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GOVERNING EQUATIONS IN TWO-PHASE FLUID NATURAL CIRCULATION FLOWS

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Abstract

This lecture presents the various models used to describe mass, momentum and energy transport processes in two-phase fluid flows related to natural circulation. By the conclusion of this lecture, the participant should have an understanding of the different types of thermal hydraulic transport models that have been implemented in nuclear reactor safety computer codes.

1. INTRODUCTION

The advent of high-speed computers has permitted analysts to develop computer models that can be applied to a wide range of complex thermal hydraulic studies. Certainly one of the more complex problems currently being studied involves the mass, momentum and energy transport in systems containing multiple phases (gas, liquid, and solids), multiple components (different chemical species) and multiple fields (continuous and dispersed geometries). This lecture provides an overview of the transport equations being applied in the study of two-phase transport behavior, particularly as related to nuclear reactor safety thermal hydraulics. Section 2 summarizes the one-dimensional differential conservation equations for two-phase flow and examines the models used in reactor safety analyses. However, before delving into the details of the various two-phase flow conservation equations, it would be useful to present a brief history of the development of nuclear reactor safety computer codes in the United States.

1.1. Brief history of U.S. nuclear reactor safety computer codes

The FLASH computer code, developed by Westinghouse-Bettis, is the earliest formally documented computer code used in nuclear reactor safety analysis in the United States [1]. As shown in Figure 1, this code used a very simple "node and branch" approach to modeling suitable for some studies of single-phase flow in PWRs. Each node consists of a control volume in which the time dependent mass and energy conservation equations are solved for a single-phase fluid. Each interconnecting line is modeled using the momentum conservation equation to take into account line resistance. Under steady-state conditions, the sum of the pressure drops around the loop would equal the sum of the buoyancy terms. The FLASH code served as the early basis for the RELAP series of computer codes.

From 1955 to 1975, numerous studies of boiling heat transfer and two-phase flow were conducted leading to significant breakthroughs in the modeling of two-phase flow. One of the pioneering efforts in the mid-1960s was Zuber's development of the drift flux model. The success of his approach led to major advancements in code development.

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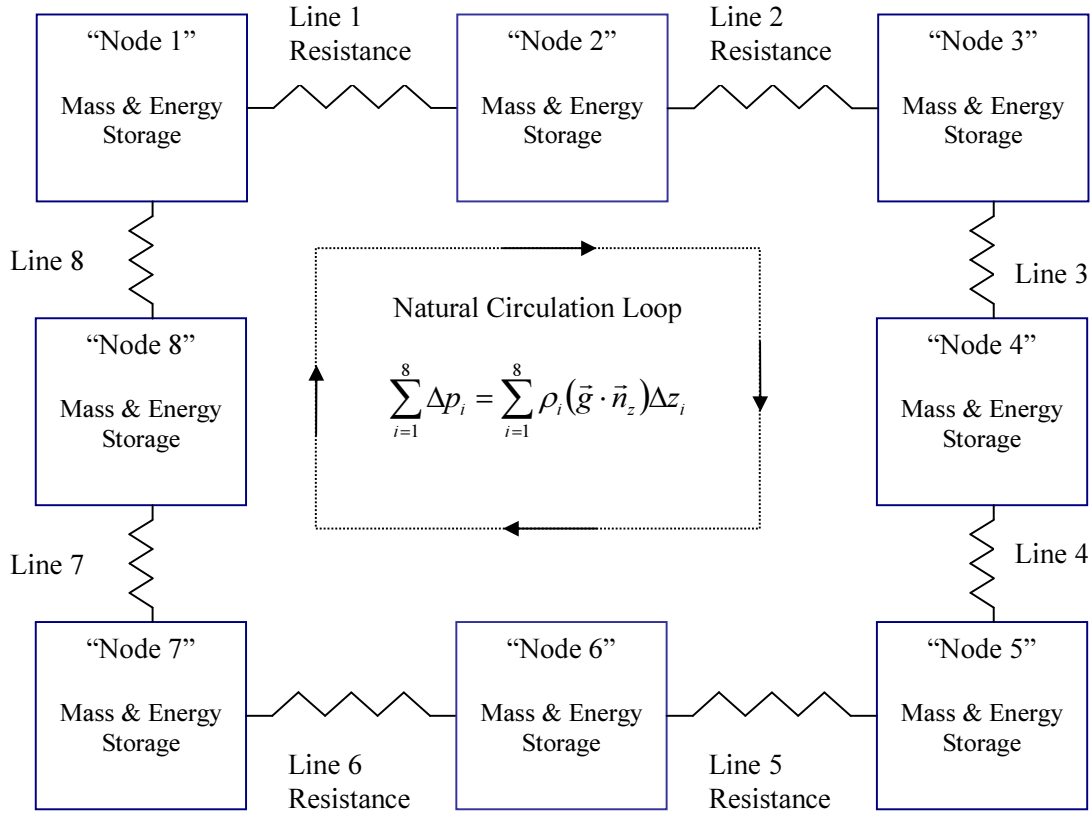


FIG. 1. Early computer modeling method used by westinghouse-bettis FLASH code to assess PWR safety.

From the early 1970s to the present, the U.S. Nuclear Regulatory Commission helped to support the development of a number of computer codes to predict Loss-of-Coolant-Accident (LOCA) phenomenon. These included RELAP2, RELAP3, RELAP3B (BNL), RELAP4, RELAP5, TRAC-PF1, TRAC-PD1, TRAC-BF1, RAMONA-3B, THOR, RAMONA-3B, RAMONA-4B, HIPA-PWR and HIPA-BWR. The Los Alamos National Laboratory held responsibility for the TRAC codes (with the exception of TRAC-BF1), the Idaho National Engineering Laboratory and Energy Incorporated (an Idaho-based consulting company) were responsible for the RELAP codes, and Brookhaven National Laboratory was responsible for THOR, RAMONA and HIPA codes.

TRAC, RELAP5 and RAMONA were recognized as the advanced simulation computer codes for PWR and BWR plants safety analysis and are currently being used for best-estimate calculations of plant behavior. In 1996, the NRC decided to produce an analysis package that combined the capabilities of RELAP5, TRAC-PWR, TRAC-BWR, and RAMONA. The software package, known as the TRAC/RELAP Advanced Computational Engine or TRACE, is currently under development by the NRC.

