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Hodge Theory and Representation Theory

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Hodge Theory and Representation Theory

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Colloquium talk given at ICTP during the Summer School and Conference on Hodge theory, June 30 - July 2, 2010. Based in part on joint work with Mark Green and Matt Kerr.

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A. Cycle spaces of Mumford-Tate domains

B. Enlargements of cycle spaces and work of Eastwood-Gindikin-Wong [EGW]

C. Automorphic version and work of Carayol [C]

I. Introduction

Hodge theory has its roots in

- algebraic geometry

(beginning early 19^{th} century - Abel, Riemann, Jacobi, ... - study of algebraic curves and abelian varieties by Hodge theoretic (complex analytic) methods)

- special functions (representation theory)

(elliptic functions, automorphic

functions, theta functions (abelian functions), theta nullwerte, ...)

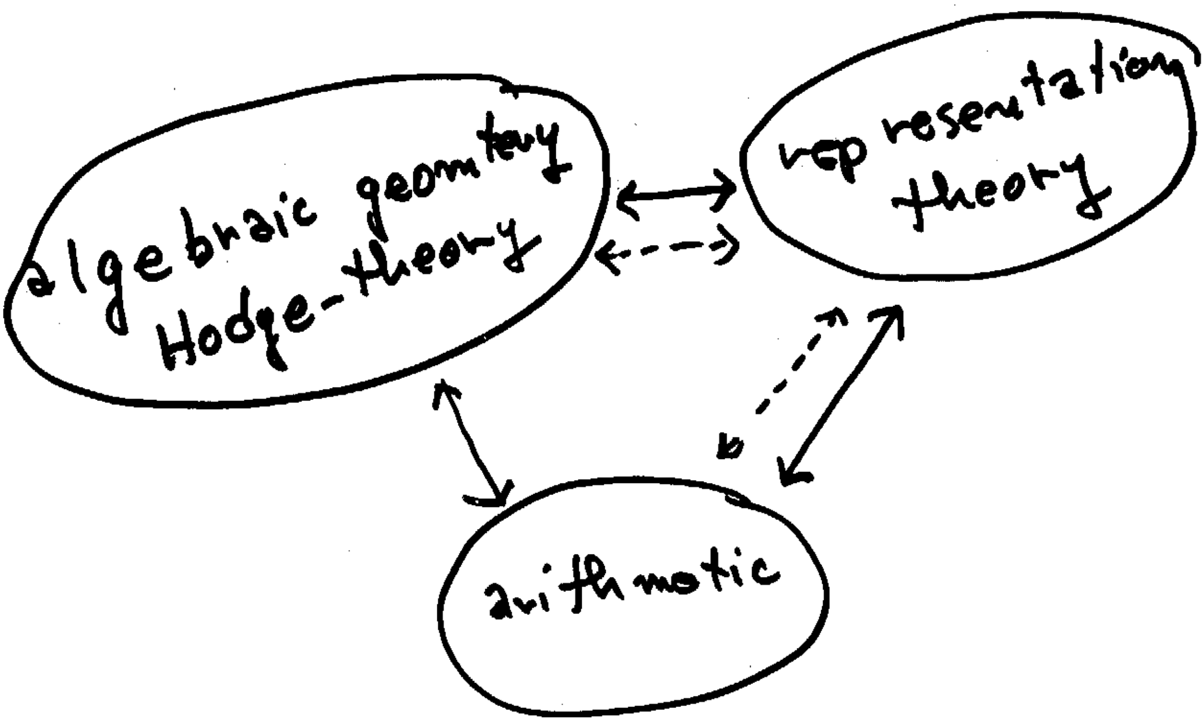
- <u>arithmetic</u>

(middle 19th century - arithmetic properties of automorphic forms, later L-functions and Galois representations)

In the 20th century and continuing through the present I would note two points

(i) In the classical case of weight one polarized Hodge structures (abelian varieties), the above interactions continue to be especially active, especially the nexus between representation theory and arithmetic in this special case - theory of Shimura varieties and part of the Langlands' program - algebraic geometry (ℓ-adic cohomology) plays a critical role

(ii) In the non-classical higher weight case, Hodge theory continues to play a central role in complex algebraic geometry. But the relation among the three areas is in its earliest stages of development



Recently there have been some hints and glimpses of where progress might be made. This is due to two factors

