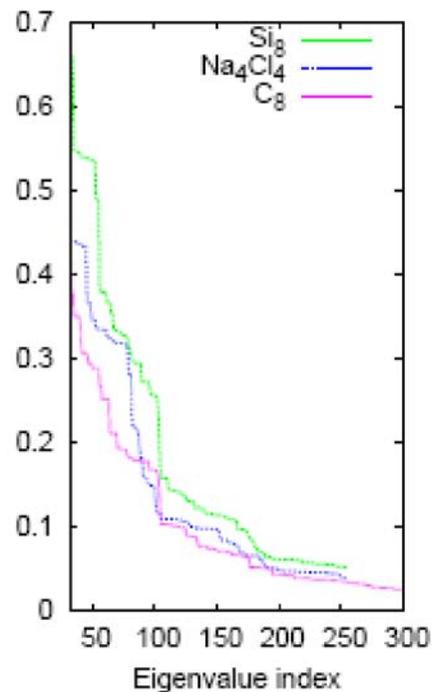


Bulk Si GW gap at Γ



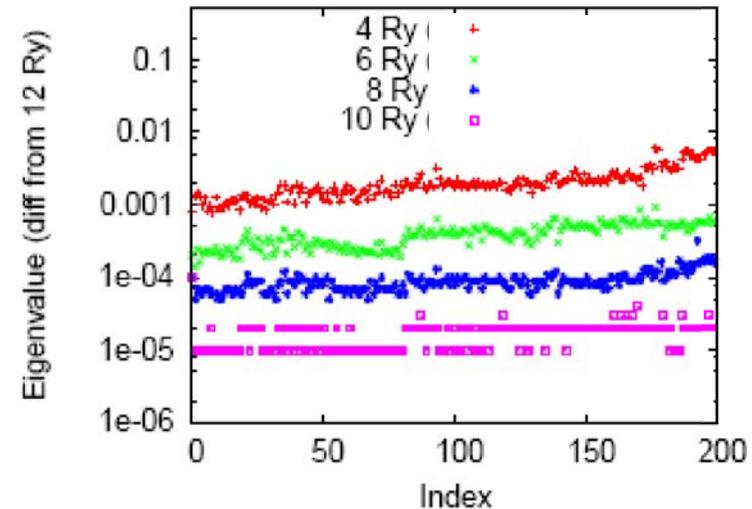
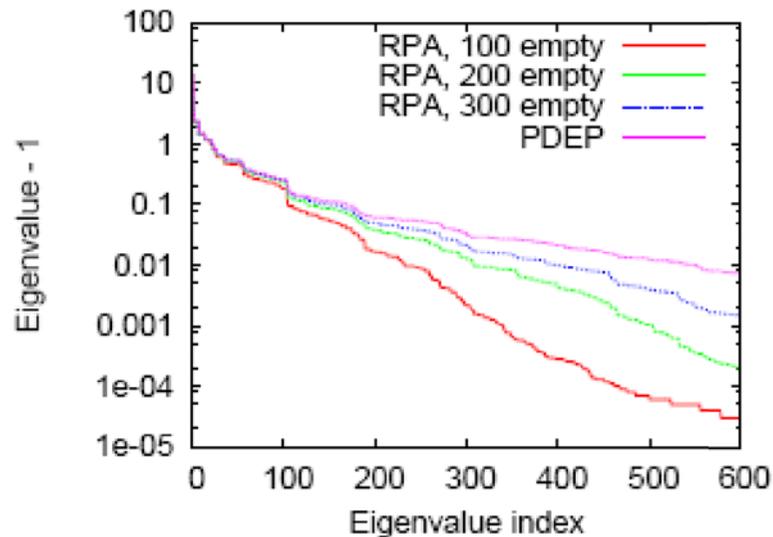
Well behaved decay of dielectric matrix eigenvalues for a broad class of systems

$$\tilde{\epsilon} = I + \sum_i \tilde{\mathbf{v}}_i^* (\lambda_i - 1) \tilde{\mathbf{v}}_i$$



Sensitivity of dielectric matrix calculations to number of empty states and energy cutoff

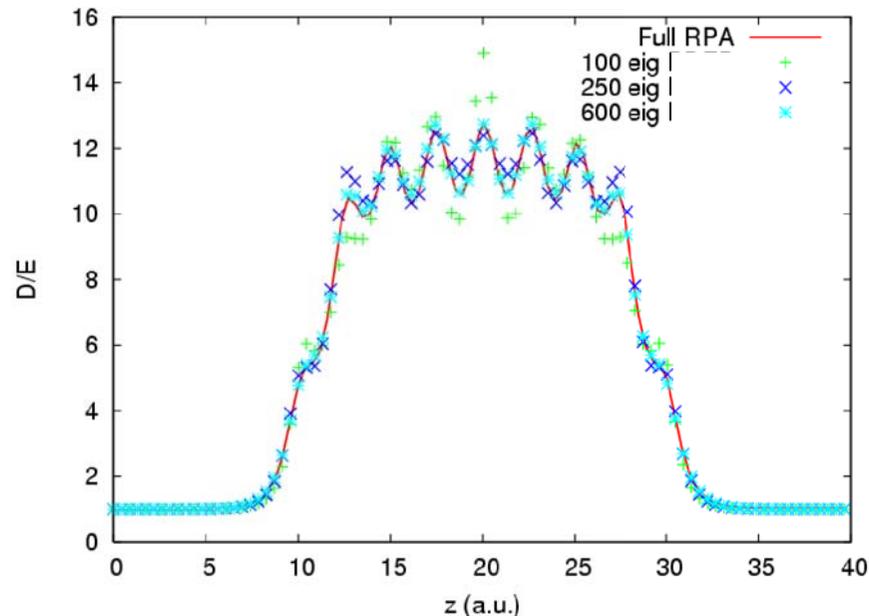
$$\tilde{\epsilon} = I + \sum_i \tilde{\mathbf{v}}_i^* (\lambda_i - 1) \tilde{\mathbf{v}}_i$$



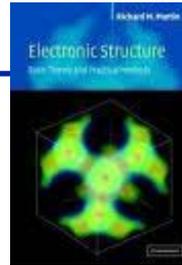
Well behaved decay of dielectric matrix eigenvalues for a broad class of systems

$$\tilde{\epsilon} = I + \sum_i \tilde{\mathbf{v}}_i^* (\lambda_i - 1) \tilde{\mathbf{v}}_i$$

Average electric field inside a hydrogenated Si slab



Outline



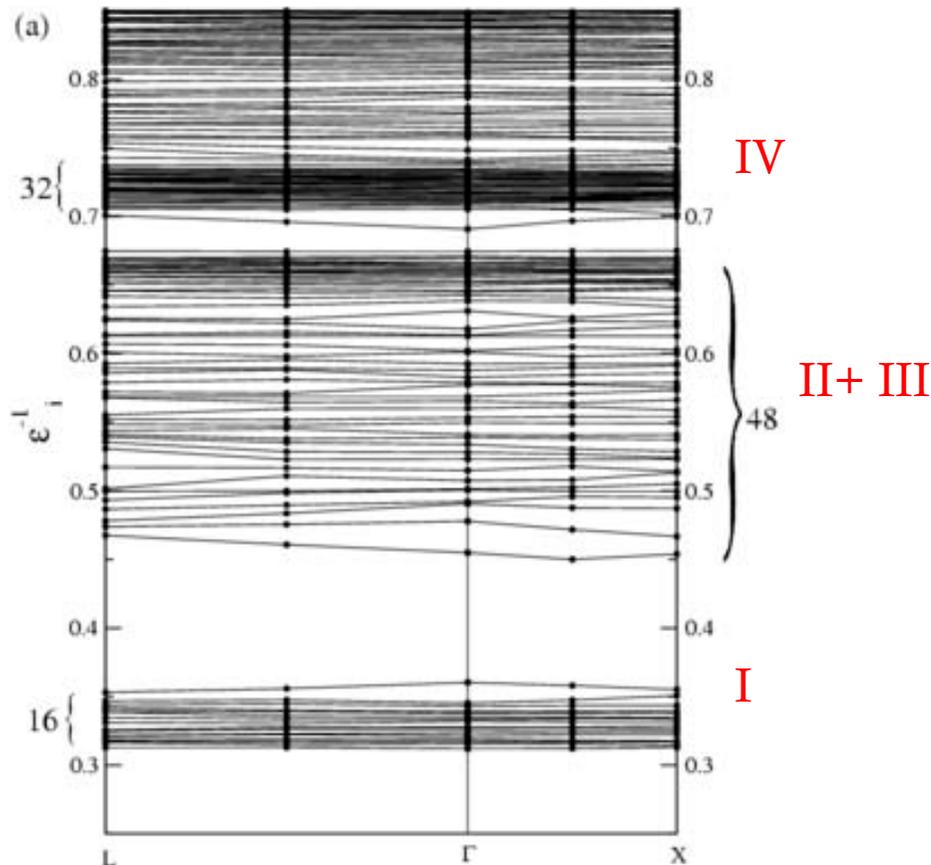
- Basic Concepts: Response functions
R.M.Martin: [Electronic Structure](#)
- Approximate yet accurate dielectric matrices from eigenvalue decomposition, using Density Functional Perturbation Theory
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- • **Interpretation of dielectric eigenmodes: water and ice**
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- Solution of Bethe-Salpeter Equations (BSE): Si clusters and charge transfer excitations
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Water: Dielectric band structure