2. TWO-PHASE FLOW TRANSPORT EQUATIONS

Accurately describing the transport of mass, momentum and energy in a two-phase fluid flowing through a complex geometry, such as a nuclear reactor core, while undergoing phase change, is a formidable task. Nonetheless, this is only part of the challenge laid before the nuclear reactor safety analyst. Coupling the thermal hydraulic behavior to the core physics (which will not be discussed here) further confounds the effort to obtain an accurate safety assessment tool. This chapter has the relatively straightforward goal of providing the reader with an overview of the different formulations of the two-phase flow transport equations as used in a variety of reactor analysis codes. The following section presents the one-dimensional two-fluid conservation equations that serve, either directly or through the development of the mixture equations, as the basis for the development of the remaining models.

2.1. One-dimensional two-fluid full non-equilibrium transport equations

This section presents the differential formulation of the one-dimensional, two-fluid, full non-equilibrium transport equations. The definition of each symbol is provided in the nomenclature. The formulation is similar to that of Todreas and Kazimi (1993) [2] with the exception of the fluid-structure transport terms.

The first transport equation considered is the conservation of mass for each phase. It is given by:

Mass Conservation Equation

$$\frac{\partial}{\partial t} \{ \rho_k \alpha_k \} + \frac{\partial}{\partial z} \{ \rho_k v_k \alpha_k \} = \Gamma_k \quad (1)$$

The subscript “k” indicates either liquid phase, when the subscript is set to “l”, or vapor phase, when the subscript is set to “v.” The first term on the left hand side of the equation represents the time rate of change of the “area averaged” mass for a given fluid phase. In this equation, and those that follow, the fluid phase parameters have been averaged over the cross-sectional flow area. That is, given a fluid phase parameter, ψ_k , area averaging is performed as follows:

$$\{ \psi_k \} \equiv \frac{1}{A} \iint_A \psi_k dA \quad (2)$$

The second term on the left hand side represents the change in mass along the flow axis. The Γ_k term represents the liquid or vapor mass generated as a result of phase change. It is the interfacial mass transfer rate per unit volume. For example, Γ_v , would represent the rate at which vapor mass is generated per unit volume as a result of liquid evaporation. Similarly, Γ_l , may represent the rate at which liquid mass is generated per unit volume as a result of vapor condensation.

The conservation of momentum for each phase is given by:

Momentum Conservation Equation

$$\frac{\partial}{\partial t} \{ \rho_k v_k \alpha_k \} + \frac{\partial}{\partial z} \{ \rho_k v_k^2 \alpha_k \} = \{ \Gamma_k \vec{v}_{ks} \cdot \vec{n}_z \} + \sum_{i=1}^N \{ \vec{F}_{wk} \cdot \vec{n}_z \}_i - \frac{\partial}{\partial z} \{ p_k \alpha_k \} + \{ \vec{F}_{sk} \cdot \vec{n}_z \} + \{ \rho_k \alpha_k \} \vec{g} \cdot \vec{n}_z \quad (3)$$

The first term on the left hand side of the equation represents the time rate of change of the area averaged momentum for a given fluid phase. The second term on the left hand side represents the change in the momentum along the flow axis. The first term on the right hand side represents the rate of momentum transfer due to phase change. The term, $\vec{v}_{ks} \cdot \vec{n}_z$, is the interface velocity of phase k along the z-coordinate. It is a scalar that can be either positive or negative. The second term on the right hand side represents the sum of the fluid phase drag forces on the structures in the flow. The third term on the right hand side represents the pressure gradient along the axis of the flow. It is typically assumed that the vapor and liquid phases have equal pressures. However, the stability studies of Ransom and Hicks (1984) [3], [4] indicate that this assumption results in an ill-posed set of equations that may result in numerical instabilities. The fourth term represents the drag forces acting on the fluid phase interface. The last term represents the gravity forces acting in the direction of the flow.

The conservation of energy for each phase is given by:

Energy Conservation Equation

$$\frac{\partial}{\partial t} \{ \rho_k u_k^o \alpha_k \} + \frac{\partial}{\partial z} \{ \rho_k h_k^o v_k \alpha_k \} = \Gamma_k h_{ks}^o - \left\{ p_k \frac{\partial \alpha_k}{\partial t} \right\} + \sum_{i=1}^N \left\{ q_k'' \alpha_k \frac{P}{A} \right\}_i - \{ \rho_k g v_k \alpha_k \} + \{ Q_{sk} \} \quad (4)$$

The energy conservation equation can be expressed in a various forms. This equation is an energy balance expressed in terms of the stagnation energy, u_k^o , and stagnation enthalpy, h_k^o , which are defined as follows:

$$u_k^o = u_k + \frac{v_k^2}{2} \quad (5)$$

$$h_k^o = u_k^o + \frac{p_k}{\rho_k} \quad (6)$$

The stagnation energy is defined as the sum of the thermodynamic internal energy and the kinetic energy of the fluid phase. The stagnation enthalpy has the usual definition, however, it is expressed in terms of the stagnation energy.

The first term on the left hand side of the energy conservation equation represents the time rate of change of the area averaged energy for a given fluid phase. The second term on the left hand side represents the change in the energy along the flow axis. The first term on the right hand side of the equation represents the rate of energy transfer due to phase change. The second term on the right represents the pressure work associated with changes in void fraction. The third term on the right represents the sum of the heat transfer between the fluid phase and structures in the flow. The fourth term on the right represent work due to gravity and the last term on the right hand side represents interfacial heat transfer.

For liquid and vapor systems, equations (1), (3) and (4) represent the six conservation equations that serve as a starting point for two-phase flow computer models. These equations are summarized in Table 1. The following section illustrates how these equations can be used to obtain a set of two-phase fluid mixture transport equations.