- the symmetry groups of Hodge theory turn out to be exactly the class of semi-simple $\mathbb{Q}$-algebraic groups M whose associated non-compact real Lie groups $M_{\mathbb{R}}$ have discrete series representations in $L^2(M_{\mathbb{R}})$. These are the factors over the place $v=\infty$

where one hopes to find cuspidal automorphic representations in

$$L^2(M(\mathbb{Q})\backslash M(\mathbb{A}))$$

(top dotted arrow)

- In some very special non-classical cases the connection between representation theory and Hodge theory / algebraic geometry has been made using Penrose-Radon transforms [EGW], and relatedly the arithmetic aspects of representation theory and Hodge theory have been investigated [C].

This has been accomplished by the use of *cycle spaces*, and their *enlargements*, associated to domains D parametrizing polarized Hodge structures whose general member has a given symmetry group. The main point is to use Penrose-Radon transform methods to map discrete series representations of the symmetry group, realized as $H^d_{(2)}(D, \mathcal{E}_{\rho\lambda})$ where, in the non-classical case, $d \neq 0$ to the holomorphic object $H^0(\mathcal{W}, \mathcal{E}_{\rho\lambda})$

where $\mathcal{W}$ is an enlargement of the cycle space associated to D.

Similarly, $H^d_{(2)}(\Gamma\backslash D, \mathcal{L}_{\rho_\lambda})$ and $H^0(\Gamma\backslash\mathcal{W}, \mathcal{E}_{\rho_\lambda})$ may be related.

In this way automorphic cohomology classes may be "evaluated" at points having Hodge theoretic properties of a special sort - e.g., CM points in $\Gamma\backslash\mathcal{W}$.

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In this talk I will try to give an overview of these developments.

II. Background concepts

A. Hodge structures

On a complex manifold X the smooth, complex-valued differential forms of degree n decompose in the so-called (p,q) types

$$\begin{cases} A^n(X) = \bigoplus_{p+q=n} A^{p,q}(X) \\ A^{p,q}(X) = \overline{A^{q,p}(X)} \end{cases}$$

where in local coordinates $z^1, \dots, z^d$ a form $\varphi \in A^{p,q}(X)$ is

$$\varphi = \sum_{\substack{|I|=p \\ |J|=q}} \varphi_{I\bar{J}}\, dz^I \wedge d\bar{z}^J \qquad \text{with}$$

$I = (i_1, \dots, i_p)$, $dz^I = dz^{i_1} \wedge \dots \wedge dz^{i_p}$, etc

If X is a compact Kähler manifold - in particular if X is a smooth projective variety - for $V = H^n(X, \mathbb{Q})$ Hodge proved that this decomposition induces

$$\begin{cases} V_{\mathbb{C}} = \bigoplus_{p+q=n} V^{p,q} \\ V^{q,p} = \overline{V}^{p,q} \end{cases},$$

and that there is

$$\begin{cases} Q : V \otimes V \to \mathbb{Q} \\ Q(v, w) = (-1)^n Q(w, v) \end{cases}$$

such that

$$(*)\quad \begin{cases} Q(V^{p,q}, V^{p',q'})=0 & p'\neq n-p \\ i^{p-q}Q(V^{p,q}, \bar{V}^{p,q})>0 \end{cases}$$

For $S^1 = \{ z : |z|=1 \}$ we define

$$(**)\quad \varphi : S^1 \to GL(V_{\mathbb{R}})$$

by

$$\varphi(z)v = z^{p-q}v, \quad v \in V^{p,q}$$

Defn: (i) A Hodge structure of weight n (V, φ) is given by $(**)$ where the weights of φ lie in $[-n, n]$. (ii) A polarized Hodge structure of weight n is given by (V, Q, φ) satisfying $(*)$ and $(**)$.