liquid water: 16 molecules

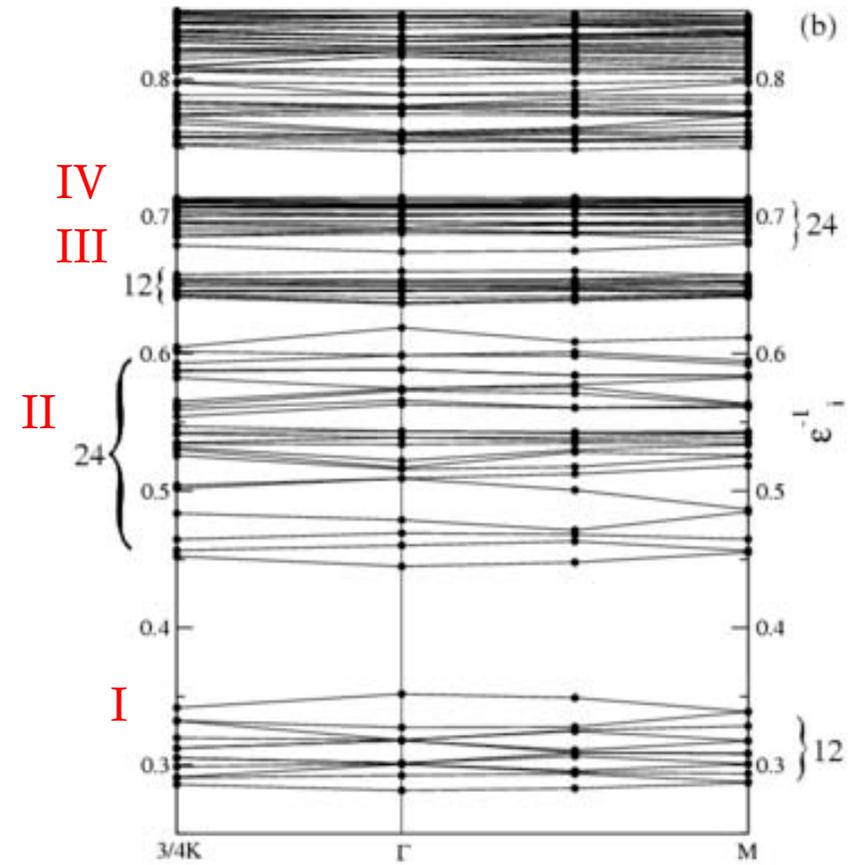
$\epsilon_M = 1.72$ (this work); $\epsilon_M \approx 1.8$ (exp)*



* Chem. Rev., 105:3197, 2005

ice Ih: 12 molecules

$\epsilon_M = 1.78$ (this work); $\epsilon_M \approx 1.7$ (exp)*

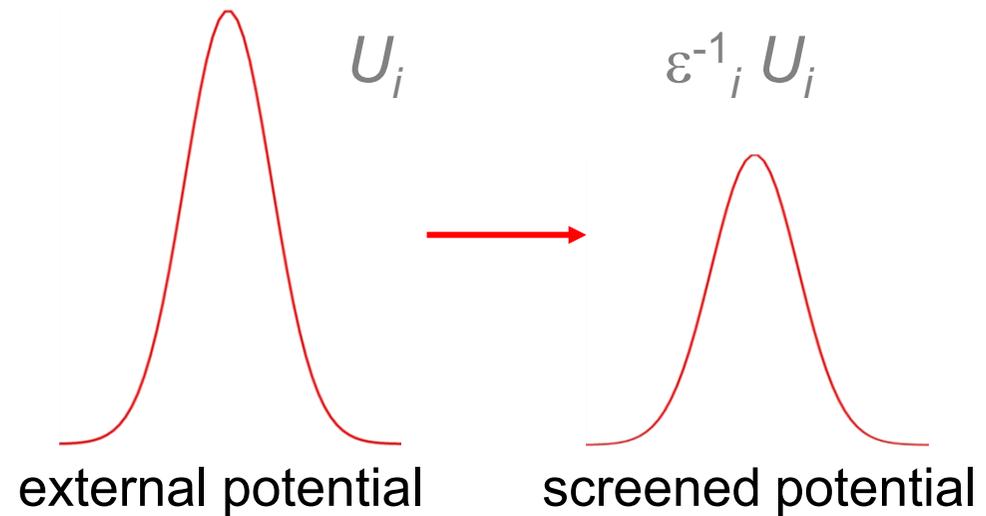


* Proc. R. Soc. Lond. A, 349:467, 1976

Localized screening modes

Eigenmode decomposition:

$$\tilde{\varepsilon}^{-1}(q) |U_i(q)\rangle = \varepsilon_i^{-1} |U_i(q)\rangle$$
$$\tilde{\varepsilon}^{-1}(q) = \sum_i |U_i(q)\rangle \varepsilon_i^{-1} \langle U_i(q)|$$



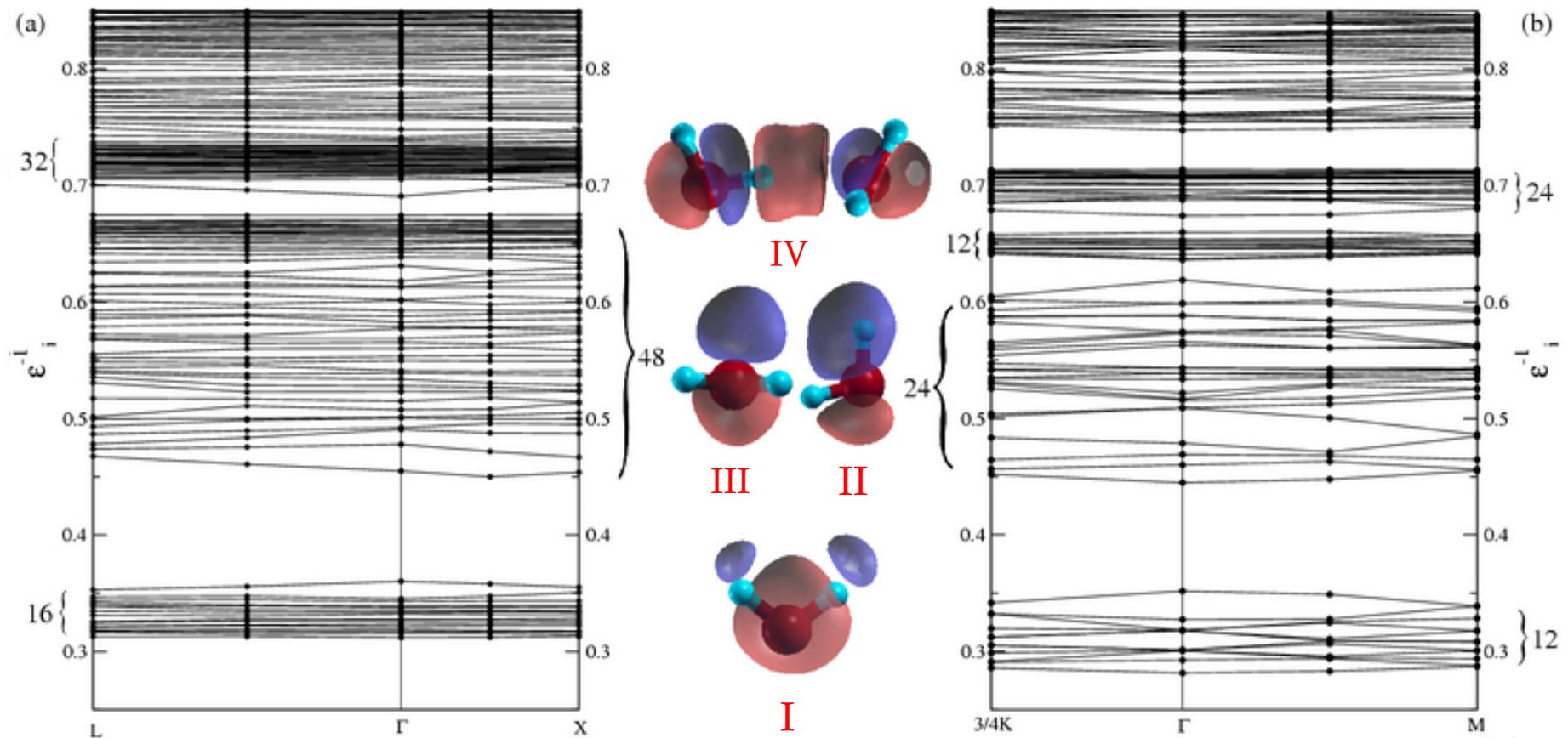
Localization:

From eigenstates extended in the periodic cell
to **maximally localized screening modes**