Table 1. One-dimensional transport equations for a two phase fluid
(Uniform Density within each Phase, Constant Axial Cross-Sectional Area)

Conservation Equations for Each Phase “k”
<p><i>Mass:</i></p> $\frac{\partial}{\partial t} \{\rho_k \alpha_k\} + \frac{\partial}{\partial z} \{\rho_k v_k \alpha_k\} = \Gamma_k$
<p><i>Momentum:</i></p> $\frac{\partial}{\partial t} \{\rho_k v_k \alpha_k\} + \frac{\partial}{\partial z} \{\rho_k v_k^2 \alpha_k\} = \{\Gamma_k \bar{v}_{ks} \cdot \bar{n}_z\} + \sum_{i=1}^N \{\bar{F}_{wk} \cdot \bar{n}_z\}_i - \frac{\partial}{\partial z} \{p_k \alpha_k\} + \{\bar{F}_{sk} \cdot \bar{n}_z\} + \{\rho_k \alpha_k\} \bar{g} \cdot \bar{n}_z$
<p><i>Energy: (Neglecting axial heat conduction and axial shear effect)</i></p> $\frac{\partial}{\partial t} \{\rho_k u_k^o \alpha_k\} + \frac{\partial}{\partial z} \{\rho_k h_k^o v_k \alpha_k\} = \Gamma_k h_{ks}^o - \left\{ p_k \frac{\partial \alpha_k}{\partial t} \right\} + \sum_{i=1}^N \left\{ q_k^o \alpha_k \frac{P}{A} \right\}_i - \{\rho_k g v_k \alpha_k\} + \{Q_{sk}\}$

2.2. Two-phase mixture transport equations

A typical approach to the analysis of two-phase flows is to use mixture equations. That is, the individual fluid phases are assumed to behave as a flowing mixture described in terms of a mixture properties. To obtain the two-phase fluid mixture transport equations using the transport equations given in Table 1, one must first recognize that interfacial transfer inside the mixture would not impact the overall mixture behavior. This is best described in terms of the following interfacial jump conditions:

Interfacial Jump Conditions:

Mass

$$\sum_{k=1}^2 \Gamma_k = 0 \quad (7)$$

Momentum:

$$\sum_{k=1}^2 (\Gamma_k \bar{v}_{ks} \cdot \bar{n}_z + \bar{F}_{sk} \cdot \bar{n}_z) = 0 \quad (8)$$

Energy:

$$\sum_{k=1}^2 (\Gamma_k h_{ks}^o + Q_{sk}) = 0 \quad (9)$$

By summing the balance equations for each phase given in Table 1 and applying the interfacial jump conditions, the six equations are reduced to three as shown in Table 2. The mixture properties have also

been defined in Table 2. These three mixture equations served as the basis for the two-phase flow mixture models developed in the following chapter.

Table 2. One-dimensional two-phase mixture transport equations
(Uniform Density within each Phase)

Conservation Equations	
Mixture Mass:	
	$\frac{\partial \rho_m}{\partial t} + \frac{\partial G_m}{\partial z} = 0 \quad (10)$
Mixture Momentum:	
	$\frac{\partial G_m}{\partial t} + \frac{\partial}{\partial z} \left(\frac{G_m^2}{\langle \rho_m \rangle} \right) = - \sum_{i=1}^N F_{wi} - \frac{\partial p_m}{\partial z} - \rho_m g \cos \theta \quad (11)$
Mixture Enthalpy: (Approximate)	
	$\frac{\partial}{\partial t} \{ \rho_m h_m - p_m \} + \frac{\partial}{\partial z} \{ G_m \langle h_m \rangle \} = \sum_{i=1}^N q_i'' \frac{P_i}{A_i} + \frac{G_m}{\rho_m} \left(\sum_{i=1}^N F_{wi} + \frac{\partial p_m}{\partial z} \right) \quad (12)$
Mixture Properties	
	$\rho_m = \{ \rho_v \alpha + \rho_l (1 - \alpha) \} \quad (13)$
	$\langle \rho_m \rangle = \frac{G_m^2}{\{ \rho_v v_v^2 \alpha + \rho_l v_l^2 (1 - \alpha) \}} \quad (14)$
	$G_m = \{ \rho_v v_v \alpha + \rho_l v_l (1 - \alpha) \} \quad (15)$
	$h_m = \frac{\{ \rho_v h_v \alpha + \rho_l h_l (1 - \alpha) \}}{\rho_m} \quad (16)$
	$\langle h_m \rangle = \frac{\{ \rho_v h_v v_v \alpha + \rho_l h_l v_l (1 - \alpha) \}}{G_m} \quad (17)$
	$(v^2)_m = \frac{\{ \rho_v \alpha v_v^2 + \rho_l (1 - \alpha) v_l^2 \}}{\rho_m} \quad (18)$
	$\langle v^2 \rangle_m = \frac{\{ \rho_v \alpha v_v^3 + \rho_l (1 - \alpha) v_l^3 \}}{G_m} \quad (19)$
	$p_m = \{ p_v \alpha + p_l (1 - \alpha) \} \quad (20)$

2.2.1. Homogeneous Equilibrium Mixture (HEM) Transport Equations

The homogeneous equilibrium mixture (HEM) model is the simplest of the two-phase fluid transport models. The HEM transport equations are derived from the two-phase mixture equations by assuming that the velocity of each fluid phase is equal (homogenous) and that both phases are at saturated conditions. The assumption of equilibrium means that the thermodynamic properties of each fluid phase can be expressed as a function of saturation pressure. Table 3 presents the HEM transport equations.