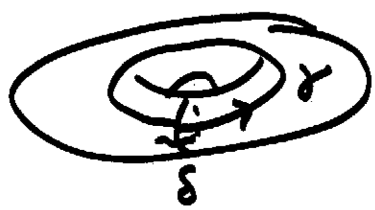
__Ex:__ For $n=1$, $\dim V = 2$ we have

$$V \cong \mathbb{Q}^2, \quad Q = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

$$V^{1,0} = \mathbb{C}\begin{pmatrix} \tau \\ 1 \end{pmatrix}, \quad \operatorname{Im} \tau > 0$$

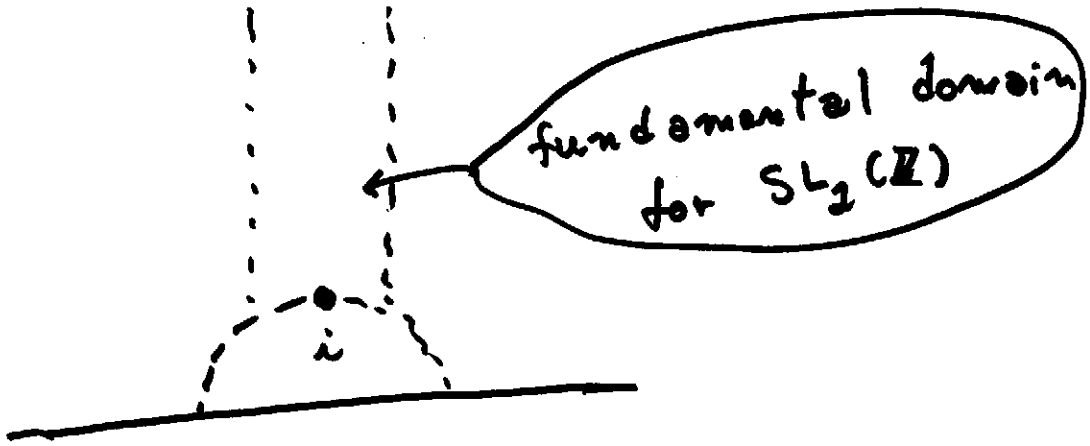
Think of $X = \mathbb{C}/\mathbb{Z} + \mathbb{Z}\tau$ and

$$\begin{cases} \tau = \int_\delta \omega \Big/ \int_\gamma \omega \\ X = \end{cases}$$



__Defn:__ The __period domain__ D is the set of polarized Hodge structures (V, Q, φ) with given $h^{p,q} = \dim V^{p,q}$.

<u>Ex (cont)</u>: $D = \mathcal{H}$ is the upper-half-plane



$$\begin{cases} \mathcal{H} = SL_2(\mathbb{R})/H_\varphi \\ H_i = SO(2) \end{cases}$$

Classically one has

$$\begin{cases} \Theta(z,\tau) & \text{theta functions} \\ f(\tau) & \text{modular forms} \end{cases}$$

with rich analytic and arithmetic properties. Very roughly speaking this story generalizes when $n=1$. For $n \geq 2$ it is a ???

In general, for $G = \mathrm{Aut}(V, Q)$

$$D = G(\mathbb{R}) / H_\varphi$$

Given (V, Q, φ) the <u>Hodge filtration</u> is

$$F^p = \bigoplus_{p' \geq p} V^{p',q'}$$

The first bilinear relation is

(4) $$Q(F^p, F^{n-p+1}) = 0$$

The set of all filtrations

$$F^n \subset F^{n-1} \subset \cdots \subset F^0 = V_{\mathbb{C}}$$

satisfying (4) is the <u>compact dual</u> $\check{D}$ of D. Then

$$D \subset \check{D}$$

is an open $G(\mathbb{R})$ orbit.

<u>Ex (cont).</u> $\mathcal{H} \subset \mathbb{P}^1$

<u>Ex</u>: For $n=2$, $h^{2,0}=2$ and $h^{1,1}=1$
we have $V_{\mathbb{C}} \cong \mathbb{C}^5$ and a quadric

$$\underline{Q} \subset \mathbb{P}^4$$

given by Q. Then $G(\mathbb{R}) = SO(4,1)$ and

- $\check{D}$ = lines in $\underline{Q}$
- D = lines on which $Q(v,\bar{v})>0$

There are no theta functions and modular forms in this case.
Rather there is the mysterious object

$$H^1_{(2)}(\Gamma\backslash D, \mathcal{L}_{\rho_\lambda})$$

Ex (mirror quintic): $n = 3$; $h^{3,0} = h^{2,1} = 1$

Then (V, Q) is a symplectic vector space and we have

$$\begin{array}{ccl} \mathbb{P}^1 \to & \check{D} & \\ & \downarrow & \\ & Gr_L(V_{\mathbb{C}}) & = \left\{ \begin{array}{c} \text{Lagrangian} \\ \text{2-planes} \\ \text{in } V_{\mathbb{C}} \end{array} \right\} \end{array}$$

with fibres the $\mathbb{P}^1$ of lines through the origin in a Lagrangian 2-plane. $D \subset \check{D}$ is defined by inequalities as before.

