The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2152-16

Joint ICTP-IAEA Course on Natural Circulation Phenomena and Passive Safety Systems in Advanced Water Cooled Reactors

17 - 21 May 2010

RELIABILITY OF PASSIVE SAFETY SYSTEMS THAT UTILIZE NATURAL CIRCULATION

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KEY WORDS

Safety, Reliability, Passive Systems, Natural Circulation

LECTURE OBJECTIVES

The objective of the course is to make the students more aware of the problem of the failure risk of passive systems that utilize natural circulation and to propose to them a methodology to evaluate the reliability of these systems and to carry out sensitivity analyses. Approaches to integrate passive system unreliability into event trees of Probabilistic Safety Assessment are proposed.

The lecture is divided in three parts:

T23: Statistical basis for Reliability Methods of Passive Systems, which presents methods for:

- Propagating the uncertainties through a T-H model
- Determining the important variables (sensitivity analysis)
- Quantifying the reliability of the system

T24: Pre-processing and Post-processing of RMPS data, which presents methods for:

- Identifying the sources of uncertainties
- Quantifying the sources of uncertainties
- Incorporating the reliability values into PSA

T31: Application of Reliability Methods for Passive Systems to the Isolation Condenser System

This lecture note is structured slightly differently, with four main Chapters:

- Chapter 1: Characterisation of the Passive system, Identification and Quantification of the sources of uncertainties, Sensitivity analysis
- Chapter 2: Propagation of uncertainty and Reliability evaluations of Passive Systems
- Chapter 3: Integration of Passive system unreliability in Probabilistic Safety Assessment
- Chapter 4: The RMPS methodology with two applications on the RP2 System and the Isolation Condenser System

The classification of the passive systems proposed by IAEA, is given in appendix.

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INTRODUCTION

Innovative reactor concepts make use of passive safety features to a large extent in combination with active safety or operational systems. Following the IAEA definitions [IAEA, 1991], a passive system does not need external input (especially energy) to operate. This is why it is expected that passive systems combine among others the advantages of simplicity, reduction of the need for human interaction, reduction or avoidance of external electrical power or signals.

Besides the open feedback on economic competitiveness, special aspects like lack of data on some phenomena, missing operating experience over the wide range of conditions, and the smaller driving forces as - in most cases - compared to active safety systems must be taken into account.

This remark is especially applicable to the passive systems of category B (i.e. implementing moving working fluid, following IAEA classification[1] [IAEA, 1991]) and in particular to the passive systems that utilize Natural Circulation (NC). These passive safety systems in these designs rely on natural forces (i.e. natural convection), to perform their accident prevention and mitigation functions once actuated and started. These driving forces are not generated by external power sources (e.g., pumped systems), as is the case in operating and evolutionary reactor designs. Because the magnitude of the natural forces, which drive the operation of passive systems, is relatively small, counter-forces (e.g., friction) can be of comparable magnitude and cannot be ignored as in usually done with pumped systems. Moreover, there are considerable uncertainties associated with factors on which the magnitude of these forces and counter forces depends (e.g., values of heat transfer coefficients and pressure losses). In addition, the magnitude of such natural driving forces depends on the specific plant conditions and configurations which could be existing at the time a system is called upon to perform its safety function. All these uncertainties affect the passive system thermal-hydraulic (T-H) performances. For this reason, the reliability of the systems that utilize NC must be assessed.

Up to now the codes used in T-H analysis, like RELAP, ATHLET and CATHARE, are best estimate codes, which don't account for the uncertainties in their input variables and in their inner correlation. But the value of the physical response predicted by a T-H computer code can significantly differ from its actual value because of:

a) Uncertainties in the value of code input variables.
b) Approximations and uncertainties in modelling the physics of the process.
c) Approximations in modelling the system geometry.

To assess the impact of uncertainties on the predicted response of the code, a large number of calculations is needed and if we consider all the sequences where the passive system studied is implied, this number of calculations can be prohibitive.

For all these reasons, it appeared necessary to use a specific methodology to assess the reliability of category B passive systems[2]

The methodology must answer the following questions:

- How to identify and quantify the sources of uncertainties and how to determine the important parameters?
- How to propagate the uncertainties through T-H codes and how to assess the unreliability (or failure probability) of the Passive System.
- How to integrate the Passive System unreliability in the Probabilistic Safety Assessment (PSA).

[1] See Annexe 1 for this classification.
[2] In order to simplify the notation, we will note thereafter Passive System instead of category B passive system.

As we can see, such a methodology will consist of several steps that will be described in this course. The first part of the methodology on the identification and the quantification of the sources of uncertainties and the determination of the important parameters will be presented in the Chapter II and the second part, propagation of uncertainties and unreliability assessment, in the Chapter III.

The third part, integration of Passive System unreliability in PSA will be described in the Part II of this course. The Part II contains also the description of two methodologies (RMPS and APSRA) and some examples of applications.

Before to start with the description of the different parts of the methodology, we have to underline the following point. Owing to the fact that the performance of a Passive System depends on the specific plant conditions and configurations which could be existing at the time a system is called upon to perform its safety function, the first-ever step should be the definition of the accidental scenarios in which the system will operate. Knowledge of these scenarios helps identify the specific failure criteria and relevant parameters and the specific quantification of uncertainties. The results obtained in the reliability and sensitivity analyses of the Passive System are thus specific to the different scenarios. A global evaluation of the Passive System is obtained by the integration of its unreliability in a Probabilistic Safety Assessment, in which all the sequences involving the Passive System are considered. This approach is preferred to conservative analyses consisting in evaluating the system reliability for the worst scenario considered or in integrating the larger variability of the uncertain parameters covering all the scenarios involving the system.

1. CHARACTERISATION OF THE PASSIVE SYSTEM, IDENTIFICATION AND QUANTIFICATION OF THE SOURCES OF UNCERTAINTIES, SENSITIVITY ANALYSIS

1.1 Characterisation of the System

The purpose of this analysis is to obtain information on the behaviour of the Passive System, in an accident scenario occurring during the life of the nuclear reactor and to identify the failure zones and conditions, if such exist. Therefore, the missions of the system, its failure modes and the failure criteria must be defined.

1.1.1 Mission of the system

The missions of the system are the goals for which the Passive System has been designed and located within the complete system. For instance, the mission of the Passive System can be the decay heat removal, the cooling of the vessel, the pressure decrease of the primary circuit… In some case, the Passive System can be designed to fulfil several missions in the same time or different missions depending on the considered scenario.

1.1.2 Failure mode

Due to the complexity of T-H phenomena and to the interaction between the Passive System and the overall system, it is not always obvious to associate a failure mode to the mission of the system. A qualitative analysis is often necessary so as to identify potential failure modes and their consequences, associated with the Passive System operation. A hazard identification qualitative method such as the FMEA (Failure Mode and Effect Analysis) can be used to identify the parameters judged critical for the performance of the Passive System and to help associate failure modes and corresponding indicators of the failure cause [Burgazzi, 2004]. This method can necessitate the introduction, in addition to mechanical components of the system (piping, drain valve, etc.), of a "virtual" component. This component is identified as natural circulation and is evaluated in terms of potential "phenomenological" factors (these include non-condensable gas build up, thermal stratification, surface oxidation, cracking, etc.), whose consequences can affect the Passive System performance.

1.1.3 Success/failure criteria

Knowledge on the system missions and failure modes allows the evaluation of the failure criteria. The failure criteria can be established as single-targets (e.g. the system must deliver a specific quantity of liquid within a fixed time) or as a function of time targets or integral values over a mission time (e.g. the system must reject at least a mean value of thermal power during the entire system intervention).

In these cases, the failure criterion is obtained by the comparison between the real performance of the system and the expected value of this performance. In some cases, it is better to define a global failure criterion for the whole system instead of a specific criterion for the Passive System. For instance the failure criterion can be based on the maximal clad temperature during a specified period. In this case, it will be necessary to have modelled the complete system and not only the Passive System.

1.1.4 Modelling of the system

Due to the lack of suitable experimental databases for Passive Systems in operation, the evaluation must rely on numerical modelling. The system analysis must be carried out with a qualified T-H system code and performing best estimate calculations. Indeed there is an increasing interest in computational reactor safety analysis to replace the conservative evaluation model calculations by best estimate calculations supplemented by a quantitative uncertainty analysis [Gläser, 2000]. Particularly in the present methodology where the objective is the evaluation of the reliability of the Passive System, it is important to calculate the Passive System

performance in a realistic and not conservative way. At this stage, calculations have to be carried out on the reference case with nominal values of the parameters characteristic of the system. The results have to be compared with experimental data if any exists. During the characterisation process, the modelling and the evaluation of the Passive System, new failure modes can be identified (such as flow oscillations, plugs phenomena due to non-condensable gases…) which must also be taken into account.

1.2 Identification of the Sources of Uncertainties

1.2.1 Overview on uncertainties related to Passive Systems

The overall uncertainty relating the T-H analysis of Passive Systems consists of two kinds of uncertainties:

- Uncertainties related to T-H code
- Uncertainties related to natural circulation performance.

Uncertainties may have different origins ranging from the approximation of the models characterizing any physical phenomena, to the approximation of the numerical solutions, to the lack of precision of the values adopted for boundary and initial conditions, and to the parameters that are the input to the phenomenological models. The amount of uncertainty that affects a calculation strongly depends upon the involved area in the technology and upon the sophistication of the adopted models and modeling techniques. This is even more relevant as far as natural circulation is concerned, due to the small engaged driving forces and the T-H phenomena affecting the system performance that may induce large uncertainties.

The potentially important contributors to uncertainty of the code results can be classified as:

- Approximations in modelling the physical process : for instance the treatment of a liquid steam mixture as an homogeneous fluid, the use of empirical correlation...
- Approximations in modelling the system geometry : simplification of complex geometry features and approximation of a three-dimensional system.
- The input variables: initial and boundary conditions, such as plant temperatures, pressures, water levels and reactor power, dimensions, physical properties, such as densities, conductivities, specific heats, and T-H parameters, such as heat transfer coefficients or friction factors.

More generally, we could say that uncertainty arises in three areas: 1) Definition of plant damage states, 2) Simulation of the problem, including event tree construction and models (computer codes) used to simulate the physical processes involved, and 3) data used to feed models. This is what classically has been considered as scenario, model and data uncertainty. Nevertheless, attending to the real origin of uncertainty, these types of uncertainty may be re-classified as aleatory and epistemic or lack of knowledge uncertainty [Devictor & Bolado, 2006].

1.2.2 Aleatory and epistemic uncertainties

Aleatory uncertainties are usually associated to parameters with some inherent variability. Aleatory uncertainties arise when an experiment is repeated several times under equivalent conditions and the results obtained differ from each other[3]. An example of a parameter affected by this kind of uncertainty is the time during which a battery provides the adequate electric current. In this case, variability comes from the set of physical and chemical processes involved.

Increasing the number of observations (experiments) doesn't make aleatory uncertainty to decrease, but will allow us to know with more accuracy the probability density function (*pdf*) followed by that parameter, i.e. the type of *pdf* and the parameters that characterise it. For the dice (the result is obvious and could be obtained without experiment), the *pdf* is discrete, with six modality having each a relative probability of 1/6. In the

[3] A very simple example is the dice. When we throw the dice (if the dice is perfectly made), the probability to obtain a given face is equal to 1/6. The uncertainty on the result is aleatory.

example of the battery, if that time follows a Weibull distribution, increasing the number of observations will allow us to know more accurately the minimum failure time, the standard deviation and the shape parameter.

Epistemic uncertainties are related to the existence of lack of knowledge about the problem. This type of uncertainty affects not only parameters, but also models and *pdf*s. A parameter will be affected by epistemic uncertainty when it isn't random, but we cannot measure it, either because it is impossible or because it is extremely expensive to do it. This type of uncertainty is completely different of the aleatory uncertainty. Parameters affected by aleatory uncertainty are fully described by their associated *pdf*s. In the case of parameters affected by epistemic uncertainty what we do is to characterise our lack of knowledge about the parameter, and we do it through *pdf*s. Those *pdf*s summarise our opinions about what values the actual value of the parameter could be close to more likely or less likely. Many parameters (coefficients) of models used in the area of Passive Systems are affected by lack of knowledge uncertainty; they are not random, but we are unable to know their values, so we have to use *pdf*s to characterise them[4].

Epistemic uncertainty affects models. Sometimes, there are several models to describe the behaviour of the system; some of them describe the behaviour of the system under some circumstances and others under other circumstances. Various methodologies have been developed in order to evaluate the overall uncertainty in the physical model predictions and some efforts have been made aimed at the internal assessment of uncertainty capability for T-H codes [Kwang-Il, 2000], [D'Auria, 2000].

However, in the methodology for Passive System reliability assessment,, the uncertainties pertaining to the code are not accounted for, focusing the attention on the uncertainties relative to the input parameters of the code, characteristic of the Passive System or of the overall system.

1.2.3 Identification of the relevant parameters

Among all the sources of uncertainties, the evaluation of the reliability of a Passive System requires the identification of the relevant parameters which really affect the accomplishment of the target of the system. This identification of the relevant parameters must be based on the Expert Opinion of the physical process and of the T-H codes.

A tool chosen here for this task could be the Analytic Hierarchy Process (AHP). The AHP [Saaty, 1980], [Zio, 2003] provides a qualitative and schematic, but rigorous and transparent, method to model the process under investigation. This method is composed of three major steps: the building of a hierarchy to decompose the problem at hand, the input of pairwise comparison judgments regarding the relevance of the considered parameters and the computation of priority vectors to obtain their ranking. The method is intended, in our case, to provide a transparent method to model the T-H process of the Passive System under investigation, so as to allow selecting the important parameters related to the target of the system design.

The building of the hierarchy is performed in three steps:

i. Define precisely the top goal of the hierarchy and place it at the top level;

ii. Build downward the hierarchy in different levels by putting in each level those factors directly influencing the elements of the level just above and directly influenced by the elements of the level just below. Directed arrows are placed to specify the interconnections between the elements;

iii. At the bottom of the hierarchy place the basic parameters.

In this way, decomposition of the problem (top goal) is provided, interactions are represented and one can describe how elements belonging to a level affect elements of the level just above.

The successive phase of the analysis is that of collecting pairwise importance judgments, through the following steps:

[4] In the case of the dice, the epistemic uncertainties could concern its quality of fabrication.

i. For each element of each level build a pairwise comparison matrix to assess the importance of the influence of the relevant entries of the level below in relation to the element under analysis. In other words, given an element *k* in level *s*, all entries of the level below, *s-1*, which affect *k* are compared in a pairwise fashion in terms of their relevance to *k*. The proper question in the pairwise comparison is of the form: 'Considering entries *X* and *Y* of level *s-1*, how much more important is entry *X* compared to entry *Y* with respect to their influence on element *k* of level *s*?'

The pairwise comparisons can be performed directly into a certain numerical scale or on a qualitative fashion and then translated into a numerical scale. Typically, the scale of integer numbers from 1 to 9 is used and the values a_{ij} obtained from the comparisons are organized in a square matrix. For example, performing qualitatively the comparison of element *A* with element *B*, the scale is the following:

1 = *A* and *B* equally important,
3 = *A* slightly more important than *B,*
5 = *A* strongly more important than *B,*
7 = *A* very strongly more important than *B,*
9 = *A* absolutely more important than *B*.

By definition an element is equally important when compared to itself so the principal diagonal of the matrix is filled with ones. The appropriate reciprocals, 1/3, 1/5,.....,1/9 are inserted where the reverse comparison, *B* vs. *A*, is required. The numbers 2, 4, 6, 8 and their reciprocals can be used to facilitate expressing judgments for intermediate situations. In other words, the expert is allowed to resort to the use of a measure of 4, for example, when making a comparison of *A* and *B* which he believes cannot exactly be expressed by 3 nor 5;

ii. For each element *k* in level *s*, determine the *potency* (*strength, priority, weight*) $w_{i(s-1),k(s)}$ with which each element *i* in level *s-1* affects element *k*. The priorities $\{w_{i(s-1),k(s)}\}$ of the entries *i* in level (*s-1*), relative to their importance for an element *k* in the next level (*s*) can be determined by solving an eigenvector problem. More precisely, it can be shown that given the matrix of pairwise comparisons for the element of interest, the principal eigenvector provides the vector of priorities, when normalized, and the maximum eigenvalue is a measure of consistency of the comparisons entered in the matrix. For complete consistency, the maximum eigenvalue, λ_{max} should be equal to the order of the matrix, *n*.

The level of consistency of a given pairwise comparison matrix can be measured by a parameter called consistency ratio, CR, defined as the ratio of the consistency index CI=$(\lambda_{max}-n)/(n-1)$ and the random index RI, which is the statistically averaged consistency index of randomly generated matrices of order *n* with entries artificially forced to be consistent. A consistency ratio of 0.10 or less is considered acceptable. Further clarification regarding the meaning of consistency in this case can be obtained with the aid of a simple example. Suppose that in doing pairwise comparisons of three elements *A, B, C,* the following relations are obtained: *A* is *4 times* more important than *B*; *A* is *8 times* more important than *C*. Then, in order for our matrix of comparisons to be consistent we expect that the judgment relating *B* and *C* will state that *B* is *twice* as important as *C*.

Although this matter might seem trivial, in practice it is quite common to encounter inconsistencies particularly when the order of the matrix is large;

iii. In case of large inconsistencies in a matrix, revise its entries by redoing the judgments on the individual pairwise comparisons or by forcing the values a_{ij} to be mathematically consistent by setting them equal to w_i / w_j, where $w_i = w_{i(s-1)k(s)}$, $w_j = w_{j(s-1)k(s)}$ are the priority values of elements *i* and *j* of level *s-1* in regards to their relevance to element *k* of level *s* immediately above. Indeed, in the ideal case of the comparisons being the results of exact physical measurements w_i , *i=1, 2, ...n*, the relation between the matrix entries and the weights is simply $a_{ij} = w_i / w_j$, for *i,j=1, 2, ...n*. Note that usually it is sufficient to revise the values for those entries a_{ij} for which $| a_{ij}-w_i/w_j |$ is maximum or for all the entries of the row for which the root mean square deviation of a_{ij} and w_i / w_j is maximum.

At this point, we can compute the priority ranking of each parameter. Once all the priority vectors are available, multiply them appropriately through the branches of the hierarchy (just like in a probability tree) to determine the overall weights of the bottom-level alternatives with regards to the previously defined top goal. For example, referring to a simple example of a three-level hierarchy with three alternatives at the bottom, as shown in Figure 1, if $w_{AY}=\{w_{AY1}, w_{AY2}, w_{AY3}, ...\}$ is the vector (1×n) of strengths of alternative *A* at the bottom level of the hierarchy and $w_Y=\{ w_{Y1}, w_{Y2} , w_{Y3} , ...\}^T$ is the vector (n×1) of priorities for the elements at the second level, then we multiply $w_A = w_{AY}. w_Y$ to get the weight of alternative *A* measuring its relative importance with regards to the top goal.

The major advantage of the pairwise comparison approach to quantification is the simple and intuitive way of expressing judgments on the relative importance of the different constituents of the hierarchy, and the possibility of checking for consistency in the judgment entries.

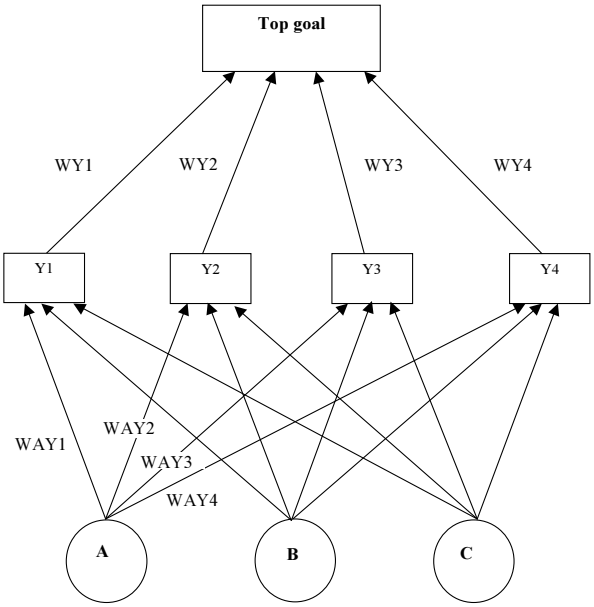

*Figure 1: **A simple three-levels hierarchy***

1.3 Quantification of the Uncertainties

A key issue in this methodology is the selection of the distributions for the input parameters. The main objective is that the selected distribution for each input parameter must quantify the state of knowledge and express the reliable and available information about the parameter. The choice of distribution may highly affect the reliability evaluations of the Passive System.

1.3.1 Different points of view for the quantification

Different points of view have to be considered for this quantification:

- The amount of data:

When the data on a parameter are abundant, statistical methods can be used such as the maximum likelihood method or the method of moments to adjust analytical density functions and different goodness-of-fit tests can be used (Chi square, Kolmogorov-Smirnov…) to find the best analytical fit to the data.

When the data are sparse or non-existent and this is generally the case when we consider the uncertainties affecting the Passive System performance, the evaluation of the probability functions of the uncertain parameters must be based on the expert judgments. Thus a subjective approach is used where the uncertainty is characterised as a *pdf* that shows the range of values where the actual value of the parameter may be and what parts of the range the analyst considers more likely than others. In the case where no preferences can be justified, a uniform distribution will be specified, i.e. each value between minimum and maximum is equally likely. These distributions are quantitative expressions of the state of knowledge and can be modified if there is new evidence. If suitable observations become available, they can be used consistently to update the distributions. As a consequence of probability distributions of input parameters, the computer code results also have a subjective probability distribution, from which uncertainty limits or intervals are derived.

- The dependence between the parameters:

If parameters have contributors to their uncertainty in common, the respective states of knowledge are dependent. As a consequence of this dependence parameter values can not be combined freely and independently. Instances of such limitations need to be identified and the dependencies need to be quantified, if judged to be potentially important. If the analyst knows of dependencies between parameters explicitly, multivariate distributions or conditional subjective probability distribution function may be used. The dependence between the parameters can be also introduced by covariance matrices or by functional relations between the parameters.

- The goal of the uncertainty analysis

It makes a difference if we are interested by the central tendency of a phenomena or its behaviour in extreme conditions. In the case of a Passive System, if we study the variation of its performance around its mean, an accurate knowledge of the *pdf* of the input parameters around their means will be sufficient. On the other hand, if we are looking for a very small value of the failure probability of the Passive System obtained in a infrequent event, it will be necessary to have a good knowledge of the tails of the *pdfs* of the input parameters.

1.3.2 Different scenarios depending on the amount of available data

The practical approach can be summarised through three scenarios [Devictor & Bolado, 2006]:

Scenario 1. If a lot of experience and data are available, the classical estimation techniques are generally used. Tools to describe sample dispersion are taken from statistics; however, their effectiveness is a function of the sample size. References [Saporta 90] describe methods that may be used to adjust a probability distribution on a sample, and then verify the adequacy of this adjusted distribution in the central part of the distribution.

Scenario 2. If data are not so much abundant, expert opinion may be used to obtain modelling hypotheses. The Bayesian analysis is used to correct a priori values established with the expert opinion as a function of observed events.

Scenario 3. If no data are available on a parameter, it is entirely reasonable to refer to expert opinion in order to model uncertainty associated with the parameter, and then to put the information obtained as a probability distribution.

"Scenario 1", where a large enough sample is available, i.e. the sample allows "characterisation of the relevant distribution with a known and adequate precision", begs the following questions:

Question 1: Is the selected distribution type relevant and justifiable? From the various statistical models available, what would be the optimal distribution choice?

Question 2: Would altering the distribution (all other things being equal) entail a significant difference in the results of the application?

Question 3: How can uncertainty associated with sample representativeness be taken into consideration (sample size, quality, etc)?

With respect to question 2, a study examining sensitivity to the used probability distribution provides information. There are two methods available for the sensitivity study:

- Moment identification-type method where it is assumed that the distribution changes while the first moments are preserved (mean, standard-deviation especially).
- Frequential or Bayesian approach where the sample is redistributed to establish the reliability data distribution parameters.

Another method is to take the uncertainty associated with some distribution parameters into consideration by replacing the parameters' deterministic value by a random variable.

Conventional criticism concerning the statistical modelling of a database concerns:

- Difficulty in interpreting experience feedback for a specific application;
- Database quality, especially if few points are available;
- Substantiation of the built probabilistic model.

The probabilistic modelling procedure should attempt answer these questions. If it is not possible to define a correct probabilistic model, it is obvious that, under these circumstances, the quantitative results in absolute value are senseless in the decision process. However, the probabilistic approach always allows results to be used relatively, notably through:

- a comparison of the efficiency of various solutions from the standpoint of reliability, availability, for example,
- or classification of parameters that make the biggest contribution to the uncertainty associated with the response in order to guide R&D works in order to reduce said uncertainty.

In fact, the probabilistic model reflects the level of knowledge of variables and models, and confidence in said knowledge. By means of sensitivity studies, this approach allows the impact of the probabilistic model choice on risk to be objectively assessed. Furthermore, in the event of new information impugning probabilistic modelling, and consequently the fractiles of a variable, the Bayesian theory, that combines objective and subjective (expertise) data, allows the probabilistic model and results of the probabilistic approach to be updated stringently.

The adjustment of a probability distribution and subsequent testing of the quality of said adjustment around the central section (or maximum failure probability region) of the distribution constitute operations that are relatively simple to implement using the available statistical software packages (SAS, Stagraphics, Splus, Statistica, etc.), for conventional laws in any case. However, the interpretation and verification of results still requires the expertise of a statistician. For example, the following points:

- the results of an adjustment based on a histogram is sensitive to the intervals width;
- the maximum likelihood or moment methods are not suitable for modelling a sample obtained by overlaying phenomena beyond a given limit of an observation variable;
- moment methods assume estimations of kurtosis and symmetry coefficients that are only usually specified for large bases (with at least one hundred values for kurtosis);
- some statistical tests are based on asymptotic results;
- in the Bayesian approach, the distribution selected *a priori* influences the result. Furthermore, the debate concerning whether or not the least informative law should be used has not been concluded.

In industrial studies, the characteristics of databases must be taken into consideration because the implementation of an adjustment can be difficult; for example the following frequently encountered scenarios:

1 the sample size is small and therefore asymptotic results need to be handled cautiously as well as approximations of more or less valid moments of an order greater than two;
2 sample data values are measured with an uncertainty;
3 sample homogeneity is not verified (mixture of samples taken from different populations, overlaying of phenomena, etc.);
4 if the area of interest is a distribution tail, it should be noted that the statistical theory and above all associated tools are less developed.

1.3.3 Classical inference methods

Classical inference methods are based on the assumptions of having a random sample and knowing the probability model from which the data come from. The most widely used methods are Maximum Likelihood method and the Method of Moments. The main shortcoming of all these methods is the need of sample sizes not easily obtainable under the restrictions of a complicated engineering facility in order to get good quality estimates.

Method of moments

This is probably the oldest inferential method to estimate the parameters of a *pdf*. K. Pearson developed the method of moments by the end of XIX century. The idea is quite simple. It consists in taking as an estimator of a parameter its equivalent sample quantity. So, the sample mean is the estimator for the mean, the sample variance is the estimator for the variance and so on.

Maximum Likelihood method

Maximum Likelihood Method is the most widely known and most powerful estimation method in the classical context. Let us assume that we wish to study a random variable X (parameter affected by uncertainty) whose type of distribution function $f(X|\theta)$ is known, but whose parameter θ is unknown. In order to estimate θ, we take a random sample $\vec{X} = (X_1, X_2, ..., X_n)$, which is supposed to be a random vector whose components are independent and identically distributed, so that its joint probability density function is

$$f(\vec{X}|\theta) = f(X_1, ..., X_n|\theta) = \prod_{i=1}^{n} f(X_i|\theta). \quad (1)$$

The objective is to determine what value, among the infinite values that θ could take, makes more likely obtaining the sample actually obtained. So, the problem is to find the value of θ for which function (1) gets its maximum value. In order to see the change in the perspective to look at the problem after getting the sample, expression (1) is usually written as

$$L(\theta|\vec{X}) = f(\theta|\vec{X}) = f(\theta|X_1, ..., X_n) = \prod_{i=1}^{n} f(X_i|\theta),$$

which means that, after sampling, the probability density function of the sample vector is changed into a function of the unknown parameter θ. *'L'* stands for 'Likelihood'. From a practical point of view, the function whose maximum is actually computed is not L, but its logarithm $l(\theta|\vec{X})$ (Both functions get a maximum in the same point since the transformation to get one from the other one is a monotonic transformation).

Selection of a distribution in practice

In practice, the criteria relevant to selecting a probabilistic model for a random variable would seem to be:

- use a family compatible with the physical properties (bound value, symmetrical or not, exhibiting exponential decay, etc),
- the result of data adjustment,
- distribution not rejected by statistical tests,
- the selection of a distribution that is least "informative" with respect to data or available information (i.e. the introduction of too many hypotheses is avoided).

Statistical tests are used to decide whether or not the adequacy of a selected distribution should be rejected a priori with a confidence level. For samples that are not very homogeneous or small in size, several distributions are frequently accepted to represent the sample or, on the contrary, no conventional distribution can be accepted.

Some criticism

To select a specific distribution, the following methods can be applied:

- either rely on selection based on expert opinion or current practice,
- or compare confidence levels taken from various statistical tests for each accepted distribution and select the distribution associated with the highest confidence level,
- or further analyze adequacy in the zone of interest and select the distribution most graphically suited in the region of interest (for example a distribution tail for a reliability problem).

When the graphical adjustment of a sample for various distribution features a "break", the rejection of all conventional distributions may be noted. For example, in Figure 2, graphically adjustment seems very good in the region of highest probability and bad at the distribution tail. These breaks can be explained by the fact that the sample is a mixture of several homogeneous sub-samples or is taken from a process combining several phenomena. Under such circumstances, if the application is a reliability calculation with an influential distribution tail, there is no possible choice: the model must be rejected. For an uncertainty propagation calculation, this model can be accepted if the number of simulations is not too high (overly frequent simulations in the distribution tail should be avoided).

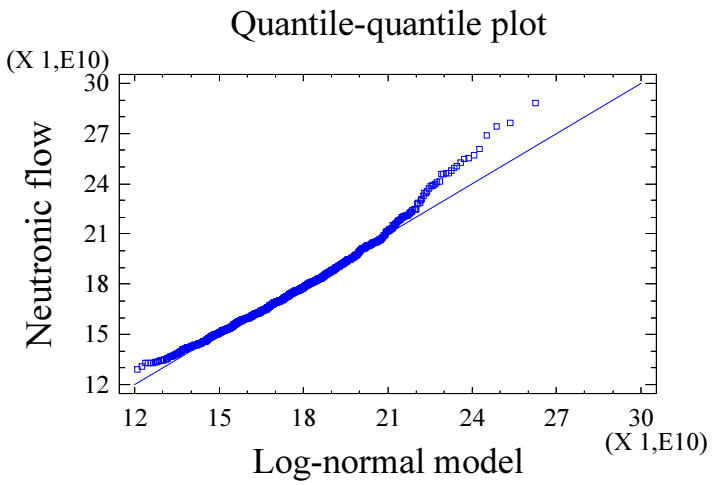

Figure 2: ***Example of a sample from a process combining several phenomena***

Practical solutions to obtain a general model include the use of mixture models and definition of a distribution through sticking two densities side by side. A mixture model is assumed to be a weighted sum of random variables, for example, in the case of two Gaussian components:

$$G_{\theta,\gamma} = p \times \Phi_{\mu 1,\sigma 1} + (1\text{-}p) \times \Phi_{\mu 2,\sigma 2}$$

where this distribution is a mixture of two normal distributions, means $\mu 1$ and $\mu 2$ respectively, standard deviations σ_1 and σ_2 respectively.

The principle of sticking two densities side by side is to build a distribution of a random variable X by rejoining two distribution functions, marked F on the left and H on the right of a threshold *u*. Hence, for an observation *x* of X, it may be written:

$$\text{if } x \le u \text{ then } X \to F \qquad \text{and} \qquad \text{if } x > u \text{ then } X \to H$$

The two laws F and H are therefore truncated. For every value *a* of [0; 1], it is possible to build a continuous density in *u* based on the value *a* at this point.

$$G_a(x) = a\frac{F(x)}{F(u)} \qquad \text{if } x \le u$$

$$G_a(x) = 1 - (1-a)\frac{1-H(x)}{1-H(u)} \qquad \text{if } x > u$$

where G_a is the distribution resulting from sticking two densities side by side.

Quality of fit in the distribution tail

In the reliability study, special interest was focused on modelling the distribution tail and hence estimation of extreme quantiles in the distribution tail, that is to say a quantile generally found outside observations. This estimation constitutes an extrapolation problem. The statistics on distribution tails is mainly characterized by a search in the "dark"; in fact, points are generally lacking or scarce in this region. In order to be able to deduce information on this zone, a position not too far from the available points must be selected. The following practical questions arise:

Question No. 1: Is it possible to have an approximation of the distribution tail distribution?

Question No. 2: If yes, what is the confidence level? (or, from a practical standpoint, up to what point do available values allow extrapolation in the dark?)

Question No. 3: How might it be possible to define complementary tests or complementary calculations to improve information quality?

The problem concerning the statistical treatment of extreme values is intrinsically a non-parametric statistical problem, however, based on tools currently available, it can only be treated parametrically. Thus, a problem specific to distribution tails is transformed into an extrapolation problem based on the hypothesis that a model obtained from an analysis in the maximum probability region is valid in "extreme" regions. Several statistical tests to establish whether or not an extreme quantile can be estimated based on an accepted distribution in the maximum probability domain have been proposed ; they use the Generalized Pareto Distribution or the Exponential Tail test, that is adapted for probability distributions exhibiting exponential decay in the Gumbel domain, for example normal, log-normal, exponential, Weibull distributions.