Table 3. Homogeneous equilibrium mixture (HEM) transport equations

<u>Restrictions Imposed on Two-Phase Mixture Equations</u>	
<ul style="list-style-type: none"> • Thermal Equilibrium ($T_l = T_v = T_{SAT}$) or Saturated Enthalpies ($h_l = h_f$ and $h_v = h_g$) • Equal Phase Pressures ($p_l = p_v = p$) • Equal Velocities ($v_l = v_v = v_m$) 	
<u>Conservation Equations</u>	
Mixture Mass:	
	$\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial z}(\rho_m v_m) = 0 \quad (21)$
Mixture Momentum:	
	$\frac{\partial}{\partial t}(\rho_m v_m) + \frac{\partial}{\partial z}(\rho_m v_m^2) = -\sum_{i=1}^N F_{wi} - \frac{\partial p}{\partial z} - \rho_m g \cos \theta \quad (22)$
Mixture Energy:	
	$\frac{\partial}{\partial t} \left\{ \rho_m \left[h_m + \frac{v_m^2}{2} \right] - p \right\} + \frac{\partial}{\partial z} \left\{ \rho_m v_m \left[h_m + \frac{v_m^2}{2} \right] \right\} = \sum_{i=1}^N \left(q_i'' \frac{P_i}{A_i} \right) - \rho_m v_m g \cos \theta \quad (23)$
<u>Mixture Properties</u>	
	$\rho_m = \{ \rho_v \alpha + \rho_l (1 - \alpha) \} \quad (24)$
	$v_m = \frac{\{ \rho_v v_v \alpha + \rho_l v_l (1 - \alpha) \}}{\rho_m} \quad (25)$
	$h_m = \frac{\{ \rho_v h_v \alpha + \rho_l h_l (1 - \alpha) \}}{\rho_m} \quad (26)$

2.3. Two-phase drift flux transport equations

The Drift-Flux Model, developed by Zuber and Findlay [5] in 1965, provides a simple, yet reasonably accurate, method of introducing the relative velocity between fluid phases into the mixture equations. The basic premise is that the relative velocity between phases, (v_r), is related to a flow regime dependent drift velocity (V_{vj}) by the equation (27) presented in Table 4. Table 4 also presents several empirical correlations developed for the drift velocity.

Table 4. Drift velocity equations

Relationship Between Relative Velocity and Drift Velocity:	
$v_r = (v_v - v_l) = \frac{V_{vj}}{1 - \{\alpha\}} \quad (27)$	
Two-Phase Flow Regimes	Drift Velocity Equations
<i>Churn-Turbulent Flow</i>	$V_{vj} = 1.41 \left(\frac{\sigma g (\rho_l - \rho_g)}{\rho_l^2} \right)^{1/4} \quad (28)$
<i>Slug Flow</i>	$V_{vj} = 0.35 \left(\frac{g (\rho_l - \rho_g) D}{\rho_l} \right)^{1/2} \quad (29)$
<i>Annular Flow</i>	$V_{vj} = 23 \frac{\Delta \rho}{\rho_l} \left(\frac{\mu_l (1 - \{\alpha\}) v_l}{\rho_v D} \right)^{1/2} \quad (30)$

The two-phase fluid mixture transport equations shown in Table 2 can be written in terms of the relative velocities between phases. This requires a significant amount of algebra that is left to the astute reader. Having rearranged the mixture equations in terms the relative velocity, equation (27) is then substituted to obtain the final result shown in Table 5.

The following chapter describes how the transport equations presented in Sections 2.1, 2.2, and 2.3 have been used to build a variety of two-phase flow models for reactor analyses.

Table 5. One dimensional two-phase drift flux transport equations
(Uniform Density within each Phase)

<u>Conservation Equations</u>	
<i>Mixture Mass:</i>	
	$\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial z}(\rho_m v_m) = 0 \quad (31)$
<i>Drift Flux Momentum:</i>	
	$\rho_m \frac{\partial v_m}{\partial t} + \rho_m v_m \frac{\partial v_m}{\partial z} + \frac{\partial}{\partial z} \left(\frac{\rho_v \rho_l \{\alpha\} V_{vj}^2}{\rho_m (1 - \{\alpha\})} \right) = - \sum_{i=1}^N F_{wi} - \frac{\partial p_m}{\partial z} - \rho_m g \cos \theta \quad (32)$
<i>Drift Flux Internal Energy:</i>	
	$\begin{aligned} \frac{\partial}{\partial t} \{\rho_m u_m\} + \frac{\partial}{\partial z} \{\rho_m u_m v_m\} + \frac{\partial}{\partial z} \left[\frac{\{\alpha\} \rho_l \rho_v (u_v - u_l) V_{vj}}{\rho_m} \right] + \\ p_m \frac{\partial v_m}{\partial z} + p_m \frac{\partial}{\partial z} \left[\frac{\{\alpha\} (\rho_l - \rho_v) V_{vj}}{\rho_m} \right] = \sum_{i=1}^N q_i'' \frac{P_i}{A_i} + v_m \left(\sum_{i=1}^N F_{wi} \right) \end{aligned} \quad (33)$
<u>Mixture Properties</u>	
	$\rho_m = \{\rho_v \alpha + \rho_l (1 - \alpha)\} \quad (34)$
	$v_m = \frac{\{\rho_v v_v \alpha + \rho_l v_l (1 - \alpha)\}}{\rho_m} \quad (35)$
	$u_m = \frac{\{\rho_v u_v \alpha + \rho_l u_l (1 - \alpha)\}}{\rho_m} \quad (36)$
	$p_m = \{p_v \alpha + p_l (1 - \alpha)\} \quad (37)$

3. TWO-PHASE FLOW MODELS FOR REACTOR ANALYSES

This chapter provides an overview of the different types of two-phase flow models that have been developed and published in the literature. Figure 2 provides a simple overview of the components that comprise a two-phase flow model. A rigorous approach to modeling two-phase flow is to implement the two-fluid, full non-equilibrium, conservation equations. This includes a mass, momentum and energy conservation equation for each phase as presented in section 2.1. Thus, it is typically referred to as the 6-equation model.

Because of the difficulty of working with a 6-equation model, a variety of simplified models have been developed. These models are derived by using mixture equations, or mixture equations in conjunction with individual phase equations, to obtain a reduced number of balance equations. Using this approach, it is possible to obtain 5-equation, 4-equation or 3-equation models for two-phase flow. Of course, the simpler models result in a reduced number of calculated parameters as dictated by the restrictions that have been imposed. One of the following three restrictions is typically imposed on the phase velocities.