B. Mumford-Tate groups and Mumford-Tate domains

Mumford-Tate groups M are the basic symmetry groups of Hodge theory. Mumford-Tate domains parametrize polarized Hodge structures whose generic Mumford-Tate group is a given M.

Definition: Given (V, Q, φ) the Mumford-Tate group M_φ is the smallest $\mathbb{Q}$-algebraic subgroup of G such that $\varphi(S^1) \subset M(\mathbb{R})$

Ex (cont): For $M_\tau \longleftrightarrow \tau \in \mathcal{H}$

- $M_\tau = \left\{ \begin{array}{l} \text{elements of norm 1 in} \\ \mathbb{Q}(\tau)^* \text{ if } \tau \text{ is quadratic} \\ \text{imaginary} \end{array} \right\}$

- $M_\tau = SL_2$ otherwise

Ex: G_2 is a Mumford-Tate group for a polarized Hodge structure when $n=2$; $h^{2,0}=2$, $h^{1,1}=3$ $\quad (G_2 \subset SO(4,3))$

Ex: $SU(2,1)$ is a Mumford-Tate group for a polarized Hodge structure when $n=4$; $h^{4,0}=1$, $h^{3,1}=2$, $h^{2,2}=2$

<u>Defn</u>: A <u>Mumford-Tate domain</u>

$$D_{M_\varphi} \subset D$$

is the $M(\mathbb{R})$-orbit of $\varphi \in D$ with Mumford-Tate group M_φ.

<u>Ex (SU(2,1)-cont)</u>: SU(2,1) acts on $\mathbb{C}^3$ preserving

$$z_0\bar{w}_0 - (z_1\bar{w}_1 + z_2\bar{w}_2)$$

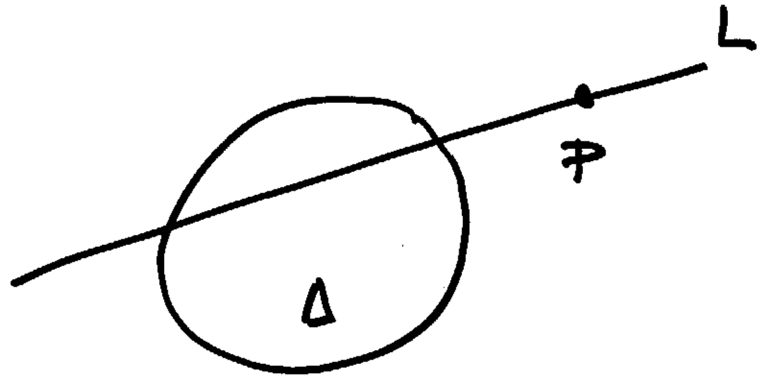
The compact dual turns out to be biholomorphic to the flag manifold (here $D = D_{SU(2,1)_\varphi}$ etc)

$$\check{D} \subset \mathbb{P}^2 \times \check{\mathbb{P}}^2$$

given by

$$\check{D} = \{(P, L) : P \in L\}$$

and $D \subset \check{D}$ is given by the picture



where $\Delta \subset \mathbb{C}^2 \subset \mathbb{P}^2$ is the unit ball.

This and the period domain when $n=2$; $h^{2,0}=2$, $h^{1,1}=1$ above are the two lowest dimensional non-classical cases (no $H^0(\Gamma\backslash D, L_{\rho_\lambda})$, only $H^1(\Gamma\backslash D, L_{\rho_\lambda})$)

III. Representation theory and the cohomology of Mumford-Tate domains

A. Discrete series representations (work of Schmid)

Let $M_{\mathbb{R}}$ be a non-compact, real semi-simple Lie group

Classical work of many people, for our purposes here especially Harish-Chandra, was concerned with decomposing $L^2(M_{\mathbb{R}})$

into irreducible unitary representations. Of particular interest are those that occur <u>discretely</u> in $L^2(M_{\mathbb{R}})$ - the so-called <u>discrete series representations.</u> A recent result that is a consequence of the classification of the Mumford-Tate groups is:

<u>The groups $M_{\mathbb{R}}$ that admit discrete series representations</u>

<u>are exactly the real Lie groups associated to semi-simple Mumford-Tate groups</u>

In somewhat more detail, given $M_{\mathbb{R}}$ with a compact maximal torus $T \subset M_{\mathbb{R}}$ having Lie algebra $\underline{t}$, to a weight $\lambda \in i\underline{\check{t}}$ satisfying certain conditions Harish-Chandra associates an irreducible discrete series representation whose character is a distribution Θ_λ on T given by an L^1-function

To the pair $(M_{\mathbb{R}}, \lambda)$ Schmid associates a homogeneous complex manifold $D_\lambda = M_{\mathbb{R}}/H$ together with a homogeneous line bundle $\mathcal{L}_{\rho_\lambda} \to D_\lambda$ such that, for $d = \dim_{\mathbb{C}} K/H$, the L^2-cohomology

$H^q_{(2)}(D_\lambda, \mathcal{L}_{\rho_\lambda}) = 0$ for $q \neq d$

and

$H^d_{(2)}(D_\lambda, \mathcal{J}_{\rho_\lambda})$ is the discrete series representation with character Θ_λ

On the other hand, it turns out that D_λ is a Mumford-Tate domain D_{M_φ}. In fact there are many different (V, Q, φ)'s with biholomorphic Mumford-Tate domains together with the infinitesimal period relation. What is important is the circle or co-character

$$\varphi : S^1 \to T \subset M(\mathbb{R})$$

This gives a complex structure together with the infinitesimal period relation on

$$D_\varphi := M(\mathbb{R})/H_\varphi$$

where $H_\varphi = Z_{M(\mathbb{R})}(\varphi(S^1))$. We shall say that λ and φ are <u>compatible</u> if the complex structure on D_λ agrees with that on D_φ. There are many φ's that are compatible with a given λ. For all such φ, $\mathcal{L}_{\rho_\lambda}$ is a Hodge bundle but which one it is depends on φ. We may summarize by saying that:

- Discrete series representation are realized as L^2-cohomology $H^d_{(2)}(D_\varphi, \mathcal{L}_{\rho_\lambda})$
- The Mumford–Tate domain has two additional structures

beyond the complex structure

(i) the choice of φ that is compatible with λ

(ii) the $\mathbb{Q}$-structure given by the $\mathbb{Q}$-algebraic group M with $M(\mathbb{R}) = M_{\mathbb{R}}$

One implication of this finer structure is that there are special arithmetically defined points in D_φ - e.g., those for which the Hodge structure is of CM type

A final remark is that the classical case is when

$$d = 0 \iff H_\varphi = K$$

These are the so-called <u>holomorphic discrete series</u>. They are very important but also very special

III. B Automorphic representations

For arithmetic purposes one is interested in discrete series representations occuring in $L^2(\Gamma \backslash M(\mathbb{R}))$ where $\Gamma \subset M$ is an arithmetic subgroup.

More precisely, one is interested in so-called <u>cuspidal automorphic representations</u> in

$$L^2(M(\mathbb{Q}) \backslash M(\mathbb{A}))$$

whose part over the place $v=0$

is a discrete series representation of $M(\mathbb{R})$. Presumably this is related to automorphic cohomology

$$H^d_{(2)}(\Gamma\backslash D_q, \mathcal{I}_{\rho\lambda}) \tag{$*$}$$

What is known is

(i) in the classical case when $d=0$, $(*)$ and its adelic counterpart is a much studied and very rich subject

(ii) when Γ is co-compact (so no L^2-condition) and $|\lambda| \gg 0$,

$H^q(\Gamma\backslash D_q, \mathcal{I}_{S\lambda}) = 0$ for $q \neq d$ and

$$\dim H^d(\Gamma\backslash D_\varphi, \mathcal{L}_{\rho_\lambda}) = C|\lambda|^N + \cdots$$

where $N = \dim D_\varphi$.

(iii) in the case $M = SU(2,1)$ there is a detailed and deep study of $H^1(\Gamma\backslash D_\varphi, \mathcal{L}_{\rho_\lambda})$ by Carayol. Here we recall that $D_\varphi \subset \mathbb{P}^2 \times \check{\mathbb{P}}^2$ and

$$\mathcal{L}_{\rho_\lambda} = \mathcal{O}_{\mathbb{P}^2}(a) \boxtimes \mathcal{O}_{\check{\mathbb{P}}^2}(b)$$

for $(a,b) \neq (0,-3)$.