1.3.4 Bayesian inference methods

Bayesian interpretation of probability makes Bayes' formula a powerful tool to update degrees of belief when new information is available about an event or a proposition. Let *H* be the knowledge of an expert, and let $\{z_i\}_{i \in I}$ be a partition of the sample space of events. The Bayesian probability provided by the expert for an event z_k is $P(z_k|H)$. The acquisition of a set of new evidence *H'* induces a change in the probability given by Bayes' formula

$$P(z_k|H,H') = \frac{P(H'|H,z_k)\cdot P(z_k|H)}{P(H'|H)}, \tag{2}$$

where $P(z_k|H,H')$ is the 'a posteriori' probability of z_k, $P(z_k|H)$ is the 'a priori' probability of z_k and $P(H'|H,z_k)$ is the likelihood of evidence conditional on knowledge H and the occurrence of event z_k. $P(H'|H)$ is the probability of new evidence conditional on previous knowledge, which may be considered a normalising factor, since the sum of expressions like (2) over the whole partition must be 1 (equivalently, the sum of the a posteriori probabilities of all the partition elements must be 1). That probability is given by

$$P(H'/H) = \sum_i P(H'/H,z_i)\cdot P(z_i/H),$$

and may be ignored in any intermediate computation. So, equation (2) may be written as

$$P(z_k/H,H') \propto P(H'/H,z_k)\cdot P(z_k/H),$$

which means that the a posteriori probability is proportional to the a priori probability and to the likelihood of evidence.

An example of a priori distribution, a posteriori distribution and likelihood function for the example of estimation of a Normal (Gaussian) random variable is presented on Figure 3. Two remarkable results are obtained from (2). If the a priori probability of an event is zero, the a posteriori probability will remain zero, even though the evidence against it could be very strong. So, much care should be taken in order not to provide a priori probabilities. Null a priori probabilities should be avoided, unless total evidence of the impossibility of the events or propositions under study is available. In English literature this is called Cromwell's statement. The second result is related to the existence of strong evidence. In that case, likelihood will be completely dominant and the a priori probability will be almost irrelevant (a posteriori probability and likelihood will be almost equal). This is the case of large sample sizes, for which relative frequencies and Bayesian probabilities will be almost equal.

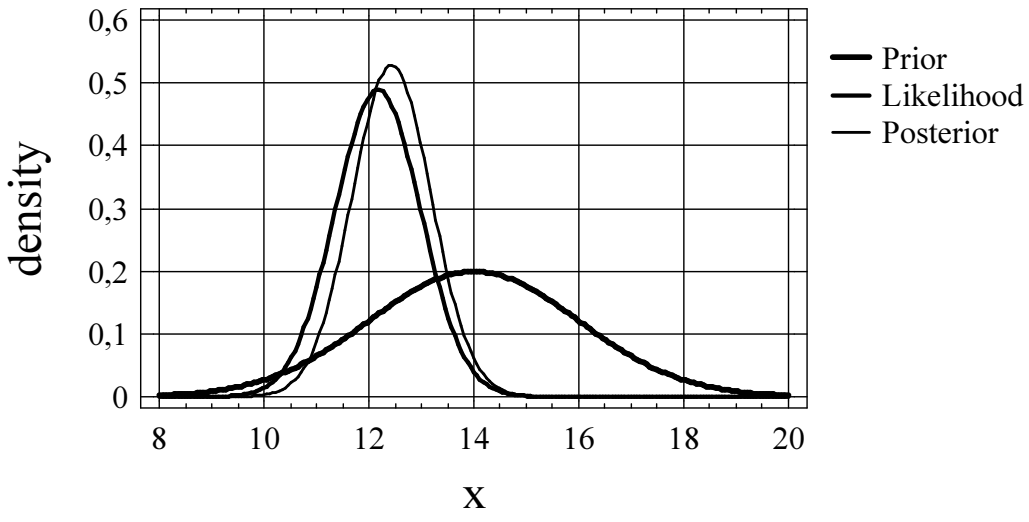

Figure 3. ***A priori distribution, a posteriori distribution and likelihood function for the example of estimation of a Normal (Gaussian) random variable.***

The validity of this estimation method comes from: 1) its consistency with the way human beings learn from experience, and from 2) its convergence to the results provided by the Maximum Likelihood Method when the sample size increases.

1.3.5 Expert judgment

The use of Expert Judgment (EJ) techniques is often unavoidable in Passive System analysis due to the lack of data about many of the involved phenomena. In some cases, it is almost impossible, from a physical point of view, to get the data we need to feed our computer codes, in other cases the cost of getting them is so high that only a few of them may be obtained. In what follows, we will list the steps of a generic EJ protocol based on most widely known protocols available in scientific literature (see for example NUREG-1150 and [Devictor & Bolado, 2006]):

1. Selection of team project
2. Definition of the questions to be studied
3. Selection of experts
4. Training
5. Tasks definition
6. Individual experts' work
7. Elicitation of experts' opinions
8. Analysis and aggregation of results
9. Documentation

Comprehensive information may be obtained in references such as [Mengolini 05] and [Simola 05].

Selection of team project

Team project consists of analysts and generalists. Analysts are in charge of organising the steps of the protocol, so, they should have a sound background in Probability and Statistics Theory, in Knowledge psychology and in elicitation techniques. Additionally, they should be skilful in working with people, since they will have to extensively interact with experts. The number of analysts needed shall depend on the extent and scope of the EJ application, though usually a couple of analysts will be enough, even for large applications. Generalists provide help to the analysts in all subjects related to the specific area of knowledge of the problem to be solved. They should be able to help experts when decomposing a problem and they should be skilful at getting information sources as needed. So, they should have a good general knowledge about the problem at hand, though they do not need to be leading experts in that field. The institution interested in the study usually provides the generalists.

Definition of the questions to be studied

Once the team project has been made, analysts and generalists must define the questions to be evaluated by the experts. The starting point for any question to be solved is usually vague. It is completely necessary to arrive at a *complete definition* of the parameters whose uncertain we want to characterise. *Complete definition* of a parameter means the full definition of the parameter, the initial conditions to evaluate it and any other implicit hypothesis under the initial conditions. The final definition should be extremely clear and accurate, with no ambiguity. It should have no problem when undergoing the clairvoyant test.

The complete definition of the question includes the way the experts should provide their answers. Since most of the uncertainties are characterised as Bayesian Probabilities, experts should provide their assessments of uncertainty through this kind of probabilities. So experts should provide probability distributions, either discrete or continuous.

After the full definition of the question, a list with all relevant sources of information should be done. Potential decompositions of the parameters could be done. The list of references to be considered in the list must show the actual state of knowledge in that area, but independence and reliability of the sources should always be kept in mind.

When experts are expected to use computer codes for their assessment, the project team should foresee the potential training of experts in uncertainty propagation techniques (sampling, response surfaces, estimation, order statistics, etc.).

Selection of experts

The only objective of this phase is to select the most qualified experts to perform the assessment. Qualified Experts are those that:

1. Have the necessary knowledge and experience to perform the assessment,
2. are willing to participate in the assessment, and
3. do not have important motivational biases.

The first step to get the final list of experts is to start with a large list of potential experts. That first list could be based on the opinion of the generalist plus a thorough search in the scientific literature about that area. A screening should be done checking the three points in the list above. If necessary, interviews should be done to check those conditions, mainly the third one. After performing the screening, a shorter list should be obtained, from which the final selection of experts will be done. In order to arrive at the final list, two criteria should be taken into account: The number of experts to assess each question should preferably be between three and five (based on Bayesian combination of opinions' criteria) and the experts should have as much diversity as possible (different background, different types of institutions, etc.).

Training

The objective of this phase is to let experts know normative aspects of EJ elicitation processes. This main objective may be decomposed as the following sub-objectives:

- Motivate experts to let them provide rigorous assessments.
- let them remember basic concepts of Probability and Statistics,
- provide them training in the assessment of Bayesian probabilities, and
- let them be aware of basic issues related to knowledge biases.

During the motivation phase the experts must get information to point out the importance of the work they are going to do. Firstly, the team project explains to the experts the study frame where their opinions will be used, stressing the part of the study where their opinions are relevant. Secondly, the necessity of EJ will be explained, letting them be aware of the concept of *Lack of Knowledge Uncertainty*, and how it links to them. Thirdly, the project team will let them know that the key issue is not to predict a single value of each parameter under study, but characterising their uncertainty, allowing others to know the actual state of knowledge in that area.

After remembering basic Probability and Statistics concepts, the experts get some training about assessing Bayesian probabilities, which includes: Accurate definition of questions to be assessed (making explicit implicit hypotheses, showing well non-well posed questions), decomposition as a way to simplify assessments (use of influence diagrams, event trees and uncertainty propagation techniques) and adequate evaluation of different evidences in order to assess probabilities (use of Bayes' theorem and concepts of independence and reliability of information sources).

The last part of the training session is dedicated to explain Knowledge biases to the experts in order to teach them to provide more reliable opinions, i.e.: representativity, availability and anchor and adjustment. Experts

should be informed of the hazard of being overconfident. A calibration exercise could be appropriate. The whole training session should not take more than one morning.

Tasks definition

This step is done through an interactive session of the team project and the experts. The issue at hand is to explain to the experts, in a detailed way, the questions to be assessed and to make a schedule of the activities to be developed by each expert. All the work developed by the team project during the *Definition of the Problem* phase should be used now. The session should start with a presentation by the generalist of the parameters to be assessed, including all relevant sources of information previously identified. Experts should provide their own view of the problem and the definition of the parameters, pointing out, if needed, further information sources, computations to be made, etc. The result of this session, eventually, would be a refined definition of the parameters under study. Common definitions to all the experts should be agreed.

The second step in this meeting is to study the possible ways to decompose each parameter. The team project should provide a seminal decomposition that should be discussed with the experts. The objective is to help the experts to develop their own decompositions. Decompositions could be quite different from one expert to another one. Expert will have to assess uncertainties of variables in the lowest levels. The analysts will usually do its aggregation. This is the point to introduce propagation of uncertainties concepts to the experts and to let them know all the potential variety of tools that the analysts could provide them to pre-process and post-process probabilistic runs of computer codes or of the simple decomposition model developed by experts.

Individual experts' work

Experts develop their analysis during this phase, according to the schedule agreed in the previous step. Each expert will write by the end of this period a report summarising the main hypothesis and procedures used during his/her work, the conclusions achieved and, if he/she wishes, a preliminary assessment of uncertainties. Whenever needed during this period, the team project should be available to each expert in order to provide statistical support or to solve any doubt about the parameters to be assessed.

At the end of this phase, the team project organises a meeting with all the experts. Each expert presents his/her work and the conclusions achieved. This meeting allows each expert to get some hints about alternative ways to tackle the problem.

Elicitation of experts' opinions

The elicitation of each expert opinion's is individual and should be done in a quiet environment, if possible without interruptions. It is convenient to have the presence of an analyst and a generalist, in addition to the expert. In a systematic way, the analyst gets the opinion of the expert for each parameter, asking for supporting reasons whenever necessary. The role of the generalist in this session is to provide additional information when needed, to provide general support and to audit the session in order to avoid irregularities (bias induction, etc.). Whenever needed, the analyst could ask questions in a different way to check potential inconsistencies. The session must be recorded as much as possible (tape recorders, video or extensive hand annotations).

The techniques used to help the expert when assessing uncertainties are quite standard: quantile assessment for continuous variables and probability estimations for discrete variables (direct or indirect methods); in the case of experts with some skills in probability other techniques like direct parameter assessment or drawings are acceptable.

Analysis and aggregation of results

Assessments provided by experts are studied in this phase. The objective is to check that there are not important biases and the logic correctness of their rationale. If biases and logic faults are not present in expert's

assessments, next step is to check if individual opinions may be aggregated to get a unique distribution for each parameter.

Before aggregating individual distributions, one condition should be checked. It is related to the overlap between distributions of different experts. If the distributions do not overlap, it means that essentially the experts disagree. In that case aggregation should be avoided. Under these circumstances a reconciliation session could be of help. An analyst should lead the session and should organise it according to the following steps:

1. Exposition of different opinions.
2. Identification of differences.
3. Discussion about the reasons for each original assessment.
4. Discussion about the different sources of information used.
5. Re-elaboration of individual opinions in posterior elicitation sessions or joint assessment (through consensus) of a common distribution, if agreed by experts.

In the case that a consensus distribution is obtained, that is the final step (before documentation). If further elicitation sessions are needed, the consistency of the opinions is checked again and aggregation is done if acceptable overlap is achieved. Otherwise, the project team should choose what opinions could be aggregated as main opinion of the group (after aggregation), and what opinions should be left as an alternative to perform sensitivity analysis. The main strategies for aggregation are the following ones:

1. Linear combination.
2. Log-linear combination.
3. Bayesian combination.

Documentation

Documentation of the application must be as complete as possible, including results and description of the ways to obtain them. The contents of the documentation should follow the order of application of the procedure, recording, in each step, *what* has been done, *why* it has been done, *how* it has been done and *Who* has done it. In order to achieve this degree of documentation, a schedule of standardised documentation activities should be made for each phase. It should always be completely to the reader what is a result assessed by an expert and what results are the outcomes of an aggregation, sensitivity analysis or any other analysis not provided explicitly by an expert.

1.4 Sensitivity Analysis

1.4.1 Objectives and preliminary concepts

An important feature of the methodology is to evaluate by sensitivity measures the importance of parameter uncertainties on the uncertainty of the Passive System performance. These sensitivity measures give a ranking of input parameters. This information provides guidance as to where to improve the state of knowledge in order to reduce the output uncertainties most effectively, to steer research and development efforts, or better understand the modelling or to obtain a good confidence in the results (potential large uncertainties). If experimental results are available to compare with calculations, sensitivity measures provide guidance where to improve the models of the computer code.

Sensitivity is becoming increasingly widespread in many fields of engineering and sciences, encompassing practically all of the experimental data processing activities as well as many computational modelling and process simulation activities. There are many methods, based either on deterministic or statistical concepts, for performing sensitivity. However, despite of this variety of methods, or perhaps because of it, a precise, unified terminology, across all methods, does not seem to exist yet. This section is devoted to present a brief review of the topic, the basic concept and the classification of the methods. After this, some specific methods for statistical global analysis will be explained. This review of methods [Ibanez, 2007] is based on the following references: [Saltelli, 2000], [Saltelli, 2004] and [Cacucci & D'Auria, 2006].

Sensitivity Analysis (SA) is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation. It aims to ascertain how the model depends upon the information fed into it, upon its structure and upon the framing assumptions made to build it. This information can be invaluable, as:

- Different level of acceptance (by the decision-makers and stakeholders) may be attached to different types of uncertainty.
- Different uncertainties impact differently on the reliability, the robustness and the efficiency of the model.

Originally, SA was created to deal simply with uncertainties in the input variables and model parameters. Over the course of time the ideas have been extended to incorporate model conceptual uncertainty, i.e. uncertainty in model structures, assumptions and specifications. As a whole, SA is used to increase the confidence in the model and its predictions, by providing an understanding of how the model response variables respond to changes in the inputs, be they data used to calibrate it, model structures, or factors, i.e. the model independent variables. SA is thus closely linked to uncertainty analysis (UA), which aims to quantify the overall uncertainty associated with the response as a result of uncertainties in the model input.

A large number of sensitivity analysis methodologies are available in the literature. Each methodology has its advantages and disadvantages. The choice of the method to adopt to perform a SA experiment on a model is therefore a very delicate step that depends on a number of factors: the properties of the model under study (linearity, additivity, monotonicity,…), the number of input factors involved in the analysis, the computational time needed to evaluate the model, and, last but not least, the objective of the analysis.

Before introducing the classification of the methods existing and describing in detail some of them, let introduce some basic terminology for the formulation of the problem in mathematical terms.

Let consider a model defined by the following relation:

$$f : R^p \rightarrow R$$

$$X = (x_1, ..., x_p) \mapsto Y = f(X)$$

where Y is the output of the model, called the *response*, and X are p independent inputs, namely, the *factors* of the response.

As was already mentioned, the notion of sensitivity or influence of one factor into the response is not a concretely defined concept, as it is usually mentioned in a variety of different context and methods. However, there are clearly two ideas that have to be distinguished, which indeed, are used to classify the methods in the two main families as will be exposed lately: the concepts of local and global sensitivity.

The objective of *local analysis* is to analyze the behaviour of the system response locally around a chosen point or trajectory in the combined phase space of parameters and state variables. On the other hand, the objective of *global analysis* is to determine all of the system's critical points (bifurcations, turning points, response maxima, minima, and/or saddle points) in the combined phase space formed by the parameters and dependent (state) variables, and subsequently analyze these critical points by local sensitivity and uncertainty analysis.

Local sensitivity

The concept of local sensitivity is defined in terms of the first derivatives of the system's response. They allow obtaining the slopes of the model's response at a given set of nominal parameter values. Thus, the exact slopes are provided by the local partial functional derivatives of the response with respect to the model parameters:

$$\partial Y / \partial X_i$$

This concept is what is called in the literature the intrinsic sensitivity or local sensitivity, and was the first approach to obtain a definition of the sensitivity of a model.

Local sensitivities can be computed exactly only by using deterministic methods that involve some form of differentiation of the system.

Global sensitivity

The global sensitivity refers to the influence of one factor over the response. Rather than being how much does the response fluctuate when the input make a small and local variation, it is the part of the uncertainty of the response explained by this factor. It is, indeed the conjunction of the input uncertainty and the model sensitivity to the factor.

This influence or global sensitivity of the factor into the response is usually expressed by means of the variance[5]. Indeed, an indicator of the importance of an input X_i could be based on what would the variance of Y be if we fixed X_i to its true value, $V(Y \mid X_i = x_i)$. This quantity is the conditional variance of Y given $X_i = x_i$. In most cases the true value of X_i is not known. To solve this problem, the average of this conditional variance under all possible values is studied, $E\left[V(Y \mid X_i = x_i)\right]$.

The prediction variance can be written as follows thanks to a general property of the decomposition of the variance:

$$V(Y) = V\left(E\left[Y \mid X_i\right]\right) + E\left(V\left[Y \mid X_i\right]\right)$$

The term $V\left(E\left[Y \mid X_i\right]\right)$ can be used as an indicator of the importance of X_i on the variance of Y, or of the sensitivity of Y to X_i as it measures the total variation of Y as X_i varies. This quantity named the variance of the conditional expectation (VCE) has an important property: the greater the importance of X_i, the greater is $V\left(E\left[Y \mid X_i\right]\right)$. The second term, $E\left(V\left[Y \mid X_i\right]\right)$, is a residual term that measures the remaining variability in Y due to other unobserved inputs or other unknown sources of variation when the X_i are fixed.

In order to have all indicators on the range 0 to 1, a normalization of the sensitivity index is performed, so that we obtain the final global sensitivity index defined by:

$$S_i = \frac{V\left(E\left[Y \mid X_i\right]\right)}{V(Y)}$$

This index is named first-order sensitivity index by Sobol, correlation ratio by McKay or importance measure by other authors. If there is no influence of X_i on Y, they would be independent, so $E(Y \mid X_i) = E(Y)$ and therefore $S_i = 0$. In the opposite case, the strongest dependency that can appears between X_i and Y would mean that $E(Y \mid X_i) = Y$, leading to a index with value $S_i = 1$. We will come to this topic later on when describing the variance-based methods.

1.4.2 Qualitative methods for sensitivity analysis

Sometimes the lack of operational experience and significant data concerning the Passive System performance forces the analysis to be performed in a **qualitative** way aimed at identifying for each failure mode, both the level of uncertainty associated with the phenomenon and the sensitivity of failure probability to that phenomenon [Burgazzi, 2000].

[5] Although some other authors propose another approach based on the entropy of the response $H(Y) = -\int f(Y)\ln(f(Y))dY$

For example, even if a phenomenon is highly uncertain (because of deficiencies in the physical modelling) this may not be important for the overall failure probability. Conversely a phenomenon may be well understood (therefore the uncertainty is small) but the failure probability may be sensitive to small variation in this parameter. The grading scheme is given in table 1.

Table 1: ***Grade Rank for Uncertainty and Sensitivity.***

	Grade	Definition
Uncertainty	H	The phenomenon is not represented in the computer modelling or the model is too complex or inappropriate which indicates that the calculation results will have a high degree of uncertainty.
	M	The phenomenon is represented by simple modelling based on experimental observations or results.
	L	The phenomenon is modelled in a detailed way with adequate validation.
Sensitivity	H	The phenomenon is expected to have a significant impact on the system failure
	M	The phenomenon is expected to have a moderate impact on the system failure
	L	The phenomenon is expected to have only a small impact on the system failure

An example of qualitative point of view analysis as outcome of expert judgment assessment is given in the table 2, regarding the failure modes of the natural circulation.

Table.2: Failure ***Modes related Uncertainty and Sensitivity.***

TOPIC	UNCERTAINTY	SENSITIVITY
Envelope failure	L	H
Cracking	L	L
Non-condensable gas	H	H
Thermal stratification	H	H
Surface characteristics modification (e.g. oxidation)	M	L

From the above qualitative table it seems that only for the structural failure modes there is a deep knowledge of the relative phenomenology. In the other cases the level of uncertainties is high or medium and in every case an effort must be devoted for their quantification. The sensitivity grades are the results of only a qualitative analysis: the natural circulation failure is very sensitive either to the loss of primary boundary or to the T-H phenomena arising during the system performance.

It's clear that the worst case is characterized by "high" rankings relative to either sensitivity or uncertainty (see e.g. non-condensable gas and thermal stratification), making the corresponding phenomena evaluation a critical challenge.

1.4.3 Quantitative methods for sensitivity analysis

Classification of the methods

The methods for sensitivity and uncertainty analysis are based on either *deterministic* or *statistical* procedures. In principle, both types of procedures can be used for either local or for global sensitivity and uncertainty analysis, although, in practice, deterministic methods are used mostly for local analysis while statistical methods are used for both local and global analysis.

The most currently used methods existing in the literature for both approaches are now presented.

Deterministic methods for Sensitivity Analysis:

The main modern deterministic methods for *local* sensitivity analysis are:

- Brute-force method
- Direct method (including the decoupled direct method)
- Green's Function method
- Forward Sensitivity Analysis Procedure (FSAP)
- Adjoint Sensitivity Analysis Procedure (ASAP)

Except for the brute-force method, which approximates the respective sensitivities by using model recalculations combined with finite differences, the other deterministic methods mentioned above employ various differentiation procedures to obtain the exact values of the local sensitivities.

On the other hand, for *global* deterministic methods, it appears that the only genuinely global deterministic method for sensitivity analysis, published thus far, is the global adjoint sensitivity analysis procedure (GASAP).

Statistical methods for Sensitivity Analysis:

For statistical sensitivity analysis methods, we can distinguish the following categories of procedures:

- Sampling-based and regression-based methods
- First and second-order reliability algorithms (FORM and SORM)
- ANOVA based methods
- "Screening" design methods

It is emphasized that all statistical uncertainty and sensitivity analysis procedures first commence with the "uncertainty analysis" stage, and only subsequently proceed to the "sensitivity analysis" stage; this path is the exact reverse of the conceptual path underlying the methods of deterministic sensitivity and uncertainty analysis, where the sensitivities are determined prior to using them for uncertainty analysis.

By comparison to deterministic methods, statistical methods for uncertainty and sensitivity analysis cannot yield exact values of the local sensitivities, but are relatively easier to develop and use. The growth of the computer and code capacity has increased its use in the latest times.

In this report indeed only the global statistical SA methods will be considered, concretely, a type of screening method, namely Morris method, and the regression-based and ANOVA-based methods.

Global Statistical Sensitivity Analysis methods

This section is devoted to explain the general ideas of some of the previously mentioned families of global statistical methods. More detail explanation of the methods should be founded in [Saltelli, 2000], [Saltelli, 2004] and [Cacucci & D'Auria, 2006]. Recall that global SA is used to quantify the contribution of each input parameter to the response variability by taking into account all the variation range of the inputs and trying to apportion the output uncertainty to the uncertainty of the input factors.

Regression-based methods for linear and monotonous models:

These techniques are based on using a sample obtained either by same sampling method (random sampling, Latin Hypercube Sampling or stratified sampling, etc) or by some data obtained after applying design of experiments concepts, and trying to identify if there is any linear or monotonic pattern between inputs and outputs by doing some regression, correlation and rank transformation analysis as an starting point for the statistical sensitivity analysis. Because some kind of sampling technique has to be applied usually as a very first step in order to begin with the statistical analysis, these methods are sometimes referred in the literature as sampling-based methods.

Considering that the response Y is a linear function of the random input variables X_i,

$$Y = \beta_0 + \sum_{i=1}^{p} \beta_i X_i$$

several coefficients can be obtained from the regression model which allows us to determine the importance of each input.

For linear models the following coefficients are considered:

Pearson correlation coefficients or linear correlation coefficients (PEAR). They measure the quality of the linear relation between the variables X_i and Y. They are given by:

$$PEAR_i = \rho(X_i, Y) = \frac{\sum_{k=1}^{n} \left(X_{ik} - E(X_i)\right)\left(Y_k - E(Y)\right)}{\left[\sum_{k=1}^{n} \left(X_{ik} - E(X_i)\right)^2\right]^{1/2} \left[\sum_{k=1}^{n} \left(Y_i - E(Y)\right)^2\right]^{1/2}}$$

If they are close to -1 or +1 the assumption of linearity between X_i and Y appears valid. They are not, however, a quantitative measure of their linearity degree, but they can reveal the linear character of some dominant parameters.

Standardized regression coefficients (SRC). They quantify the effect of varying each input variable away from its average by a fixed fraction of its variance. If the input variables are independent, the sum is one, and therefore they obey to a decomposition of the variance of the response. They are used to identify and quantify linear relations between the inputs and the outputs and are given by:

$$SRC(Y, X_i) = \beta_i \sqrt{\frac{Var(X_i)}{Var(Y)}}$$

Partial correlation coefficients (PCC). The partial correlation coefficient between the output variable Y and the input variable X_j is defined as the correlation coefficient between $Y - \hat{Y}$ and $X_j - \hat{X}_j$:

$$PCC(Y, X_i) = \rho(Y - \hat{Y}, X_i - \hat{X}_i)$$

where

$$\hat{Y} = b_0 + \sum_{h \neq j} b_h x_h, \; \hat{X}_j = c_0 + \sum_{h \neq j} c_h x_h$$

When the input factors are correlated, the SRCs indexes are taking into account as well the effect of the correlated factors. In this case, it is more suitable to use the PCCs, which provide a measure of the linear relation between the input and the response when the linear effects of all of the other parameters are removed. In other words, the partial correlation coefficient provides a measure of the importance of a single parameter under exclusion of the effects of: (i) the other parameters, (ii)

the assumed distribution for the input variable, and (iii) the magnitude of the impact of an input parameter on the response. The partial correlation coefficient between an individual parameter and the response is obtained by considering a sequence of regression models.

In all the cases, the sign of the coefficient indicates if the response increases (+) or decreases (-) when the variable increase.

When the model in not linear but monotonous the previous analysis has to be transformed to a rank analysis, to improve the construction of the respective regression model. The conceptual framework underlying rank transformation involves simply replacing the parameters by their respective ranks, and then performing the customary regression analysis on the ranks rather than the corresponding parameters. Thus, if the number of observations is M , then the smallest value of each parameter is assigned rank 1, the next largest value is assigned rank 2, etc., until the largest value, which is assigned rank M ; if several parameters have the same values, then they are assigned an averaged rank.

The regression analysis is then performed by using the ranks as input/output parameters, as replacements for the actual parameter/response values. This replacement has the effect of *replacing the linearized parameter/response relationships by rank-transformed monotonic input/output relationships* in an otherwise conventional regression analysis. In practice, a regression analysis using the rank transformed (instead of raw) data may yield better results, but only as long as the relationships between parameters and responses are monotonically nonlinear. Otherwise, the rank-transformation does not improve significantly the quality of the results produced by regression analysis.

The same indexes are calculated as in the linear case, which become the followings:

Spearman Rank correlation coefficient (SPEAR). They are the same as the Person coefficient but calculated over the ranks. Thus, they identify monotonic relations. They are calculated by:

$$SPEAR_i = \rho^S(X_i, Y) = 1 - \frac{\sum_{k=1}^{n}\left(r_{X_i}^k - r_Y^k\right)^2}{N(N^2-1)}$$

Standardised Rank Regression coefficients (SRRC). The SRC calculated over the rank values.

Partial Rank Correlation Coefficient (PRCC). The PCC calculated over the rank values.

In all cases, the validity of the indices obtained relies on the quality of the linear (or monotonic) approach adopted (as well as the independency between the input parameters). If such hypothesis is not right, other more suitable techniques should be used. To validate this hypothesis, it important to calculate the determination coefficient of the regression model, the R^2 (or equivalently the coefficient of determination based on the rank, R^2*):

$$R^2 = \frac{\sum_{i=1}^{N}\left(\hat{y}_i - \bar{y}\right)^2}{\sum_{i=1}^{N}\left(y_i - \bar{y}\right)^2}$$

where $\hat{y}_i$ denotes the estimate of y_i obtained from the regression model. The coefficient represents the fraction of variation compared to the average explained by the regression model, i.e. the variance percentage of output variables Y explained by the regression model $\hat{y}$. Thus, R^2 provides a measurement of how well the linear regression model can reproduce the actual output y, so it is a global measure of the goodness of fit. The closer R^2 is to unity, the better is the model performance, and so we can use the standardized regression coefficients as

sensitivity indices. Moreover, the R^{2*} will be higher than the R^2 in case of non-linear models, and the difference between R^2 and R^{2*} can be a useful indicator of the non-linearity of the model.

Screening methods

Screening design methods refer to preliminary numerical experiments designed to identify the parameters that have the largest influence on a particular model response. The objective of screening is to arrive at a *short list* of important factors. In turn, this objective can only be achieved if the underlying numerical experiments are judiciously designed.

An assumption often used as a working hypothesis in screening design is the assumption that the number of parameters that are truly important to the model response is small by comparison to the total number of parameters underlying the model. This assumption is based on the idea that the influence of parameters in models follows Pareto's law of income distribution within nations, characterized by a few, very important parameters and a majority on non-influential ones.

Since screening designs are organized to deal with models containing very many parameters, they should be economical computationally. There is an inevitable trade-off, however, between computational costs and information extracted from a screening design. Thus, computationally economical methods often provide only qualitative, rather than quantitative information, in that they provide a parameter importance ranking rather than a quantification of how much a given parameter is more important than another.

This section presents a brief survey of the most relevant methods. Only a particular case referring to the global OAT screening method will be described with more detail, as it will be used later on for the benchmark exercise analysis.

Classical OAT methods. Falling within the simplest class of screening designs are the so-called *one-at-a-time* (*OAT*) experiments, in which the impact of changing the values of each parameter is evaluated in turn. *OAT* designs can be classified into five categories, as follows: (i) *standard OAT* designs, which vary one factor from a standard condition; (ii) *strict OAT* designs, which vary one factor from the condition of the last preceding experimental run; (iii) *paired OAT* designs, which produce two observations and, therefore, one simple comparison at a time; (iv) *free OAT* designs, which make each new computation under new conditions; and (v) *curved OAT* designs, which produce a subset of results by varying only one parameter that is easy to vary.

The classical *OAT* experiments yield information about the system's response *local* behaviour, since they cannot provide information about interactions between parameters, so the model's behaviour can only be assessed in a small interval around the "control" scenario. Therefore, the results of a classical *OAT* experiment are meaningful only if the model's input-output relation can be adequately represented by a first order polynomial in the model's parameters. If the model is affected by nonlinearities (as is often the case in practice), then parameter changes around the "control" scenario would provide drastically different "sensitivities", depending on the chosen "control" scenario.

Several alternative designs have been proposed to alleviate the severe limitation of the classical OAT designs. Among the most popular alternatives are the *global OAT design methods*, as the one proposed by Morris, which aims to obtain a global sensitivity ranking, and the *group screening design methods*, proposed to identify interactions between the parameters.

Global OAT methods: the Morris method. *The global OAT* design method, proposed by Morris, intends to cover the entire space in which the parameters may vary, independently of the specific initial "control" scenario one may commence the experiment with. A global *OAT* design assumes that the model is characterized by a large number of parameters and/or is computationally expensive (in computational time & computational resources) to run. For large-scale models, therefore, the *OAT* design of Morris requires a relatively high computational effort. We will now describe the Morris method with some detail.

Elementary effects. Let consider that the range of each factor or input parameter is standardized to the unit interval, and that each component is considered to take *n* values so that the factor space $[0,1]^p$ is discretized in a p-dimensional n-level grid:

$$\left\{0, \frac{1}{n-1}, \frac{2}{n-1}, ..., 1\right\}^p$$

Let Δ be a predetermined multiple of $1/(1-n)$. Then, an elementary effect of the *i*th-factor is the model variation over the *i*th-direction of the parameter space is defined as:

$$d_i(x) = \frac{y(x_1, x_2, ..., x_{i-1}, x_i + \Delta, x_{i+1} ..., x_p) - y(x_1, ..., x_p)}{\Delta} = \frac{y(x + \Delta e_i) - y(x)}{\Delta}$$

where e_i stands for the canonical basis and such that the point $x + \Delta e_i$ is still within the region of experimentation.

Sensitivity measures and interpretation. A finite distribution F_i of elementary effects for the *i*th-parameter is obtained by sampling the factor's region of experimentation r times. The distribution F_i is then characterized by its *mean* and *standard deviation*:

$$\mu_i^* = E(|d_i|), \ \sigma_i = \sigma(d_i)$$

These measures are indeed global sensitivity indices, as they are obtained as average quantities over the full region. Its interpretation is moreover very simple. If a plot (μ, σ) is considered for all the factors, then the graphical interpretation is as follows:

- μ_i^* is a measure of the sensitivity of the model to the ith-factor. Thus, a high mean indicates a parameter with an important overall influence on the response, and a small value indicates that the effect can be rejected.
- σ_i is a measure of the non-linearity of the interactions. A small standard deviation indicates that the elementary effects are significantly identical, which means that they are linear with respect to the response and with no interactions with other factors. On the other hand, a high standard deviation indicates either a parameter interacting with other parameters or a parameter whose effect is nonlinear.

Computational effort. An important point is the selection of the parameters n (number of levels for the discretization of the factor space) or equivalently Δ (interval length for the discretization), and *r* (number of repetitions of the experiment). The election of Δ or n has no direct effect into the computational time, although it affects the granularity of the exploration of the factor space and therefore the idea of measuring more local variations when computing the elementary effects. In contrast, the election of the number of repetitions *r* affects directly the computational effort required, which is $n^* = (p+1) \cdot r$.