1. Homogeneous Flow. This assumes that the velocity of both phases is equal. That is;

$$v_l = v_v = v_m \quad (38)$$

2. Phase Slip. This assumes that there is a relative velocity between the phases. The ratio of the vapor velocity to the liquid velocity is defined as the slip ratio, given a flow dependent correlation. Thus the slip ratio, S , equals:

$$S = \frac{v_v}{v_l} \quad (39)$$

3. Drift Flux. This assumes that there is a relative velocity between the phases. The relative velocity is determined from a known drift flux correlation, v_{vj} , that is flow regime dependent. That is,

$$\{v_v - v_l\} = \frac{v_{vj}}{\{1 - \alpha\}} \quad (40)$$

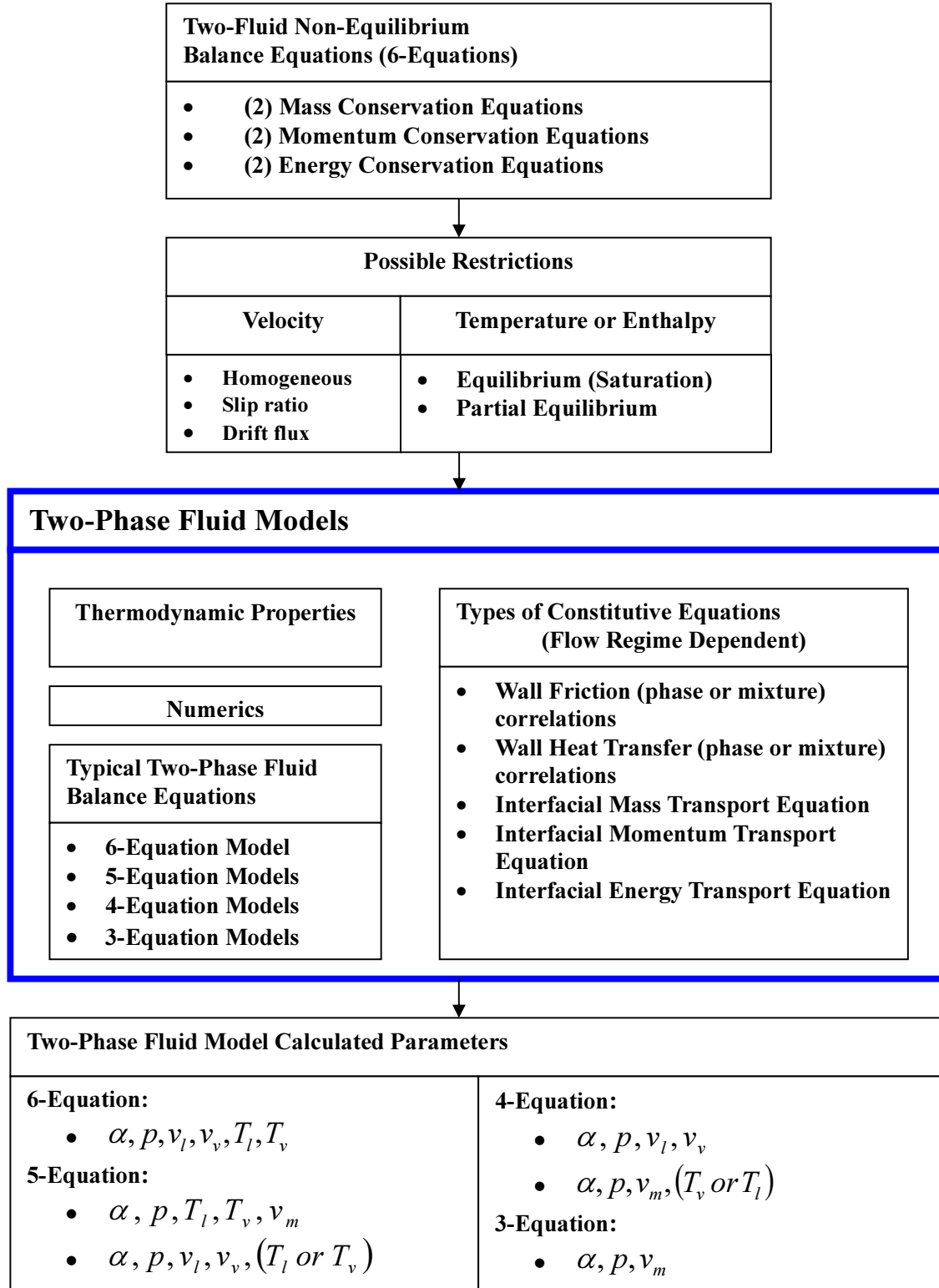


FIG. 2. Types of two-phase flow models used in nuclear reactor safety analyses.

One of the following two restrictions is typically applied to the phase temperatures or enthalpies.

4. Full Thermal Equilibrium. This refers to the assumption that both of the fluid temperatures are equal and at the saturation temperature corresponding to the local pressure. That is:

$$T_l = T_v = T_{SAT}(p) \quad (41)$$

5. Partial Thermal Equilibrium. This assumes that one of the two phases is at the saturation temperature corresponding to the local pressure. That is:

$$T_l \text{ or } T_v = T_{SAT}(p) \quad (42)$$

Figure 2 also shows that in addition to the balance equations, a two-phase flow model requires constitutive equations and thermodynamic property equations or tables to obtain a closed set of equations. The constitutive equations include fluid-structure and interfacial transport equations. These are as follows:

- A wall friction correlation for a mixture or for each of the phases. One such equation is needed for each momentum balance equation;
- A wall heat transfer correlation for a mixture or for each of the phases. One such equation is needed for each energy balance equation;
- An interfacial mass transport equation;
- An interfacial momentum transport equation;
- An interfacial energy transport equation.

Although not emphasized in Figure 2, nor expanded upon in this lecture, the numerical methods used to solve the equations play a major role in the accuracy and stability of the predictions. The topic can easily fill an entire course.

Lastly, the number of parameters calculated by each model depends on the number of balance equations. The 6-equation model can be used to calculate 6-unknowns; $\alpha, p, v_l, v_v, T_l, T_v$.

Table 6 provides the details of different types of two-phase flow models that have been used for nuclear reactor safety analyses. Eleven different models are presented, including their restrictions, types of constitutive equations and calculated parameters. It is important to recognize that, for any model formulation, a mixture conservation equation plus a phase conservation equation can be used in place of the two conservation equations for the phases. An example is provided in Figure 3.