<(o)>

The intermediate step is to convert H^d into an H^0, to which we turn

IV. Enlargements of cycle spaces of Mumford-Tate domains and realization of cohomology by holomorphic data

A. Cycle spaces of Mumford-Tate domains

The basic observations are

(i) D_φ contains compact, complex submanifolds Y of dimension d

(ii) $\mathcal{L}_{\rho_\lambda}\big|_Y$ is negative, so that

$H^q(\mathcal{L}_{\rho_\lambda} \otimes \mathcal{O}_Y) = 0$ for $q \neq d$ and

$H^d(\mathcal{L}_{\rho_\lambda} \otimes \mathcal{O}_Y)$ is "big" for $|\lambda| \gg$

(iii) the $M(\mathbb{C})$- translates of Y that remain in D_φ form an open set $\mathcal{U}$ in the Hilbert scheme of Y in $\check{D}_\varphi$ - in fact

$$\mathcal{U} \subset M(\mathbb{C})/K(\mathbb{C})$$

(iv) this suggests mapping

(4) $$H^d(D_\varphi, \mathcal{L}_{\rho_\lambda}) \to H^0(\mathcal{U}, \mathcal{E}_{\rho_\lambda})$$

where $\mathcal{E}_{\rho_\lambda} \to \mathcal{U}$ is a bundle with fibres $H^d(\mathcal{L}_{\rho_\lambda} \otimes \mathcal{O}_Y)^*$

In fact one has

<o>

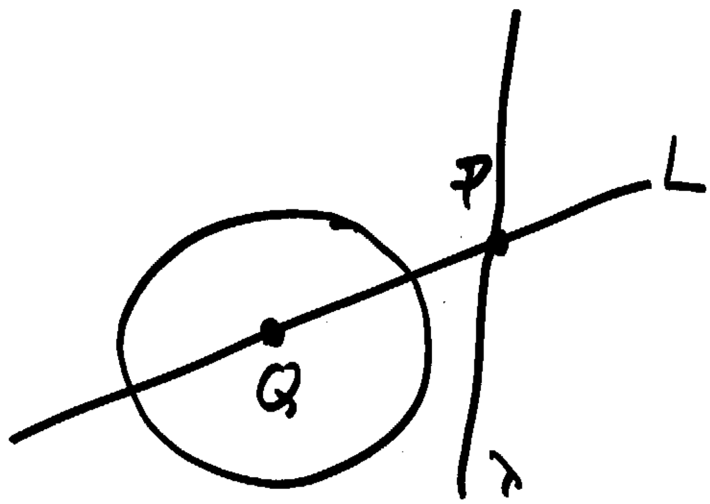
The map (4) is a Penrose-Radon transform

<u>Theorem</u> (Fels - Huckleberry - Wolf):

(a) $\mathcal{U}$ is a Stein manifold

(b) the map (4) is injective for $|\lambda| \gg 0$

<u>Ex</u> ($SU(2,1)$ cont'd): In the picture

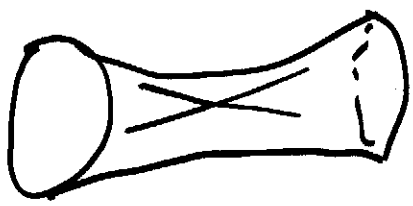


for $Q \in \Delta$, $\lambda \in \check{\Delta}$ we obtain a $\mathbb{P}^1 \subset D_\varphi$ given by the locus of (P, L) in the above picture - thus $\mathcal{U} \cong \Delta \times \check{\Delta}$

Ex ($n=2$; $h^{2,0}=2$, $h^{1,1}=1$ cont) In this case $\mathcal{U}$ has the description

$$\mathcal{U} = \{ E \in \mathbb{P}\check{V}_{\mathbb{C}} : Q(E, \bar{E}) > 0 \}$$

Then $\underline{Q} \cap \mathbb{P}E = \underline{Q}_E$ is a quadric in $\mathbb{P}^3$ and the compact subvariety in D is $\mathbb{P}^1 \sqcup \mathbb{P}^1$ given by the two families of rulings on $\underline{Q}_E$