Group screening design methods. In addition to screening designs that consider each parameter individually, the originally individual parameters can be clustered into groups that are subsequently treated by *group screening designs*. Perhaps the most efficient modern group screening designs techniques are the *iterated fractional factorial design* (*IFFD*), and the *sequential bifurcation* (*SB*).

Each type of design has its own advantages and disadvantages. The advantages of OAT designs are: (i) no assumption of a monotonic input-output relation; (ii) no assumption that the model contains only a "few important" parameters; and (iii) the computational cost increases linearly with the number of parameters. The

major disadvantage of *OAT* designs is the neglect of parameter interactions. Although such an assumption drastically simplifies the analysis of the model, it can rarely be accepted in practice.

This simplifying assumption is absent in the *global OAT* design of Morris, which aims at determining the parameters that have (i) negligible effects, (ii) linear and additive effects, and (iii) nonlinear or interaction effects. Although the *global OAT* is easy to implement, it requires a high computational effort for large-scale models, and provides only a qualitative (but not quantitative) indication of the interactions of a parameter with the rest of the model; it cannot provide specific information about the identity of the interactions, and individual interactions among parameters cannot be estimated As a final conclusion, it has to be remarked that all of these methods are computationally very intensive, which severely limits the amount of reliable information that can be extracted from a screening design.

ANOVA-based methods

In statistics, the analysis of the variance, among other indicators, is used as a measure of the importance of a parameter in contributing to the overall uncertainty in the response.

The concept of variance as a measure of the importance of a parameter also underlies the conceptual foundation of three additional methods for statistical uncertainty and sensitivity analysis, which are grouped under the name of variance-based or ANOVA-based methods. These three well-know techniques are:

- correlation ratio-based method
- Fourier Amplitude Sensitivity Test (FAST)
- Sobol's method

It is important to note that, in contrast to the regression-based methods already presented, the correlation ratio, the *FAST*, and Sobol's methods do not make the a priori assumption that the input model parameters are linearly related to the model's response. For this reason, among global sensitivity analysis techniques, these variance-based methods are most often used. The correlation ratio-based methods is the cheapest of these techniques in computational terms, at least for models with many input parameter, but it allows to estimate only the contribution of main effects. The two other methods are indeed the most common used, although they are very expensive in computational terms, specially the Sobol method, as a good estimation of the sensitivity indices requires a big number of calculations. We recall that the main idea of these methods is to decompose the total variance of the response to express sensitivity through variance, and to evaluate how the variance of such an input or group of inputs contributes into variance of the output. While the FAST method calculate these contributions, namely, the sensitivity indices, through the Fast Fourier Transformation, the Sobol method calculated them by Monte-Carlo simulation, thus its more expensive cost. In the following, the basis of both methods will be exposed.

Sobol method

The Sobol method has its roots in a theorem by Kolmogorov which states that any multivariate function defined in the unit n-dimensional cube can be written as a linear superposition of univariate functions. The Kolmogorov expansion of a multivariate function has inspired the development of the Sobol theorem, which refers to the expansion of an multivariate integrable function into summands of increasing dimensionality in the n-dimensional unit hypercube K^p .This theorem, called the 'expansion of a function into summands of different dimensions' or 'High Dimensional Model Representation (HDMR)' is the basis of the Sobol' method to calculate the sensitivity indexes, as it is applied to obtain a decomposition of the variance.

Sobol's theorem for High Dimensional Model Representation

Let be f an integrable function over $[0,1]^p$. Then the function can be decomposed into summands of increasing dimensionality of the form:

$$f(x_1,...,x_p) = f_0 + \sum_{i=1}^{p} f_i(x_i) + \sum_{1\le i<j\le p} f_{ij}(x_i,x_j) + ... + f_{1,...,p}(x_1,...,x_p)$$

This decomposition is unique and has the following properties:

- The integrals over any of its own variables are zero, i.e.

$$\int_0^1 f_{i_1 i_2 \ldots i_p}(x_{i_1},\ldots,x_{i_p})dx_{i_m} = 0, \forall 1 \le m \le p$$

- The summands are mutually orthogonal, i.e.

$$\int_{[0,1]^p} f_{i_1 i_2 \ldots i_p} f_{j_1 j_2 \ldots j_m} dx = 0, \text{ if } (i_1, i_2, \ldots, i_p) \neq (j_1, j_2, \ldots, j_m)$$

- The term f_0 is constant.

Sobol's decomposition of the Variance

By squaring the equation and integrating the result over the unit cube the following relation is obtained for the decomposition of the total variance of $Y = f(X)$:

$$V(Y) = \int_{[0,1]^p} f^2(x)dx - f_0^2 = \sum_{i=1}^{p} D_i + \sum_{1 \le i < j \le p} D_{ij} + \ldots + D_{1,\ldots,p}$$

where the partial variance of Y are defined as:

$$D_i = V\left(E\left[Y \mid X_i\right]\right)$$

$$D_{ij} = V\left(E\left[Y \mid X_i, X_j\right] - E\left[Y \mid X_i\right] - E\left[Y \mid X_j\right]\right)$$

$$D_{ijk} = V(E\left[Y \mid X_i, X_j, X_k\right] - E\left[Y \mid X_i, X_j\right] - E\left[Y \mid X_i, X_k\right] - E\left[Y \mid X_j, X_k\right] + E\left[Y \mid X_i\right] + E\left[Y \mid X_j\right] + E\left[Y \mid X_k\right])$$

etc.

As all inputs are assumed to be independent, we have that:

$$D_i = V\left(E\left[Y \mid X_i\right]\right)$$

$$D_{ij} = V\left(E\left[Y \mid X_i, X_j\right]\right) - D_i - D_j$$

$$D_{ijk} = V(E\left[Y \mid X_i, X_j, X_k\right]) - D_{ij} - D_{ik} - D_{jk} - D_i - D_j - D_k$$

etc.

Sobol's Sensitivity Indexes

From this decomposition and dividing all terms by the total variance of the response to normalise all the equation, we obtain the Sobol sensitivity indexes. The first order indices are defined by:

$$S_i = \frac{D_i}{V(Y)}$$

second-order sensitivity indexes by:

$$S_{ij} = \frac{D_{ij}}{V(Y)}$$

and so on. Or in a more general formulation:

$$S_{i_1 i_2 \ldots i_m} = \frac{D_{i_1 i_2 \ldots i_m}}{V(Y)}, 1 \le i_1 < i_2 < \ldots < i_m \le p, m = 1, \ldots, p$$

The first-order sensitivity index, S_i, for the parameter X_i indicates the fractional contribution of X_i to the variance of the output Y taken independently, the second-order sensitivity index, S_{ij} measures the part of the variation in Y due to X_i and X_j that cannot be explained by the sum of the individual effects of X_i and X_j, and so on. The sum of all order indexes is equal to 1 if all inputs variables are independent. In this case, index values are easy to interpret: the greater an index value, the greater is the importance of a variable or a group of variables.

Total Sensitivity Indexes

In order to rank the random variables according to their influence on the response, we need to evaluate the total contribution of each parameter. For a model with p inputs, the number of all order indices is $2^p - 1$. As this number can be very large when p increases, total sensitivity indices have been introduced by Saltelli. For an input X_i, the total sensitivity index S_{T_i} is defined as the sum of all indices relating to X_i (first and higher order):

$$S_i^{tot} = \sum_{i \in K} S_K$$

They represent therefore the part of the variance that is explained by the contributions of all the factors.

In practice, it is not needed to calculate the indexes of all the orders to calculate the total order indexes. Let define $S_{\sim i}$ is the sum f all order indices $S_{i_1 i_2 \ldots i_m}$ for which the index i is not presented, so that the total order index can be represented as:

$$S_i^{tot} = 1 - S_{\sim i}$$

Usually, the first order and/or second order indexes are sufficient to calculate $S_{\sim i}$ and therefore the total indexes.

Estimation of the indexes

Sobol introduced one method of estimation of these indices based on Monte-Carlo simulation. After, Saltelli extended this method for a best use of the model evaluations, and also for lower cost. The Monte-Carlo method requires a very significant number of simulations, typically N_s=10000 for the estimation of each index (S_i, S_{ij}, …or S_{tot}) and for each input parameter and one output response variable.

The Sobol methods is the most general method from the ones based on the decomposition of the variance, as it allows to obtain the sensitivity indexes of order 1 and of any other order (which allows to study the interaction between the factors), and the total order indexes (to study the total influence of one factor). Moreover, the fact that it relies on Monte-Carlo simulation has the benefit of being able to compute confidence intervals on the Sobol indices by repeating these index calculations, and thus obtaining more realistic results. It is however very expensive computationally, as it relies on Monte-Carlo simulations. A good estimation of the indices requires a great number of calculations. This justifies the use of response surface or surrogate models (see section 3.2.2) to evaluate the output of the model for all the simulations required, to minimize the computing times. It has to be highlighted that, if input factors are correlated or non-independent, then the functions of Sobol decomposition are not orthogonal, and so a new term appears in the variance decomposition. This term implies that the sum of all order sensitivity indexes is not equal to 1. This is because the variabilities of two correlated variables are linked, and so when we quantify sensitivity to one of these two variables, we also quantify a part of sensitivity due to the other variable. In order to take this into account, it is necessary to regroup the correlated variables and to apply group sensitivity methods.

1.5 Summary of the Chapter 1

The first step of the analysis is the definition of the accidental scenario in which the system will operate. Then the system can be characterized. The goal of this analysis is to obtain information about the Passive System behaviour, in an accident occurring during the life of the complete system and to identify the failure zones and conditions, if they exist. For that, the missions of the system, its failure modes and the failure criteria are defined. Due to the lack of suitable experimental databases for Passive Systems in operation, the evaluation relies on qualified T-H system code performing best estimate calculations.

The method requires the identification of the potentially important contributors to uncertainty of the code results. In the present study the uncertainties pertaining to the code itself are not accounted for, focusing the attention on the uncertainties relative to the input parameters of the code, characteristic of the Passive System or of the complete system.

Among all the sources of uncertainties, the evaluation of the reliability of a Passive System requires the identification of the relevant parameters which really affect the accomplishment of the target of the system. The tool here chosen for this task is the Analytic Hierarchy Process.

An important feature of the methodology is the measure of the importance of parameter uncertainties for the uncertainty of the Passive System performance. These sensitivity analyses give a ranking of input parameters and provide guidance as to where to improve the state of knowledge in order to reduce the output uncertainties most effectively. In the methodology, the use of linear and non linear sensitivity indices is proposed.

2. PROPAGATION OF UNCERTAINTY AND RELIABILITY EVALUATIONS OF PASSIVE SYSTEMS

Once the different sources of uncertainties have been identified and modelled, the problem is how to propagate these uncertainties through the T-H code and how to assess the system reliability. It is beyond the purpose of the course the full integration between uncertainty methods (adopted for best-estimate T-H code predictions) and reliability methods (object of the present research activity). Nevertheless, starting from the consideration of some methods used for assessing the uncertainty of T-H system code predictions, an attempt is made hereafter to present 'support methods' that can be helpful for the evaluation of the reliability of Passive Systems.

In the studies of uncertainty propagation, we are interested with the uncertainty evaluation of a response Y according to uncertainties on the input data $X = (X_i, i=1,..,p)$ and on the functional relation g connecting Y to X. In chapter 3.1, the statistical bases of some methods where the uncertainty on the response are evaluated either in the form of an uncertainty range, or in the form of a *pdf* are outlined.

The reliability of the system is often defined as the probability that an output parameter $Y(X)$ exceeds a threshold or a critical value Y_c. In the chapter 3.2, we will describe the methods used to evaluate the reliability of the system: the simulation methods and the approximate methods FORM/SORM.

2.1 Propagating the Uncertainties

Different methods are used for the propagation of uncertainty or for determining the uncertainty that characterises the results of T-H system code calculations [D'Auria, 2002]. Hereafter reference is made to some of these methods that are based on the use of statistics.

In a Passive System, a large number of variables influences the performance of the system, e.g. temperatures, pressures, water levels, non-condensable fraction… The performance of the system is evaluated by a *best-estimate* code. Suppose performance criterion Y of the system depends on the inputs variables $X_1, X_2, ..., X_n$. Although the actual response Y is a function of the input variables, i.e., $Y=g(X_1,...,X_n)$, this function is generally unavailable in closed form. In order to get information about the uncertainty of Y, a number of code runs have to be performed. For each of these calculation runs, all identified uncertain parameters are varied simultaneously.

According to the exploitation of the result of these studies, the uncertainty on the response can be evaluated either in the form of an uncertainty range, or in the form of a *pdf*.

2.1.1 Uncertainty range

Ref. [Glaeser, 2000], [Wilks, 1941]

A two-sided tolerance interval $[m,M]$ of a response Y, for a fractile α and a confidence level β is given by:

$$P\{P(m \le Y \le M) \ge \alpha\} \ge \beta$$

Such a relation means that one can affirm, with at the most $(1-\beta)$ percent of chances of error, that at least α percents values of the response Y lie between the values m and M.

To calculate the limits m and M, the technique usually used is a method of simulation combined with the formula of Wilks. The formula of Wilks determines the minimal size N of a sample to be generated randomly according to the values of α and β.

For two-sided statistical tolerance intervals the formula is:

$$1-\alpha^N - N(1-\alpha)\alpha^{N-1} \geq \beta$$

The minimum number of calculations can be found in table 3.

Table 3: Minimum ***number of calculations N for one-sided and two-sided statistical tolerance limits****.*

	One-sided statistical tolerance limit			Two-sided statistical tolerance limit		
β/α	0.90	0.95	0.99	0.90	0.95	0.99
0.90	22	45	230	38	77	388
0.95	29	59	299	46	93	473
0.99	44	90	459	64	130	662

The bounds m and M of the confidence interval of Y are obtained by retaining the minimal and maximal values of the sample $\{Y_j, j = 1,...,N\}$.

This method supposes that the function g is continuous and that all uncertainties on the input data X_i are distributed according to continuous laws.

The advantage of using this technique is that the number of code calculation needed is independent of the number of uncertain parameters.

However for reliability evaluation, this method is not very useful because it is difficult, indeed impossible to interpret the two levels of probability (α and β) in term of reliability value for the system.

2.1.2 Density of probability

The uncertainty evaluation in the form of a *pdf* gives richer information than a confidence interval. Once the *pdf* of the system performance is determined, the reliability of the system can be directly obtained if a failure criterion is given. But the determination of this distribution can be expensive in computing times. The following paragraphs describe the various methods available for this evaluation.

Monte Carlo method [Rubinstein, 1981]

The method of Monte-Carlo is used to build *pdf*, but also to assess the reliability of components or structures or to evaluate the sensitivity of parameters. Monte Carlo simulation consists of drawing samples of the basic variables according to their probabilistic characteristics and then feeding them into the performance function. In this way, a sample of response $\{Y_j, j = 1,..,N\}$ is obtained.

The *pdf* is obtained by fitting a law on the sample $\{Y_j, j = 1,..,N\}$. This fitting is a problem well known and many tests exist and are adapted to the law tested (Chi-square, Kolmogorov-Smirnov, Anderson-Darling...]). Degrees of confidence can be associated to the fitting.

It is obvious that the quality of the fitting depends on the number of simulations carried out and on the good repartition of these simulations in the random space, especially if the tails of distributions are one of the interests of the study. It is necessary to notice that no rule exists, when there is no a priori knowledge on the type of *pdf*, to determine the number of simulations necessary to obtain this distribution with confidence.

Method of moments

This method allows to calculate the first four moments by using the Gauss integration method and then to fit a distribution of probability to these moments by using the Pearson or Johnson methods (cf. [Pearson 65] and [Baldeweck 97]). The first objective is to evaluate the first moments of the random response *Y*. The expectation of *Y* can be calculated by:

$$E(Y) = E(g(X)) = \int \cdots \int g(x) f_X(x) dx_1 \cdots dx_p$$

where f_x is the joint density distribution of the input uncertain parameter vector X.

This equation can be evaluated by a squaring method of Gauss. This method allows the integration of a continuous function with the desired accuracy. It consists in the discretisation of the interval of integration in a number of *X*-coordinates x_i to which a weight w_i is associated. The number of *X*-coordinates is a function of the desired accuracy. For a continuous function g(x):

$$\int_a^b W(x) g(x) dx \approx \sum_{i=1}^{N} \omega_i g(x_i)$$

Practically, a set of order j orthogonal polynomials $\{p_j(x)\}_{j=0,1,2...}$ are associated to the weight function W(x). These polynomials verify the following relations:

$$\int_a^b W(x) p_i(x) p_j(x) dx = 0 \qquad \text{if } i \neq j; \qquad \int_a^b W(x) p_i(x)^2 dx = 1$$

The *N* X-coordinates of a squaring formula with a weight function *W(x)* are the zeros of the polynomial $p_N(x)$, which has exactly N zeros in the interval [a, b]. Relations of recurrence generally define these polynomials. The weights are calculated by solving the system of linear equations:

$$\sum_{j=1}^{N} p_i(x) \omega_j = \int_a^b W(x) p_i(x) \qquad i = 1 \cdots N$$

Then the average is evaluated from:

$$E(Y) = E(g(X)) \approx \sum_{i=1}^{N} \omega_i \times g(u_i)$$

and the moment of order *k* from:

$$M_Y(k) = \int [g(x) - \mu_Y]^k f_X(x) dx \approx \sum_{i=1}^{N} \omega_{i,X} \times [g(u_{i,X}) - \mu_Y]^k$$

Pearson and al. ([Pearson 65]) show that one can define in an approximate way a density of probability from the average, the standard deviation and two additional coefficients called coefficients of Fisher:

The coefficient of symmetry $\gamma_1 = \sqrt{\beta_1} = \frac{|\mu_3|}{\sigma^3}$ and the coefficient of flatness $\gamma_2 = \beta_2 = \frac{\mu_4}{\sigma^4}$

where β_1 is called the Skewness and β_2 the Kurtosis.

The four-parameter Pearson family of distributions

$$f(x) = f(x_0)(x - a_1)^{p_1}(a_2 - x)^{p_2}$$

where a_1, a_2, p_1 and p_2 are the parameters that define each specific distribution and $f(x_0)$ is a constant introduced to the distribution to integrate 1 over the whole range, can approximate a great number of continuous distributions. The curves can have several shapes (bell-shaped curve, curved in J, curved in U), conditional on the values of the four parameters (a_1, a_2, p_1 and p_2). This method is efficient to estimate a *pdf* if the number of random variables is small.

2.1.3 Conclusion on the methods for propagating uncertainties

We have presented some of the methods used to propagate the uncertainty through a T-H code and usable for the reliability assessment of a Passive System. These methods are based on the estimation of the *pdf* of the system performance. Once the *pdf* of the system performance is determined, the reliability of the system can be directly obtained if a failure criterion is given.

The method of Monte-Carlo is used to build the *pdf*. The main advantage of the method of Monte-Carlo, is that this method is valid for static, but also for dynamic models and for probabilistic model with continuous or discrete variables. The main drawback of this method is that it requires often a large number of calculations and can be prohibitive when each calculation involves a long and onerous computer time. To avoid this problem, it can be interesting to build a response surface, which approximates the complex physical phenomena. But this response surface must be qualified. The practical problems encountered by the use of the response surface method are in the analysis of strongly non-linear phenomena where it is not obvious to find a family of adequate functions and in the analysis of discontinuous phenomena.

Another approach consists in calculating the first four moments of the system performance and then to fit a *pdf* to these moments. However this approach can be also very time consuming.

2.2 Evaluating the unreliability (or failure probability)

Different methods can be used to quantify the reliability of a Passive Systems once given a best estimate T-H code and a model of the system. We present now specific methods usable for a direct evaluation of the Passive System reliability, without the need of defining the *pdf* of the system performance.

The performance function of a Passive System according to a specified mission is given by:

$$M = performance\ criterion - limit = g(X_1, X_2, \ldots, X_n)$$

in which the X_i $(i=1,\ldots,n)$ are the n basic random variables (input parameters), and $g(.)$ is the functional relationship between the random variables and the failure of the system. The performance function can be defined such that the limit state, or failure surface, is given by $M = 0$. The failure event is defined as the space where $M < 0$, and the success event is defined as the space where $M > 0$. Thus a probability of failure can be evaluated by the following integral:

$$P_f = \int\int \ldots \int f_{X(x_1,x_2,\ldots,x_n)}\, dx_1, dx_2, \ldots, dx_n \qquad (3)$$

where f_X is the joint density function of $X_1, X_2, \ldots, X_n$, and the integration is performed over the region where $M < 0$. Because each of the basic random variables has a unique distribution and they interact, the integral (3) cannot be easily evaluated. Two types of methods can be used to estimate the probability of failure: the Monte Carlo simulation with or without variance reduction techniques and the approximated methods (FORM/SORM).

2.2.1 Reliability evaluations using Monte-Carlo simulation

Direct Monte-Carlo simulation techniques (Rubinstein, 1981), (Sundarajan 95)

Using the Monte Carlo method to compute the probability of failure of process consists of applying the method as usual, using as an output variable a performance function. The value of this variable is compared for each run against a failure criterion (Figure 4). If the failure criterion is fulfilled, this run is considered a failure. The estimate of the probability of failure is

$$\hat{P}_f = N_f / N$$

where N_f is the number of runs in which the failure criterion is fulfilled, and N the total number of runs. As N approaches infinity, $\hat{P}_f$ approaches the true probability of failure. The accuracy of the estimation can be evaluated in terms of its variance computed approximately as:

$$Var(\hat{P}_f) \cong (1 - \hat{P}_f)\hat{P}_f / N$$

The smaller the variance, the better the accuracy of the estimated probability of failure. It is obvious that as N approaches infinity, $Var(\hat{P}_f)$ approaches zero. For a small number of runs, the variance of $\hat{P}_f$ can be quite large. Consequently, it may take a large number of runs to achieve a specific accuracy. The amount of computer time needed for the direct Monte Carlo method is large, specially in the case where each run involves a long calculation performed by a T-H code. Variance reduction techniques can be of use to reduce computational costs.

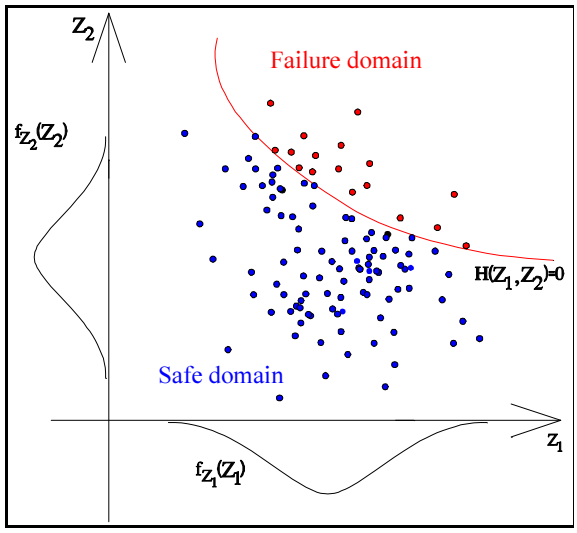

Figure 4. ***Reliability assessment by Monte-Carlo simulation***

Variance reduction techniques

In many actual applications, applying simple Monte Carlo may be prohibitive, conditional on the code used. In those cases, variance reduction techniques is useful to reduce the computational costs. Variance reduction techniques allow the user to get the same accuracy with a lower computational cost. Main techniques are Latin Hypercube sampling (LHS), stratified sampling, control variates, importance sampling and antithetic variates, among others ([Rubinstein 81]). In the following pages we discuss about most relevant variance reduction techniques.

Stratified sampling

Stratified sampling is based on the fact that the variance of any random variable, once it has been divided in strata, may be decomposed into two contributions: the variability within each stratum and the variability between different strata, which means

$$\sigma_y^{\,2} = \sum_{i=1}^{h} \omega_i \cdot \sigma_{yi}^{\,2} + \sum_{i=1}^{h} \omega_i \cdot (\mu_{yi} - \mu_y)^2 ,$$

where the first summand represents the variability within the h considered strata and the second one represents the variability between different strata. ω_i stands for the probability of stratum i, and μ_{yi} and σ_{yi} stand for the mean and the standard deviation of (output) variable Y also in stratum i. If we estimate the mean within each stratum and combine them in the appropriate way, we avoid an estimation error to the second contributor , so that more accurate estimates may be obtained.

Once the sample size has been chosen, there are two problems to be solved: 1) the way to build up the strata and 2) the sample size within each stratum. There is no clear rule to build up the partition of the sample space. But, when no additional information is available about the system model, the most common strategy is to build a net of hypercubes all of them with the same probability, usually *1/n*, where *n* is the sample size. Regarding the sample size per stratum, there are several options, though two of them are the most used by far. The first one consists in, assuming a partition like the one considered at the beginning of this paragraph, to take one observation at random per hypercube. The second one consists in, assuming a generic partition, to take, in each stratum, a sample size proportional to the probability and the standard deviation of the output variable in that stratum (figure 5).

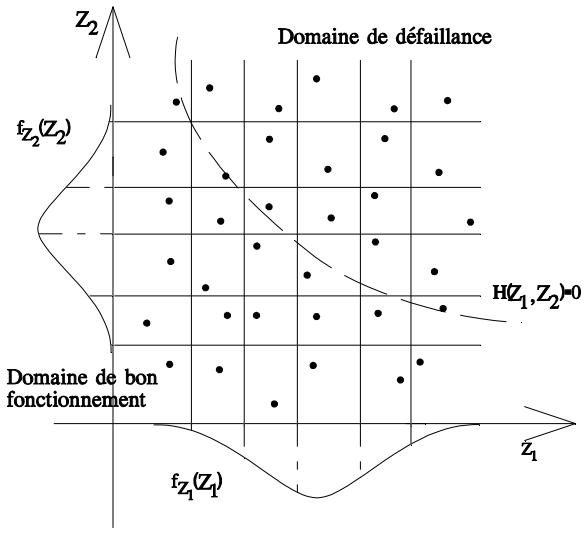

Figure 5. ***Stratified sampling***

Latin Hypercube Sampling (LHS)

Latin Hypercube Sampling, LHS from now on, is an extension of stratified sampling that benefits from some advantages taken from the theory of Design of Experiments. In order to get a sample of size n, the procedure is the following one: 1) a stratified sample is obtained for each input variable (n strata with probability 1/n each one and a sample of size 1 per stratum), 2) get a permutation of each one of the samples of each input variable and 3) combine the first observations of all the variables (after permutation) to get the first observation of the input vector, combine the second observations of all the variables (after permutation) to get the second observation of the input vector and so on. Figure 6 shows the way to generate a sample of size 5 through this method for a bivariate random vector. A necessary condition to get an estimation error for the sample mean smaller that in the case of random sample is that the model be monotonic in all its input variables. Under any condition, the Latin Hypercube Sampling reduces the variance relative to a simple random sampling in a relevant asymptotic sense.

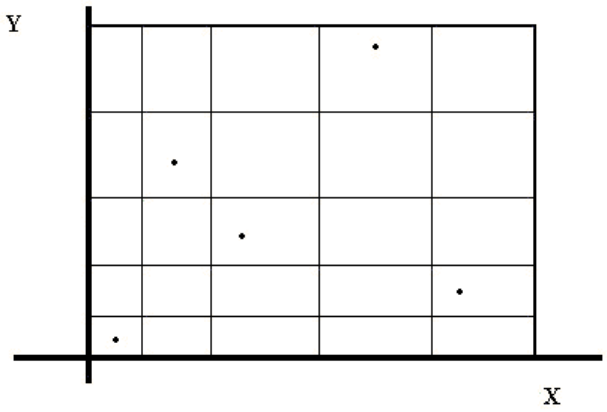

Figure 6*:* ***LHS sample of size 5 for two variables. Each stratum has the same probability***

Importance sampling

Importance sampling is based on substituting the variable under study by another one that definitely has the same mean but a smaller variance. In order to get this, let us observe that the mean of the output variable in the system model (*Y*) may be written in this way

$$\mu_y = \int_{S_x} y(\mathbf{x}) \cdot f(\mathbf{x}) \cdot d\mathbf{x} = \int_{S_x} y(\mathbf{x}) \cdot \frac{f(\mathbf{x})}{g(\mathbf{x})} \cdot g(\mathbf{x}) \cdot d\mathbf{x} \quad ,$$

so that the variable whose mean is actually studied is *Y'*=y´(x)=y(x)·f(x)/g(x), where the only requirement on the function g(x) is that it must be a *pdf*. Under these conditions, the variance of *Y'* is

$$Var(y') = \int_{S_x} (y(\mathbf{x}) \cdot \frac{f(\mathbf{x})}{g(\mathbf{x})} - \mu_y)^2 \cdot g(\mathbf{x}) \cdot d\mathbf{x} \quad ,$$

so that the more constant (stable over the values of x) the function *y(x)·f(x)/g(x)*, the smaller the variance). This means that, in order to reduce the estimation error, the larger is the probability of a region of the input space and the larger are the values of the output variable in that region, the more we have to sample it. It is important to notice that, since by construction the means of *Y* and *Y'* are the same, we will estimate the mean of *Y'* because

we are able to do it in a more accurate way, and this estimation will also be valid for *Y*. Figure 7 shows the idea behind importance sampling.

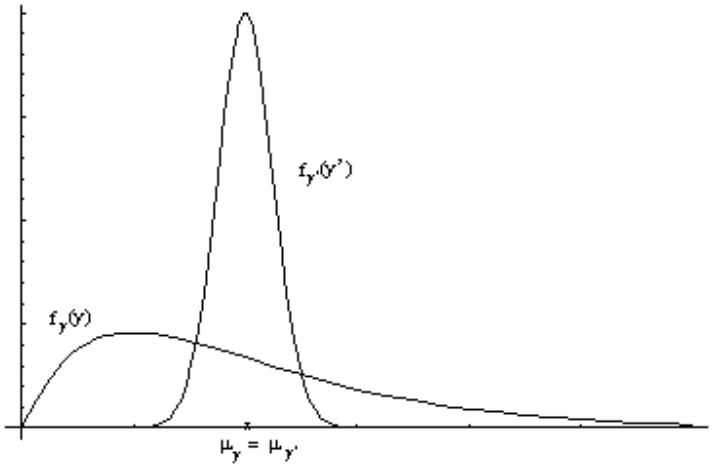

Figure 7. ***Intuitive idea behind importance sampling***

2.2.2 Reliability evaluation using of surrogate models

The T-H codes are very expensive in computational terms, so that, in many cases, it is unthinkable to run them as many times as needed to get accurate probabilistic results. In those cases it could be of interest to seek for good predictors that could be used as surrogate models (or synonym "response surface" or "metamodel") to be run instead of the real models.

Principles of the response surfacemethod (RSM)

The response surface method (RSM) [Box, 87] is used to build a function which simulates the behaviour of a physical phenomenon in the field of variation of the influential parameters, starting from a certain number of experiments. It was originally proposed as a statistical tool, to find the operating conditions of a chemical process at which some response were optimized. Subsequent generalizations led to these methods being used to develop approximating functions that surrogate for long running computer codes [Sacks 89], [Devictor 00]. These surrogate response surface models fit data that are generated by running the simulation model at selected points in the parameter space. Building such response surface aims at obtaining a mathematical model representative of the studied software, having good capacities of prediction, and for which computing time to evaluate an output variable is negligible. It will be thus efficient for the uncertainty and sensitivity analyses requiring several thousands of simulations.

There are different issues involved in selecting runs for uncertainty analysis and correlation coefficients computation compared with selecting runs for building a response surface ([Sacks 89]). For this latter purpose, one can often do better than random selection by making hypotheses on the response surface model and on the interactions between parameters. There are multiple response surface families [Hastie 02]: polynomials, splines, interpolating radial functions, kriging, generalized linear models (GLM), partial least squares, neural networks, regression trees, support vector machines, etc.

In general, to build a response surface, it is necessary to have the following elements:

- a sample ***D*** of points (**x**(i), z(i)), where **x** is the vector (x_1, …, x_p), z=*f*(**x**);
- *P(**X**,Z)* the probability law of the random vector *(**X**,Z)* (unknown in practice);
- a family *F* of response surface functions *f(**x**,**c**)*, where **c** is either a parameter vector or an index vector that identifies the different elements of *F*.

The aim of a RSM is, in consideration of the sample ***D***, to determine the best representing function f_0 in ***F***. It is then the function f_0 that minimizes the risk function:

$$R(f) = \int L(z, f(\mathbf{x}, c))dP(\mathbf{x}, z)$$

If assumptions of normal distributions and constant standard deviations are done, the loss function is a quadratic one:

$$L(z, f(\mathbf{x}, \mathbf{c})) = \varepsilon(\mathbf{x},f)^2 = [z(\mathbf{x}) - f(\mathbf{x}, \mathbf{c})]^2$$

In practice an empirical risk function is used:

$$R_E(f) = \frac{1}{N}\sum_{i=1}^{N}[z(i) - f(\mathbf{x}(i), \mathbf{c})]^2$$

Choice of design points

In case of surrogate model of simple shape, building points or sample **D** are often chosen from classical design of experiments. In case of polynomial surrogate models, main designs used are 2^k, 3^k, star designs and other composite designs. According of the characteristics of the code, other experimental designs are available in the literature, as D-optimal designs, etc. During the last years other options have been explored, mainly seeking for designs that could detect discontinuities. As the use of surrogate models introduces bias errors, these models should be used only if the reference model is too much time consuming.

Validation of a response surface

Before any use of response surface, it is necessary to qualify it for the foreseen utilisation. This qualification keeps a part of subjectivity. The characteristics « good approximation » is subjective and depends on the use of the response surface. The use could introduce additional constraints. For example, constraints like "conservatism", "bound on the remainder", "a better accuracy in an interest area (distribution tail…)" can be needed. For reliability studies, a good representation of the domain of maximum of failure probability is often sufficient and it is not necessary to seek a good quality of approximation in the entire field of variation of the input parameters. If the response surface is used in a problem where the knowledge of uncertainties is inaccurate, it is not judicious to seek response surfaces explaining 99.9% of the variability.

The statistical accuracy of the response surfaces can be assessed by cross-validation or bootstrap techniques [Hastie, 02]. The cross validation method allows a good estimation of the theoretical prediction error associated with the response surface, while the bootstrap method is especially useful when the size of the data sample is small. With a sufficient size of data sample, a simpler method can be chosen:

- the quality of approximation is given by statistical analyses carried out on the bases of points used to build the surface (this set of points is called here "training set");
- the quality of prediction is obtained by statistical analyses carried out on points not belonging to the building base (this set of points is called the "base of test").