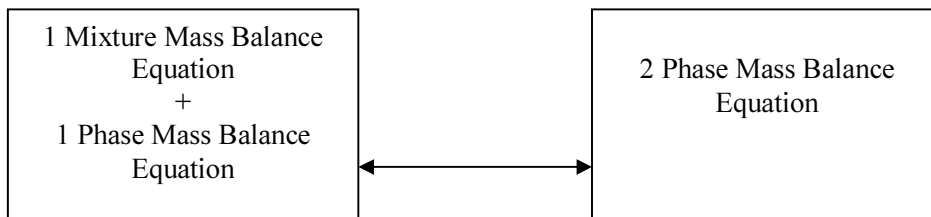


FIG. 3. Equivalent approaches to developing model mass balance equations.

Table 6. Two –phase flow models with equal phase pressures ($P_v = P_L = P$)

6-Equation Model			
Conservation Equations	Restrictions¹	Constitutive Laws²	Calculated Parameters
<u>Two-Fluid Non-Equilibrium</u> (2) Mass Phase Balance (2) Momentum Phase Balance (2) Energy Phase Balance	None	(2) Phase wall friction (2) Phase heat flux friction (1) Interfacial mass (1) Interfacial momentum (1) Interfacial energy	α, p, v_l v_v, T_l, T_v
5-Equation Models			
<u>Two-Fluid Partial Non-Equilibrium</u> (2) Mass Phase Balance (2) Momentum Phase Balance (1) Mixture Energy Balance	$T_l = T_{SAT}$ <i>or</i> $T_v = T_{SAT}$	(2) Phase wall friction (1) Mixture wall heat flux (1) Interfacial mass (1) Interfacial momentum	α, p, v_l, v_v $(T_l \text{ or } T_v)$
<u>Two-Fluid Partial Non-Equilibrium</u> (1) Mixture Mass Balance (2) Momentum Phase Balance (2) Energy Phase Balance	$T_l = T_{SAT}$ <i>or</i> $T_v = T_{SAT}$	(2) Phase wall friction (2) Phase heat flux friction (1) Interfacial mass ³ (1) Interfacial momentum (1) Interfacial energy	α, p, v_l, v_v $(T_l \text{ or } T_v)$
<u>Slip or Drift Non-Equilibrium</u> (2) Mass Phase Balance (1) Mixture Momentum Balance (2) Energy Phase Balance	Slip or Drift Velocity	(1) Mixture wall friction (2) Phase heat flux friction (1) Interfacial mass (1) Interfacial energy (1) Slip velocity or Drift flux	α, p, T_l T_v, v_m
<u>Homogeneous Non-Equilibrium</u> (2) Mass Phase Balance (1) Mixture Momentum Balance (2) Energy Phase Balance	Equal Velocity $v_l = v_v = v_m$	(1) Mixture wall friction (2) Phase heat flux friction (1) Interfacial mass ³ (1) Interfacial energy	α, p, T_l T_v, v_m

1. Restrictions imposed on fluid phase velocities or temperatures (or enthalpies in lieu of temperatures).
2. Minimum number of constitutive laws. For example, for N structures in the flow, N structure heat flux and N wall friction correlations may be required.
3. Interfacial mass transfer is required to determine interfacial momentum or interfacial energy transfer.

Table 6. (continued) Two-phase flow models with equal phase pressures ($P_v = P_l = P$)

4 -Equation Models			
Conservation Equations	Restrictions ¹	Constitutive Laws ²	Calculated Parameters
<u>Two-Fluid Equilibrium Model</u> (1) Mixture Mass Balance (2) Momentum Phase Balance (1) Mixture Energy Balance	$T_l = T_v = T_{SAT}$	(2) Phase wall friction (1) Mixture heat flux friction (1) Interfacial mass ³ (1) Interfacial momentum	α, p, v_l, v_v
<u>Drift Partial Non-Equilibrium</u> (2) Mass Phase Balance (1) Mixture Momentum Balance (1) Mixture Energy Balance	Drift Velocity $T_v \text{ or } T_l = T_{SAT}$	(1) Mixture wall friction (1) Mixture wall heat flux (1) Interfacial mass (1) Drift flux correlation	$\alpha, p, v_m, T_v \text{ or } T_l$
<u>Slip Partial Non-Equilibrium</u> (1) Mixture Mass Balance (1) Mixture Momentum Balance (2) Phase Energy Balance	Slip Ratio $T_v \text{ or } T_l = T_{SAT}$	(1) Mixture wall friction (1) Mixture wall heat flux (1) Interfacial mass (1) Drift flux correlation	$\alpha, p, v_m, T_v \text{ or } T_l$
<u>Homogeneous, Partial Non-Equilibrium:</u> (1) Mixture Mass Balance (1) Mixture Momentum Balance (2) Phase Energy Balance	$u_l = u_v = u_m$ $T_v \text{ or } T_l = T_{SAT}$	(1) Mixture wall friction (2) Phase wall heat flux (1) Interfacial mass ³ (1) Interfacial energy	$\alpha, p, v_m, T_v \text{ or } T_l$
3-Equation Models			
<u>Homogeneous Equilibrium (HEM):</u> (1) Mixture Mass Balance (1) Mixture Momentum Balance (1) Mixture Energy Balance	$u_l = u_v = u_m$ $T_l = T_v = T_{SAT}$	(1) Mixture wall friction (1) Mixture wall heat flux	α, p, u_m
<u>Slip or Drift Equilibrium:</u> (1) Mixture Mass Balance (1) Mixture Momentum Balance (1) Mixture Energy Balance	Slip or Drift Velocity $T_l = T_v = T_{SAT}$	(1) Mixture wall friction (1) Mixture wall heat flux (1) Slip velocity or Drift flux	α, p, u_m

1. Restrictions imposed on fluid phase velocities or temperatures (or enthalpies in lieu of temperatures).
2. Minimum number of constitutive laws. For example, for N structures in the flow, N structure heat flux and N wall friction correlations may be required.
3. Interfacial mass transfer is required to determine interfacial momentum or interfacial energy transfer.