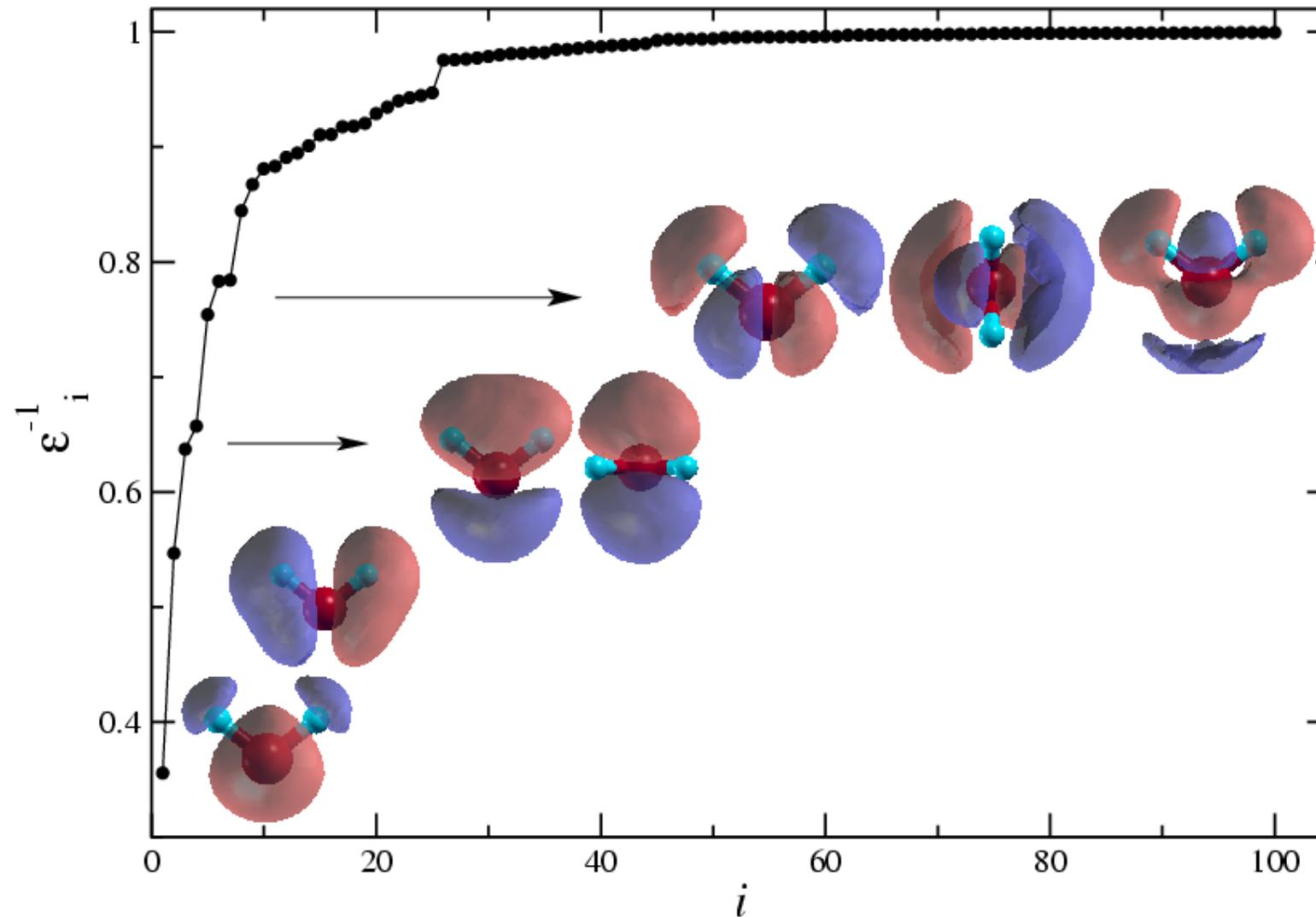


Dielectric band structure and localized screening modes

Are the lowest Maximally Localized Dielectric Modes (MLDM) related to the Dielectric Eigenmodes of (DEM) of the gas phase molecule?

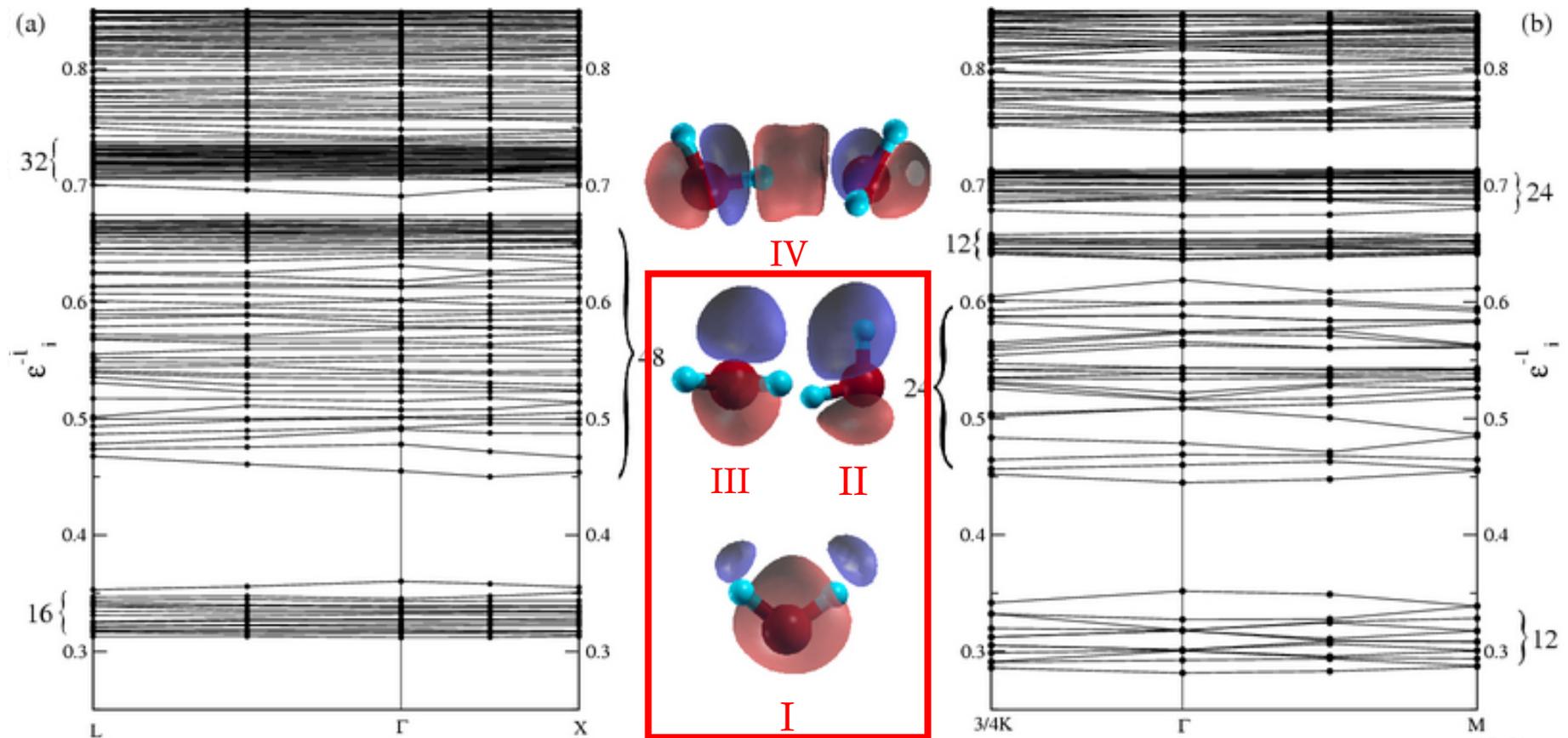


The dielectric eigenmodes of the water molecule



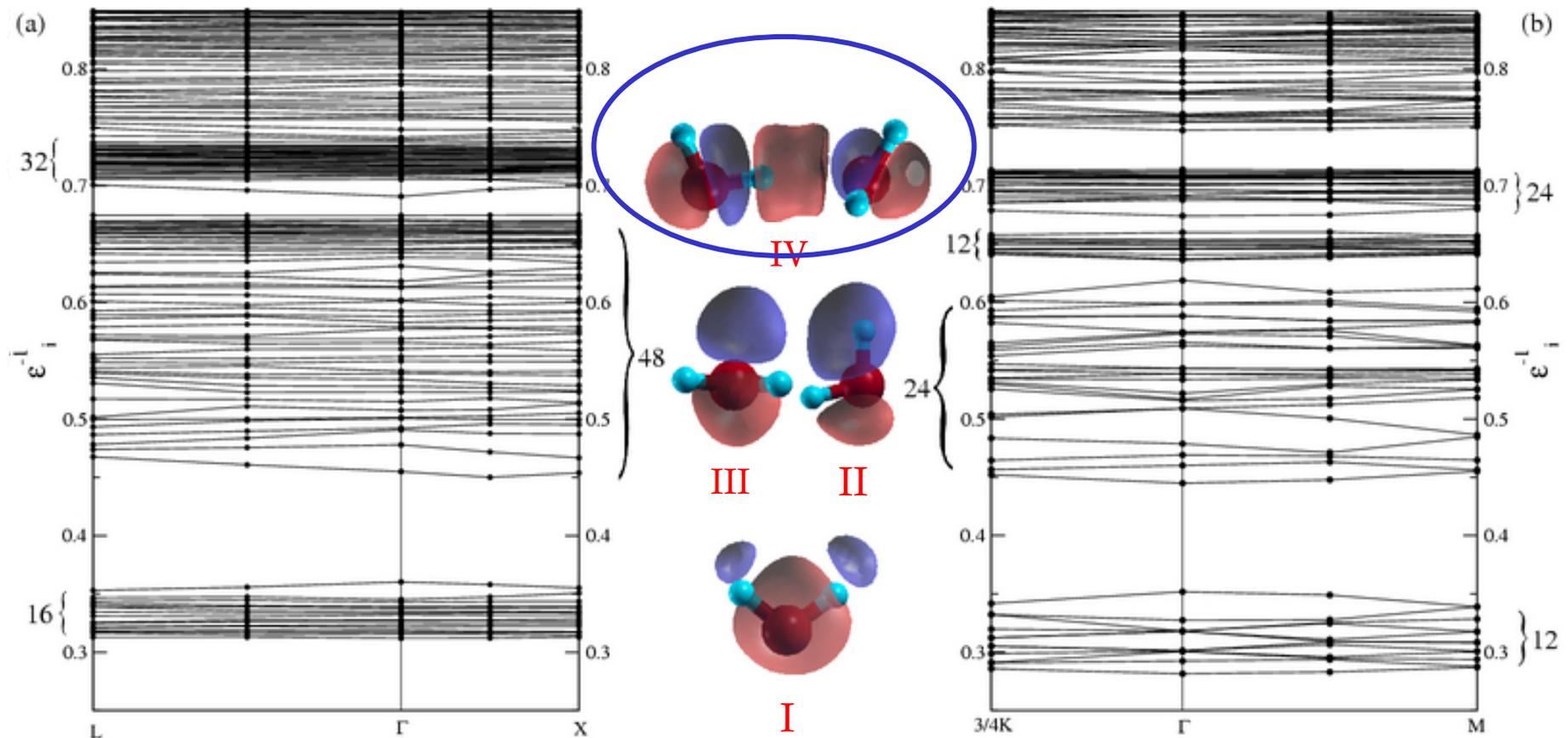
Dielectric band structures and localized screening modes