The following general methodology can be used to validate the response surface:

- Initially some indicators obtained from the response surface with those obtained directly with software are compared on the two databases: average, standard deviation, minimum and maximum.
- A regression analysis (figure 8) allows determining the share of variability of the output variable explained by the fitted model. Two statistics, which give global measurements of correlation between two data sets, have been particularly studied: the Pearson correlation coefficient and the coefficient of determination R^2. These statistics between the software response and the response surface are calculated. The values of these statistics have to be approximately equal using the points of the database of construction and those of the database of prediction.
- The previous criteria are global and it is possible that the adjusted data are not homogeneous. This is the case when the studied variable covers a broad range of variations with multiple orders of magnitude. In this case, the contributions of the low values to the R^2 measurement are negligible. To cure this problem, the study of residual statistics gives some indications of the regression accuracy. The residuals $\varepsilon(\mathbf{x},f)$ have to follow a Gaussian distribution of mean zero, with a constant standard deviation small compared to the standard deviation of Z (results of the computer code).
- The average and the standard deviation of the relative residuals $\varepsilon(\mathbf{x},f)/z(x)$ are also examined. The response surface is valid on all the field of variation if the average and standard deviation of these relative residuals are small.

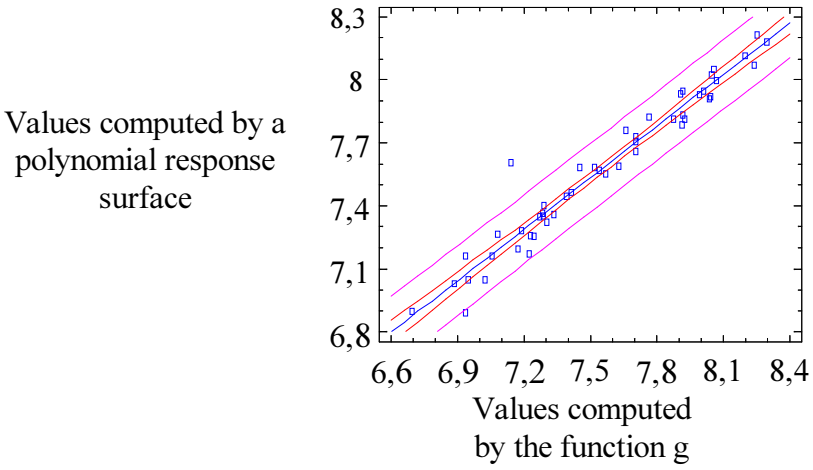

Figure 8. ***Linear relationship between the outputs of the function g and the response surface***

Feedback on the response surface

This approach is now mature; there are a lot of applications. Explicit response surface functions are more understandable for physicists, which is not the case of neural networks (NNRS). NNRS and GLM are more suitable for continuous and complex models, but only users with a lot of experience are able to apply them successfully.

But this approach is not the panacea. Additional R&D works are needed to solve particular problems:

- mainly in case of a lot of random inputs and very high time consuming code, when it is necessary to develop specific experimental design;
- the analysis of strongly non-linear phenomena where it is not obvious to find a family of adequate functions (multi-layer neural networks can bring a solution);

- the analysis of discontinuous phenomena: a solution is to build a preliminary function of classification and to calculate response surfaces for each branch of the bifurcation.

Some improvements of the method are on-going by the development of a iterative strategy: at the end of each iteration, the analysis of the database and the built surrogated model could help to define new points that are suitable to improve the quality of the surrogated model.

Another method is kriging [Santner, 03]. This technique is based on studying the characteristics of autocorrelation of output results. Some design concepts are used to select points of the input space to run the model and get the needed information, like maximum entropy, minimum integrated mean square error, minimax and maximin. This approach is one of the most efficient and it has the advantage to give physically interpretable model.

One question often encountered is : is the size and the quality of the database sufficient ?

A simple analysis consists in studying the convergence of some statistics (mean, variance or other) when the database is truncated. Figure 9 gives an example. Convergence of the variance and estimation of the bias could be assessed by the use of Bootstrap method. [Devictor, 00] gives a theoretical example.

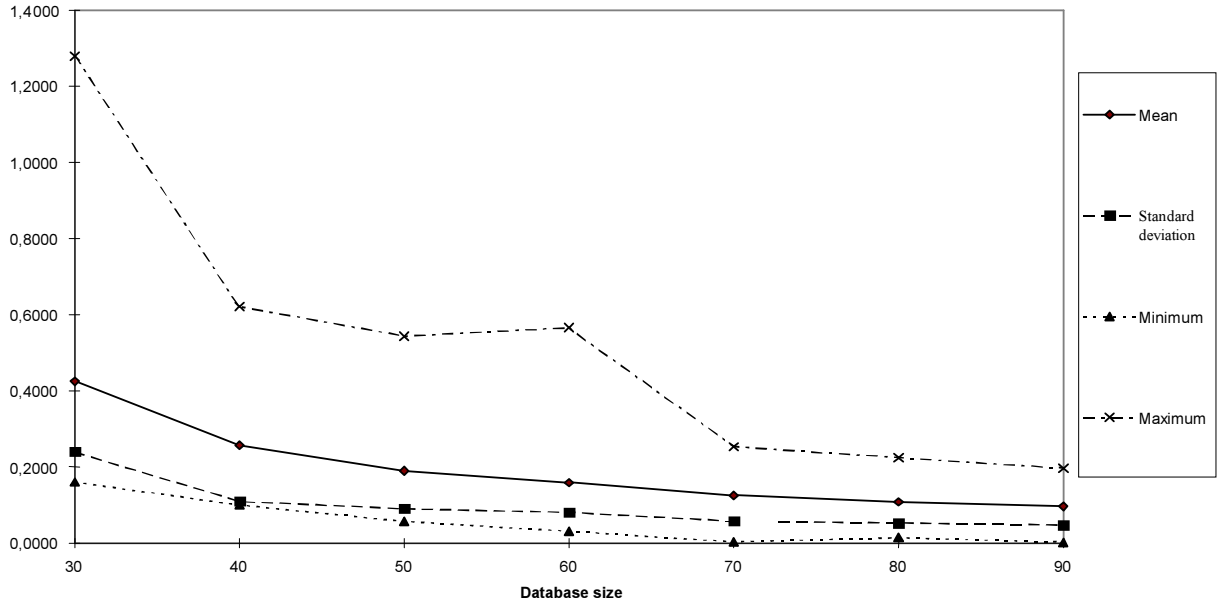

Figure 9: ***Convergence of statistics with the size of the database***

Bootstrap method

When a model is built from a learning statistical process, like neural networks or a generalized linear model, an important step is the estimation of the risk function of the model. From the points of the data set, the risk function is not « learnable » by definition. To have an estimation of the risk function, the usual method in neural networks is based on a cross validation. The dataset is divided into two parts. The first part used for the training process provides the learning error, the second part allows to calculate the generalization error using samples not

used during the learning process. The estimation of the risk function is given by the generalization error. The Bootstrap method has been applied to improve the estimation of the bias between learning and generalization error. The Bootstrap method is a « *computer-based method for assigning measures of accuracy to statistical estimates* » [Efron 93]. Bootstrap method allows to estimate the sensitivity of the trained model *f* in relation to available data. The method consists of generating from dataset ***D*** a large number of *bootstrap samples* X^{*1}, X^{*2}, ...,X^{*B}. Each element of X^{*i} is obtained by a random sampling with replacement in *D* ; the size of each X^{*i} is *N*.

Corresponding to each bootstrap sample X^{*b} is a bootstrap replication of the model namely $f(X^{*b})$. An analysis of the set of these model samples $\{f(X^{*b})\}_{b=1, \ldots,B}$ allows to estimate the sensitivity of the model in relation to data. If the number *N* is big, an under-sampling of data points ($cardinal(X^{*b}) < N$) could be used. For each model $f(X^{*b})$, estimates of the empirical risk function ε^{*b} (learning error) using data points in X^{*b} and of the risk function ε^b (generalization error) using data points in *X* are computed. Indeed, there is a ratio of 1/e on average of the number of original data points which are not member of the bootstrap samples X^{*b}. The Bootstrap estimates of the average and the standard deviation of the set of bias between risk function and empirical risk function, being estimated using samples of the differences δ^b between ε^b and ε^{*b}, allow to characterize the model and are given from the bootstrap replications:

$$\delta_{average} = \frac{1}{B}\sum_{b=1}^{B}\left(\varepsilon^b - \varepsilon^{*b}\right) \tag{33}$$

$$\sigma_\delta = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\delta^b - \delta_{average}\right)^2} \tag{34}$$

When *B* is large (*B* greater than 50), good estimations of these statistics are obtained.

About the impact of response surface error [Devictor 04]

The use of a response surface or a surrogate model in an uncertainty and sensitivity analysis generally implies a bias or an error on the results of the uncertainty and sensitivity analysis, because the difference between the two models is rarely negligible. Usual questions are:

- What is the impact of this "error" on the results of an uncertainty and sensitivity analysis made on a response surface?
- Is it possible to deduce results on the "true" function from results obtained from a response surface?

If the variance of the result and sensitivity analysis defined by variance analysis are considered, the difficulty of the question can be figured. The "residual function" $\Delta(x_1, \ldots, x_p)$ between the response surface SR and the studied function *f* is:

$$\Delta(x_1, \ldots, x_p) = SR(x_1, \ldots, x_p) - f(x_1, \ldots, x_p)$$

Let's assume that a good approximation of the residual function is available. Then Δ-*SR* appears as a good approximation of the true function. Let's assume that all the X_i are independent, and sensitivity analysis have been done on both *SR* and Δ, and $S_{SR,i}$ and $S_{\Delta,i}$ are respectively the computed sensitivity indices. The assessment of $V(E(f(X_1, \ldots, X_p)/X_i))$ from $S_{SR,i}$ and $S_{\Delta,i}$ is obtained from:

$$S_{f,i} = S_{SR,i} \times \frac{V\left(SR\left(X_1,\cdots,X_p\right)\right)}{V\left(f\left(X_1,\cdots,X_p\right)\right)} + S_{\Delta,i} \times \frac{V\left(\Delta\left(X_1,\cdots,X_p\right)\right)}{V\left(f\left(X_1,\cdots,X_p\right)\right)} + \frac{2\,cov\left(E\left[SR\left(X_1,\cdots,X_p\right)\middle|X_i\right], E\left[\Delta\left(X_1,\cdots,X_p\right)\middle|X_i\right]\right)}{V\left(f\left(X_1,\cdots,X_p\right)\right)}$$

The computation of the covariance term requires a Monte-Carlo simulation. Then it is generally impossible to deduce results on the "true" function from results obtained from a response surface.

Only cases where results can be deduced are:

- *SR* is a truncated model obtained from a decomposition of f in an orthogonal functional basis;
- The residual function Δ is not very sensitive of the variables $X_1, \ldots, X_p$, then Δ can be considered as a random variable independent of $X_1, \ldots, X_p$ and $f(X_1, \ldots, X_p)$.
  Then $S_{f,i} = V(E(f(X_1, \ldots, X_p)/ X_i)) / V(Y)$ is estimated by
  $S_{SR,i} / (V(\Delta(x_1, \ldots, x_p)) + V(SR(x_1, \ldots, x_p)))$

Discontinuous models

For discontinuous functions, no usual response surface family is suitable. In practice, discontinuous behaviour generally means that more than one physical phenomenon is implemented in the function *f*. Then, to avoid misleading in the interpretation of results of uncertainty and sensitivity analysis, discriminant analysis should be used to define areas where the function is continuous. Statistical analyses are led on each continuous area. Different methods are available in statistical tools (like R or WEKA) or mathematical softwares:

- neural networks with sigmoid activation function,
- GLM models with a logit link or logistic regression,
- vector support machine,
- decision tree and variants like random forest…

2.2.3 Reliability evaluations by FORM/SORM methods

The first- and second-order reliability methods (FORM/SORM) [Rackwitz, 1979], [Madsen 1986], [Melcher, 1999] consist of 4 steps (Figure 10):

- the transformation of the space of the basic random variables $X_1, X_2, \ldots, X_n$ into a space of standard normal variables,
- the search, in this transformed space, of the point of minimum distance from the origin on the limit state surface (this point is called the design point),
- an approximation of the failure surface near the design point,
- a computation of the failure probability corresponding to the approximating failure surface.

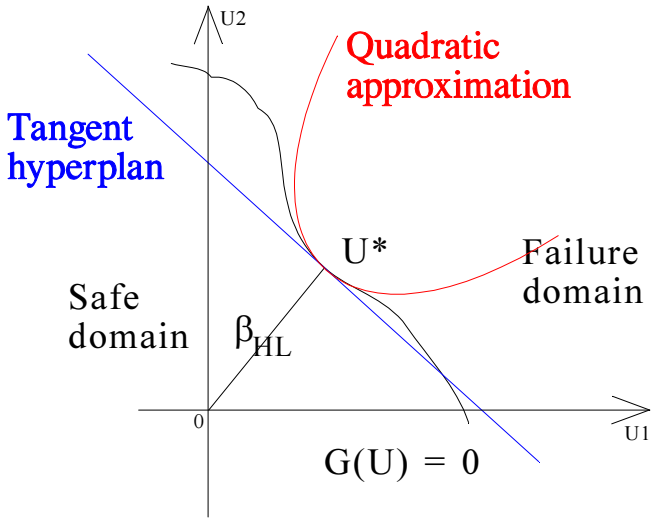

Figure 10: ***Reliability assessment with FORM/SORM methods***

FORM and SORM apply to problems where all the basic random variables are continuous.

Transformation of space

The choice of the transformation to be used depends on the characteristics of the joint density of random input vector *X*. The most current transformations are the transformation of Rosenblatt when the joint density is known, and the transformation of Nataf, when the probabilistic model is only made up of the marginal densities and of the matrix of covariance. In the Gaussian space, the surface of failure is defined by $G(u) = g[T^{-1}(u)]$.

Design point research

The Hasofer-Lind indice β_{HL} is defined as the minimal distance between the failure surface and the origin of the Gaussian space. The calculation of β_{HL} consists in solving the following problem of optimisation under constraint:

$$\min_{G(u)=0} \sqrt{u^t u}$$

The point associated with this minimal distance is often called the design point. It corresponds to the point of maximum of probability of failure and is denoted u^*. Indeed, in the Gaussian space, the joint density of the vector U is symmetrical in rotation with respect to the origin and thus involves that the design point coincides with the most probable point of failure.

The probability of failure can be calculated by:

$$P_f = P\left[G(\bar{U}) \le 0\right] = \int_{g(\bar{x})\le 0} f_{\bar{x}}(\bar{x})\, d\bar{x} = \int_{G(\bar{U})\le 0} \varphi_N(\bar{u})\, d\bar{u}$$

where φ_N is the multi-normal standard distribution of dimension *N*.

FORM method

The approximate method FORM (First Order Reliability Method) consists in approaching the surface of failure by a hyper plane tangent to the surface of failure at the design point. Then an estimate of the probability of failure is obtained by:

$$P_f = \Phi(-\beta_{HL})$$

where Φ is related to cumulative distribution of the standard normal law. The precision of this approximation depends on the non-linearity of the failure surface.

The knowledge of this design point makes it possible to determine the most influential variables on reliability. By supposing that there is a single design point and that the index of reliability defined by Hasofer and Lind β_{HL} is positive, the vector directional cosine unit α is defined by $\alpha = u^*/\beta_{HL}$.

The components α_i are also called factors of sensitivity and the factors α_i^2 are interpreted as factors of importance associated to the variables U_i. A variable associated with one significant α_i is regarded as having a significant influence on the probability of failure.

Note: the area of maximum of probability of failure is often different from the area around the average. Consequently the most influential variables are often different for these two domains. In consequence, the

factors of importance have a different interpretation that the sensitivity indices (Standardized regression Coefficients, Partial Correlation Coefficients…) calculated in global sensitivity analyses.

SORM method and variants

If the linear approximation is not satisfactory, more precise evaluations can be obtained from approximations to higher orders of the failure surface at the design point.

The approximation by a quadratic surface at the design point is called the SORM method (Second Order Reliability Method).

The corresponding formula uses the knowledge of the (q-1) principal curves κ_i of the failure surface at the design point:

$$P_f \approx \Phi(-\beta_{HL}) \prod_{i=1}^{N-1} \frac{1}{\sqrt{(1+\beta_{HL}\kappa_i)}}$$

This result is known as asymptotically exact, in the sense that the approximation of the probability of failure is better for large indices of reliability. The computing time is influenced by the calculation of the matrix of the second-order derivative, or Hessian matrix.

Another idea to contribute to the validation of a result FORM or to improve the precision of the result is to use a method of simulation in the vicinity of the design point *u**. The methods of importance sampling and directional simulation are the most used.

For an importance sampling (Figure 11) around the design point, the probability of failure is estimated by:

$$P_f = \int_F \frac{\phi_n(\mathbf{u})}{h(\mathbf{u})} h(\mathbf{u}) d\mathbf{u} \approx \frac{1}{n} \sum_{i=1}^{n} 1_{H(\mathbf{u})\le 0}(\mathbf{u}_i) \frac{\phi_n(\mathbf{u}_i)}{h(\mathbf{u}_i)}$$

where *h(u)* is the density of importance defined by a multinormal distribution centered on the point of design.

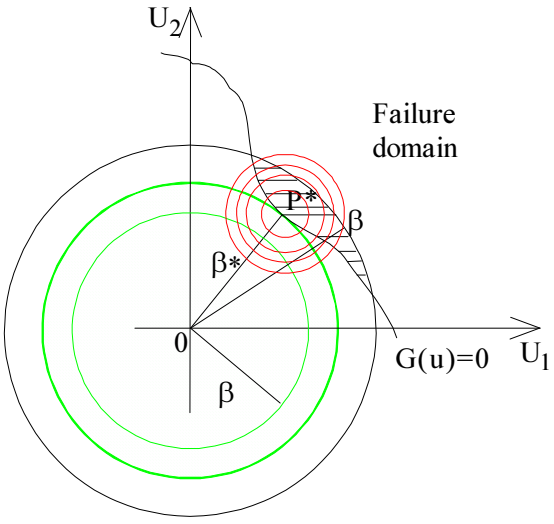

Figure 11: ***Conditional Importance Sampling***

If several local minima exist, the field of failure can be approached, for example, by the union of sub-fields of failure obtained by the linearization of the function of failure at each minimum. The methods multi-FORM and multi-SORM allow estimating the probability of failure.

Characteristics of FORM/SORM methods

FORM and SORM are approximate methods, and their accuracy is generally good for small probabilities. The analytical properties enable the methods to yield relatively inexpensive sensitivity factors. The basic random variables must be continuous, and the failure function must be continuous. With the optimisation procedures presently used in most cases, the failure functions should be smooth.

Accuracy: The methods are approximate, but yield generally accurate results for practical purposes, in particular for small range probabilities (10^{-3}-10^{-8}).

Efficiency: For small order probabilities FORM/SORM are extremely efficient as compared to simulation methods. The CPU-time is for FORM approximately linear in the number of basic variables n, and the additional CPU-time for a SORM computation grows approximately with n^2. The absolute computation time depends on the time necessary to evaluate the failure function. This time may in fact depend on the actual values of the basic variables. Extreme values may take longer due to increased non-linearities in the problem. The CPU-time is independent of the probability level, assuming a constant time for the evaluation of the failure function.

Restrictions and further developments: When the failure surface is not sufficiently smooth, the most likely failure point cannot be identified by efficient mathematical programming methods applying the gradient of the function. In this case, it would be advantageous to fit the exact failure function by a differentiable function. If the function is extremely CPU-time costly to evaluate, a simpler function may be used. It was suggested to use response surface methods. In particular, linear and quadratic response surfaces are well suited in combination with FORM/SORM, since the exact result with respect to the response surface is extremely efficiently computed.

New approached methods are under development for time-dependent problems.

Directional simulation

From some ideas of the FORM/SORM, a specific variance reduction technique has been developed. If the density f_X is continuous, then there is a one to one transformation $T : X \rightarrow U$, where U is a random vector of p independent random variables distributed according to a standard normal law. The space of the vectors U is called Gaussian space. As the transformation is one to one, the value of G at the point U is equal to the value of the function of failure g as in point $\boldsymbol{x}=T^{-1}(\boldsymbol{u})$. Thus the surface of failure in the Gaussian space is defined by $G(u) = g(T^{-1}(u))$. The probability of failure P_f is expressed by:

$$P_f = P\left[G\left(\overline{U}\right) \leq 0\right] = \int_{g\left(\overline{x}\right)\leq 0} f_{\overline{x}}\left(\overline{x}\right) d\overline{x} = \int_{G\left(\overline{U}\right)\leq 0} \varphi_N\left(\overline{u}\right) d\overline{u}$$

where φ_N is the standard normal density with N dimensions.

The idea of the method of directional simulation is to generate directions resulting from the origin of standard space and to calculate P_f conditionally with these directions. The Gaussian vector U is expressed in the form $U = R\,A\ (R \geq 0)$ where R^2 follows a Chi-square law χ_p^2 with p degrees of freedom independently of the random vector A uniformly distributed on the sphere unit. An estimator of P_f is obtained starting from N simulations a_i of the vector A. If R_i is solution of $G(R_i \times a_i) = 0$, and by noting $p_i = P(G(R_i \times a_i) \leq 0)$, an un-biased estimator is obtained by a Monte Carlo simulation:

$$\hat{E}[P_f] = \frac{1}{N}\sum_{i=1}^{N} p_i$$

$$\hat{D}[P_f]^2 = \frac{1}{N(N-1)}\sum_{i=1}^{N}\left(p_i - \hat{E}[P_f]\right)^2$$

The selection of the directions of search influences the efficiency of this technique. It is sometimes interesting to combine this technique with a method of importance sampling. To concentrate the directions drawn towards the area from the field of failure nearest to the origin allows the acceleration of the convergence. To determine this area is not obvious; methods FORM/SORM make possible to determine this area.

Coupling response surface method and FORM/SORM

FORM/SORM methods are often efficient to compute the reliability of a component or a system. But if the model is complex, the evaluation of the failure function can lead to expensive computation costs and the direct coupling (Figure 12) can be inadequate.

In order to solve the reliability problem, efficient coupling techniques (Figure 13) between complex codes (such T-H codes) and reliability codes using response surfaces have already been proposed. In practice, this coupling consists in determining a response surface locally and then performing a reliability study in replacing the failure function by this response surface.

The validation of these reliability results requires that the response surface is a good approximation of the failure surface in the region of maximum probability. The approximation quality depends on the choice of the points in the experimental design and must be verified by statistical criteria.

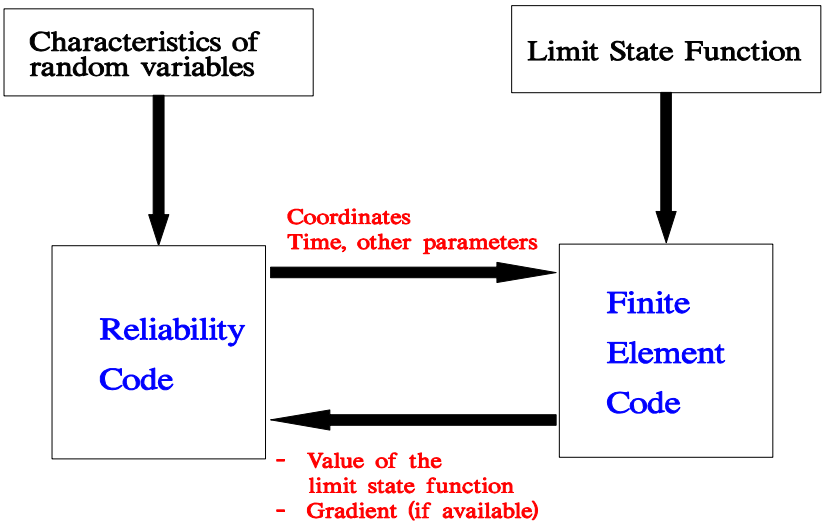

Figure 12: ***Direct coupling between reliability code and T-H code***

It can be interesting to couple methods FORM/SORM with quadratic response surfaces. Indeed methods FORM/SORM aim at determining the area of maximum of probability of failure and then to estimate the probability of failure in this area. For many problems, a quadratic response surface is sufficient to represent the continuous function in the vicinity of the design point. Thus, if such a response surface is validated in the

vicinity of the point of maximum of probability of failure, then it is possible to validate a calculation FORM or a calculation SORM with the hypothesis that the surface of failure is either an hyper plan or a quadratic surface in the vicinity of this point. An iterative method called RSAED (Response Surface with Adaptive Experimental Design) coupling FORM/SORM and quadratic response surfaces has been proposed [Devictor, 97].

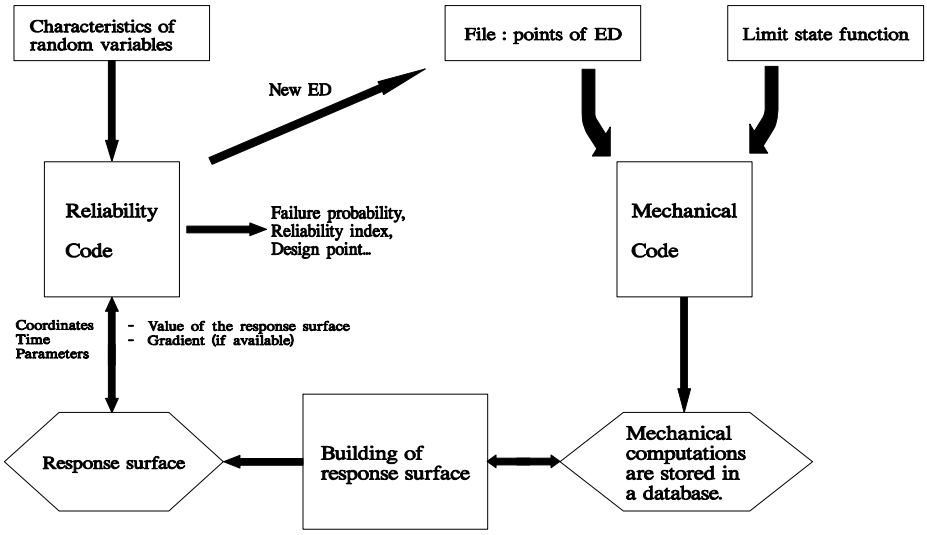

Figure 13: ***Coupling by use of response surface***

Comparison of Monte Carlo methods and FORM/SORM

The principal advantage of the Monte-Carlo method, but also of the majority of the variance reduction methods, is that they are valid for static, but also for dynamic models and for probabilistic models with continuous or discrete variables. Furthermore, there is no requirement on the failure functions – only the sign of the failure function is being used. In case of large computing times, it can be interesting to build an approximate mathematical model called response surface.

FORM and SORM are analytical and approximate methods, and their accuracy is generally good for small probabilities ($< 10^{-3}$). The analytical properties enable the methods to yield relatively inexpensive sensitivity factors. The basic random variables must be continuous, and the failure function must be continuous. With the optimisation procedures presently used in most cases, the failure functions should be smooth.

For small order probabilities FORM/SORM are extremely efficient compared to simulation methods, if the number of random variables is not too high. The CPU-time is for FORM approximately linear in the number of basic variables n, and the additional CPU-time for a SORM computation grows approximately with n^2. The absolute computation time depends on the time necessary to evaluate the failure function. This time may in fact depend on the actual values of the basic variables. The CPU-time is independent of the probability level, assuming a constant time for evaluation of the failure function. Table 4 summarizes the advantages and drawbacks of Monte-Carlo simulation and FORM/SORM methods.

Table 4: **Comparison of the characteristics of reliability methods**

Simulations	**FORM/SORM**
Results	***Results***
Failure probability Error on the estimation Probability distribution of the response	Failure probability Most influential variables (on the probability) Efficiency (depends on the number of random variables)
Assumptions	***Assumptions***
No assumptions on the random variables (discrete, continuous, dependency, etc.) No assumptions on the limit state function	Continuous random variables Continuous limit state function (more suitable for optimization step)
Drawbacks	***Drawbacks***
Computation costs (depends on the probability level)	No error on the estimation Global minimum is required, but it is necessary to obtain all the minima of the optimization problem.

2.4 Summary of the chapter 2

The reliability of a Passive System that utilizes NC can be assessed in introducing uncertainties in a T-H code reputed to give a best estimation of the system performances. Then, once the different source of uncertainties have been identified and modelled, the problem is how to propagate these uncertainties through the T-H code and how to evaluate the reliability of the Passive System.

The uncertainty in the physical response of the T-H code can be evaluated by a confidence interval or by a probability density function (*pdf*). Methods giving an uncertainty range of the system performance are not very useful for reliability estimation. On the other hand, the *pdf* of the system performance can be directly used for reliability estimation once a failure criterion is given.

For the evaluation of the *pdf*, the existing methods are generally based on Monte-Carlo simulations. These methods require a large number of calculations and can be often prohibitive when each calculation involves a long and onerous computer time. To avoid this problem, two approaches are possible: the variance reduction techniques in Monte-Carlo methods or the use of response surfaces where a simpler mathematical model no leading to expensive computation costs replaces the physical model. In order to estimate the reliability of a system, it is also possible to use approximate methods such as First and Second Order Reliability Methods (FORM/SORM).

3. INTEGRATION OF PASSIVE SYSTEM UNRELIABILITY IN PROBABILISTIC SAFETY ASSESSMENT

The treatment of Passive Systems within Probabilistic Safety Assessment (PSA) models is a difficult and challenging task. The main concern arises from the nature of the Passive Systems whose predominant operating principles are based on physical phenomena rather than on active components. Up to now, in the existing PSA of innovative nuclear reactor projects, only are taken into account the failures of the Passive System components (valves, pipes…), but not the failure of the physical phenomena on which the system is based. No commonly accepted practices exist for the treatment of this aspect of the Passive System failure in the PSA models.

This chapter provides different approaches how to integrate reliability evaluation of the Passive Systems into PSA models of accident sequences. The chapter is divided into three sections. Section 4.1 provides general information on PSA and its structure. Section 4.2 discusses a number of possibilities to develop interface between PSA model and Passive Systems reliability methodology and specifies the requirements for output of reliability calculations. The section 4.3 presents two practical approaches for the integration of Passive System unreliability in PSA studies.

3.1 Overview of PSA

This chapter provides general information on PSA [Kopustinskas, 2003].

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The first comprehensive application of the PSA dates back to 1975, to the United States Nuclear Regulatory Commission's (U.S. NRC) Reactor Safety Study [WASH-1400, 1975]. Since that pioneering study, there has been substantial methodological development, and PSA techniques have become a standard tool in the safety evaluation of the nuclear power plants (NPPs) and industrial installations in general. Due to historical reasons, the PSA sometimes is called PRA.

3.1.1 Levels of PSA.

As the most important area of PSA projects remains nuclear power plants, mainly due to the specific features of the nuclear installations, three levels of PSA have evolved:

Level 1: The assessment of plant failures leading to core damage and the estimation of core damage frequency. A Level 1 PSA provides insights into design weaknesses and ways of preventing core damage. In the case of other industrial assessments, Level 1 PSA provides estimates of the accidents frequency and the main contributors.

Level 2: As possible releases are additionally protected by containment in most NPPs, PSA at this level provides estimates of off-site\ release frequencies, based on the containment response and severe accident management possibilities. The results obtained in Level 1 are the basis for Level 2 quantification. In the case of other industrial assessments, Level 2 PSA might be fully covered by Level 1, as containment function is rather unique feature and is not common in other industries.

Level 3: The assessment of off-site consequences leading to estimates of risks to the public. Level 3 incorporates results from both previous levels.

3.1.2 PSA structure

Level 1 PSA is the most important level and creates the background for further risk assessment, therefore it will be presented in detail. The structure of the other levels is much more application-specific, and will be discussed only in general.

The typical phases of a PSA project are shown in Figure 14.

During the planning phase, scope of the study, as well as assumptions, limitations, level of detail and other boundary conditions have to be defined. Physical boundaries of the system have to be determined, which parts are to be included in the analysis and which are not? Operational state of the system have to be fixed, at which capacity the system is analyzed (full or reduced), what are the equipment states (valves open or closed etc.)? What external factors are to be analyzed (e.g. earthquake, extreme wind etc.)? The level of detail is also an important issue, e.g. is it enough to identify the reason as a "pump failure" or the detailed classification is required as pump failure at start, while running, due to cooling failure or oil leak and so forth. The level of detail is often restricted by the amount of information available.

The model construction consists of selection of initiating events (IE), modeling of accident sequences by FT/ET and quantification of the model components (initiating events, basic events, success criteria).

The calculations is a broad area of PSA work. It includes calculations of top event probabilities, minimal cut sets, importance, sensitivity and uncertainty analysis, quantification of sequences and consequences.

The last phase is to draw conclusions, provide recommendations and support for safety improvement decisions.