There is a similiar description for the mirror quintic case

B. Enlargements of cycle spaces and the work of [EGW]

For many purposes a better object in an enlargement $\mathcal{W}$ of the cycle space. This means we have

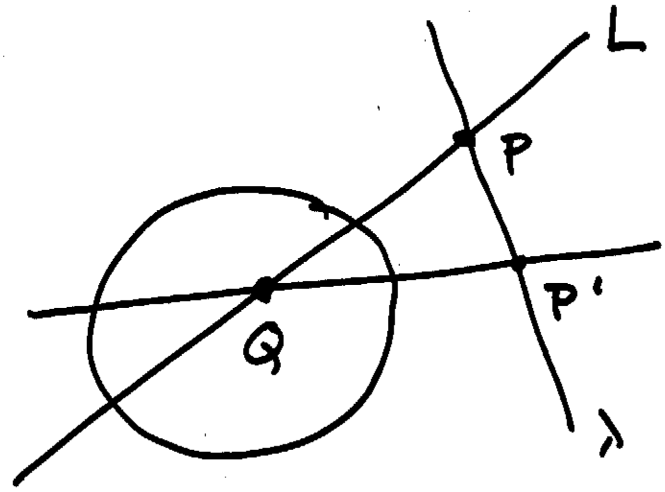
(i)
$$\begin{tikzcd}[column sep=small] & \mathcal{W} \subset D \times D \arrow[dl] \arrow[dr] & \\ D & & D \end{tikzcd}$$

where $\mathcal{W}$ is Stein and the fibres of the projections are contractible and Stein

(ii)
$$\begin{tikzcd} \mathcal{W} \arrow[d] \\ \mathcal{U} \end{tikzcd}$$

where the fibres are Stein

<u>Ex (SU(2,1) cont)</u>: Then $\mathcal{W}$ is given by the picture



$$\mathcal{W} = \left\{ (P, L; P', L') : L \cap L' \in \Delta,\ \overline{PP'} \cap \bar{\Delta} = \phi \right\}$$

The map $\mathcal{W} \to \mathcal{U}$ is

$$(P, L; P', L') \longrightarrow (Q, \lambda)$$

<u>Ex</u> <u>($n=2$, $h^{2,0}=2$, $h^{1,1}=1$ cont)</u> Then $\mathcal{W}$ is

$$\{ (F, F') \in D \times D : F \cap F' = (0) \}$$

The map $\mathcal{W} \to \mathcal{U}$ is $(F, F') \to F + F'$.

The mirror quintic case is similiar.

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The above enlargement in the SU(2,1) case was introduced in [EGW] for the purpose of realizing the non-classical discrete series representations by holomorphic data. For this they used the following general result (motivated by earlier work on Penrose-Radon transforms): Given

$$\begin{cases} \pi : \mathbb{X} \to \mathbb{Y} \\ \mathcal{E} \to \mathbb{Y} \end{cases}$$

where $\mathbb{X}$ and $\mathbb{Y}$ are complex manifolds, π is a holomorphic submersion with Stein fibres and $\mathcal{E} \to \mathbb{Y}$ is a holomorphic vector bundle, then

$$\boxed{H^n(\mathbb{Y}, \mathcal{E}) \cong H^n_{DR}(\Gamma(\Omega^{\bullet}_{\mathbb{X}/\mathbb{Y}}(\mathcal{E}_\pi), d_\pi))}$$

The RHS is the relative de Rham cohomology - it is a global, holomorphic object. In the situation where there are Lie groups acting as above, the RHS can be expressed as Lie algebra cohomology and then using the Cartan-Killing form the notion of holomorphic harmonic forms ($= \ker \delta_\pi \cap \ker d_\pi^*$) can be used to represent the RHS by holomorphic sections of appropriate bundles. For the $SU(2,1)$ example and