The lowest Maximally Localized Dielectric Modes (MLDM) are related to the Dielectric Eigenmodes of (DEM) of the **monomer** and **dimer**

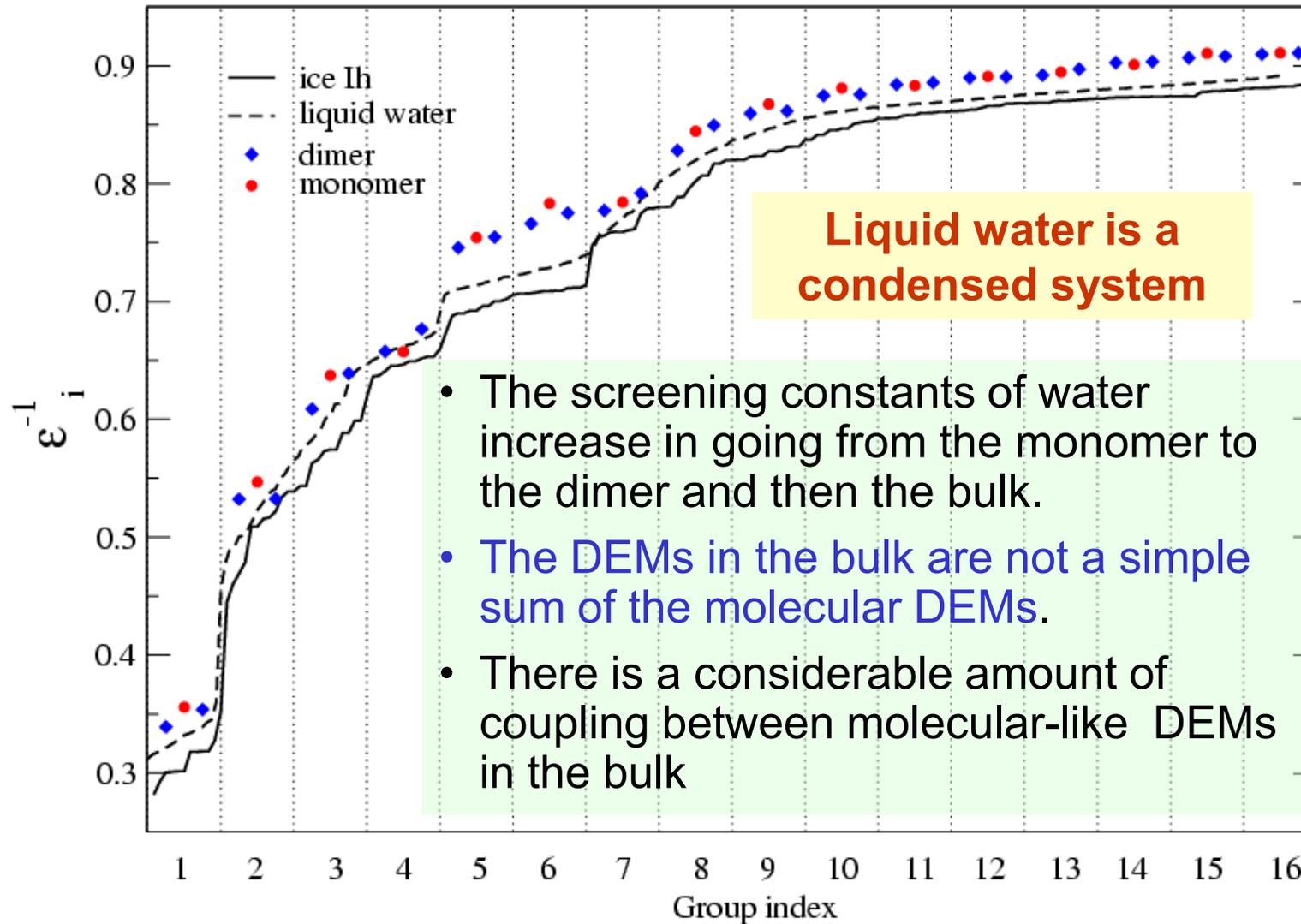


Dielectric band structures and localized screening modes

The lowest Maximally Localized Dielectric Modes (MLDM) are related to the Dielectric Eigenmodes of (DEM) of the **monomer** and **dimer**



Water: Dielectric response in the gas phase and in the bulk



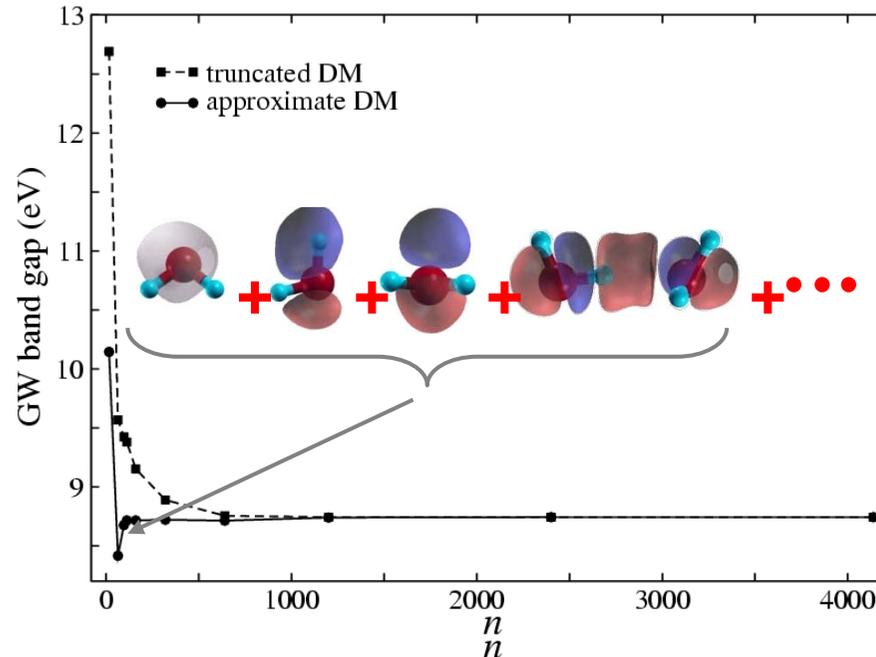
Approximate dielectric matrices for GW calculations

$$\tilde{\epsilon}^{-1}(q) = \sum_{i=1}^n |U_i(q)\rangle \epsilon_i^{-1} \langle U_i(q)| + \sum_{i=n+1}^N |V_i(q)\rangle \epsilon_i^{-1} \langle V_i(q)|$$

exact DEMs model dielectric response

D. Penn, *Phys. Rev.*, 128: 2093,1962.

Plasmon Pole model: W. von der Linden and P. Horsch, *PRB*, 37, 8351,1988



$$E_n = \epsilon_n^{KS} + \langle \Sigma_{GW}(E_n) \rangle_n - \langle V_{xc} \rangle_n$$

$$\Sigma(r, r', \omega) = \frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega' G(r, r', \omega') W(r, r', \omega - \omega')$$

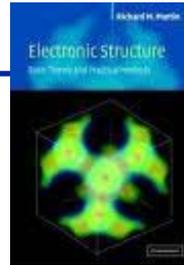
QP band gap: 8.7 eV(this work);
8.6 eV (theory)¹; 8.7±0.5 eV (exp)²

[1] Garbuio *et al.*, *Phys. Rev. Lett.*, 97:137402, 2006.
[2] Bernas *et al.*, *Chem. Phys.* 222:151, 1997.

D.Lu, F.Gygi and G.Galli, *Phys Rev. Lett.* 2008



Outline



- Basic Concepts: Response functions
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Correlation energy in terms of response functions

- Exact expression for E_c

$$E_{xc} = E_x + E_c$$

$$E_c = -\frac{1}{2\pi} \int_0^1 d\lambda \int_0^\infty du \operatorname{Tr} \{v_c[\chi_\lambda(iu) - \chi_0(iu)]\}$$

- Kohn-Sham response function

$$\chi_0(\mathbf{r}, \mathbf{r}'; iu) = \sum_{i,j} (f_i - f_j) \frac{\phi_i^*(\mathbf{r}) \phi_j(\mathbf{r}) \phi_j^*(\mathbf{r}') \phi_i(\mathbf{r}')}{\epsilon_i - \epsilon_j + iu}$$

- Dyson-like equation

$$\chi_\lambda = \chi_0 + \chi_0[\lambda v_c + f_\lambda^{xc}] \chi_\lambda$$

$f_\lambda^{xc}(iu)$ unknown!