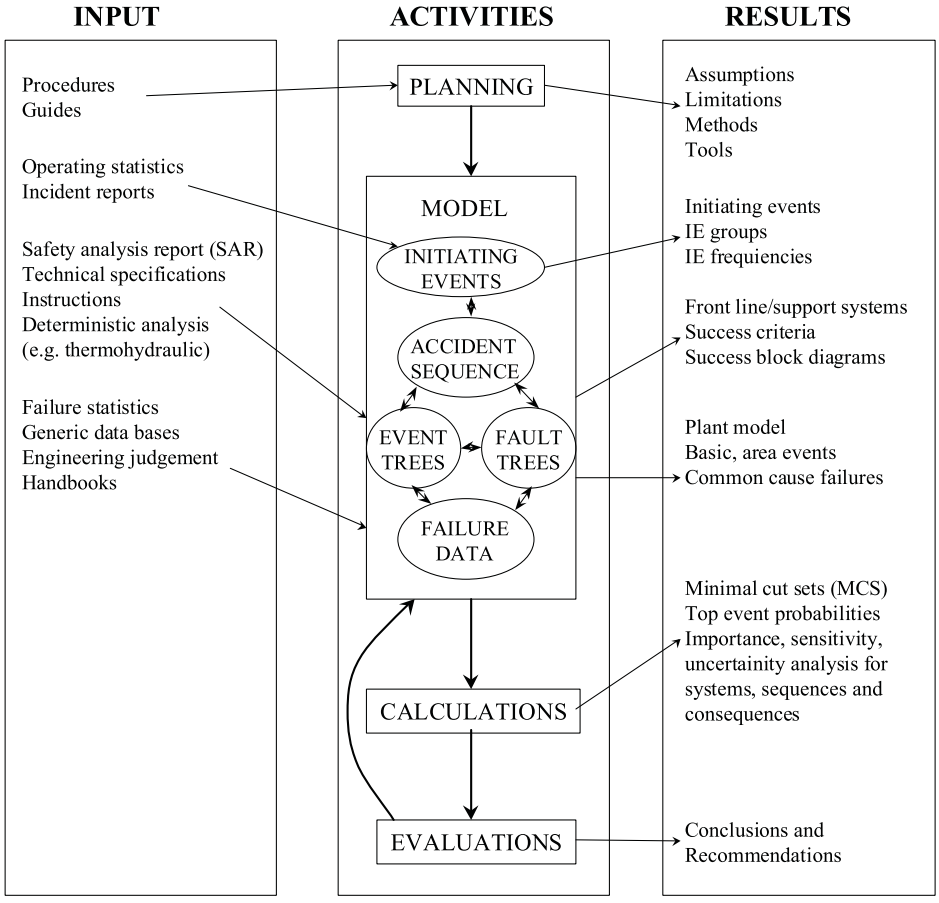


Figure 14: ***PSA structure and phases****.*

The model

The initial step in the construction of the model is to select initiating events. The initiating event is an event (e.g. equipment failure, transient) that can lead to the accident if no protective actions are taken. The protective

actions can be either automatic (most safety systems are actuated in this way) or manual (operator intervention is required). Identification of the IE is usually performed by combination of several approaches, like failure mode and effect analysis (FMEA), operation experience, incidents reports.

For each selected IE, detailed examination of the accident progression has to be made and accident sequences as logical combinations of success/failure conditions of functions or systems are identified. Each accident sequence ends with certain consequence, which also have to be defined. Consequences in the case of Level 1 PSA of NPPs are usually defined as degrees of reactor core damage, including 'safe' state and 'severe' accident state.

Event trees are used for the graphical and logical presentation of the accident sequences. An example of an event tree is shown in Figure 15. The logical combinations of success/failure conditions of functions or systems (usually safety systems, also called front-line systems) in the event tree are modeled by the fault trees.

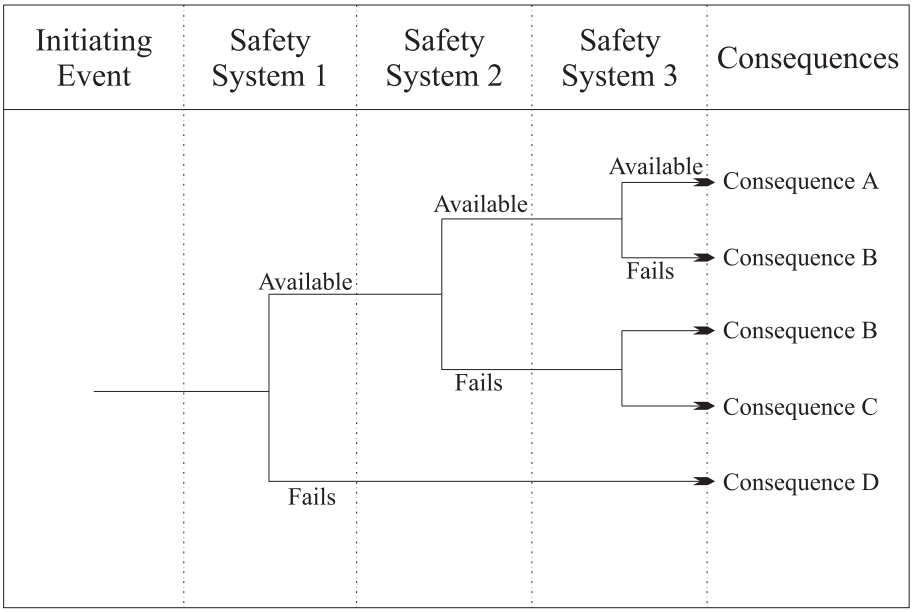


Figure 15: ***Example of an event tree****.*

A fault tree logically combines the top event (e.g. complete failure of a support system) and the causes for that event (e.g. equipment failure, operator error etc.). An example of the fault tree is shown in Figure 16. The fault tree mainly consists of the basic events (all possible causes of the top event that are consistent with the level of detail of the study) and logical gates (OR, AND, M out of N and other logical operations). Other modeling tools, like common cause failures, house or area events are also used in the fault trees. All front-line and support systems are modeled by the fault trees and then combined in the event trees depending on the initiating event.

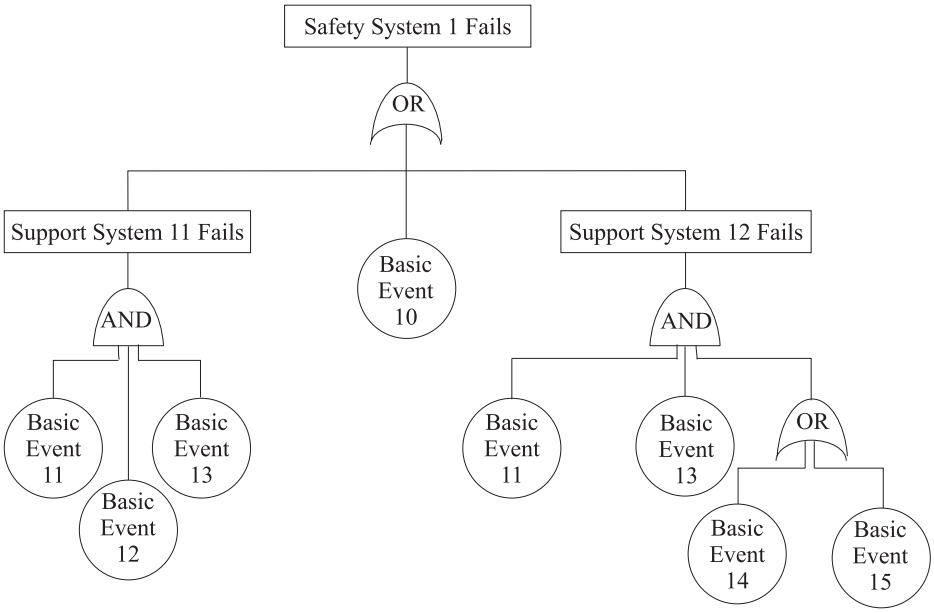


Figure 16: ***An example of a fault tree****.*

A fault tree is capable to include rather special cases, usually identified in complex systems. This include system and components dependencies, called common cause failures (simultaneous failures of several components due to the same reason), area events (usually fire, flood etc., which damages groups of components in certain rooms), human actions (operator errors or mitigation actions).

Data in PSA

Quantification of the model involves data analysis of the generic and plant-specific failure data. Data analysis should produce the following input to the model:

- Initiating event frequencies,
- Basic event failure parameters (failure probability per demand, failure rate, repair rate, test intervals, mission time),
- Human error probabilities,
- Data for common cause failures,
- Uncertainty data.

Calculations in PSA

A number of computer codes were developed to visualize PSA model construction and perform various calculations. The most popular of them are Risk Spectrum PSA (Sweden), IRRAS, CAFTA (USA), CAFTAN (Norway) and others.

Many kinds of different calculations can be performed with the model depending on the purpose. The most important of them are the following:

- Minimal cut sets,
- Probabilities of top events, sequences and consequences,
- Importance analysis,
- Uncertainty analysis,
- Sensitivity analysis.

A fault tree contains valuable information about possible combinations of faults that can result in a critical failure defined by the top event. Such a combination is called a *cut set*, which in fault tree terminology is defined as a set of basic events whose (simultaneous) occurrence ensures that the top event occurs. We are most interested in *minimal cut sets* which are cut sets that can not be reduced without loosing their status as cut sets.

Several algorithms are available to determine minimal cut sets in large FT, e.g. MOCUS. Minimal cut set analysis highlights the main contributors to the top event, which can be ranked on a quantitative basis. Quantitative as well as qualitative analysis of the cut sets provide insights of the system weaknesses and dominating sequences.

Top event probability shows unreliability of the system or function for which the top event is constructed. Probabilities of the accident sequences and consequences provide likelihood estimates of certain scenarios. As the logical structure of the fault and event tree is known, there should not be any problems to calculate top event or consequence probability using simple operations of Boolean algebra. However, even for simple systems, exact computational methods such as inclusion-exclusion, pivotal decomposition or ERAC may be cumbersome and time-consuming. Realistic applications may however contain thousands of basic events and hundreds of fault and event trees. Therefore, a number of approximate methods were developed for mathematical treatment of large FTs, many of them are based on the inclusion-exclusion principle.

The importance analysis is aimed at identification of the components that, by being improved, would increase the reliability of the system the most. It is obvious that reliability importance of a component in a system depends on two factors:

- The location of the component in the system,
- The reliability of the component in question.

A number of different reliability importance measures are proposed: Birnbaum's, Vesely-Fussell's, critical importance, improvement potential, risk achievement worth, risk reduction worth and others. However, different measures lead to different rankings, since the measures are defined differently. To identify the component that should be improved to increase system reliability, Birnhaum's and improvement potential are the most appropriate. To identify the component that has the largest probability of being the cause of system failure, the critical importance and Vesely- Fussell's measures are used.

The uncertainty analysis is an important task of a PSA. Any model of a large scale facility inevitably contains uncertainties, usually grouped into three categories:

- ***Completeness.*** This part of uncertainty reflects the lack of confidence in completeness of accident sequences and initiating events analysed in the study. There is no guarantee that other scenarios are impossible. This is the most difficult kind of uncertainty for assessment and quantification.
- ***Modelling adequacy.*** Even those scenarios and sequences that have been identified do not precisely represent reality because of simplifying assumptions, idealizations, mathematical models, numerical approximations or computational limits. Sensitivity studies are usually performed to assess importance of the key assumptions.
- ***Input parameters uncertainties.*** The parameters of various failure models are not exactly known because of lack of data, variability within the samples or assumptions. This category of uncertainties is quantified assuming that PSA result as well as input parameters (e.g. failure rates) are random variables. The probability distribution functions are assumed for each parameter and then by propagation through the model the uncertainty of the results is obtained. The most widely used technique for propagating uncertainties is a Monte-Carlo simulation.

The uncertainty analysis is still considered by many scientists and many scientific papers show increased interest in this field. The most recent area of uncertainty analysis is to assess uncertainties of the deterministic calculations, which play also essential role in the determination of success criteria in a PSA.

The purpose of sensitivity analysis is twofold: (1) to determine the sensitivity of the system failure frequency to possible dependencies among basic events and (2) to analyze those modeling assumptions suspected to have a potentially significant impact on the results. The general practice of the realistic models shows that results should not be too sensitive to any of the model parameters.

3.1.3 Benefits of PSA

The PSA is a powerful tool that can be used in many different ways to assess, understand and manage risk. Its primarily objectives are the following:

- estimate risk level of the facility,
- identify dominant event sequences affecting safety of the facility,
- identify systems, components and human actions important for safety,
- assess important dependencies (among systems or man-machine
- interactions),
- provide decision support in various application areas.

The growing area of PSA use is extensive support of probabilistic results in risk management and decision-making processes. The main areas of the PSA applications are assessment of design modifications and back-fitting, risk informed optimization of the Technical Specifications, accident management, emergency planning and others. Several modern tools of risk management are also based on the PSA model, such as risk monitoring, precursor analysis and others.

3.1.4 Limitations and drawbacks of PSA

Despite its popularity among the risk assessment tools, the PSA has a number of imitations and drawbacks. The main limitations of the PSA model are the following:

Binary representation of the component state. Only two states are analyzed: failed state or fully functioning state. However, this is not always realistic, as intermediate states are also possible. The same limitation exists for the redundant systems with certain success criteria - system is in failed state (success criteria is not satisfied) or in full power. The intermediate states for redundant systems are even more important.

Independence. In most cases, the components are assumed to be independent (except modelled by CCF), however there are many sources of dependencies, not treated by the model.

Aging effect. The aging effect is ignored because of the constant failure rate assumption. The only conservative possibility to treat the aging impact is to perform sensitivity study.

Time treatment. The FT/ET model is not capable to treat time explicitly during the accident progression. This is one of the major drawbacks of the methodology. In realistic systems, many parameters and functions depend on time and this is not encountered in the model and only approximate chronological order is assumed.

Uncertainty *of the calculations.* Uncertainties are inevitable in the PSA results and calculations and therefore direct treatment of the quantitative PSA estimates might be misleading. Due to the fact of uncertainties, the qualitative PSA results (identification of dominant accident sequences, comparison of different safety modifications) are of greater importance than quantitative.

3.2 Interface between PSA and Passive System Reliability Model

Passive Systems designed are devoted to plant safety, acting as safety features both to stop the progress and mitigate the consequences of an accident. This implies the evaluation of their unavailability within the plant probabilistic safety studies, with regard to the accident sequence definition and assessment, since they are required to perform safety-related functions in the event of undesired transients and accidents. Because these systems rely mostly on natural forces for their operation, their unavailability due to hardware failure and human error is significantly smaller than that of active systems. Their reliability addresses mainly the likelihood of occurrence of phenomenological factors that challenge the natural forces (e.g. gravity) and physical principles (e.g. heat conduction, natural convection), and degrade the system function to accomplish the safety goal.

Classically in PSA studies, accident scenarios are modelled through the ET (Event Tree) technique (see Section 1.1.2), which allows identifying all the different chains of accident sequences deriving from the initiating events. ET development implies each sequence represents a certain combination of events,

corresponding to failed or operating safety or front-line systems: thus ETs, starting from the initiators, branch down following success or failure of the mitigating features, which match the ET headings, providing therefore a set of alternative consequences.

There is always a nonzero likelihood of the occurrence of physical phenomena leading to pertinent failure modes, once the system enters into operation. The proneness of the system to the failure is dependent on the initial and boundary conditions and the mechanisms needed for starting and maintaining the intrinsic phenomena and thus on the accident progress in terms of physical parameters evolution: for this reason the unavailability figures to be assigned to the reference Passive System will depend on each single accident sequence, developed in the ET.

Two ways how to integrate Passive System unreliability into the ETs of PSA model have been proposed:

- directly in the ET as a single basic event,
- in developing a separate fault tree.

The first approach (developed in section 4.3.1) consists in using directly the probability of failure of the system to match the heading of the event tree: the reliability figure is derived by T-H assessment by code simulation, with reference to each accident sequence individuated by the ET (considering each sequence can influence the system behaviour differently). The underlying methods to evaluate the failure probabilities of the Passive System have been presented in Chapter 3 of the Part I of this course.

The second approach (developed in section 4.3.2) consists in developing a separate FT in order to evaluate the failure probability of the Passive System. . The system fault tree in a case of Passive Systems would be very simple, consisting of several basic events, representing failure of physical phenomena (natural circulation) and failure of activating valve or other means of initial system activation, connected together by “OR” gate, as shown in Figure 8.

3.3 Approaches for the integration of Passive System unreliability in PSA studies

3.3.1 Direct use of Passive System failure probabilities in the ET

This approach has been studied by CEA and TECHNICATOME, within the framework of the RMPS project [Marques et al., 2004]. It is based on a representation of the accidental scenarios in the form of static ET. This method was applied to a fictive PWR reactor equipped with two types of safety Passive Systems. The failures analyses performed on this reactor allowed the characterisation of the technical failures (valves, heat exchanger pipes…) and the ranges of variation of the uncertain parameters that affect the physical process. A simplified PSA was performed starting from a single initiating event. The majority of the sequences addressed by this ET were analysed by deterministic evaluations using envelope values of the uncertain parameters. For some sequences where the definition of envelope cases was impossible, basic events corresponding to the failure of the physical process were added to the event tree and quantitative reliability evaluations, based on Monte Carlo simulations and on a TH code were carried out to evaluate the corresponding failure probability. The failure probabilities obtained by these reliability analyses were fed into the corresponding sequences. This approach allows the evaluation of the influence of the Passive System on the accidental scenario. In particular, on the studied example, a new design basis of the system has been proposed in order to fully satisfy the global safety objective assigned to the reactor. The approach applied to the Residual Passive heat Removal system on Primary circuit (RP2) of PWR reactors is described in details in Section 5.3.1. In this application, the Passive System acts as an ultimate system in the management of the scenario. For these conditions, the characteristics of the current Level 1 PSA models are suitable. For the other conditions, complementary R&D is necessary and leave open number of questions discussed below. This choice of static ET might seem unsuitable because it does not appear to consider the dynamic aspects of the transient progression including dynamic system interactions, T-H induced failure, and operator actions in response to system dynamics. In fact, in the example treated, the overall reactor, including the safety systems and in particular the Passive System, is modelled by the T-H code. This leads to the fact that the dynamic system interactions are taken into account by the T-H calculation itself. In

addition, we have not considered human intervention during the studied sequences, which is coherent with the usual utilization of the Passive Systems in innovative reactors. So, for a first approach, the ET presentation seems a good and simple representation for the assessment of accident sequences, including the Passive Systems. However in order to generalise the methodology, it is important to take into account the dynamics aspects differently than by their alone modelling into the T-H code. Indeed in complex situations where several safety systems are competing and where the human operation cannot be completely eliminated, this modelling should prove to be impossible or too expensive in computing times. It is thus interesting to explore other solutions already used in the dynamic PSA like the method of the dynamic ET.

3.3.2 Development of separate FT for the Passive System

This approach has been developed by the ENEA and applied to the Isolation Condenser (IC) system of a Boiling Water Reactor [Burgazzi, 2006].

With regard to the IC system two kinds of system malfunction are to be expected, to be considered as ET headings and to be assessed by specific fault trees:

1. Failure to start-up (e.g. drain valve failure to open), which addresses mainly the component malfunction, rather than the initial conditions and mechanisms needed for starting the system operation
2. Failure to continue operating (e.g. natural circulation stability), which addresses the natural circulation stability, and the proneness of the system to the failure is dependent on the boundary conditions and the mechanisms needed for maintaining the intrinsic phenomena rather than on component malfunctions.

In figure 17, a general schematic of the ET including the Passive System is shown, while in figure 18, a typical scheme of the FT model referred to both start-up and operating condition is reported. The first calls for well-engineered safety components with at least the same level of reliability of the active ones. The second aspect is concerned with the way the physical principles (gravity and density difference) operate in terms of T-H parameters evolution (i.e. characteristic parameters as flow rate and exchanged heat flux) and depends on the surrounding conditions related to accident development. This could require not a unique unreliability figure, but the unreliability to be re evaluated for each sequence following an accident initiator, or at least for a small group of bounding accident sequences, enveloping the ones chosen upon similarity of accident progress and expected consequences: with this respect T-H analysis of the accident is helpful to estimate the evolution of the parameters during the accident progress.

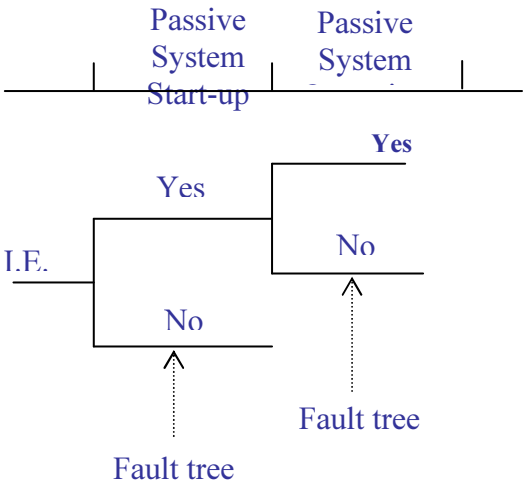


Figure 17: ***Event Tree Development for Passive System***

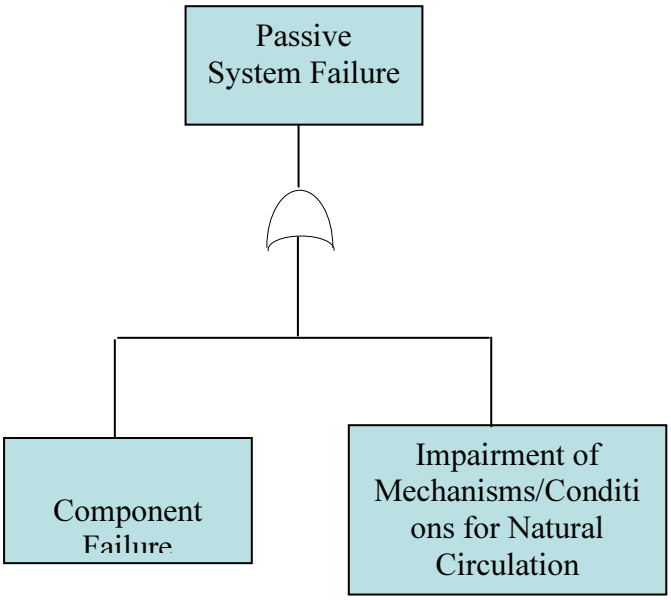


Figure 18: ***Fault Tree Model of the Passive System***

With focus on the latter aspect, the Passive Systems reliability assessment should consist of these two main steps:

- identification of all the relevant failure modes,
- unavailability quantification, in the form of probability of occurrence of the different failure modes.

As regards the former item, a previous qualitative analysis allowed to identify, by means of hazard identification used structured procedures as FMEA (Failure Mode and Effect Analysis) and HAZOP (HAZard and OPerability study), a set of relevant critical parameters direct indicators of the system physical failures, representing a consistent basis for further unavailability assessment.

Among these there are:

- Non-condensable fraction
- Undetected leakage
- POV (Partially Opened Valve) in the discharge line
- Heat loss
- HX plugged pipes.

As stated before, FT technique seems to be the most suitable mean to quantify the Passive System unavailability, once introduced the failure modes in the form of critical parameters elementary basic events, linked following the Boolean algebra rules (AND et OR), or in the form of sub-fault trees.

In fact, since the failure of the physical process is addressed, the conventional failure model associated with the basic events (i.e. exponential, $e^{-\lambda t}$, λ failure rate, t mission time), commonly used for component failure model, is not applicable: each pertinent basic event will be characterized by defined parameters driving the failure mechanisms - e.g. non-condensable fraction, leak rate, partial opening of the isolation valve, heat exchanger plugged pipes, etc. - and the associated failure criterion. Thus each basic event model pertaining to the relevant failure mode requires the assignment of both the probability distribution and failure range of the correspondent parameter.

In order to evaluate the overall probability of failure of the system, the single failure probabilities are combined according to:

$$Pe_t = 1.0 - ((1.0 - Pe_1)*(1.0 - Pe_2)*...*(1.0 - Pe_n)) \quad (1)$$

where:

- Pe_t = overall probability of failure
- Pe_1 through Pe_n = individual probabilities of failure pertaining to each failure mode, assuming mutually non-exclusive independent events

Thus for a Passive System with n mutually independent failure modes, the total failure probability is computed as for a series system with n critical elements so that each failure sums up to the system failure probability.

It is important to note that a mutual statistical independence between the basic random variables is assumed in this analysis for simplicity. However, in general, if some correlation between variables exists and the analyst knows of dependencies between parameters explicitly, multivariate distributions or conditional probability distributions may be used. The dependence between the parameters can be also introduced by covariance matrices or by functional relations between the parameters.

The failure model relative to each single basic event i is given by:

$$Pe_i = \int p_i(x)\, dx \qquad x<x_O \quad (2)$$

- $p_i(x)$ probability distribution function of the parameter x
- x_O value of the critical parameter at which the probability of failure is estimated.

Once the probabilistic distributions of the parameters are assigned, the reliability of the system, with regard to each mode of failure, can be directly obtained from (2).

Unfortunately difficulties arise in assigning the range and the probability density functions relative to the important characteristic parameters defining the failure modes, in addition to the definition of a proper failure criterion, because of the unavailability of a consistent experimental and operating data base: consequently the adoption of expert/engineering judgement procedure is helpful to address the set of uncertainties, mainly of epistemic character, associated with the issue.

Nevertheless, in order to overcome these difficulties, an approach implying the evaluation of components designed to assure the best conditions for passive function performance (e.g. vent valves opening for removal of non-condensables, piping integrity, heat exchanger for heat transfer process), could be considered in order to simplify the problem. Thus failures that defeat or degrade the natural mechanism upon which the Passive System relies are treated as an unavailability of components which challenge the boundary conditions or mechanisms needed for assuring the passive operation]. The drawback related to this approach is that the range of failure modes the system may potentially undergo is not fully covered.

Application of the approach

Let's consider three of the main parameters emerging from the previous qualitative analysis likely to affect the performance of the IC system: coefficient of closure of the isolation valve, heat exchanger plugged pipes, heat loss and non-condensable gas build-up. For each one the choices of the ranges are elaborated here below.

Valve closure coefficient

The opening of the valve is required in order to start-up the system from the stand-by condition allowing the steam flowing to the isolation condenser and, once condensed, establishing the natural circulation through the system. Partial closure of the valve (or failure to remain fully open) represents the presence of an abnormal loss in the pressure drop of the Passive System, hence a direct degradation of the natural circulation. The parameter is supposed to vary its range from 0, valve completely opened, in nominal operating conditions, to 0,5: this

value is considered the upper limit above which it's judged unlikely the occurrence of holding the valve partially open.

HX plugged pipes

Heat exchanger plugged pipes are detrimental for the heat transfer process in the pool: the lower limit being 0, corresponding to the nominal condition, the upper limit being 15%, corresponding to the condition requiring the replacement or refurbishment of the component.

Heat loss (Kw)

Heat dissipations along the piping due to thermal insulation degradation or inaccurate material assembly cause a reduction of the heat convection thus challenging the natural circulation flow. The parameter range is estimated 5-100 kw.

Non-condensable gas build-up

The presence of non-condensable gas at the inlet of the isolation condenser piping reduces the heat transfer rate and challenges the system's mission accomplishment. The non-condensable fraction is assumed to range from 0,01 to 0,8.

The limits have been chosen in order to exclude unrealistic values or those values representing a limit zone for the operation demand of the Passive System. Finally the truncated normal distribution, over the estimated range, is considered the most suitable for describing the expected distribution of the physical quantities and thus it is selected to construct the natural circulation failure probability distributions pertaining to each failure mode. The normal distribution parameter values (i.e. mean, variance) are derived from the range of variation assignment and the discrete values selection for each physical variable, via a structured "expert"/"engineering" judgement procedure. As a general rule, a central pivot has been identified, and then the range has been extended to higher and lower values, if applicable. The pivot value represents the nominal condition for the parameter.

The truncated normal distribution, over the estimated range, is considered for its simplicity and familiarity to engineers. It represents a good approximation in case the standard deviation is small as compared to the mean value.

$$f_{DTN}(x) = \begin{cases} 0, & -\infty \leq x \leq x_L \\ f(x)/[F(x_R) - F(x_L)], & x_L \leq x \leq x_R \\ 0, & x_R \leq x \leq +\infty \end{cases} \quad (3)$$

where f(x) and F(x) are respectively, is the probability density function and the cumulative function of the normal (gaussian) distribution.

This kind of distribution is shown in figure 19.

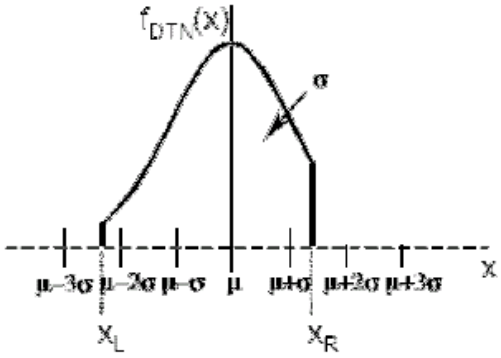


Figure 19: ***Doubly-truncated Normal Distribution***

Based on the normal *pdf*s defined above for each failure mode, a natural circulation failure probability distribution can be constructed.

Each probability of failure P_i in equation 1, is being assessed, in general, by solving according to equation 2:

$$Pi(x0) = \int_{x<xo} p_i(x)\, dx \tag{4}$$

- $x0$ = value of the parameter at which the probability of failure is calculated
- $p_i(x)$ = truncated normal pdf of the parameter as (3)

The values of the normal cumulative distribution functions at various values of the parameters, utilizing the relationship (4), are reported in table 5.

Finally the results allow defining a probabilistic criterion for the failure of the system operation in terms of the critical parameter threshold values for which the system performance is expected to decrease consistently, i.e. above which the probability of failure becomes relevant. Results are conditional on the assumptions concerning the parameter ranges and distribution selection, which are based on the author's experience and are rough estimates for the real values, since neither data bases nor operating data for the natural circulation system are available for statistical inference. Notwithstanding this, the results are physically realistic and comply well with the expected physical behaviour of the system under off-normal conditions. According to the equation (1), the final reliability figure will depend clearly upon the occurrence and the combination of the natural circulation failure modes and the parameter (e.g. HX plugged pipes, valve position and non-condensable gases build-up) evolution of the system stability during the transient/accident. Finally, from the above relationship one infers that the concurrence of various impairing factors results in a relevant value for the probability of failure of the system.

Table 5: ***Failure Probabilities at Different Parameter Values for Each Failure Mode***

Parameter		Unavailability
Partially Closed Valve (fraction)	0	0
	0,03	0,03
	0,05	0,06
	0,08	0,11
	0,10	0,26
	0,20	0,37
	0,30	0,62
	0,40	0,84
	0,50	1
HX Plugged Pipes (fraction)	0	0
	0,02	0,086
	0,05	0,29
	0,08	0,54
	0,10	0,58
	0,12	0,85
	0,14	0,96
	0,15	1
Heat Loss (KW)	5	0
	10	0,03
	20	0,13
	30	0,24
	50	0,52
	60	0,64
	80	0,86
	100	1
Non condensable gas (fraction)	0,01	0
	0,1	0,1
	0,2	0,24
	0,3	0,40
	0,5	0,72
	0,6	0,84
	0,7	0,94
	0,8	1

3.4 Summary of the chapter 3

The objective of this part of the methodology is the development of a consistent approach for introducing Passive System unreliability in an accident sequence of Probabilistic Safety Assessments (PSA).

The new element in the probabilistic modelling of the Passive System is the methodology to quantify reliability of the physical process, represented as a single basic event, from T-H modelling calculations.

There is two different ways how to integrate Passive System unreliability model into the whole plant PSA model. It could be done directly in the ET of relevant accident sequence as a single basic event, or a separate FT could be developed.

The former one accounts directly the failure probability/frequency value and is based on the T-H assessment by code simulation, through unreliability quantification methods presented in the part I.

The latter considers the system fault tree model based on the probabilistic estimation of the single failure modes.

Finally this choice of the ET presentation might seem unsuitable because it does not appear to consider the dynamic aspects of the transient progression including dynamic system interactions, T-H induced failure and operator actions in response to system dynamics.

Additional research and investigations, with focus for instance on the development of the dynamic event tree, are required in order to add credit to the whole methodology and to reach a more consistent approach.

4. THE RMPS METHODOLOGY

4.1 Overview

Within the European 5th Framework Programme, the RMPS methodology has been developed to evaluate the reliability of passive systems characterized by a moving fluid and whose operation is based on physical principles, such as the natural circulation [Marquès, 2005]. The reliability evaluation of such systems is based in particular on the results of thermal-hydraulic (T-H) calculations. This methodology (Figure 20) can be structured in three parts:

- Identification and quantification of the sources of uncertainties;
- Reliability evaluations of passive systems;
- Integration of passive system reliability in Probabilistic Safety Assessment.

Identification and quantification of the sources of uncertainties (see Chapter II for more details)

The methodology is applied to a specific accident scenario for which a particular Passive safety System is required. Having specified the scenario to be examined, the first step, **identification of the system**, requires fully characterizing the system under investigation. That is, specifying the goals of the system, the modes by which it can fail, and providing a definition of system failure, (i.e., success/failure criteria). **Modelling the system** is also required. This is done using best-estimate (B-E) computer codes. The numerous sources of uncertainties present in the modelling process have to be identified. This includes approximations in modelling the physical process and system geometry and uncertainties in the input variables such as initial and boundary conditions. **Identifying the most important thermal hydraulic phenomena and parameters** for the system that have to be investigated is an important part of the methodology. This can be done using an expert panel having a good understanding of the systems functions, B-E code calculations and a method for developing a relative ranking of the phenomena. The ranking technique implemented in the RMPS project was the Analytical Hierarchy Process (AHP). Having identified the important thermal hydraulic parameters, the next step is to **quantify their uncertainties**. In case of lack of experimental data, this requires expert judgement to identify the range of uncertainty and to select an appropriate probability density function for a given set of variables. The methodology then incorporates a **sensitivity analysis** which will determine, among all the uncertain parameters, the main contributors to the failure risk of the system.

Reliability evaluations of passive systems (see Chapter III for more details)

The second part of the methodology requires evaluating the uncertainty in the expected performance of the passive system as predicted by the T-H code and according to the studied scenario and the uncertainties' modelling. This uncertainty evaluation can be performed using a confidence interval or a probability density function. The RMPS study found that methods giving an uncertainty range of the system performance are not very efficient for reliability estimation. Therefore the use of a probability density function was the approach that was implemented. Then the probability density function of the system performance can be directly used for reliability estimation, once a failure criterion is given. The existing methods for this **quantitative reliability evaluation** are generally based on Monte-Carlo simulations. Monte Carlo simulations consist in drawing samples of the basic variables according to their probabilistic density functions and then feeding them into the performance function evaluated by the T-H code. An estimate of the probability of failure can be found in dividing the number of simulations leading to the failure of the system, by the total number of simulation cycles. Monte-Carlo simulations require a large number of calculations. As a consequence, these calculations can be prohibitively long. To avoid this problem, two approaches are possible: the variance reduction techniques in Monte-Carlo methods or the use of **response surfaces**. It is also possible to use approximate methods such as First and Second Order Reliability Methods (FORM/SORM).