3.1. Advancements in two-phase flow modeling

This section touches briefly on some of the ongoing progress in the area of two-phase flow modeling in the United States. The first area discussed is the current effort to model the structure of two-phase flow regimes using an interfacial area concentration transport equation as opposed to a static flow regime map. The second area discussed is the U.S. Nuclear Regulatory Commission's effort to merge their current reactor analysis codes into a single multi-purpose code.

3.1.1. Interfacial Area Transport Model

The 6-equation, two-fluid model requires constitutive equations for the interfacial mass, momentum and energy transfer rates. These constitutive laws are quite important because the interfacial transfer rate govern the rate of phase change and the degree of mechanical and thermal non-equilibrium between phases. The interfacial transfer rate is related to the driving potential between the phases and the amount of interfacial area per unit volume of the mixture. This latter quantity is known as the interfacial area concentration.

Present day computer models for two-phase flows implement constitutive laws for interfacial transport that are based on *static flow regime maps*. That is, the computer codes include semi-empirical transition criteria to determine the two-phase flow regime present for a given combination of vapor and liquid flows and corresponding fluid properties. Having determined the two-phase flow regime, the code uses the criteria to select appropriate heat transfer and drag correlations. This approach has been relatively successful. However, efforts are underway to develop the tools for *dynamic flow regime modeling*.

Recent efforts by Kocamustafaogullari and Ishii (1995) [6], Wu, et al., (1998) [7], and Wu, Ishii and Uhle (1998) [8] have led to the development of the two-group interfacial area transport equations. The concept of two-group transport is, of course, not new. The application of multi-energy group models for neutron transport in reactors is well known. However its use in two-phase flows is relatively new. Reyes (1989) used a Boltzmann transport equation in his statistical derivation of the conservation equations for fluid particle flows. The novelty of this approach was the inclusion of breakage and coalescence “scattering integrals” that produce changes in the particle size distributions as the flow evolves. The interfacial area transport model extends that work by considering particle interfacial area as opposed to particle number density.

Two bubble groups are currently used. Group I consists of the spherical/distorted bubble group and Group II consists of the cap/slug bubble group. The two-group bubble number density transport equations can be derived from the Boltzmann transport equation to obtain:

Group I Number Density Transport:

$$\frac{\partial n_1}{\partial t} + \nabla \cdot n_1 \vec{v}_{pm,1} = \sum_j (S_{j,1} + S_{j,12}) + (S_{ph,1} + S_{ph,12}) \quad (43)$$

Group II Number Density Transport:

$$\frac{\partial n_2}{\partial t} + \nabla \cdot n_2 \vec{v}_{pm,2} = \sum_j (S_{j,2} + S_{j,21}) + (S_{ph,2} + S_{ph,21}) \quad (44)$$

where S_j is the net rate of change in the number density function due to the particle breakup and coalescence processes and S_{ph} is the net rate of change in the number density function due to phase change. Both terms have been integrated from the minimum to maximum particle volume. $S_{j,12}$ and $S_{j,21}$ are the inter-group particle exchange terms. The subscript “j” represents the different breakup and coalescence processes. The following expression is used to develop the interfacial area transport equations:

$$n_k = \psi_k \left(\frac{a_{i,k}^3}{\alpha_k^2} \right) \quad (45)$$

where, $a_{i,k}$ is the interfacial area concentration, α is the void fraction and ψ_k is the bubble shape factor. The subscript “k” represents the bubble group. Substituting equation (45) into (43) and (44) yields the following two-group interfacial area transport equations:

$$\frac{\partial a_{i,1}}{\partial t} + \nabla \cdot a_{i,1} \vec{v}_{i,1} = \frac{1}{3\psi_1} \left(\frac{\alpha_1}{a_{i,1}} \right)^2 \left[\sum_j (S_{j,1} + S_{j,12}) + (S_{ph,1} + S_{ph,12}) \right] + \left(\frac{2a_{i,1}}{3\alpha_1} \right) \left[\frac{\partial \alpha_1}{\partial t} + \nabla \cdot \alpha_1 \vec{v}_{i,1} \right] \quad (46)$$

$$\frac{\partial a_{i,2}}{\partial t} + \nabla \cdot a_{i,2} \vec{v}_{i,2} = \frac{1}{3\psi_2} \left(\frac{\alpha_2}{a_{i,2}} \right)^2 \left[\sum_j (S_{j,2} + S_{j,21}) + (S_{ph,2} + S_{ph,21}) \right] + \left(\frac{2a_{i,2}}{3\alpha_2} \right) \left[\frac{\partial \alpha_2}{\partial t} + \nabla \cdot \alpha_2 \vec{v}_{i,2} \right] \quad (47)$$

A significant effort is underway to develop the appropriate coalescence and breakage models for the two groups. Initial comparisons to data, including bubbly to slug flow transitions are very encouraging, Hibiki and Ishii (2000) [9].

3.1.2. *TRACE reactor system analysis code [10]*

The U.S. Nuclear Regulatory Commission (USNRC) is in the process of developing a modern code for reactor analysis. It is an evolutionary code that merges RAMONA, RELAP5, TRAC-PWR and TRAC-BWR into a single code. The reason for merging the codes, as opposed to starting new, is to maintain the sizable investment that exists in the development of input models for each of the codes. The consolidated code is called the TRAC/RELAP Advanced Computational Engine or TRACE.

TRACE is a component-oriented code designed to analyze reactor transients and accidents up to the point of fuel failure. It is a finite-volume, two-fluid, compressible flow code with 3-D capability. It can model heat structures and control systems that interact with the component models and the fluid solution. TRACE can be run in a coupled mode with the PARCS three dimensional reactor kinetics code. Typical TRACE reactor models range in size from a few hundred to a few thousand fluid volumes. TRACE can be run in parallel. TRACE has been coupled to CONTAIN through its exterior communications interface (ECI) and can be coupled to detailed fuel models or CFD codes in the future using the ECI. TRACE has been coupled to as user-friendly front end, SNAP, that supports input model development and accepts existing RELAP5 and TRAC-P input. Figure 4 illustrates the TRACE /SNAP environment architecture.