In RPA, $f_\lambda^{xc}(iu) = 0$



Adiabatic Connection

- Fictitious Hamiltonian

$$H(\lambda) = T + V_\lambda + \lambda V_{ee} \quad n_\lambda(\mathbf{r}) \equiv n(\mathbf{r}) \quad \forall \lambda \in [0, 1]$$

$$\text{KS system: } V_{\lambda=0} = V_{KS} = V_{ext} + V_H + V_{xc} \quad \text{Real system: } V_{\lambda=1} = V_{ext}$$

- Hellmann-Feynman theorem

$$\frac{dE(\lambda)}{d\lambda} = \langle \Psi^\lambda | V_{ee} | \Psi^\lambda \rangle + \langle \Psi^\lambda | \frac{\partial V_\lambda}{\partial \lambda} | \Psi^\lambda \rangle$$

- Integrating over λ

$$\begin{aligned} E_H + E_{xc} &= \int_0^1 d\lambda \langle \Psi^\lambda | V_{ee} | \Psi^\lambda \rangle \\ &= \frac{1}{2} \int_0^1 d\lambda \int d\mathbf{r} d\mathbf{r}' \frac{e^2}{|\mathbf{r} - \mathbf{r}'|} n_2^\lambda(\mathbf{r}, \mathbf{r}') \end{aligned}$$



Fluctuation dissipation theorem

- Fluctuation-Dissipation Theorem

$$- \int \frac{du}{2\pi} \chi_\lambda(\mathbf{r}, \mathbf{r}'; iu) = n_2^\lambda(\mathbf{r}, \mathbf{r}') - n(\mathbf{r})n(\mathbf{r}') + \delta(\mathbf{r} - \mathbf{r}')n(\mathbf{r})$$

- Exchange-correlation energy

$$E_{xc} = -\frac{1}{2} \int_0^1 d\lambda \int d\mathbf{r}d\mathbf{r}' \frac{e^2}{|\mathbf{r} - \mathbf{r}'|} \left\{ \int_0^\infty \frac{du}{\pi} \chi_\lambda(\mathbf{r}, \mathbf{r}'; iu) + \delta(\mathbf{r} - \mathbf{r}')n(\mathbf{r}) \right\}$$

- Kohn-Sham exact exchange

$$\begin{aligned} E_x^{KS} &= -\frac{1}{2\pi} \int d\mathbf{r}d\mathbf{r}' \frac{e^2}{|\mathbf{r} - \mathbf{r}'|} \left\{ \int_0^\infty du \chi_0(\mathbf{r}, \mathbf{r}'; iu) + \delta(\mathbf{r} - \mathbf{r}')n(\mathbf{r}) \right\} \\ &= -\frac{e^2}{2} \int d\mathbf{r}d\mathbf{r}' \frac{|\sum_i^{occ} \phi_i^*(\mathbf{r})\phi_i(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} \end{aligned}$$

- Correlation energy

$$E_c = -\frac{1}{2\pi} \int_0^1 d\lambda \int d\mathbf{r}d\mathbf{r}' \frac{e^2}{|\mathbf{r} - \mathbf{r}'|} \int_0^\infty du \{ \chi_\lambda(\mathbf{r}, \mathbf{r}'; iu) - \chi_0(\mathbf{r}, \mathbf{r}'; iu) \}$$



Correlation and dielectric response function

RPA Correlation energy in terms of dielectric matrix eigenvalues:

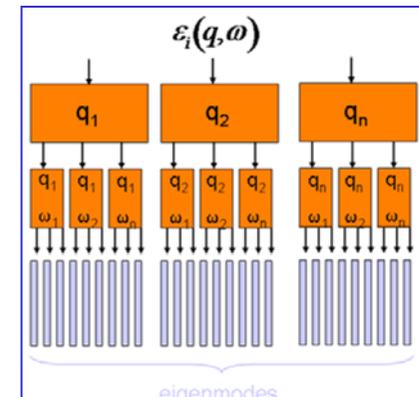
$$\varepsilon = 1 - v_c \chi_0$$

Scaling ($N_v^2 N_{eig} N_{pw}$) enables calculation for large systems.

$$E_c = \frac{1}{2\pi} \int_0^\infty du \sum_q \sum_i \{ \text{Log}[\varepsilon_i(q, iu)] - \varepsilon_i(q, iu) + 1 \}$$

Solve frequency dependent Sternheimer equation to obtain eigenvalues ε_i — use a so called BiConjugate Gradient Stabilized (Bi-CGSTAB) technique¹

Efficient parallelization is possible



¹H. A. van der Vorst, "Bi-CGSTAB: A Fast and Smoothly Converging Variant of Bi-CG for the Solution of Nonsymmetric Linear Systems", SIAM J. Sci. Stat. Comput., 13, 631-664, (1992)—thanks to Z. Bai.



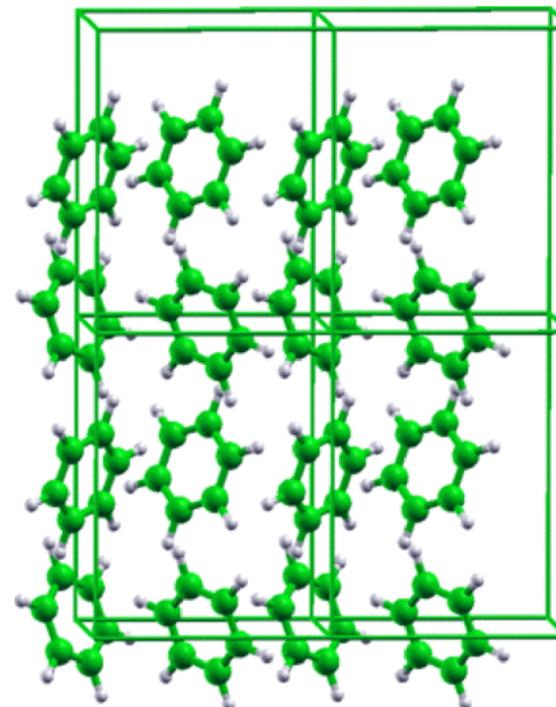
Correlation energy calculations for molecular crystals and SAM on surfaces

- No integration over coupling constant needed
- No extrapolation as a function of energy cutoff and k-pts sampling
- No empty state calculation
- No dielectric matrix inversion



Benzene Crystal

- One of the simplest organic crystals bound by vdW forces
- Experimental lattice constants (4K) [1]:
 - **Four monomers per cell (120 e⁻)**
 - Pbc₂v symmetry
 - a=7.360 Å, b=9.375Å, c=6.70 Å, α=β=γ=90°
- **LDA** overestimates the equilibrium density by **13%**; **PBE** underestimates the equilibrium density by **27%** and severely under-binds^[2].

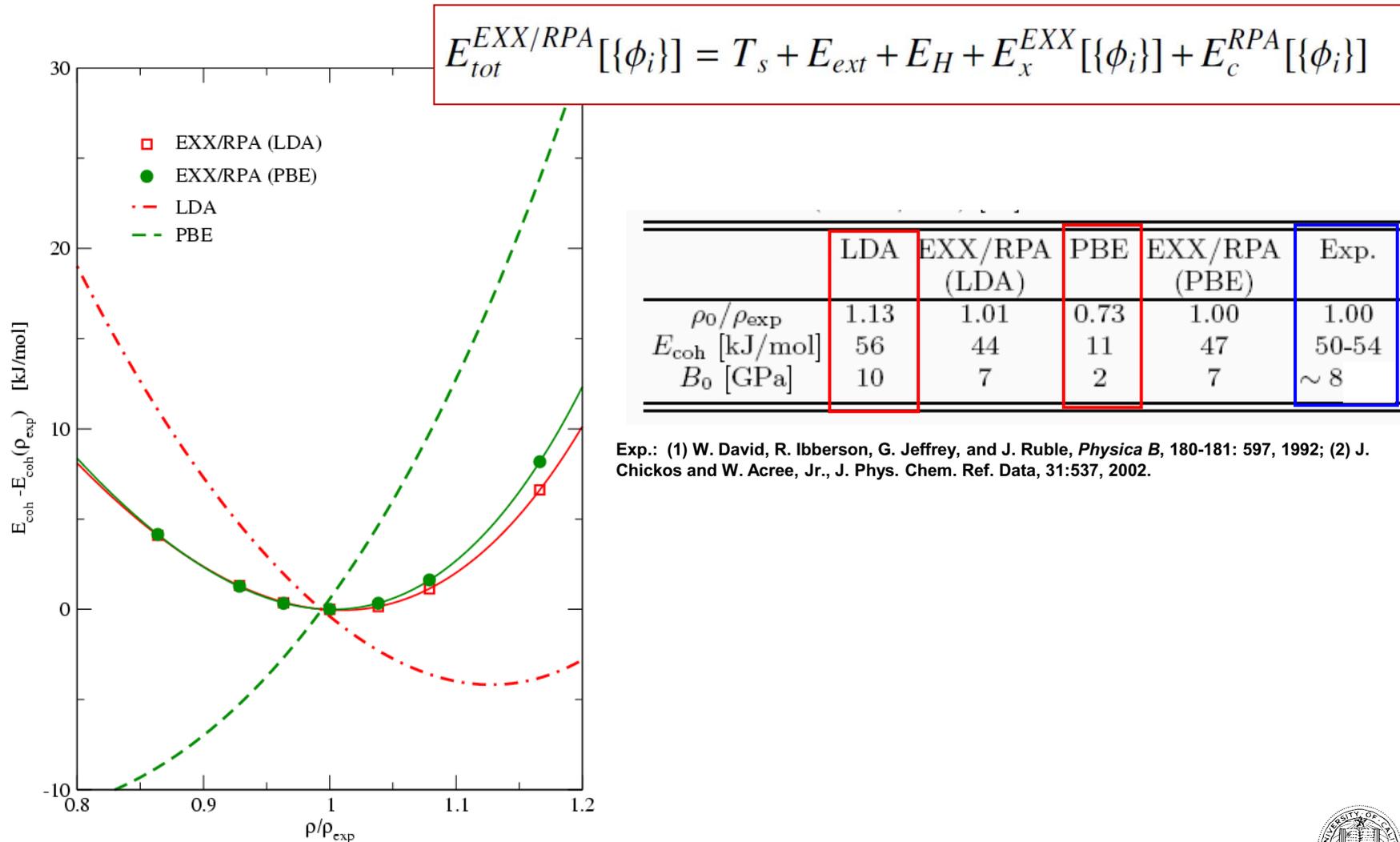


[1] W. David, R. Ibberson, G. Jeroy, and J. Ruble, *Physica B*, 180-181, 597, 1992.

$$E_{tot}^{EXX/RPA}[\{\phi_i\}] = T_s + E_{ext} + E_H + E_x^{EXX}[\{\phi_i\}] + E_c^{RPA}[\{\phi_i\}]$$



Benzene crystal binding curve



Exp.: (1) W. David, R. Ibberson, G. Jeffrey, and J. Ruble, *Physica B*, 180-181: 597, 1992; (2) J. Chickos and W. Acree, Jr., *J. Phys. Chem. Ref. Data*, 31:537, 2002.