Integration of passive system reliability in a Probabilistic Safety Assessment (PSA)

The objective of this last part of the methodology (not included in Figure 20) is the development of a consistent approach for these quantitative reliability evaluation of the Passive System and for its introduction in the accident sequence of PSA. Up to now, in the existing PSA of innovative nuclear reactor projects, are only taken into account the failures of the passive system components (valves, pipes…), but not the failure of the physical phenomena on which the system is based. Different possibilities have been discussed within the framework of the RMPS project, but no real consensus between partners was obtained. In coherence with the current standards of Level 1 PSA models, the approach studied by CEA and TECHNICATOME is based on a representation of the accidental scenarios in the form of static ET. The ET technique allows the identification of all the different chains of accident sequences deriving from an initiating events and describing the different basic events corresponding to the failure or to the success of the safety systems. This method was applied to a fictive PWR reactor equipped with two types of safety passive systems. The failures analyses performed on this reactor allowed the characterisation of the technical failures (valves, heat exchanger pipes…) and the ranges of variation of the uncertain parameters that affect the physical process. A simplified PSA was performed starting from a single initiating event. The majority of the sequences addressed by this event tree were analysed by deterministic evaluations using envelope values of the uncertain parameters. For some sequences where the definition of envelope cases was impossible, basic events corresponding to the failure of the physical process were added to the event tree and quantitative reliability evaluations, based on Monte Carlo simulations and on a T-H code were carried out to evaluate the corresponding failure probability. The failure probabilities obtained by these reliability analyses were fed into the corresponding sequences. This approach allows the evaluation of the influence of the passive system on the accidental scenario. In particular, on the studied example, a new design basis of the system has been proposed in order to fully satisfy the global safety objective assigned to the reactor.

This approach for integration of passive system reliability in a PSA has been applied to the Residual Passive heat Removal system on Primary circuit (RP2) of PWR reactors. In this application, the thermal-hydraulic passive system acts as an ultimate system in the management of the scenario. For these conditions, the characteristics of the current Level 1 PSA models are suitable. For the other conditions, the partners of the RMPS project consider that complementary R&D is necessary and leave open number of questions discussed below.

Conclusion on the RMPS methodology

The developed methodology participates to the safety assessment of reactors equipped with thermal-hydraulic passive systems (T-H) and is an indispensable tool for the designers who define the architecture of safety systems and for the regulatory authorities in the safety evaluation of passive system.

It should first of all be noted that the concept of intrinsic reliability of a component which exists for an active system or structural component is not directly applicable to a T-H passive system. Indeed, the probability or the frequency value associated with the T-H passive system is linked to a given scenario (state of the other systems on the circuit, environmental conditions...); the concept of functional margin has been introduced. A methodology to incorporate these reliability data into a single characteristic value, representative of all the possible situations, has not been found to date.

The results of the analyses made show that, in spite of the inherent characteristics of Passive Systems, which are a priori considered as advantages (simplicity, decrease of the need for human interaction, reduction or avoidance of external electrical power or signals), the decision for the designers to replace an active safety system by a passive system is not easy from a safety point of view. Moreover before making a final decision on this selection, other points which have not been addressed within the framework of the RMPS project, due to limited time and resources, have to be studied in future work. In particular, a very important issue concerns the human factors, which play an important role in the inspection and the maintenance of the Passive Systems.

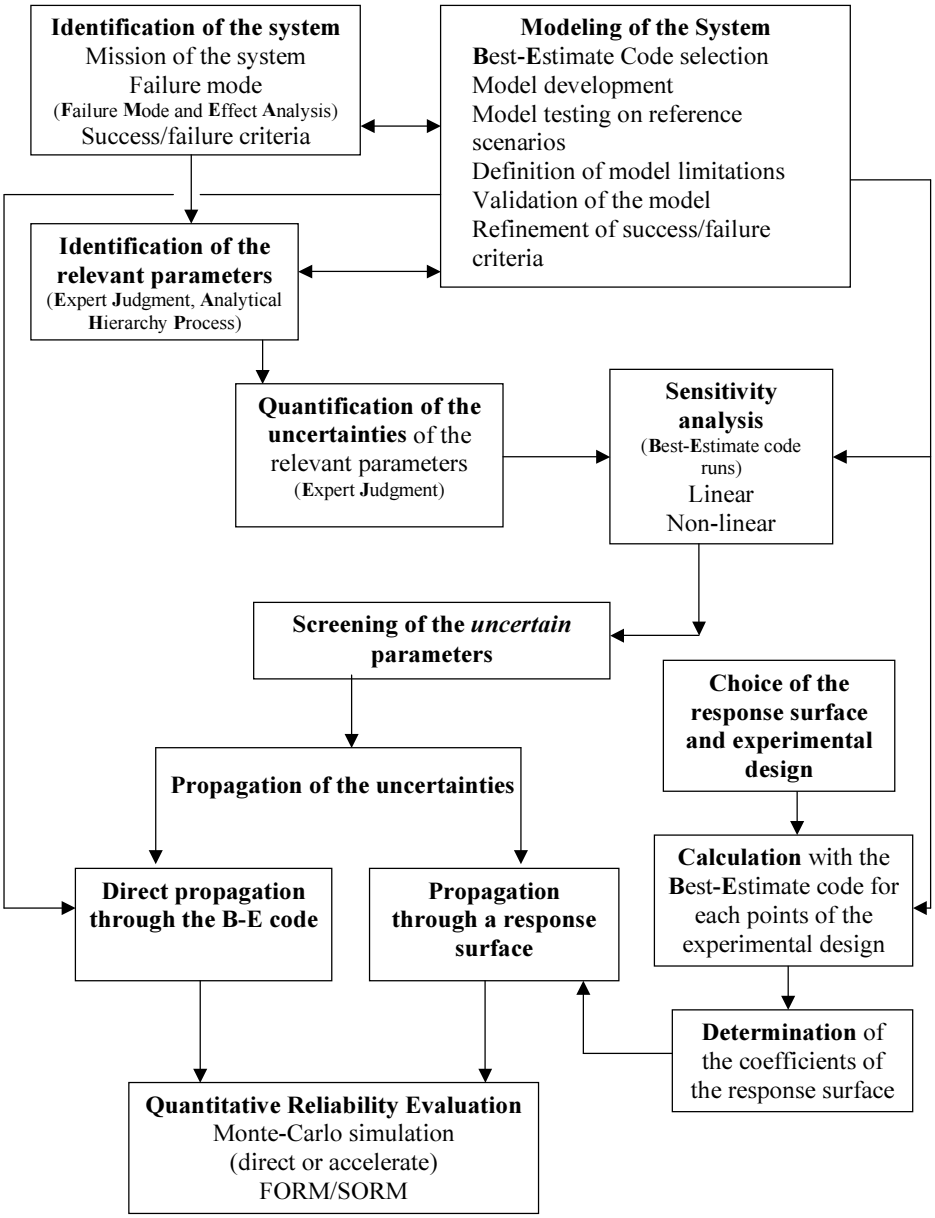


Figure 20: ***RMPS methodology roadmap****.*

4.2 Open questions

In addition to the improvement of different topics of the RMPS methodology and in order to answer to the question of the choice between active and passive systems, it is necessary to take into account other points of view that the reliability, such as the efficiency, the simplicity, the robustness, the human factor and economic evaluations, and to develop a tool for helping the systems design's optimisation.

1) About the improvement of the RMPS methodology, two items of the methodology roadmap deserve closer attention: the identification of the relevant parameters and the quantification of uncertainties. In RMPS, the identification of the input parameters is not based on strict rules. Rules, which guarantee a rationale approach to the problem and, which demonstrate that the procedure is based on realistic assumptions, would justify the choice of the uncertain parameters and moreover should convince the designer. In the selection of the relevant input parameters, a clear distinction for the various kinds of uncertainties should be introduced distinguishing between modelling uncertainties on one side and uncertainties dealing with the state of knowledge about the passive systems and their characteristic parameters on the other side.
2) Another important item of improvement is the integration of the passive systems reliability in the PSAs. The first attempts performed within the framework of RMPS have taken into account as well the failures of the components of the passive system as the failures of the physical process involved like basic events in static event trees.
   This last choice might seem unsuitable because it does not appear to consider the dynamic aspects of the transient progression including dynamic system interactions, T-H induced failure, and operator actions in response to system dynamics. In fact, we have treated in the RMPS project, examples where the overall reactor, including the safety systems and in particular the passive system, is modelled by the T-H code. This leads to the fact that the dynamic system interactions are taken into account by the T-H calculation itself. In addition, we have not considered human intervention during the studied sequences, which is coherent with the usual utilization of the passive systems in innovative reactors. So, for a first approach, the event tree presentation seems a good and simple representation for the assessment of accident sequences, including the passive systems.
   However in order to generalise the methodology, it is important to take into account the dynamics aspects differently than by their alone modelling into the T-H code. Indeed in complex situations where several safety systems are competing and where the human operation cannot be completely eliminated, this modelling should prove to be impossible or too expensive in computing times. It is thus interesting to explore other solutions already used in the dynamic PSA like the method of the dynamic event trees.
3) A very important issue is on the human factors, which play an important role in the reliability assessment of a passive system. Indeed the periodic maintenance and inspection of such systems introduce particular constraints; unlike an active system that can be more easily isolated or inspected during the shutdown periods, a passive system requires to be tested under its real physical conditions of utilization, and this can generate new specific implementation in the global architecture and safety problems. In addition, the question of whether it is an advantage or a disadvantage that passive systems do not allow operator intervention during its operation, should be investigated.
   Before comparing a passive and an active system on the same mission, it is necessary to make sure that the passive system design is optimised in terms of performance. Methods have to be developed to ensure the optimisation.
4) Technical-economical evaluations of the systems must be carried out to provide information that is essential for the comparison between passive and active systems.

4.3 Applications

Within the RMPS European Project, the methodology was applied to 3 Passive Systems: the Isolation Condenser System (ICS) of BWR reactors, the Residual Passive heat Removal system on Primary circuit (RP2)

of PWR reactors and the Hydro-Accumulator (HA) system of PWR, VVER type reactors [RMPS Final Report, 2004].

Within the Coordinated Research Project of IAEA, "*Natural circulation Phenomena, modeling and reliability of passive systems that utilize natural circulation"*, an application of the RMPS methodology is in progress on the Passive Residual Heat Removal System (PRHRS) of the Argentinean CAREM reactor project.

Within the framework of Generation IV reactors development, 2 applications of the RMPS methodology are in progress, one concerning the Decay Heat Removal system (DHR) of a Gas-cooled Fast Reactor (GFR) and the other, the decay heat removal system by radiation and conductivity of a Very High Temperature Reactor (VHTR).

In this chapter, we will present the application on the RP2 system and the two applications concerning the Generation IV reactors, each application illustrating different aspects of the methodology.

4.3.1 RP2 passive system

Description of the RP2 system

The Residual Passive heat Removal system on the Primary circuit (RP2) system is composed of three circuits dedicated to the heat removal, each one being connected on a loop of the primary circuit (Figure 21) [Gautier, 1999]. Each circuit includes an exchanger immersed in a cooling pool located inside the containment, and a valve to allow its starting. For the study in progress, this valve was put on the cold leg of the system, downstream the exchanger. The exchanger is located higher than the main piping of the primary circuit to allow a natural convection between the core and the exchanger. On criterion of emergency shutdown of the reactor, the valve opens and the natural convection starts. The residual power produced by the fuel is transferred to the cooling pool via the RP2 exchanger.

This system is quite similar to the PHRS from AP600, but its missions are different. AP600 only rely upon passive systems for Design Basis Accidents. RP2 has been designed within the framework of a new management principle, termed "Base Operation Passive Heat Removal" (BOPHR), where the residual power is removed jointly by active and passive systems, immediately after emergency shutdown.

Characterisation of the RP2 system

The transient of Total Loss of the Power Supplies (or Blackout) was selected as reference accident for the reliability evaluation of the system.

The objective of the safety systems is to avoid the core fusion in pressure. Thus the mission of the RP2 system is double, on the one hand to depressurise the primary circuit, and on the other hand to avoid the fusion of the core.

For the exercise, the duration of accidental calculation was fixed arbitrarily at 12 hours, relatively long time where no human intervention is simulated. The failure of the system is obtained if the maximum temperature of the clad or the temperature of the fluid at the core output go beyond respectively the values of 500°C and 450°C, in less than 12 hours.

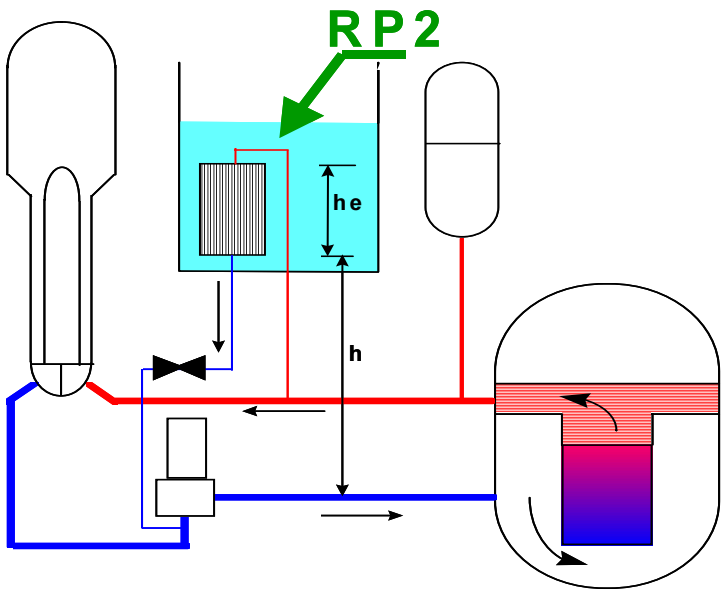


Figure 21: ***BOPHR/RP2 system***

Modelling of the RP2 system

A modelling with CATHARE (1.5a Mod 3.1) [Barre, 1990] of a complete pressurized water reactor PWR 900 MWe with the 3 independently simulated primary/secondary loops has been carried out. Each loop is equipped with the RP2 system with its exchanger immersed in a pool. The 3 cooling pools are modelled independently. Each system RP2 is connected to a primary loop between the hot and cold legs. Steady state calculation consists in carrying out the regulations of the characteristic parameters retained in the study with their target values. Once the regulations finish (primary circuit flow rate in the loops, Steam Generator levels, pools levels and feedwater flow rate of the Steam Generators) and all the values of the uncertain parameters imposed, the transient can start with all the physical parameters with the desired values. The following assumptions are taken into account for the Blackout reference calculation:

- Shutdown of the pumps of the primary circuit;
- Curve ANS 100% NP (2775 MW) for the decay of residual power;
- Loss of the Feedwater Flow Control System and the Auxiliary Feedwater System;
- Core Power at 100% NP: 2775 MW;
- Primary pressure: 15.5 MPa;
- Level of the pressurizer: 7.3 m;
- Initial level of fluid in the Steam Generators: 12.78 m;
- Secondary pressure: 5.8 Mpa;
- 3 RP2 available;
- Initial temperature of the water of the pool: 30°C.

With an aim of differentiating the various effects, the accumulators were not modelled.

Identification of the sources of uncertainties of the RP2 System

A set of 24 parameters likely to be more or less uncertain at the time of the RP2 passive system start-up and influencing significantly the performances of the system was identified by expert judgment. These parameters are called hereafter the characteristic parameters and are listed below.

For each of the three BOPHR/RP2 systems (i = 1,3):

- I_i: instant of opening of the isolation valve of the RP2;
- X_i: rate of incondensable at the inlet of the RP2 exchanger;
- L_i: initial pool level;
- T_i: initial temperature of the water of the pool;
- C_i: fouling of the tubes of RP2 exchanger;
- R_i: number of broken tubes of RP2 exchanger.

For the primary circuit:

- PUI: percentage of the nominal power of the core;
- PP: pressure in the pressurizer;
- ANS: decay of residual power according to the ANS law.

For the secondary circuit (i = 1,3):

- NGVi: real secondary level in the three steam generators.

Quantification of the sources of uncertainties of the RP2 System

The chosen probabilistic model is presented in table 6. The choice of the ranges of variation and *pdf* of the parameters was based on expert judgment. We detailed these choices for two parameters: the instant of opening of the RP2 valve and for the non-condensable fraction at the inlet of the RP2 exchanger.

Instant of opening of the RP2 Valve (minutes): I_1, I_2, I_3

The RP2 is started by opening a valve placed on the cold leg of the system, downstream the exchanger. This valve is supposed to be a pneumatic valve opened by default of power supply (a default of power supply implies the closure of the valve of compressed air supply which causes the opening of the pneumatic valve). We suppose that the failure of opening of the pneumatic valve is due to the failure of closure of the valve of compressed air supply. After half an hour, we suppose that the action of an operator is possible in case of non-opening of the valve. We have considered only two states for the valve, completely open or completely close.

The state of the valve (open/close) is then modeled by :

- a discrete variable with two values, giving the state of the valve at the initial time just after the black-out :

$$O_{t=0} : P(O_{t=0}) = 0.95$$
$$F_{t=0} : P(F_{t=0}) = 0.05$$

- a continuous variable giving the instant of opening of the valve after the time t= 30 mn , in the case the valve fails to open at the initial time:

$$P(O_{t>30}/F_0) = \mathrm{Log}(1.0607t + 0.809) \text{ (--> P=1 à t=5heures)}$$

Non-condensable fraction at the inlet of the RP2 exchanger: X_1, X_2, X_3

(In the exchanger tubes and in the inlet pipes from a level corresponding to the lower part of the exchanger)

The presence of non-condensables could be caused e.g. by an incorrect maintenance of the passive system piping or by gas accumulation during the nominal operational life of the system.

Sensitivity analysis of the RP2 system

88 samples of the 24 random variables have been generated and for each sample a CATHARE calculations have been performed.

Among these 88 calculations, we have obtained 7 failures of the system considering the failure criterion: fluid temperature at core output greater than 450 °C in less than 12 hours. All these 7 failure cases correspond to cases with one tube rupture in one of the RP2's. Depending on the cases, the limit core output temperature is reached between 4100s and 7100s. All the other calculations are a success for the system mission.

In a second step, we have decided to suppress the possibility of tube rupture at the RP2 startup. We can imagine that probability of tube rupture could be included in the failure of mechanical components of the system in a Probability Safety Analysis (See below *Approach used for integration in PSA*). In this case, we have 21 random variables.

85 samples of the 21 random variables have been generated and for each sample a CATHARE calculations have been performed. All these cases lead to a success of the system mission considering the failure criterion (fluid temperature at core output greater then 450 °C in less than 12 hours).

Table 6: ***Probabilistic model of the input characteristic parameters****.*

Variable	**Distribution**	**Average**	**Standard deviation**	$\mathbf{X_{min}}$	$\mathbf{X_{max}}$	λ	μ
I_1, I_2, I_3	Composed						
X_1, X_2, X_3	Exponential					182	0
L_1,L_2,L_3	Truncated normal	4.5	0.6	2	5		
T_1, T_2, T_3	Truncated normal	303	20	280	368		
C_1, C_2, C_3	Truncated normal	15	5	0	30		
R_1, R_2, R_3	Exponential					7	0
PUI	Truncated normal	100	1	98	102		
PP	Truncated normal	155	4	153	166		
ANS	Truncated normal	10	5	0	20		
NGV_1, NGV_2, NGV_3	Truncated normal	12.78	0.30	12.08	13.91		

Criterion of performance of the system

In order to analyse the performance of the system, we have considered the following performance criterion:

$$Ratio = \frac{\text{Sum of the energy extracted by each RP2 during the transient (42000s)}}{\text{Energy of the core during the transient}}$$

Sensitivity analysis

In order to perform sensitivity analysis on the performance of the system, we have try to fit different types of mathematical model between the 21 input parameters X_i and the output value Y (ratio) calculated by CATHARE.

We have first fitted polynomial response surface of the form:

$$Y = a + \sum_{i=1}^{21} b_i X_i + \sum_{i=1}^{21} c_i X_i^2 + \sum_{i=1}^{21} d_i X_i^3$$

The table 7 gives for each types of polynomial, the coefficient of determination obtained.

Table 7: ***Coefficients of determination R2 obtained for each polynomial fitted***

Polynomial	R^2
1st degree	0.77
2nd degree	0.90
3rd degree	0.93

The figure 22 compares the values of the ratio obtained by CATHARE and predicted by the 3rd degree polynomial. We have also tried responses surfaces obtained by Neural Network. Among different NN tested, the network 21_30_1 gives the best result with a R^2 equal to 0.90. The figure 23 compares the values of the ratio obtained by CATHARE and predicted by the NN 21_30_1.

We have performed two types of sensitivity analysis: a) a sensitivity analysis with linear coefficients (Standardized Regression Coefficients) even if the model is not fully linear ($R^2 = 0.77$) and b) with SOBOL indices calculated in using the response surface (Neural Network 21-30-1) and in performing 10000 simulations.

The results of the calculation of the Standardised Regression Coefficients are presented in the figure 24 and the results of the calculation of the SOBOL indices in the figure 25. They give both the same indications: the most important variables are *ANS*, the residual power decay which is mainly due to the state of the fuel in the core when the transient occurs and I_1, I_2, I_3, the instants of opening of the RP2 valves which govern directly the duration of the heat exchange time in the RP2's.

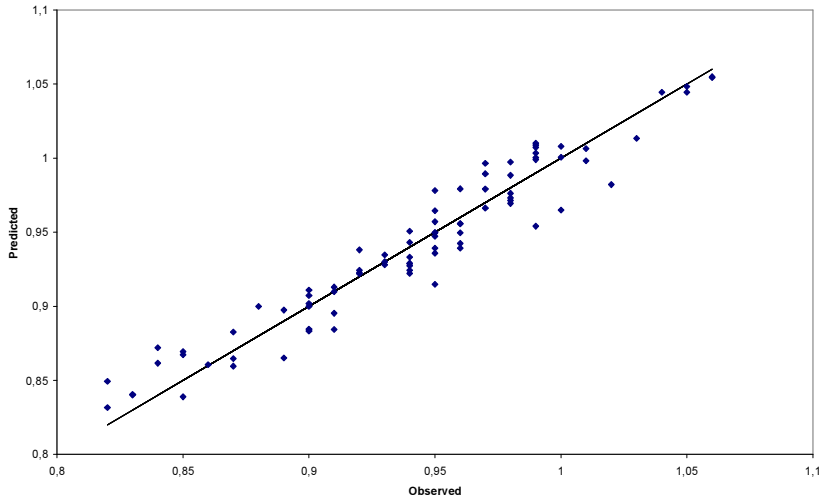


Figure 22: ***Comparison the values of the ratio obtained by CATHARE and predicted by the 3rd degree polynomial.***

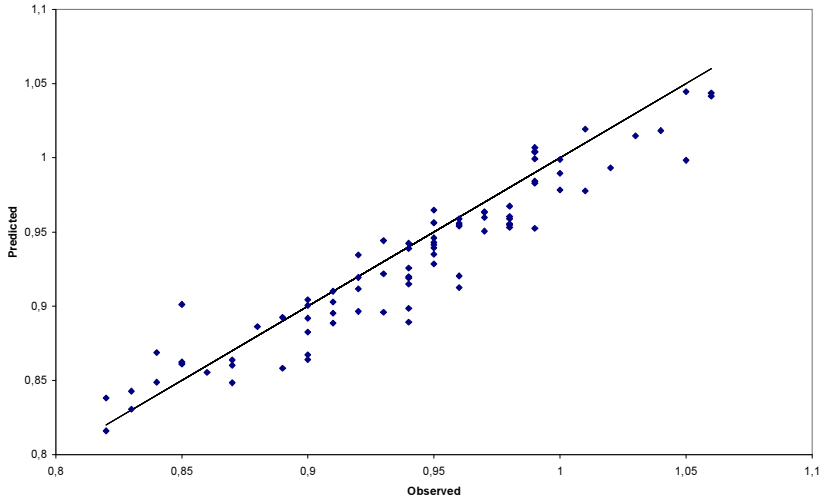


Figure 23: ***Comparison the values of the ratio obtained by CATHARE and predicted by the response surface model (Neural Network 21_30_1).***

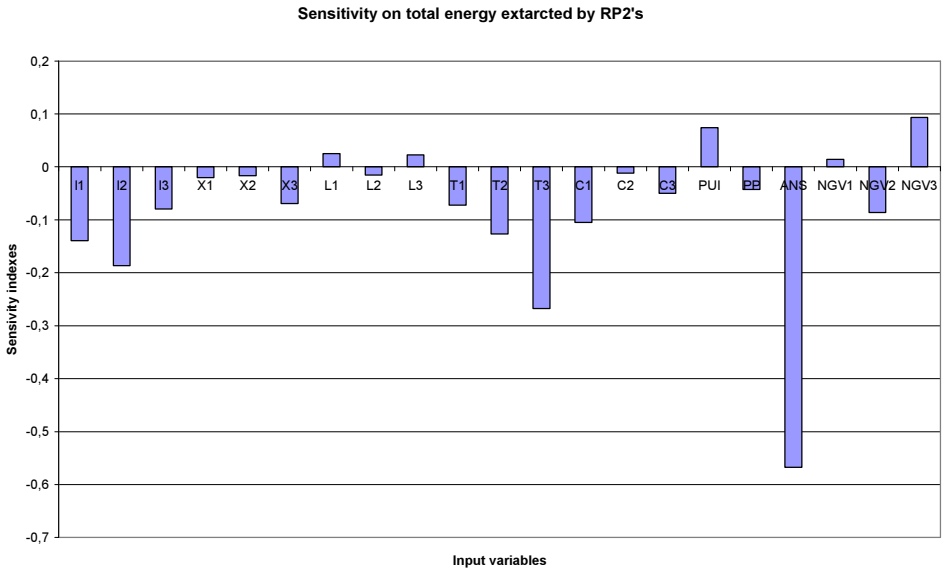


Figure 24: ***Standardised Regression Coefficients (SRC)***

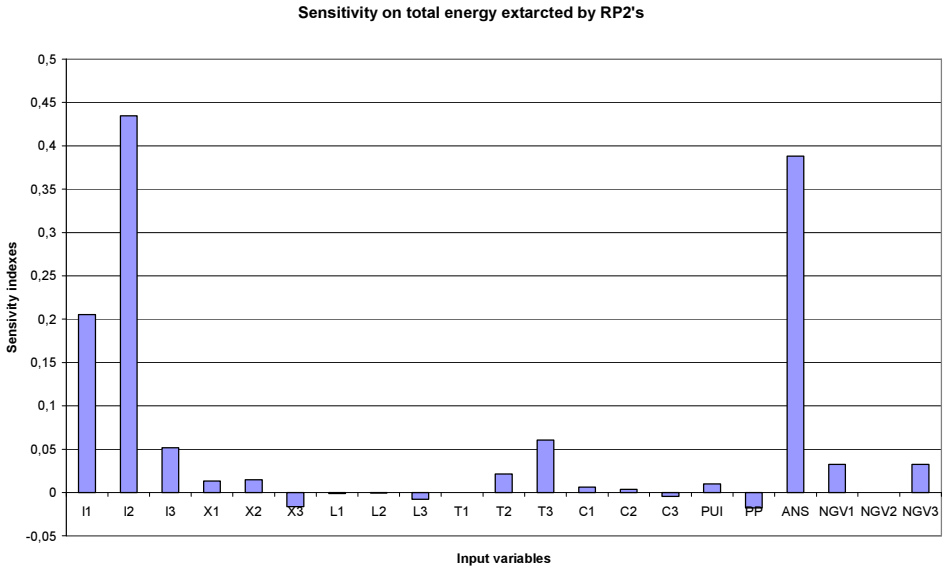


Figure 25: ***SOBOL indices***

Approach used for integration in PSA

In a first approach, applied to a simplified PSA carried out on a fictive reactor equipped with two types of safety passive systems, we have chosen an Event Tree (ET) representation of the accidental scenario. ET techniques allows identifying all the different chains of accident sequences deriving from the initiating events. ET development implies each sequence represents a certain combination of events, corresponding to failed or operating safety or front-line systems: thus ETs, starting from the initiators, branch down following success or failure of the mitigating features, that match the ET headings, providing therefore a set of alternative consequences.

The failures analyses performed on this reactor have allowed the characterisation of the technical failures (on valves, tubes in exchanger and safety injection check valves) and the ranges of variation of uncertain parameters which influence the physical process.

The majority of the sequences of this event tree have been analysed by deterministic evaluations with envelope values of the uncertain parameters. For some sequences where the definition of envelope cases was impossible, basic events corresponding to the failure of the physical process have been added and uncertainty analyses have been performed. For this purpose the thermal-hydraulic code has been coupled to a Monte-Carlo simulation modulus. The failure probabilities obtained by these reliability analyses have been integrated in the corresponding sequences. This methodology allows the probabilistic evaluation of the influence of the passive system on an accidental scenario and could be used to test the interest to replace an active system by a passive system on specific situations.

Specific reliability analyses for the PSA integration

Within the framework of the integration of the system reliability in a PSA in order to test the influence of the passive system on different accidental situations, specific ranges of variation and specific probabilistic density functions of the characteristic parameters have been identified for the studied sequences. Specific reliability and sensitivity analyses have been carried out for these sequences. For two sequences where the definition of envelope cases was impossible, uncertainty analyses have been performed to evaluate the corresponding probability of failure. These sequences were: the sequence with 2 RP2 available and no broken tube and the sequence with 2 RP2 available and one broken tube.

We present here the example of the sequence with two RP2 available and no broken tube in the RP2 exchanger. In this case, the number of characteristic parameters was reduced to 14 (there is only two RP2 systems and the number of broken tubes and the valve failure are no more considered in the uncertainty analysis, but taken into account in the event tree of the PSA). In addition, a monitoring system was supposed to be implemented on the RP2 system, in order to verify continuously that the RP2 loops are available when they are solicited. This led to narrower ranges of variation for the levels and the temperatures of the two pools. The choice of the probabilistic model presented in table 4 was based on engineer judgment.

Table 8: ***Probabilistic model of the 14 parameters***

Variable	Distribution	Par 1	Par 2	X_{min}	X_{max}
X_1, X_2	Truncated log-normal	0.12	0.43	0	1
L_1, L_2	Truncated normal	4.5	0.5	4	5
T_1, T_2	Truncated normal	30	20	10	50
C_1, C_2	Truncated log-normal	12	0.4	0	30
PUI	Truncated normal	100	2	98	102
PP	Truncated normal	155	2	153	157
ANS	Truncated log-normal	6	0.4	0	20
NGV_1, NGV_2, NGV_3	Truncated normal	12.78	0.70	12.08	13.91

The results of the sensitivity analysis performed in calculating the Standardized Regression Coefficient and the Partial Correlation Coefficient show that the more influential parameters on the performance ratio are L_1 and L_2 the initial pool levels and the *ANS* curve. The objective of the uncertainty calculations was to evaluate the probability *p1* corresponding to the failure of the physical process, considered as a basic event in the event tree, when only two RP2 are available. We have carried out 76 calculations with CATHARE with values of the input variables randomly generated in considering this probabilistic model. Among these 76 calculations, we obtained

18 cases of failure, leading to a rough estimation of the failure probability p_1 to 0.24. This failure probability is conditional with respect to the sequence considered and has to be multiplied by the probabilities of all the basic events involved in the sequence in order to determine the failure probability of the sequence. In the same way, we determine a failure probability p_2 equal to 0.04 for the RP2 passive system in the sequence with 2 RP2 available and one broken tube.

Integration of the unreliability of the RP2 Passive System in PSA

This study consisted in:

- carrying out a global quantitative evaluation of the reliability of the RP2 passive system,
- including this evaluation in a simplified PSA of PWR reactor,
- carrying out calculations of a set of CATHARE transients,
- including the CATHARE results in the PSA,
- evaluating the yearly occurrence frequency of core damage for the reactor equipped with safety passive systems, in case of transient of Total Loss of the Power Supplies (or Blackout),
- identifying complementary CATHARE calculations to test and validate the methodology.

The reliability analysis of the RP2 passive system underlines the existence of two types of failures which could affect the system:

- Failures on passive system components, which lead, directly or indirectly, to the loss of the system,
- The occurrence of an initial configuration of the passive system, which is not standard and leads to the loss of the system, mainly because of thermal-hydraulic reasons. The not standard configuration of the passive system does not allow to guarantee the good working of the system in case of demand.

For this second type of failures, we can consider two possibilities:

- A monitoring system can detect, before the occurrence of the blackout, the existence of the not standard configuration of the passive system. It is also considered that, as soon as the not standard configuration is detected, the automatic safety systems or the operators shutdown the reactor in safety state (shutdown of the reactor by instruction). The occurrence of this type of configuration lies in the failure of monitoring systems. For the reliability analyses, the initial configuration of the passive system to be considered is the one corresponding to the limit conditions generating the shutdown of the reactor in safety state (shutdown of the reactor by instruction).
- No monitoring system can detect, before the occurrence of the blackout, the existence of a not standard configuration of the passive system. It is not considered that operators shutdown the reactor in safety state. The occurrence of this type of configuration lies in the existence of a set of characteristic parameters, corresponding to an initial not standard configuration of the passive system.

The table 9 gives the failure probabilities obtained, for each of the 3 types of failures previously identified. Concerning the first and the second types of failures, the analysis consists in taking into account the failure in form of a probability of occurrence. The probability of failure per demand for each RP2 loop is estimated through a failure analysis. The values adopted for the failure probabilities leading to the failure of the RP2 system are evaluated by analogy with similar components existing on PWR reactors.

The accidental transient of Total Loss of the Power supplies (reactor in full power) has been chosen, as it was the reference transient having been used for the dimensioning of the safety systems dedicated to residual power removal. The probability of occurrence of this initiating event is 10^{-5}/year. The analysis is carried out through the method of event tree, integrating the RP2 loops and the safety injection by accumulators. The event tree is presented on figure 26. In order to simplify the event tree representation, the different numbers of RP2 loops available are presented in the same event tree.

Table 9: ***Failure probabilities***

FAILURE TYPE	FAILURE PROBABILITY (/demand)
Non opening per demand of the RP2 valve (defined for each RP2 loop)	3.10^{-3}
Broken tubes in the RP2 exchanger (for at least 1 of the 3 RP2 loop)	3.10^{-3} / hypothesis : 10^{-3} per RP2 loop
Failure of the pool level measure (captor, calculator)	3.10^{-3}
Failure of the pool water temperature measure (captor, calculator)	3.10^{-3}
Failure of the steam generator level measure (captor, calculator)	Negligible (reactor safety system)
Failure of the primary pressure measure	
T-H process failure (p1) with 1 RP2 loop not available and no broken tubes	0.24
T-H process failure (p2) with 1 RP2 loop not available and broken tubes	0.04
Failure of safety injection by accumulators	10^{-3}

The core damage frequency, after a blackout, is estimated at $7.5.10^{-8}$/year. This frequency corresponds to the sum of the probabilities of each accident sequence leading to the core melt in pressure for the transient of blackout. The main accident sequence (sequence 5) represents 96% of the core damage frequency. This sequence corresponds to a T-H process failure when 1 RP2 loop has failed. This frequency is at the limit of the acceptability, as it does not respect the probabilistic objectives 10^{-7}/year for all the transient families, which corresponds for a transient family to 10^{-8}/year. This result does not affect the conception of the RP2 system which is efficient to avoid the high pressure core melt.