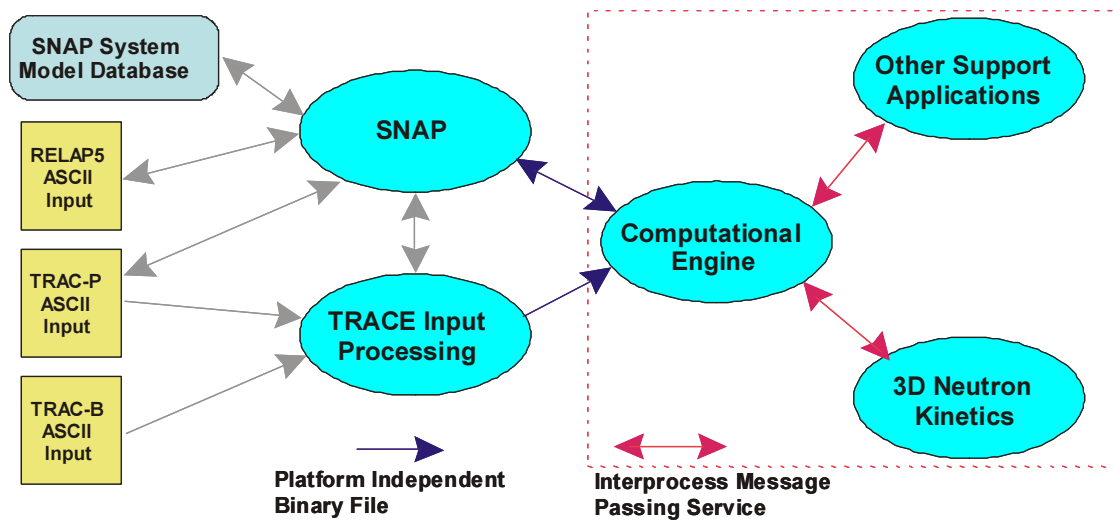


FIG. 4. TRACE /SNAP environment architecture [10].

Table 7 presents the conservation equations and constitutive equations for the TRACE computer code. It consists of a 3-D, 6-equation, non-equilibrium model that can be coupled to additional equations to solve for the non-condensable gas pressure, boron concentration and heat structure temperatures.

Table 7. Trace field equations [10]

<p><u>Conservation Equations</u></p> <p><i>Mixture Mass:</i></p> $\frac{\partial}{\partial t}(\rho_v \alpha + (1 - \alpha) \rho_l) + \nabla \cdot (\rho_v \vec{v}_v \alpha + \rho_l \vec{v}_l (1 - \alpha)) = 0$ <p><i>Vapor Mass:</i></p> $\frac{\partial}{\partial t}(\rho_v \alpha) + \nabla \cdot (\rho_v \vec{v}_v \alpha) = \Gamma_v$ <p><i>Liquid Momentum:</i></p> $\frac{\partial \vec{v}_l}{\partial t} + \vec{v}_l \cdot \nabla \vec{v}_l = -\frac{1}{\rho_l} \nabla p + \frac{c_i}{(1 - \alpha) \rho_l} (\vec{v}_v - \vec{v}_l) \vec{v}_v - \vec{v}_l - \frac{\Gamma_{Cond}}{(1 - \alpha) \rho_l} (\vec{v}_v - \vec{v}_l) + \frac{c_w}{(1 - \alpha) \rho_l} \vec{v}_l \vec{v}_l + \vec{g}$ <p><i>Gas Momentum:</i></p> $\frac{\partial \vec{v}_v}{\partial t} + \vec{v}_v \cdot \nabla \vec{v}_v = -\frac{1}{\rho_v} \nabla p + \frac{c_i}{\alpha \rho_v} (\vec{v}_v - \vec{v}_l) \vec{v}_v - \vec{v}_l - \frac{\Gamma_{Boiling}}{\alpha \rho_v} (\vec{v}_v - \vec{v}_l) + \frac{c_{wv}}{\alpha \rho_v} \vec{v}_v \vec{v}_v + \vec{g}$ <p><i>Mixture Energy:</i></p> $\frac{\partial}{\partial t}(\rho_v \alpha e_v + \rho_l (1 - \alpha) e_l) + \nabla \cdot (\rho_v \alpha e_v \vec{v}_v + \rho_l (1 - \alpha) e_l \vec{v}_l) = -p \nabla \cdot (\alpha \vec{v}_v + (1 - \alpha) \vec{v}_l) + q_{wl} + q_{div}$ <p><i>Vapor Energy:</i></p> $\frac{\partial}{\partial t}(\rho_v \alpha e_v) + \nabla \cdot (\rho_v \alpha e_v \vec{v}_v) = -p \frac{\partial \alpha}{\partial t} - p \nabla \cdot (\alpha \vec{v}_v) + q_{wv} + q_{dv} + q_{iv} + \Gamma_v h_v$	
<p><u>Constitutive Equations:</u></p> <ul style="list-style-type: none"> • Equations of State • Wall Drag • Interfacial Drag • Wall Heat Transfer • Interfacial Heat Transfer • Static Flow Regime Maps 	<p><u>Additional Equations:</u></p> <ul style="list-style-type: none"> • Non-condensable Gas • Dissolved Boron • Control Systems • Reactor Power
<p><u>Calculated Parameters:</u></p> <ul style="list-style-type: none"> • Vapor Void Fraction • Steam Pressure • Non-condensable Gas Pressure • Liquid Velocity and Temperature • Vapor Velocity and Temperature • Boron Concentration • Heat Structure Temperatures 	

NOMENCLATURE

a	interfacial area concentration
A	cross-sectional area
D	pipe diameter
e	total energy
F	force
g	gravitational constant
G	mass flux
h^o	stagnation enthalpy
\vec{n}	outward normal vector
p	pressure
q''	heat flux
Q_{sk}	interfacial heat transfer rate
S	velocity slip ratio
S_j	Interaction term for change in bubble number density due to bubble coalescence and breakage
S_{ph}	Interaction term for change in bubble number density due to phase change
T	temperature
u^o	stagnation energy
v_k	phase velocity
V_{vj}	drift velocity

Greek Symbols

α	vapor void fraction
Γ_k	mass generation rate per unit volume
ρ	density
σ	surface tension
ψ	bubble shape factor

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