D. Lu, Y. Li, D. Rocca and G. Galli, *Phys. Rev. Lett.*, 102, 206411, 2009.



Benzene crystal

1 kJ/mol ~ 10 meV

| | EXX/RPA@LDA | EXX/RPA@PBE | QC methods | Exp. |
|----------------------------|-------------|-------------|------------|-------|
| E_{coh} (kJ/mol) | 43 | 47 | 51-56 | 50-54 |
| ρ_0/ρ_{exp} | 1.01 | 1.00 | -- | 1.0 |

Exp. values [J. S. Chickos *et al.*, JPC Ref. Data 31, 537 (2002)] have been corrected for

- **Temperature effects** (2RT=4.2 kJ/mol at 250 K)
- **Zero-point energy** (2.8 kJ/mol). [M. Nakamura *et al.*, JCP. 51, 3146 (1969).]

QC results: see Li *et al.*, JPCA 114, 1944 (2010) for details.

- EXX/RPA greatly improves over LDA and PBE results.
- Cohesive energy from EXX/RPA@LDA is smaller (by 3kJ/mol) than EXX/RPA@PBE.
- **EXX/RPA** has similar accuracy as **quantum chemistry methods**.

EXX/RPA (PBE) results for methane

| config. | EXX/RPA | PBE+D ^a | PBE+D ^b | MP2 ^c | exp. |
|------------------|---------|--------------------|--------------------|------------------|---|
| 1 a_0 | >6 | 5.84 | 5.80 | 6.00 | plastic phase |
| E_{coh} | - | 10.8 | 11.35 | 10.3 | |
| 2 a_0 | 5.80 | 5.50 | - | - | $a_0 \sim 5.84^d$ $E_{\text{coh}}=10.4-11.4^e$ |
| E_{coh} | 8.8 | 14.9 | - | - | |

a: This work from PBE ground state. Dispersion corrections were evaluated with Grimme2006 parametrization scheme. [S. Grimme, *J. Comp. Chem.* 27, 1787 (2006)].

b: T. Kerber, M. Sierka, and J. Sauer, *J. Comput. Chem.* 29, 2088 (2008).

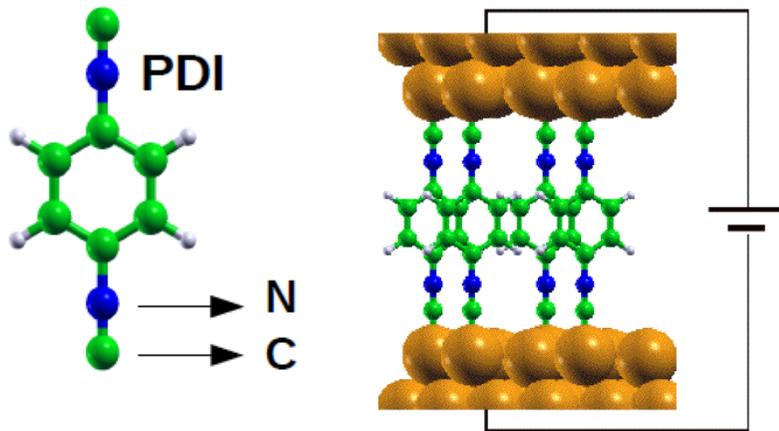
c: A. B. Kunz, *J. Phys. : Condens. Matter.* 6, L233 (1994)

d: $a_0 = 5.84 \text{ \AA}$ was measured at 29K.

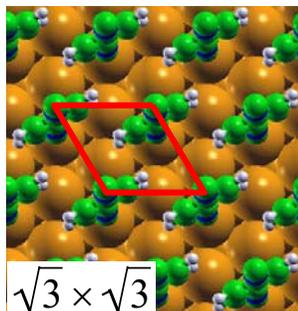
e: The measured heat of sublimation (in kJ/mol) is corrected for temperature effects ZPE is not included.



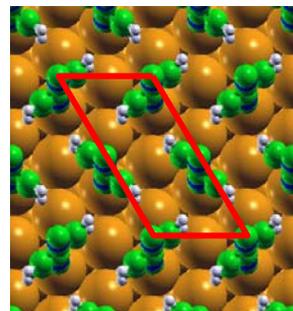
PDI SAM on Au(111) surface: Is Isocyanide group a good linker?



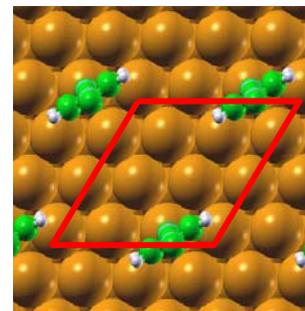
- The presence of the triple bond in the isocyanide group may effectively connect the **p π electrons in the aromatic moiety** and the **d π electrons on the gold surface**.
- However, **no ordered monolayers of PDI or BPDI has been observed** in either AFM or STM experiments.
- DFT (LDA or GGA) does not describe well vdW interactions among SAMs.



full coverage:
1 PDI / 3 Au



full coverage:
Herringbone

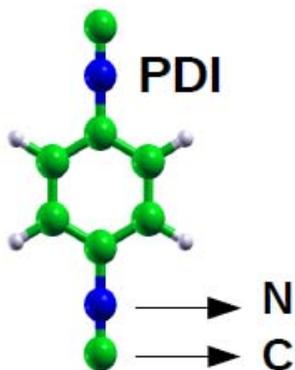
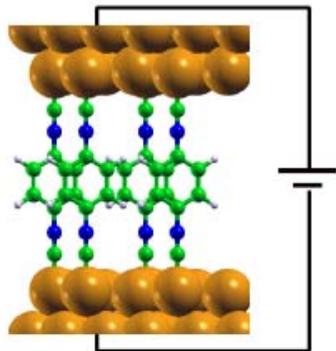


low coverage (1/3):
1 PDI / 9 Au

Which arrangement is more stable?



Self-assembled monolayers on metal surfaces



- Experimentally, no closely-packed SAM structures of PDI (or PDI derivatives) have been observed.
- DFT/PBE calculations predicted repulsive inter-molecular interactions for full-coverage PDI/Au(111).
Y. Li *et al.*, JPCCC 112, 6413 (2008).
- MP2 calculations predicted more attractive interactions between the PDI dimer than the benzene dimer.
R. Steele *et al.*, PCCP 12, 82-96 (2010).

Is there a significant contributions of dispersion interactions to the cohesive energy of PDI SAMs on Au(111) ?

RPA vdW energy in PDI SAM

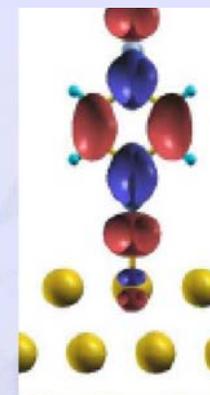
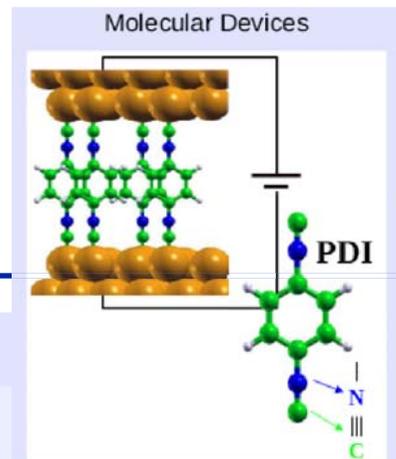
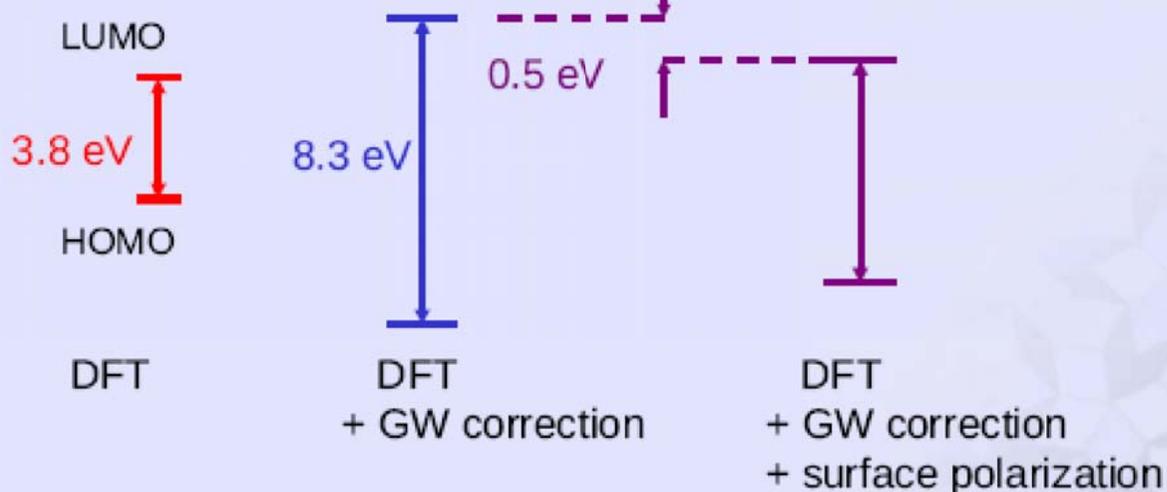
- **EXX/RPA greatly reduces the under-binding found in PBE for PDI SAMs at high packing density.**
- **Contrary to the case of methane, PBE+D agrees reasonably well with EXX/RPA.**
- **The adsorption energy of the gas phase and the high coverage herringbone structure are nearly identical.**
- **The lack of ordering in experiments may arise from the close competition between vdW interactions at high density and stronger surface binding at low density.**