As result of this exercise, we concluded that it would be desirable to re-examine the dimensioning of the RP2 system, in order to obtain a well-running process, when 1 RP2 loop is in failure. The probabilistic objective to reach is 0.03 for the T-H process failure in case of 2 RP2 loops available. This value would allow to reach a yearly core damage frequency of 10^{-8} regarding the high pressure core damage for the studied transient family (blackout). The RP2 passive system, dimensioned in this way should allow to reach the probabilistic safety objectives for a reactor integrating passive safety systems. These results underline the importance to take into account the T-H process failure probability to evaluate the reliability of a safety passive system.

The current analysis was an illustrating exercise, dedicated to the test and the validation of the reliability method applied to the passive system. This exercise implies some specific limits in the analysis. In any case, these limits should be analysed in a PSA. These specific limits are:

- The analysis concerns only one initiating event, the Total Loss of Power supplies, even if this transient is the transient of reference having been used for the dimensioning of the safety systems dedicated to the residual power removal; other initiating events have to be analysed,
- The consequences of a system failure, when it is not in demand (valve opening, valve leak, rupture of a primary nozzle) are not considered (i.e. the initiating events created by a failure of the RP2 are not taken into account), even if this failure could have a potential effect on the safety,

- No aggravating event is considered, relative to the initiating event of Total Loss of Power supplies, else than the RP2 passive system failures (component failures or T-H process failure) and the safety injection,
- Human factor (operator errors) are not explicitly taken into account (the presence of a crisis team allows limited error possibilities),
- No "mechanical" common cause failure between the 3 RP2 loops have been considered. Only the "thermal-hydraulic" common cause failure has been taken into account through the global CATHARE modelling of the 3 RP2 loops. Thus, the mechanical failure of 1 RP2 loop has no consequence on the operation of the others,
- The common cause failure between the monitoring systems of the RP2 loop are considered as negligible,
- No common cause failure is considered between the RP2 passive system and the safety injection,
- Only 1 failure is considered for each RP2 loop.

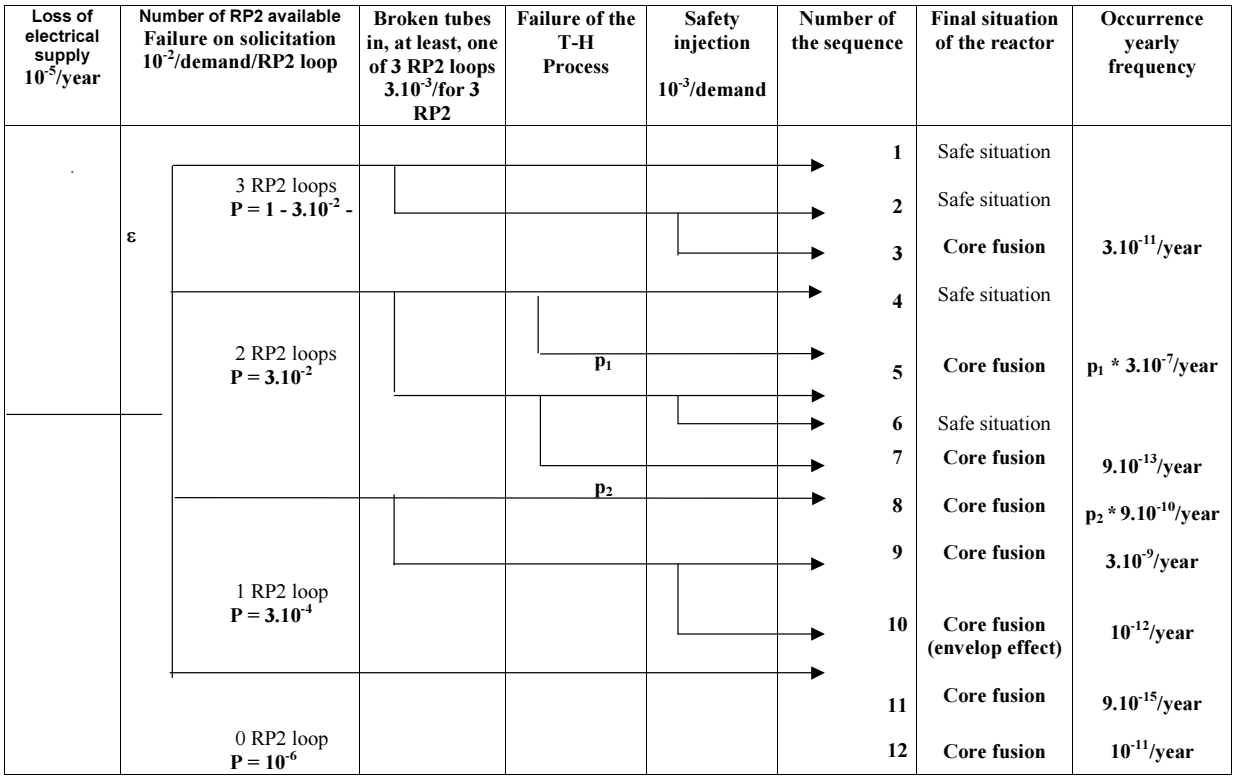


Figure 26: ***Simplified Even Tree of Loss of Electrical Supply including Passive Systems***

4.3.2 Isolation Condenser System (ICS)

Description of the ICS

The aim of the ICS is the decay heat removal during accidental operation using passive systems and not active as is presently the case in standard reactors.
A pipe, which is connected to the reactor pressure vessel, in BWR type application (Fig.27) or to the Steam Generator of the PWRs, is used to direct the steam to a heat exchanger, which is immersed in a cooling pool. In every configuration, the pool is placed higher than the element to be cooled (vessel or SG) in order to ensure a flow in natural convection, which guarantees a minimum of human of mechanical interventions.

The steam side connection between the vessel and the IC is normally open and the condensate line is normally closed. This allows the isolation condenser and drain piping to fill with condensate, which is maintained at a sub-cooled temperature by the pool water during normal reactor operation.
The IC is started into operation, after opening a valve, by draining the condensate to the reactor, thus causing steam from the reactor to fill the tubes, which transfer heat to the cooler pool water.
The selected configuration, as well as the operating conditions, are typical of those expected for the operation of the Isolation Condenser, that is part of the SBWR design. Nominal/reference/initial and boundary conditions for the system operation are assumed to be available.
The transient operation starts with the opening of the valve on the drain line of the condenser.

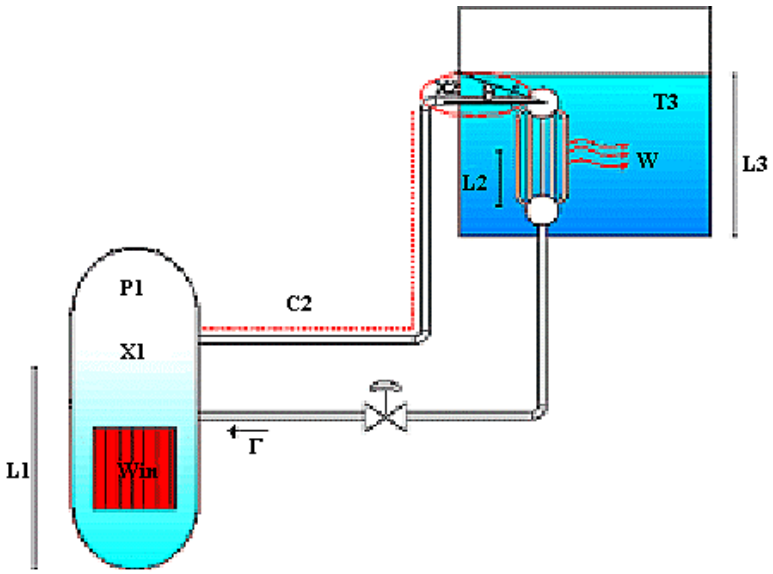


Figure 27: Standard Isolation Condenser as studied within the BWR

Qualitative analysis of the system

The reliability assessment of the ICS requires, as a basic step, the identification of all the relevant failure modes: starting point of the analysis is the identification of the main sources of physical failure.
A qualitative analysis is required in order to identify all the potential failure modes affecting the system and their consequences associated with the passive system operation [Burgazzi 2002]. Thus well structured procedures, commonly used for hazard identification in risk analysis, like FMEA and HAZOP are considered, as suitable means for such an analysis. These approaches are presented and discussed and the relative results compared.
In general the reliability of the ICS should be seen from two main aspects:

- systems/components reliability (e.g. piping, valves)
- physical phenomena reliability (e.g. natural circulation stability)

The first calls for well engineered safety components with at least the same level of reliability of the active ones.
The second aspect is concerned with the way the natural physical phenomena operate in a particular system and the long term effect of the surrounding on their performance/stability. It calls for the identification of the relative modes/causes of failures.
As consequence the qualitative approach has implied the introduction of a "virtual" component, that is the physical principle, for instance natural circulation, upon which the system relies, and its evaluation with reference to the "phenomenological" factors likely to impair the system performance.

Failure Mode and Effect Analysis

FMEA is a bottom-up procedure conducted at component level by which each failure mode in a system is investigated in terms of failure causes, preventive actions on causes, consequences on the system, corrective/preventive actions to mitigate the effects on the system and eventually classification according to its severity.
As highlighted before, the FMEA approach application calls for the examination, in addition to mechanical components of the system (piping, drain valve, etc.), of a "virtual" component identified as Natural Circulation and its evaluation in terms of potential "phenomenological" factors (the list of these includes e.g. non-condensable gas build-up, thermal stratification, surface oxidation, cracking, etc.), the consequences of which can impair or stop the natural circulation, and the identification of the relative critical parameters (non-condensable fraction, undetected leakage, heat loss, etc.).
Failure modes are ranked according to a severity index related to the estimated range of frequency of occurrence of the fault, according to table 10, which categorizes the events by likelihood of occurrence.
The introduction of this "virtual" component claims the concept of "functional reliability", that is the probability of failure of the physical principle, in this case the natural circulation (that is gravity and density difference) upon which the system is relying.
In table A3.1 (Appendix 3), FMEA sheets are reported, together with the assessed critical parameters associated to the failure modes and the tentative severity ranks, based on subjective/engineering judgement.
The analysis pointed out several factors leading to disturbances in the Isolation Condenser system;
the list of these includes:

- Unexpected mechanical and thermal loads, challenging the primary boundary integrity
- HX plugging
- Mechanical component malfunction, i.e. drain valve
- Non-condensable gas build-up
- Heat exchange process reduction: surface oxidation, thermal stratification, piping layout, etc.

Finally, as previously assumed, this qualitative analysis allows to detect a set of critical parameters affecting the natural convection reliability and to be accounted for in further probabilistic analysis:

- Non-condensable fraction
- Undetected leakage
- POV(Partially Opened Valve) in the discharge line
- Heat loss
- Piping inclination
- HX plugged pipes

Hazard and Operability (HAZOP) studies

The HAZOP approach is taken into consideration in order to compare and "endorse" the results coming from the FMEA analysis. This procedure considers any parameters characteristic of the system (among pressure, temperature, flow-rate, heat exchanged through the HX, opening of the drain valve) and by applying a set of "guide" words, which imply a deviation from the nominal conditions as for instance undesired decrease or increase, determines the consequences of operating conditions outside the design intentions.
In table A3.2 (Appendix 3), HAZOP tables are reported.
The main difference between the two approaches consists in the fact that while the former is performed at component level, the latter is conducted on a parameter basis (e.g. flow, temperature), thus making the FMEA analysis more suitable for the assessment of the phenomena likely to degrade the system.

Looking at the tables the HAZOP analysis confirms what assessed in the previous FMEA analysis: in fact in general the possible causes of the deviation of the characteristic parameter from the nominal conditions, which result in the natural circulation impairment, imply the occurrence of the aforementioned failure modes individuated by the previous analysis.
The qualitative analysis of a thermal-hydraulic passive system, by means of the FMEA approach, has enabled to identify the most relevant failure modes related to passive system performance and provided as output a set of critical parameters (i.e. physical quantities) that characterize the system failure and thus to be considered in the relative assessment process.
The FMEA approach had to be modified and tailored on the specific argument by introducing the typical phenomenology of the passive system (i.e. natural circulation flow impairing, heat transfer degradation, etc.), thus resulting in a sort of "phenomenological FMEA". Expert judgement has been used as viable path to identify those typical phenomenology for the passive system.
HAZOP analysis, conducted on the main physical parameters, has been helpful to qualify and eventually confirm the outcome of the earlier study.

		FREQUENCY OF OCCURRENCE
I	Operational Events	More than one per year
II	Likely Events	$1 \div 10^{-2}$/y
III	Unlikely Events	$10^{-2} \div 10^{-4}$/y
IV	Extremely Unlikely Events	$10^{-4} \div 10^{-6}$/y
V	Hypothetical Sequences	Less than 10^{-6}/y

Table 10: ***Event Categorization by Likelihood of Occurrence***

Modeling of the system

The need arose to simulate the transient performance of this system by different thermal-hydraulic codes available: RELAP5, ATHLET and CATHARE 2.

Sketches of the system nodalisation are given in Fig. 28 for RELAP5, in Fig. 29 for CATHARE2 and in Fig. 30 for ATHLET.

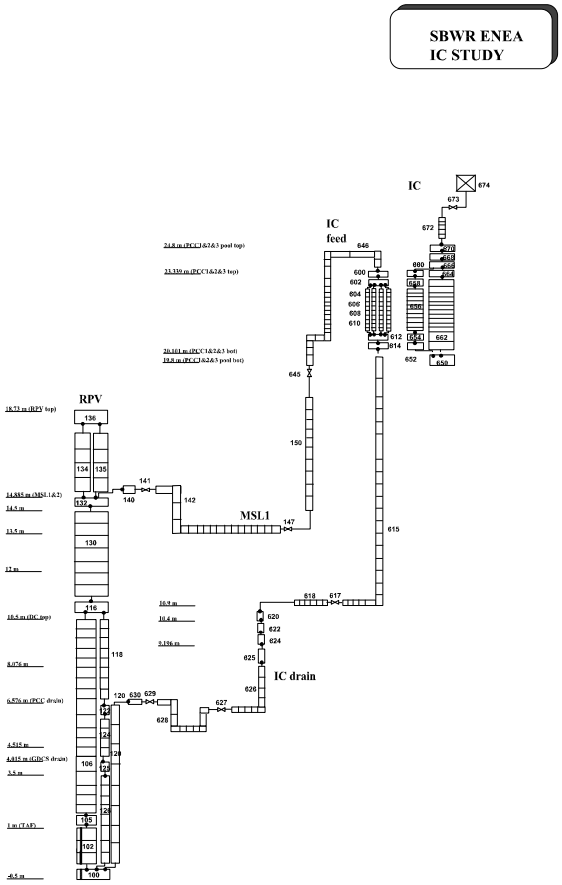


Fig. 28: ***Nodalisation of the passive system by the Relap5 code.***

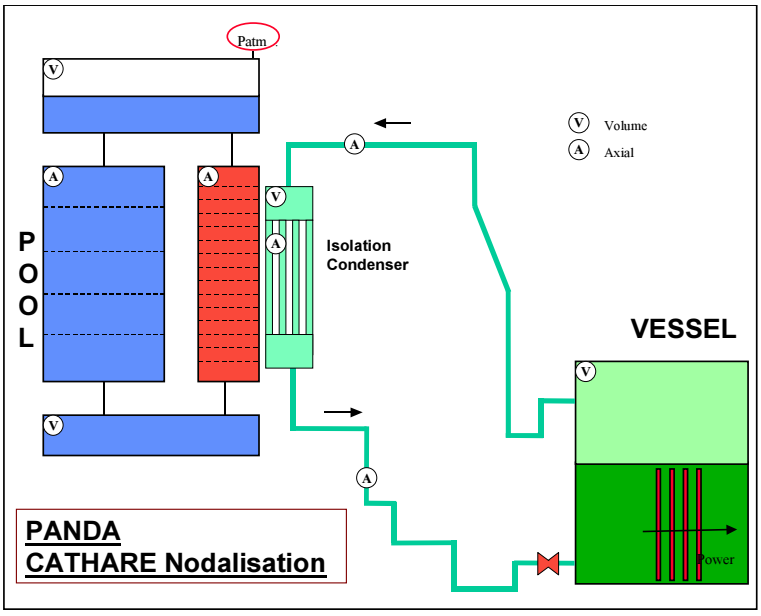


Fig. 29: ***ICS modeling with CATHARE.***

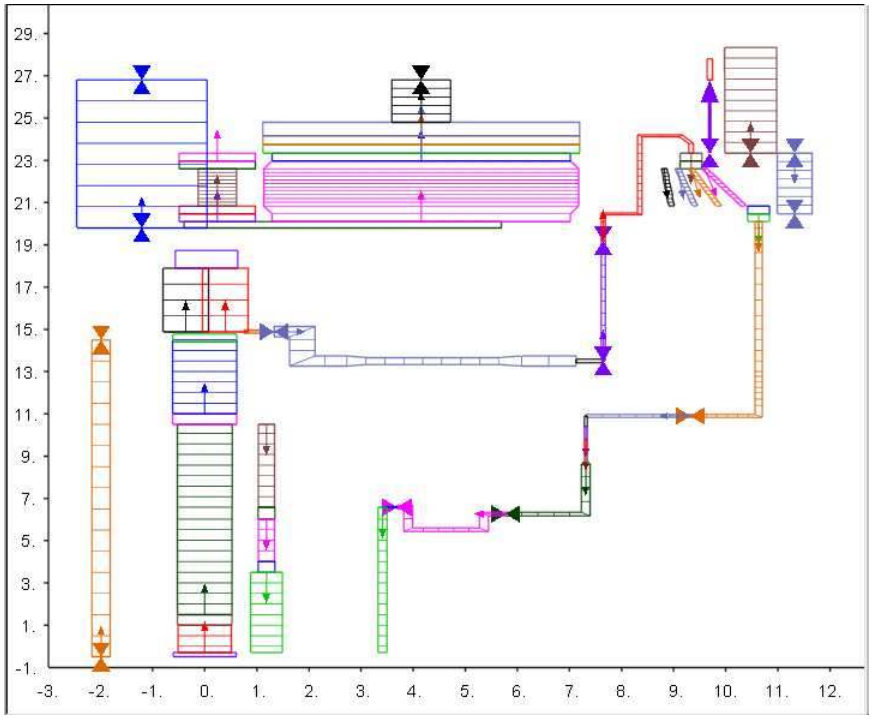


*Fig. 30: **ATHLET nodalisation of the ICS***

Identification and quantification of the sources of uncertainties [Appendix 2]

a) Selection of the parameters

Once defined the System Mission, i.e. the rejection of the core decay power to an external heat sink in this case study, a series of discussions on the main phenomena involved in the physical behaviour of the passive system has been carried out. That led to the identification of those quantities having the capability to degrade, to hamper or to prevent the fulfillment of the mission [D'Auria, 2000]. Two categories have been defined for the parameters related to those two items: the *Design* and the *Critical* categories.

The *Design Parameters* refer to the quantities directly connected to the design of the system, to the quantities representing the physical links of the passive system to the whole system into which it is inserted (e.g. the primary system), and to the parameters identifying the status or behavior of the passive system during its functioning, in nominal or operational conditions.

The *Critical Parameters* refer to the quantities that could represent a direct source of "failure" for the passive system, i.e. they could leave the mission unfulfilled. Therefore the Critical Parameters are directly connected to the physical phenomenology involved in the passive system behaviour, that could impair it.

In practice, the Design and Critical parameters refer to Initial Conditions and Boundary Conditions for the Passive System, thus identifying a specific configuration of the system, at the beginning and all along the mission time.

b) Range of the parameters

The procedure for the reliability evaluation foreseen the characterization of the passive system behavior in all possible conditions. This task is performed by investigating the passive system response in different system configurations. The configurations are set up by composing different values for the Design and Critical Parameters, to be selected within specific range of values for each parameter.

These ranges identify the bounding values within which the parameters could fall, during nominal, anticipated, abnormal or accident conditions for the whole system.

c) Probability distribution for the parameters and corresponding ranges

In order to duly compose the system configurations, to be analysed for the passive system characterization, a stochastic selection has to be performed. For this reason, a probability distribution must be associated to each range of variation for the design and critical parameters. The distribution represents the probability of occurrence of the values, assumed by the parameter when the passive system is on demand (I.C.) or during its operation (B.C.).

The configuration set is made up via a Monte Carlo selection of the parameter values.

Criteria adopted in the case study and justification for the choises

According to the general procedure previously depicted, the following apply to the case study.

a) The attention has been mostly paid on two items:

- the heat transfer capability of the passive system,
- the natural circulation flow rate.

These two general features have been judged as the most important for the fulfillment of the mission by the passive system. Since the goal is the rejection of the decay heat to an external heat sink, the capability of the system to perform its mission relies mainly on its heat transfer features and on the degree of natural circulation the system could maintain during operation.

Therefore the design and critical parameters selected for the analysis are directly linked to these two items.

b) In order to simplify both the identification of the ranges and their corresponding probabilities, discrete values have been selected.

As a general rule, a central pivot has been identified, then the range has been extended to higher and lower values, if applicable.

The pivot value represents the nominal condition for the parameter.

The limits have been chosen in order to exclude unrealistic values or those values representing a limit zone for the operation demand of the passive system.

c) Once the discrete ranges have been set up, discrete probability distributions have been associated.

The general rule adopted is that the higher probability of occurrence corresponds to the nominal value for the parameter.

Then lower probabilities have been assigned to the other values, as much low the probability as much wide the distance from the nominal value, as in a sort of Gaussian distribution.

Failure criteria and indicators of system performance

Acceptability or design limits for the system operation must be known in order to assign failure criteria and to define indicators of system performance. Those limits are specific for the system and connected with its mission. Several acceptability limits and even a larger number of failure criteria can be identified. A systematic evaluation of acceptability limits and of failure criteria does not constitute the purpose of the present activity. However, parameters connected with acceptability or design limits are selected and failure criteria are defined as a consequence.

The following parameters are assumed to be connected with the acceptability or design limits of the system and are the output of the reference code calculation:

W_2 = thermal power exchanged across the IC,

L_1 = 0.9 times the initial value for the RPV collapsed liquid level $L_1(0)$,

Γ_2 = mass flow rate at the IC inlet,

T_{rod} = surface temperature of fuel rods installed into the RPV.

A unique failure criterion is defined as a function of time:

$$\frac{Z - Z_{ref}}{Z_{ref}} < (-0.2) \qquad (1)$$

continuously valid for a time interval greater than 100 s, where Z may be either W_2 or Γ_2 and "ref" is related to the reference code calculation. It may be noted that:

- The criterion is a function of time (the target for the system performance is a function of time),
- L_1 and T_{rod} do not affect the failure criteria. Variations of L_1 from its initial value are expected to be of minor relevance. The fuel rod model is not part of the input deck; therefore no information is available in relation to Trod.

The following indicators of system performance are defined:

- Failure time, Ft (s): time during the calculation when the condition (1) is verified.
- Ratio Y/Yref where Y is the integral of W_2 and of Γ_2 performed between time 0 (start of the system operation) and the time of the calculation end.
- Ratio between the failure time and the time of the calculation (fixed to 15000s).

Uncertainties calculations

This chapter presents the results of the calculation obtained in performing deterministic selection of the input parameters or in generating probabilistic samples of the input parameters from the discrete probability functions defined in Appendix 2.

Deterministic selection

Six combinations of the input parameters have been selected deterministically. These are described in the table 11. In the last column of the table, a probability value is associated with each selected transient. This is derived by the multiplication of individual probability values that characterize each system status or each critical parameter. It must be noted that:

- The probability value associated with each status has a relative (i.e. not absolute) meaning;
- The deterministically based selection has not the purpose to bound the results expected from the statistically based selection;
- The main purpose of the deterministically based selection is to get the sensitivity in relation to the effect of each relevant parameter;
- Three groups of transients can be distinguished in the above table, based upon the probability of occurrence. The first group includes the transient No. 1 characterized by a high relative probability. The second group includes the transients Nos. 2 to 5 characterized by an average relative probability. The third group includes the transient No. 6 characterized by a very low relative probability.
- Initial liquid level in the primary side of the tubes of the IC is assumed equal to 100% (liquid only occupies the tubes) in all cases when non-condensable gases are not present in the steam line. When non-condensable gases are present in the steam line, the initial value of the level may vary between 0 and 100%.

No.	Main Parameter Studied	P1	L1	L3	Tp	Syst. Geom	X1	X2	θ	C2	L2	UL	POV	P
1	**Nominal conditions**	7	8.7	4.3	303	Nominal	0.	0.	0.	5.	100.	0.	100	***2.06e-2***
2	**Pressure & Level**	0.2	12	4.3	303	Nominal	0.	0.	0.	5.	100.	0.	100	***6.20e-4***
3	**Gas**	7	8.7	4.3	303	Nominal	0.01	0.5	0.	5.	50.	0.	100	***1.82e-5***
4	**Leakage**	7	8.7	4.3	303	Nominal	0.	0.	0.	5.	100.	5.e-5	100	***2.32e-4***
5	**Valve**	7	8.7	4.3	303	Nominal	0.	0.	0.	5.	100.	0.	10.	***2.32e-4***
6	**'Extreme'**	0.2	5	2	368	Nominal	0.01	0.5	0.	20.	0.	1.e-5	50.	***4.5e-12***

Table 11 – Deterministically selected system status.

Statistic selection

Sixty-nine cases have been generated statistically making reference to the discrete probability distributions. The number of calculations is voluntarily limited due to the long computation time (for instance a CATHARE calculation on the 15000s of transient last 4 hours on a UNIX computer.).

Uncertainty calculations with CATHARE

Seventy-five calculations have been performed with CATHARE in considering the 11 random variables.

The calculation 1 corresponds to the nominal case, which is also the reference case with an initial pool temperature of 30°C.

Calculation failures

Five calculations (cases 2, 12, 24, 68, and 75) fail in the permanent stage. For these cases, the regulation of the system is impossible for the sample of physical variables.

- The case 2 corresponds to a low vessel pressure (2 bars) and a high vessel level (12m).
- The case 12 corresponds to a low vessel pressure (2 bars) and a low pool level (2m).
- The case 24 corresponds to important heat losses piping (20 kW) and 10% of incondensable at the inlet of the IC piping.
- The case 68 corresponds to 10% of incondensable in the vessel and 1% of incondensable at the inlet of the IC piping.

Plug phenomena

When the fraction of non-condensable in the vessel is high (more then 10%), plug phenomena appears (cases 43, 49, 54, 59 and 66):

- The natural circulation stops and restarts at regular times
- In two cases, the natural circulation is completely blocked at the beginning of the transient: for case 54 until 10000s and for case 66 until 4000s.

- The vessel pressure exceeds 90 bars (158 bars in the case 43, 179 bars in the case 66).
- The vessel level reaches 15,3 m in the case 54.
- The vessel temperature reaches 346°C is reached in the case 43 and 315°C in the case 54.
- The power ratio is between 0.45 and 0.86

The cases with plug phenomena must be considered as cases leading to the failure of the system even if the exchanged power is significant, because the limits of pressure, temperature or level are exceeded.

Other functioning problems
In many cases, the vessel pressure increases at the end of the transient. Often this increase is not significant (a few percents), but sometimes it is, cases 7, 58...
The vessel level remains more or less in the range of variation, except in some cases, 22,31,33… the pool level is underneath the minimum limit (2m).
In some cases, 11,25,41… the pool temperature exceeds 95 °C (99,5°c in the case 41).

Oscillations
In many cases, oscillations are observed on the drain liquid flow. All these cases are cases with non-condensable gas, most of the time at the inlet of the IC piping and some time in the vessel.
These oscillations do not affect the exchanged power and then the performance of the system (power ratio is between 0.88 and 1.04).

Comparison of the results (global criterion)

The thermal power ratio $Z = Y/Y_{ref}$ was used as global criterion for this comparison on the results obtained by the four codes (RELAP/mod3.1 and 3.2, ATHLET and CATHARE) on the deterministic cases (6 cases) and on the statistically generated cases (69 cases).

The figure 31 presents the global criterion obtained by each codes. The figures 32 and 33 compares the relative differences on the global criterion with reference to RELAP/mod3.2 and CATHARE respectively.
From these comparisons, we can notice a general good agreement between the results of the codes, with some differences: RELAP5/mod3.1 provides in general the largest amount of exchanged power and RELAP5 the lowest. Between these two bounds, CATHARE calculates in general larger amount of exchanged power than ATHLET. These results are coherent with the observations made on the reference case.

The scattering on the relative difference is around 60% if we consider the four codes. The results of ATHLET are more scattered. The scattering on the relative difference is around 40% if we consider only CATHARE and RELAP5/mod3.1 and mod3.2.

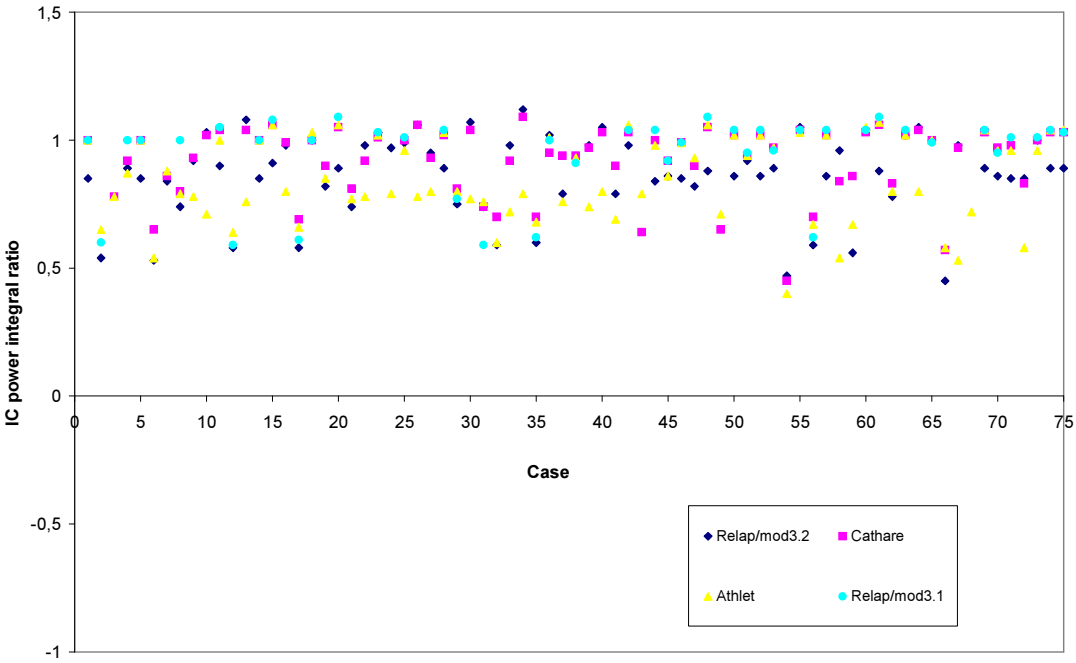


Figure 31: Thermal power ration obtained with RELAP5/mod3.1 and mod3.2, ATHLET and CATHARE.

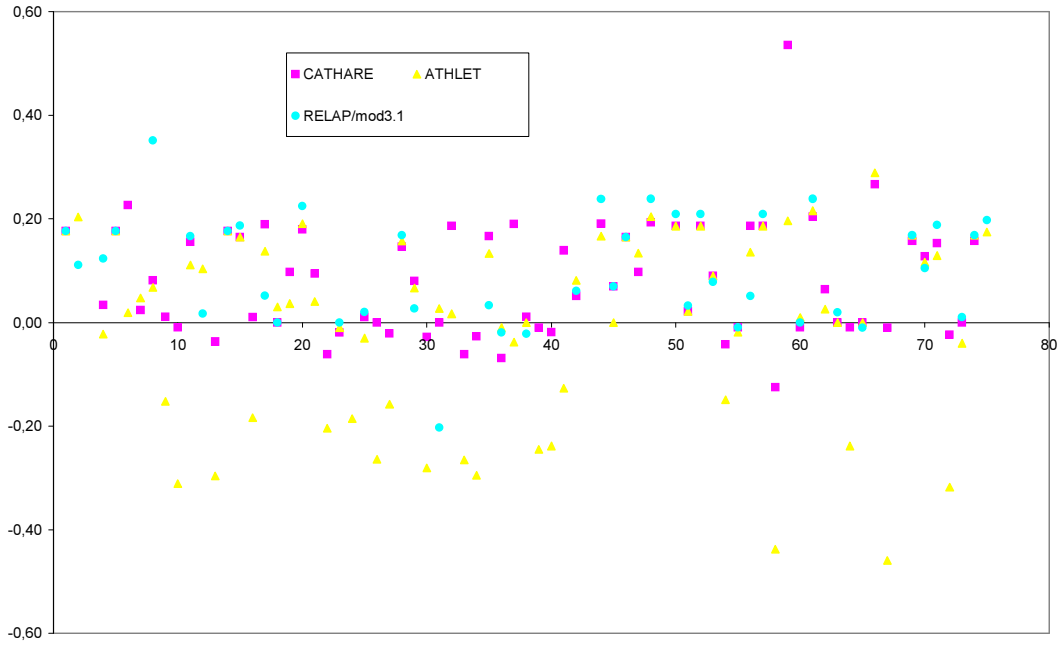


Figure 32: Relative difference between CATHARE, ATHLET or RELAP5/mod3.1, and RELAP5/mod3.2.