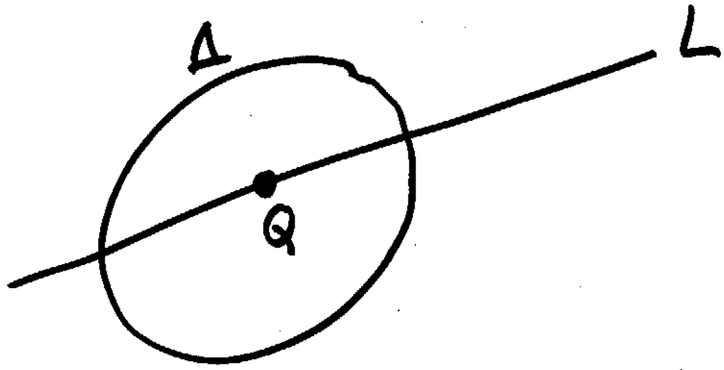
$$\mathcal{L}_{\rho_\lambda} = \mathcal{O}_{\mathbb{P}^2}(r) \boxtimes \mathcal{O}_{\check{\mathbb{P}}^2}(t), \qquad r+t \leqq -2$$

they are able to explicitly "see" the Harish-Chandra module $H^1(D_\varphi, \mathcal{L}_{\rho_\lambda})$ in terms of holomorphic objects.

C. Automorphic version and the work of Carayol

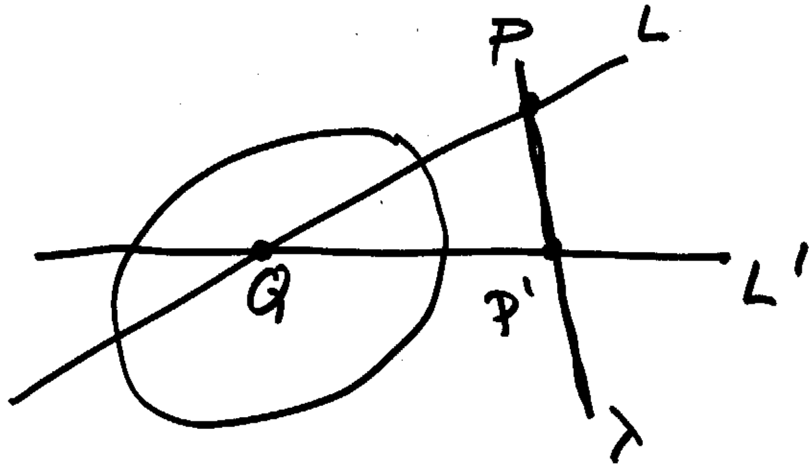
For the purposes of algebraic geometry, given by period mappings, and of arithmetic one wants to "factor" the preceeding discussion by an arithmetic subgroup $\Gamma \subset M$. I will very briefly describe the work [C] which, for the first time to my knowledge, allows one to explicitly "see" automorphic cohomology. Carayol's idea is the following:

(i) a "classical" open orbit D' of $SU(2,1)$ on $\check{D} \subset \mathbb{P}^2 \times \check{\mathbb{P}}^2$ is given by the picture



Here, classical means that D' fibres over ~~the~~ the Hermitian symmetric domain Δ with $\mathbb{P}^1$ as fibres.

(ii) recalling that $\mathcal{W}$ is



we have

$$(44)\qquad \begin{tikzcd} & \mathcal{W} \arrow[dl, "\pi"'] \arrow[dr, "\pi'"] & \\ D & & D' \end{tikzcd}$$

where $\pi'(P,L;\ P',L') = (Q,L') \in D'$

(iii) we may factor (44) by Γ to have

$$\begin{tikzcd} & \Gamma\backslash \mathcal{W} \arrow[dl] \arrow[dr] & \\ \Gamma\backslash D & & \Gamma\backslash D' \end{tikzcd}$$

and Carayol shows that [EGW] applies to this situation

(iv) classically, one has

$$H^0_{(2)}(\Gamma\backslash D', \mathcal{L}'_{\rho'_\lambda})$$

(Picard modular forms), and then [EGW] together with quite explicit calculations lead to

$$\boxed{H^0_{(2)}(\Gamma\backslash D', \mathcal{L}(n,t)) \xrightarrow{\sim} H^1_{(2)}(\Gamma\backslash D, \mathcal{L}(n+t+1,-t-2))}$$

where $\mathcal{L}(n,t) = \mathcal{O}_{\mathbb{P}^2}(n) \boxtimes \mathcal{O}_{\check{\mathbb{P}}^2}(t)$ and $(n,t) \neq (0,-$

<u>Conclusion</u>: The groups that arise in Hodge theory and in representation theory (discrete series, cuspidal automorphic representations) are the same. The relations between the two quite different aspects of the same class of groups has been extensively explored in (very special) classical case, but is only in its earliest stage in the non-classical case.

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