Organics on metals are usually `difficult` cases for DFT/GGA

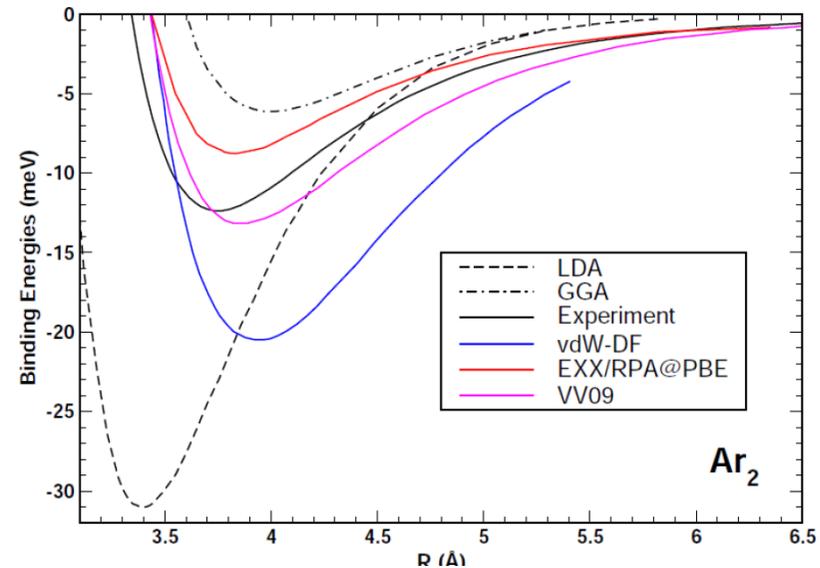
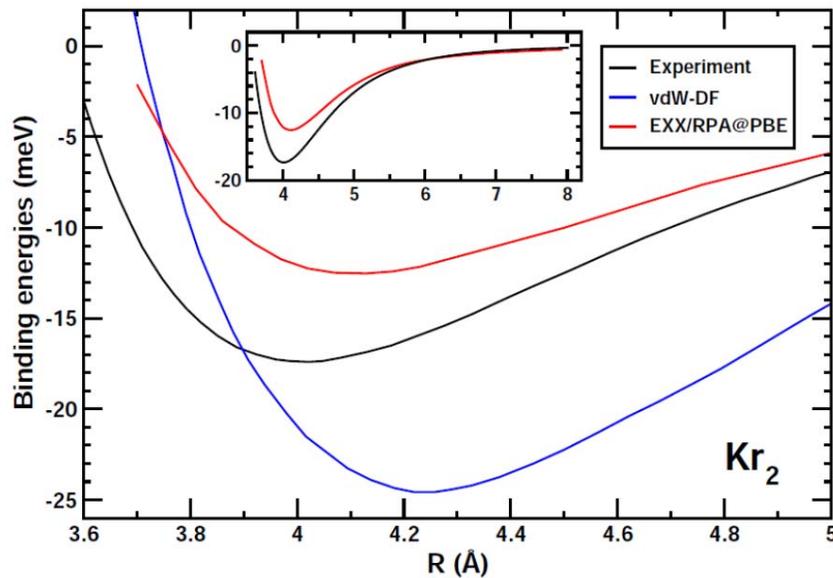
Define the GW-corrected adsorbate energy level as

$$\mathcal{E}_{GW}^{Ad}(j) = \mathcal{E}_{DFT}^{Ad}(j) + \Delta\Sigma_{GW}^{SAM} + \int_{z>z_L} [V_{im}(z) - V_{xc,DFT}(z)] \rho_{DFT}^{SAM}(j)$$

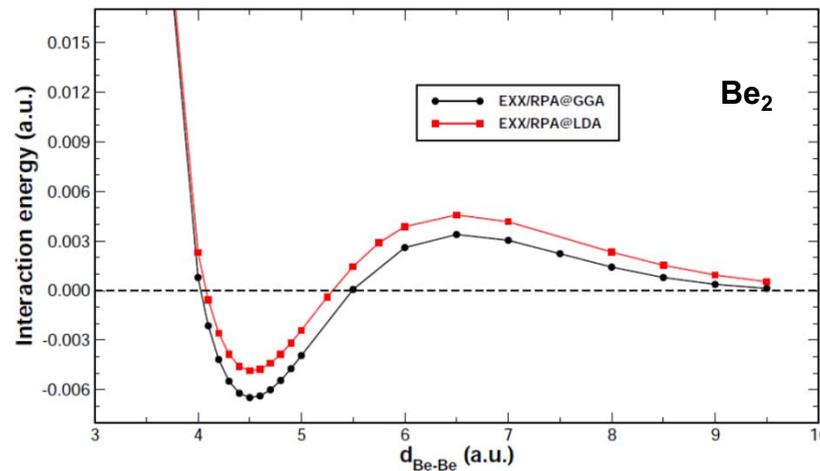


The energy of the lowest unoccupied molecular level is shifted upward by 1.5-2.0 eV in GW calculations, compared with DFT results
→ this qualitatively changes the picture of electron transport

EXX/RPA and van der waals energy functionals



Lack of self-consistency may be a serious issue in Exx/RPA calculations



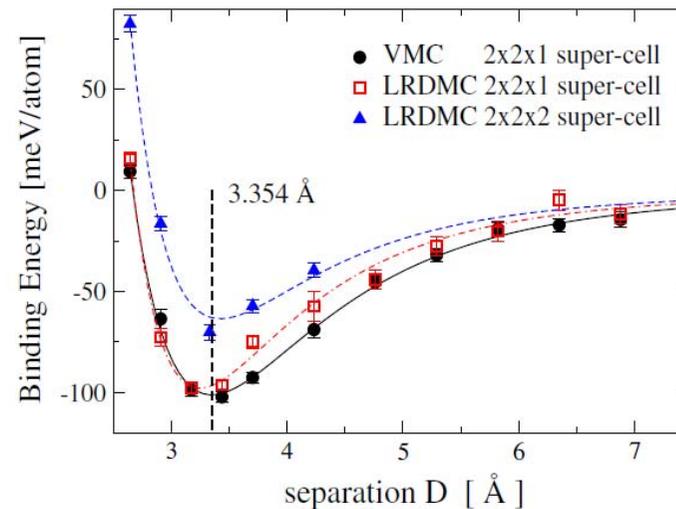
V.Nguyen and
GG JCP 2010



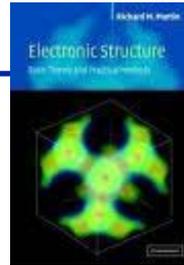
Graphite: Quantum Monte Carlo calculations

- Good agreement with most recent experiments for graphite binding energy (56 +/- 5 meV): phonon contributions taken into account at the DFT level.
- At large interlayer separations: $E \sim D^{-4.2} \rightarrow$ conventional van der Waals interaction from the semiconducting sp^2 bands dominate the unusual asymptotic form arising from in plane polarization of the single semi-metallic band.

Benchmark for other theoretical methods developed to describe dispersion forces



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[Dario Rocca](#) and [Deyu Lu](#)



Perturbative approach using density matrices

Quantum Liouville equation

$$i \frac{d\hat{\rho}(t)}{dt} = [\hat{H}(t), \hat{\rho}(t)] \longrightarrow \rho(\mathbf{r}, \mathbf{r}', t) = \sum_v \phi_v^*(\mathbf{r}, t) \phi_v(\mathbf{r}', t)$$

$$\hat{H}(t)\phi(\mathbf{r}, t) = \left[-\frac{1}{2}\nabla^2 + v_H(\mathbf{r}, t) + v_{ext}(\mathbf{r}, t) \right] \phi(\mathbf{r}, t) + \int \Sigma(\mathbf{r}, \mathbf{r}', t) \phi(\mathbf{r}', t) d\mathbf{r}'$$

Apply external electric field, use first order perturbation theory and linearize equation with respect to external potential

$$v'_{ext}(\mathbf{r}, \omega) = -\mathbf{E}(\omega) \cdot \mathbf{r}$$



Perturbative approach using density matrices

1st Order perturbation theory; linearized Quantum Liouville equation

$$\frac{d\hat{\rho}'(t)}{dt} = \mathcal{L} \cdot \hat{\rho}'(t) + [\hat{v}'_{ext}(t), \hat{\rho}^0],$$
$$\mathcal{L} \cdot \hat{\rho}'(t) = [\hat{H}^0, \hat{\rho}'(t)] + [\hat{v}'_H[\hat{\rho}'](t), \hat{\rho}^0] + [\hat{\Sigma}'[\hat{\rho}'](t), \hat{\rho}^0],$$



Fourier Transform

$$(\omega - \mathcal{L}) \cdot \hat{\rho}'(\omega) = [\hat{v}'_{ext}(\omega), \hat{\rho}^0].$$

Solve for $\hat{\rho}'(\omega)$, \rightarrow dipole moment : $d_i(\omega) = \sum_j \alpha_{ij}(\omega) E_j(\omega) = \text{Tr}(\hat{r}_i \hat{\rho}'(\omega))$

Solve equation using techniques based on Density Functional Perturbation Theory; obtain polarizability \rightarrow **absorption coefficient proportional to the Trace of the polarizability tensor.**



Many body perturbation theory to obtain optical spectra

Quantum Liouville equation

$$i \frac{d\hat{\rho}(t)}{dt} = [\hat{H}(t), \hat{\rho}(t)] \longrightarrow \rho(\mathbf{r}, \mathbf{r}', t) = \sum_v \phi_v^*(\mathbf{r}, t) \phi_v(\mathbf{r}', t)$$

$$\hat{H}(t)\phi(\mathbf{r}, t) = \left[-\frac{1}{2}\nabla^2 + v_H(\mathbf{r}, t) + v_{ext}(\mathbf{r}, t) \right] \phi(\mathbf{r}, t) + \int \Sigma(\mathbf{r}, \mathbf{r}', t) \phi(\mathbf{r}', t) d\mathbf{r}'$$

$$\Sigma(\mathbf{r}, \mathbf{r}') = v_{xc}(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') \quad \text{TDDFT in the adiabatic approximation}$$

$$\Sigma_{HF}(\mathbf{r}, \mathbf{r}', t) = - \sum_v \phi_v(\mathbf{r}, t) \phi_v^*(\mathbf{r}', t) v(\mathbf{r}, \mathbf{r}') \quad \text{TD-HF}$$

$$\Sigma_{COH}(\mathbf{r}, \mathbf{r}') = \frac{1}{2} \delta(\mathbf{r} - \mathbf{r}') W_p(\mathbf{r}', \mathbf{r}) \quad \text{Statically screened Bethe-Salpeter Equation (BSE)}$$

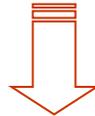
$$\Sigma_{SEX}(\mathbf{r}, \mathbf{r}', t) = - \sum_v \phi_v(\mathbf{r}, t) \phi_v^*(\mathbf{r}', t) W(\mathbf{r}', \mathbf{r})$$

↑ Screened Coulomb interaction



Optical properties of solids and nanostructures

- We have solved the BSE by replacing the KS exchange-correlation potential by **screened exact exchange**, in **the quantum Liouville equations** (1st order perturbation theory)
- We have used the techniques proposed by B.Walker et al. (PRL 06) and D.Rocca et al. (JCP 08) to solve the time dependent equations.
- We have computed the **static dielectric screening iteratively**, by using eigenvalue decompositions (H.Wilson et al., PRB 08 and PRB 09).

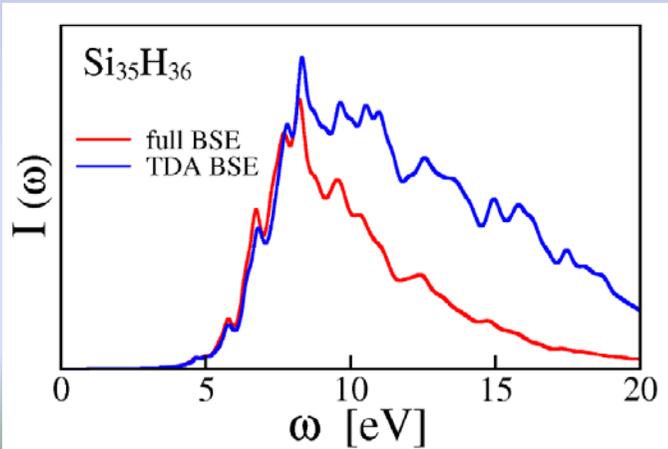


- **Full spectrum at a computational cost comparable to several Hartree-Fock ground state calculations**
- **Calculations of empty electronic states, full dielectric matrix, inverse of dielectric matrix (and basis set approximations) avoided**
- **Method suitable for large basis sets and large systems**

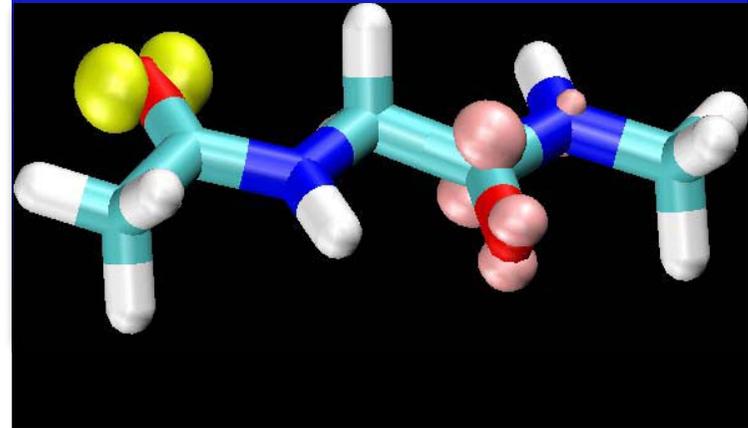


Optical properties of solids and nanostructures, including excitonic effects

Systems of size comparable to experiments, e.g. Si dots in the few nm range



Charge transfer excitations:
e.g. in dipeptides



Iterative solutions of the Bethe-Salpeter Equation:

- Full spectrum at a computational cost comparable to several Hartree-Fock ground state calculations
- Calculations of empty electronic states, full dielectric matrix, inverse of dielectric matrix (and basis set approximations) avoided
- Method suitable for large basis sets and large systems

D.Rocca, D.Lu and G.G.,
submitted (2009)

Open Source Codes

Qbox: <http://eslab.ucdavis.edu/software/qbox>

ESlab : <http://eslab.ucdavis.edu>

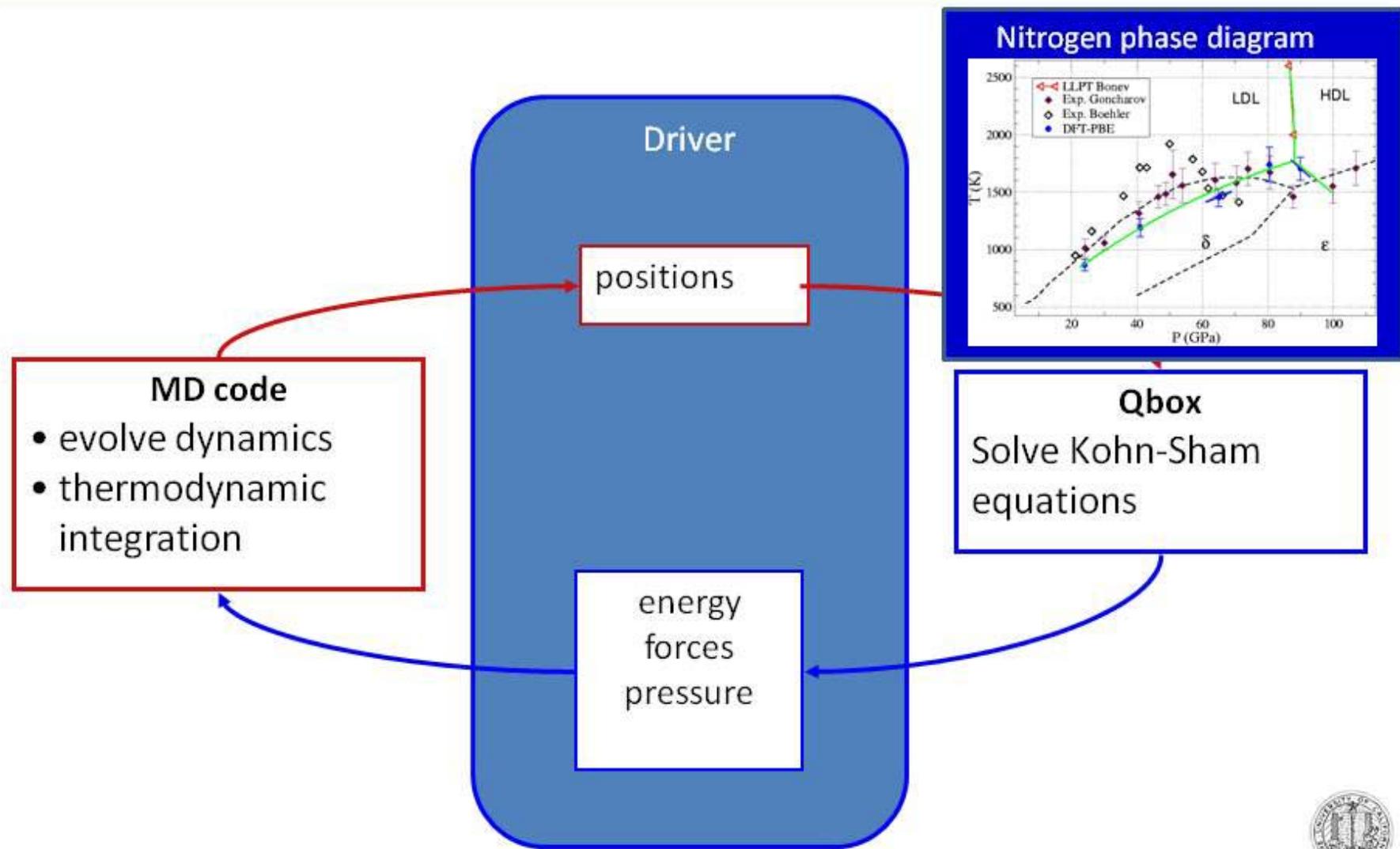
The database of pseudopotentials: <http://fpmd.ucdavis.edu>

The XML standards site for electronic structure:
<http://www.quantum-simulation.org>

Quantum Espresso: <http://quantum.espresso.org>



Use of Qbox in client-server mode



<http://eslab.ucdavis.edu/software/qbox/>



Exciting opportunities in theoretical and computational materials science

- Development of **advanced materials** is a **key** element for creating new technologies for, e.g. **efficient and sustainable energy supplies**
- It is an exciting time to make a difference!

-Agreement with experiment is never a sufficient condition to judge simulation results; in certain instances it may be misleading

-Distinction between theory and numerical techniques is essential and increasingly overlooked



- **Verification and validation**
- **Theoretical and algorithmic advances**
- **Connection with experiment**





<http://angstrom.ucdavis.edu/>

*Ab-initio Nanoscience Group
for Simulations and Theoretical Research On Materials*



Deyu Lu



Yan Li



Dario Rocca



Hugh Wilson



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