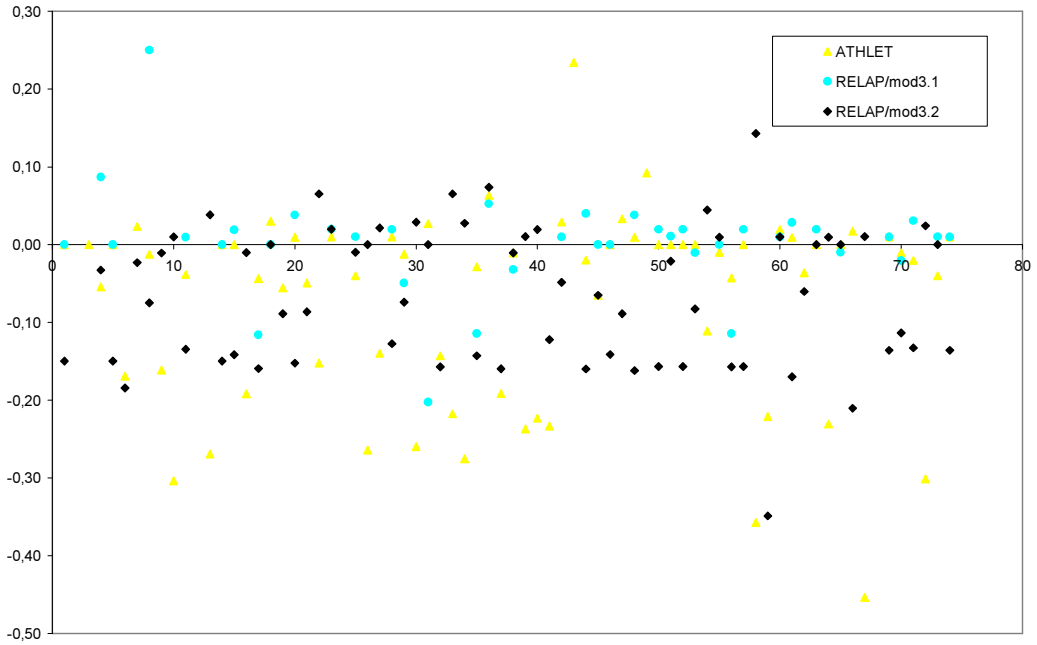


Figure 33: ***Relative difference between ATHLET, RELAP5/mod3.1 or RELAP5/mod3.2, and CATHARE***

Finally on the figure 34, we have compared the empirical cumulative distribution obtained on the 69 statistical cases. We observe on this sample that the range of variation of the thermal power ratio is included between 0.4 and 1.12. The "optimistic" distribution (high probability for high values of the thermal power exchanged) is given by RELAP5/mod3.1 and the "pessimistic" distributions (high probability for low values of the thermal power exchanged) are given by ATHLET and RELAP5/mod3.2. The scattering between the curves is larger in the range of thermal power ratio: [0.8 , 1].

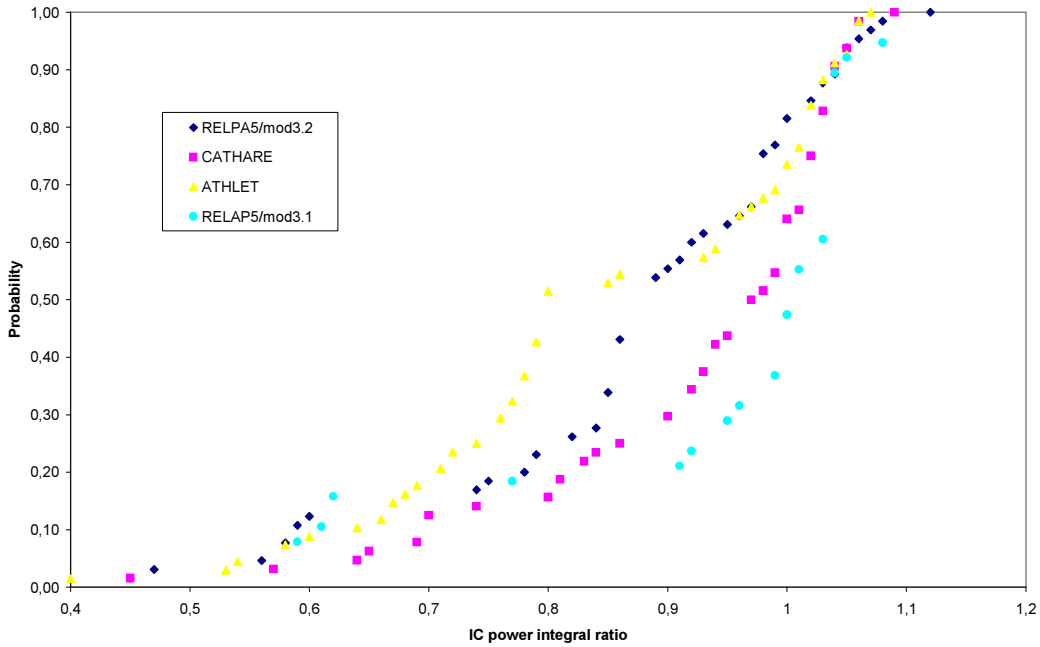


Figure 34: ***Emprical cumulative distribution of the thermal power ratio obtained with RELAP5/mod3.1 and 3.2, ATHLET and CATHARE.***

Sensitivity analysis on the CATHARE results

The thermal power ratio $Z = Y_2/Y_{ref}$ was used as global criterion for this sensitivity analysis.
The sixty-nine results obtained by CATHARE in considering discrete probability distributions for the input parameters (chapter 2.7.2) were used to evaluate different sensitivity coefficients:
a) Standardized Regression Coefficients (SRC) and Partial Correlation Coefficients (PCC):
These coefficients have been previously defined. They are based on linear assumption of the thermal-hydraulic model. To validate this hypothesis, it is important to calculate the model coefficient of determination. In our case $R^2 = 0.87$, that means that the linear model fit quite well the data.
b) Standardized Rank Regression Coefficients (SRRC) and the Partial Rank Correlation Coefficients (PRCC):
To avoid the problem of non-linearity, rank transformations are frequently employed. Rank can cope with non-linear, albeit monotonic, relations between the input and the output distributions. The rank transform is a simple procedure, which involves replacing the data with their corresponding ranks.
The coefficient of determination based on the raw data is equal 0.83. No improvement is obtained in considering a non-linear but monotonous model.
The table 12 and figure 35 gives the sensitivity coefficients obtained for each random input variable.

	SRC	PCC	SRRC	PRCC
P1	0,67	0,86	0,81	0,86
L1	0,25	0,53	0,42	0,68
L3	0,14	0,36	0,13	0,22
Tp(0)	0,03	0,07	0,03	0,05
X1	-0,49	-0,76	-0,62	-0,66
X2	-0,07	-0,16	-0,03	-0,04
□	-0,04	-0,09	0,09	0,18
C2	-0,02	-0,06	-0,16	-0,26
L2(0)	0,06	0,11	0,19	0,22
UL	0,03	0,06	0,00	0,00
POV	0,06	0,14	0,00	0,00

Table 12: ***SRC, PCC, SRCC and PRCC sensitivity coefficients obtained on CATHARE results.***

We obtain the following decreasing order of importance: P1, X1, L1, L3, X2, L2, POV, θ, UL, C2, Tp(0) for the SRCs and P1, X1, L1, L3, X2, POV, L2, θ, Tp, C2, UL for the PCCs. For the two coefficients, the order of the parameter is the same for the five more important parameters P1 (RPV pressure), X1 (RPV non-condensable fraction), L1 (RPV collapsed level), L3 (pool level) and X2 (non-condensable fraction at the inlet of IC piping). Three variables are very important X1, P1 and L1.

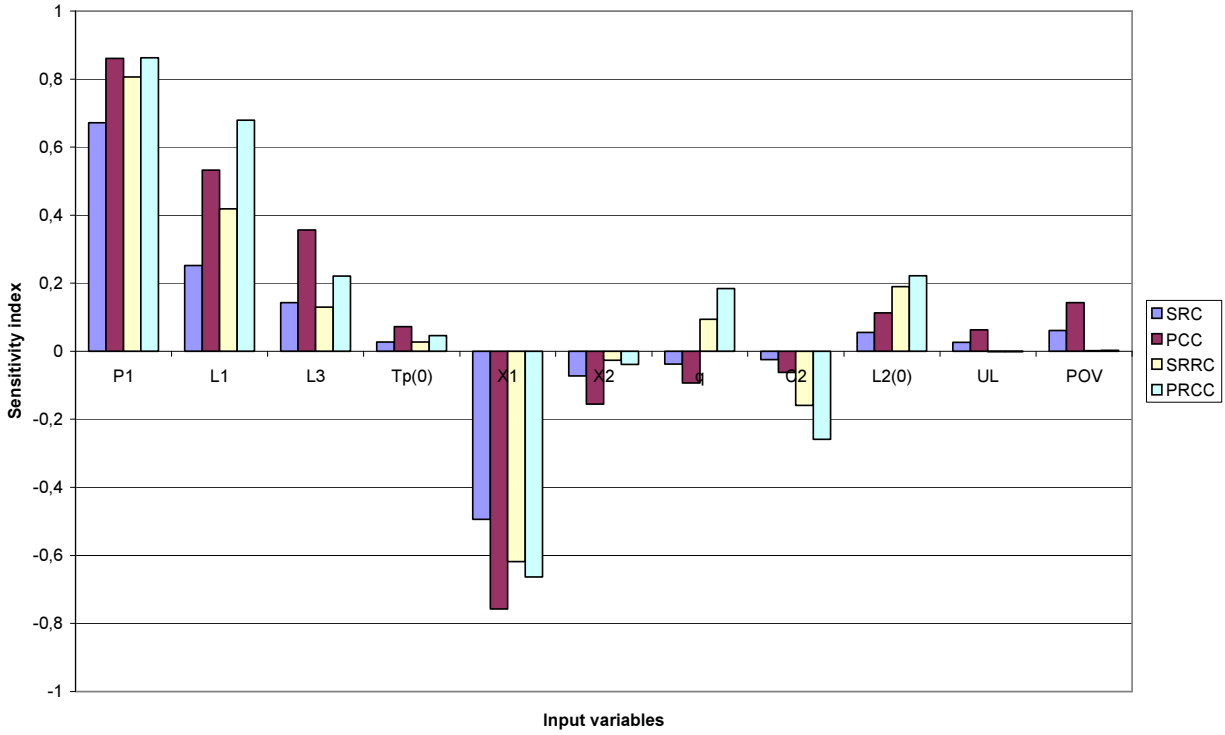


Figure 35: ***Sensitivity index on W2/W2ref (CATHARE calculations)***

Same types of results were obtained with the RELAP5 code [Ricotti, 2002].

Conclusions on the sensitivity analysis

The identification and evaluation of the sources of uncertainties in the IC passive system operation have been performed based on thermal-hydraulic calculations of 69 different input scenarios stochastically selected, according to the discrete probability distributions assigned to the parameters, using the RELAP5mod3.2 and CATHARE computer codes.The choice of the design and critical parameters of the passive system, their nominal values, the ranges of variation and the selected initial values for the analysis has been based on the preliminary engineering judgment (see Appendix 2) and by Analytical Hierarchy Process

Acceptability or design limits for the system operation must be known in order to assign failure criteria and to define indicators of system performance. Those limits are specific for the system and connected with its mission. The ratio of W_2 and $\square_2$ to their reference values, assumed to be connected with the acceptability or design limits of the system, have been taken as output variables. In the cases examined, these quantities have been seen to somehow correlate each other, highlighting some redundancy in their usefulness. The importance of input variables on the transient evolution has been analyzed by performing a sensitivity analysis under the hypothesis of linearity between the input and the output variables. Three different coefficients of importance have been calculated for each parameter, and a coherent general ranking was assessed.

The results of the sensitivity analysis (with the criterion ratio of W_2 to its reference) obtained by RELAP and CATHARE have been compared and good accordance was found. The behaviour of this criterion with respect to the input parameters, is however more linear on the results obtained by CATHARE than on those of RELAP.

The hypothesis of linearity may not always apply so that a non linear analysis tool should be used to obtain more realistic results. The degree of correlation between input parameters should also be taken in account.

Estimation of the reliability of the ICS on the CATHARE results (Marquès, 2002]

The power ratio: $Z = \frac{Y}{Y_{ref}}$ $with$ $Y = \int_{t=0}^{t=end\ of\ calculation} W_2$ and W_2 the thermal power exchanged across the Isolation Condenser was used as global criterion (or response) for this reliability analysis.
Ninety two calculations were performed with CATHARE in considering discrete probability distributions for the eleven input parameters considered as uncertain.

Response surfaces evaluations

To avoid the problem of long computer time in the method of Monte-Carlo, it can be interesting to build an approximate mathematical model called response surface.
Experiments are conducted with input variables $X_1, X_2, \ldots, X_n$ a sufficient number of times to define the response surface to the level of accuracy desired. Each experiment can be represented by a point with coordinates $x_{1j}, x_{2j}, \ldots, x_{nj}$ in an n-dimensional space. At each point, a value of y_i is calculated. The basic response procedure is to approximate the physical model $g(X)$ by a simple mathematical model $\tilde{g}(X)$ with undetermined coefficients. Thermal-hydraulic analysis is performed at various points x_i, in order to determine the unknown coefficients of the model $\tilde{g}(X)$ so as to minimize the error in the domain of interest.
When a response surface has been determined, the passive system reliability can be easily assessed with Monte Carlo simulation, in using the approximate mathematical model.
Such a response surface can be useful not only for uncertainty propagation but also for reliability evaluation and sensitivity studies.

Different types of response surfaces have been fitted on these ninety two results. We present these different fittings in the following, giving in each case the value of the coefficient of determination R^2 defined as:

$$R^2 = \frac{\sum_{k=1}^{m} (\hat{y}_k - \bar{y})^2}{\sum_{k=1}^{m} (y_k - \bar{y})^2},$$

where y_k are the response obtained by CATHARE, $\hat{y}_k$ denotes the estimate of y_k, k=1,2,3.....,m, obtained from the response surface model, $\bar{y}$ is the average of y_k and m is the number of observations (92 in our case). R^2 provides a measure of how well the linear regression model can reproduce the actual output y; it represents the fraction of the variance of the output explained by the regression model. The closer R^2 is to unity; the better is the model performance.

- **<u>Linear response surface</u>**

The following linear model was obtained:

*Ratio= 0.4388 + 0.2928E-01*P1 + 0.1608E-01*L1 + 0.1909E-01*L3 + 0.2806E-03*Tp - 0.9784 * X1 - 0.1747 * X2 + 0.2994E-02 *θ - 0.2915E-03 * C2 + 0.1637E-03 *L2 + 0.4563E-01*UL -0.9456E-04*POV*

The coefficient of determination R^2 is equal to *0.75*. This means that the thermal power ratio is not fully linear with regard to the input parameters.

The figure 36 compares the values of the thermal power ratio obtained by CATHARE and predicted by the linear model.

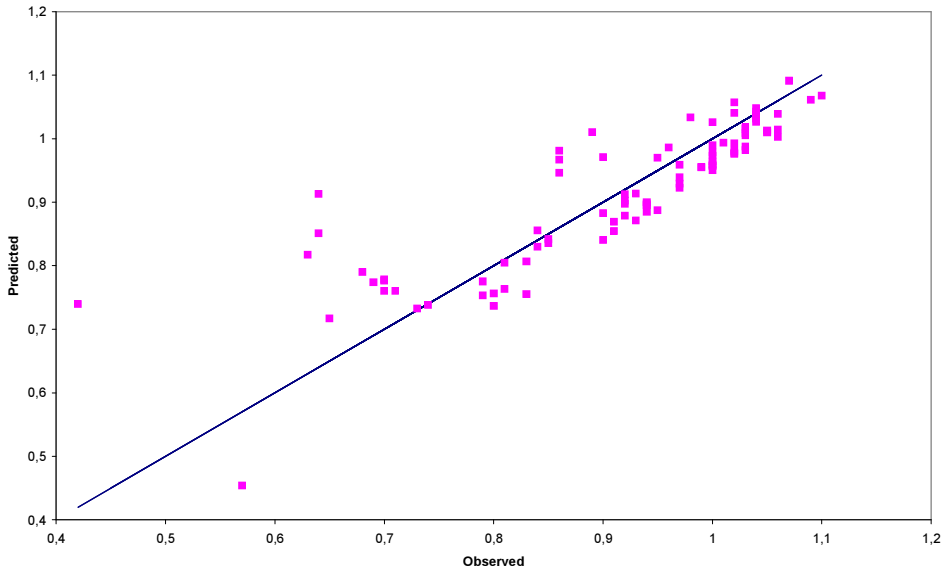


Figure 36: ***Comparison of the thermal power ratios obtained by CATHARE and predicted by the response surface model (linear model).***

Note that a linear model with only the following 5 independent variables : P1,L1,L3,X1,X2 gives a R^2 equal to *0.73*.

- **<u>Nonlinear Regression</u>**

We have tried different non linear models. The best fit is obtained with the following model with only seven independent variables (P1,L1,L3,Tp,X1,X2,L2).

*Ratio = 0,410304 + 0,0726057*P1 + 0,0134634*L1 + 0,0225096*L3 + 0,000175838*Tp*
*+ -4,35566*X1 + -0,139538*X2 +0,000367615*L2 -0,00440591*P1*P1 + 7,18601*X1*X1*

The R^2 statistic indicates that the model as fitted explains *94 %* of the variability in Ratio.

The figure 37 compares the values of the thermal power ratio obtained by CATHARE and predicted by this non linear model.

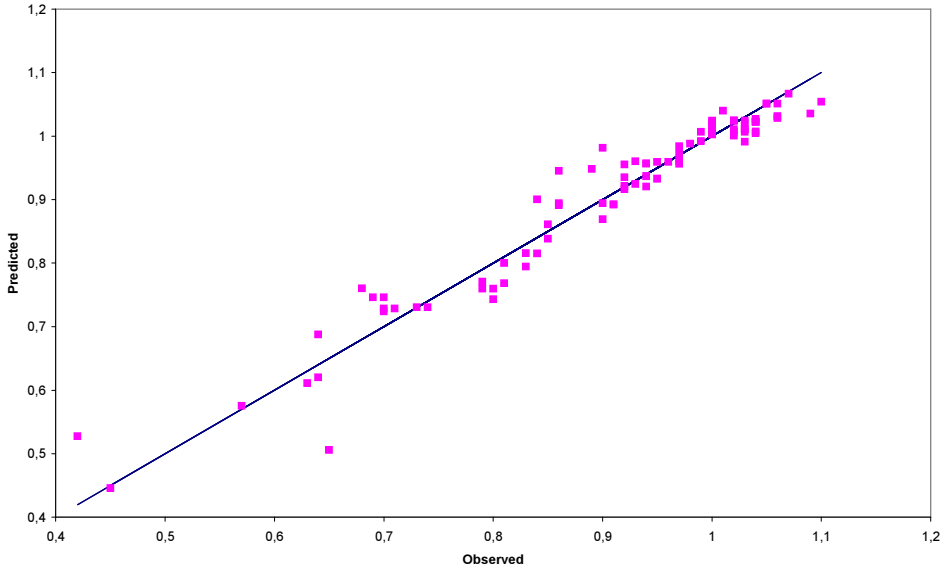


Figure 37: ***Comparison of the thermal power ratios obtained by CATHARE and predicted by the response surface model (non linear model).***

- **Neural network response surface**

We have tried different response surfaces obtained by Neural Network (NN)
A network consists of units (or cells) and directed, weighted links (connections) between them. Depending on their function in the net, one can distinguish three types of units: the units whose activations are the problem input for the net: the input units (the input parameters in our case); the units whose output represent the output of the net: the output units (the responses in our case); the remaining units are called hidden units, because they are not visible from the outside.
More details on neural networks will be given in the deliverable 6 "Development of complementary methods".
We have considered different types of NN: the network type *X-Y-Z* means *X* input units, *Z* output units and *Y* hidden units.
The table 13 gives different NN used with the number of bootstrap calculations performed and the coefficient of determination R^2 and the approximation error obtained. The approximation error gives an indication of the accuracy of the model when it is used with other values that the ones used to build it.

Among the different NN tested, the network *11-15-1* gives the best result with a R^2 equal to *0.96* and an approximation error equal to *0.10.*
The figure 38 compares the values of the thermal power ratio obtained by CATHARE and predicted by the *NN 11-15-1.*

Number of bootstrap calculations	Network type	R^2	Approximation error
10	11-20-1	0.90	0.14
20	11-10-1	0.97	0.11
20	11-15-1	0.96	0.10
20	11-25-1	0.94	0.14

Table 13: ***Different Neural Network used to fit the data***

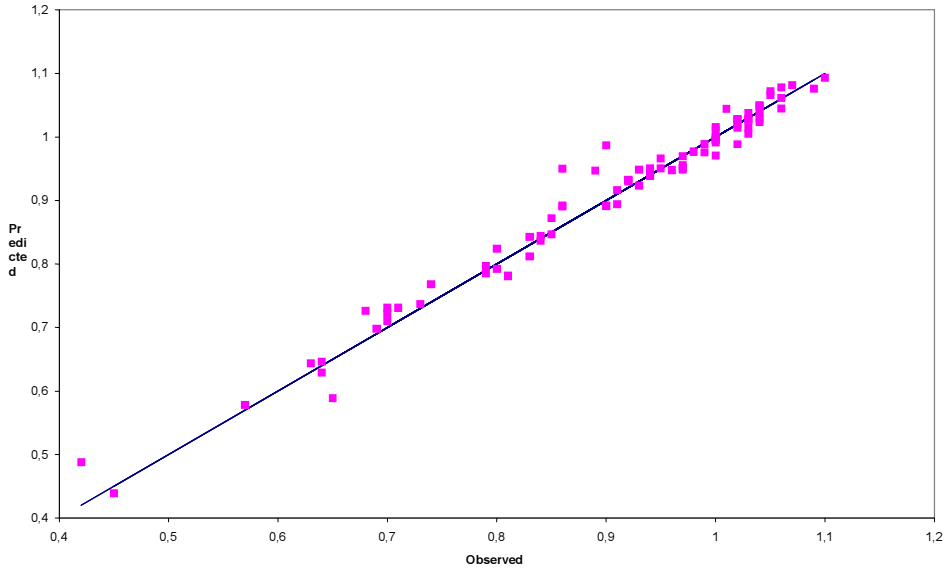


Figure 38: Comparison of the thermal power ratios obtained by CATHARE and predicted by the response surface model (Neural Network 11-15-1).

The figure 39 compares the empirical cumulative distributions of the power ratios obtained from the results of CATHARE (observed), and the from the different response surfaces. The NN response surface gives the better representation of the observed empirical cumulative distribution.

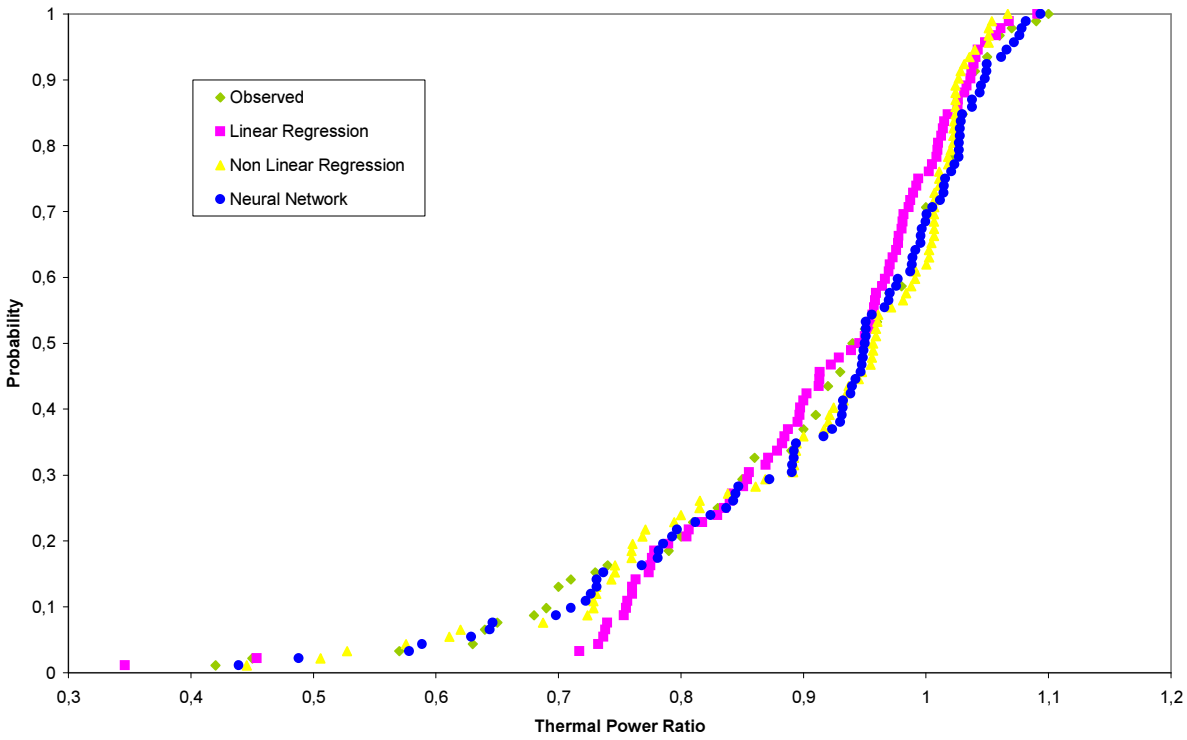


Figure 39: Empirical cumulative distributions observed and obtained by response surfaces.

Reliability evaluation of the ICS by Monte Carlo simulation on the response surfaces

For the reliability evaluation of the ICS, we have defined a failure criterion consisting in the comparison of the thermal power ratio to a given limit α.

$$\frac{Y}{Y_{ref}} \leq \alpha$$

The mission of the system is supposed fulfilled if the thermal power ratio is greater or equal the limit α, or in other words if the system has sufficiently exchanged heat to assure a sufficient decay heat removal during the accidental transient.
The limit α has not yet been fixed for the ICS and it is beyond the purpose of the present project to supply this limit. We can imagine that this limit could be fixed by the Safety Authorities and could be different with regard to the mission given to the system.
So in order to perform reliability evaluation of the system, we have performed different calculations in considering different values of the limit α.
The failure probability or unreliability of the passive system is given then by:

$$P_f = P\left(\frac{Y}{Y_{ref}} \leq \alpha\right)$$

The table 14 gives the failure probabilities obtained in using the non linear responses surface and the NN 11-15-1 and in performing 10000 Monte Carlo runs.

Criterium α	Failure probability Pf	
	Non Linear Regression	Neural Network
0.05	0.0	
0.10	4.00E-4	
0.15	3.30E-3	
0.2	6.50E-3	
0.25	9.50E-3	
0.3	1.26E-2	0.0
0.35	2.00E-2	1.00E4
0.4	3.21E-2	1.06E-2
0.45	5.94E-2	4.41E-2
0.5	6.97E-2	8.29E-2
0.55	8.26E-2	1.03E-1
0.6	9.49E-2	1.22E-1
0.65	1.36E-1	1.37E-1
0.7	1.68E-1	1.57E-1
0.75	2.09E-1	1.99E-1
0.8	2.71E-1	2.66E-1
0.85	3.10E-1	3.33E-1
0.9	3.81E-1	4.24E-1
0.95	4.64E-1	5.20E-1
1	6.45E-01	7.77E-1
1.05	9.44E-01	9.24E-1
1.1	9.90E-01	9.96E-1

Table 14: ***Failure probabilities obtained by 10000 Monte Carlo runs using responses surfaces.***

The figure 40 compares the failure probability obtained in using the two response surface models. The results are very close

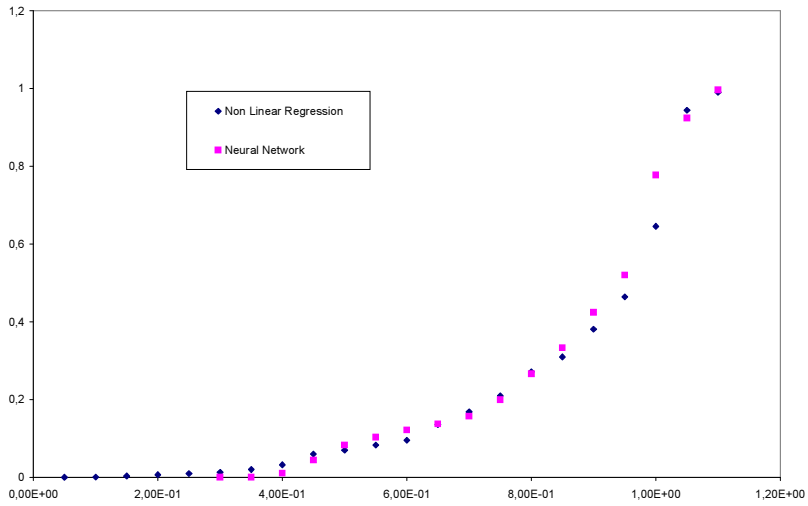


Figure 39: ***Comparison the failure probabilities obtained in using two response surface model.***

Reliability evaluation of the ICS by the FORM method on the response surfaces

In order to use FORM /SORM methods for the evaluation of the reliability of the ICS system, it was necessary to adjust analytical continuous probabilistic models to the eleven random variables.
The fitting of continuous models was performed on the discrete cumulative functions defined in table ?. The objective was to find a continuous cumulative function approximating for the best the empirical discrete cumulative function.
The fitting have been performed by non linear regression. Only two types of analytical cumulative functions were used for modelling the 11 parameters:

- Truncated normal distribution with parameters m, σ, X_{min} et X_{max}, with probability density and cumulative functions defined on $[X_{min}, X_{max}]$, by:

$$f_t(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-m)^2}{2\sigma^2}\right) \frac{1}{F(X_{max}) - F(X_{min})}$$

$$F_t(x) = \frac{F(x) - F(X_{min})}{F(X_{max}) - F(X_{min})}$$

where F is the cumulative function of the normal distribution $N(m,\sigma)$.

- Exponential distribution with parameters λ *(>0),* μ *defined by:*

$$f(x) = \lambda . exp[-\lambda.(x-\mu)]$$

The distributions obtained are given in the table 15. These continuous distributions introduce a bias with regard to the discrete distributions. This can be seen in the table 16 and in the figure 41 comparing the results of the Monte-Carlo simulations performed on the non linear response surface model in using either the discrete distributions or the continuous distributions. This bias is mainly due to the difference of probabilistic modelling of the variables X1 and X2, the non-condensable fractions. The continuous distributions generate lower values of X1 and X2 that the discrete distributions. The results obtained by the FORM method are presented in the table 16 and in the figure 41. They are close to the results of the Monte-Carlo simulations, when both used the same input continuous analytical distributions.

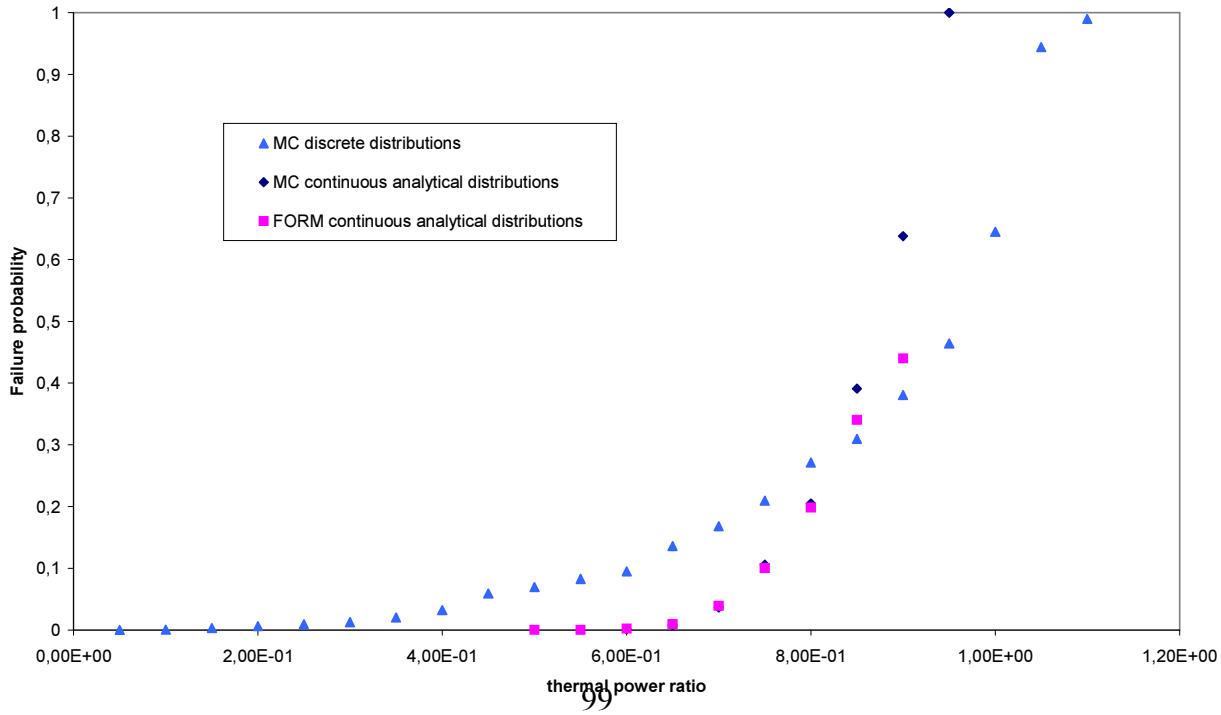
Variable	Distribution	Average	Standard deviation	X_{min}	X_{max}	λ	μ
P1	Truncated normal	4.26	2.6	0.2	9		
L1	Truncated normal	8.54	1.42	5	12		
L3	Truncated normal	2.58	1.32	2	5		
TP(0)	Truncated normal	140	40	280	368		
X1	Exponential					182	0
X2	Exponential					177	0
Theta	Exponential					2.3	0
C2	Truncated normal	1	1	0	100		
L2(0)	Truncated normal	102	40.8	0	100		
UL	Exponential					459521	0
POV	Truncated normal	123	39.4	1	100		

Table 15: ***Continuous analytical distribution constructed on the discrete cumulative functions.***

	MC lois discrètes	MC lois continues	FORM lois continues
5,00E-02	0		
1,00E-01	4,00E-04		
0,15	3,30E-03		
0,2	6,50E-03		
0,25	9,50E-03		
0,3	1,26E-02		
0,35	2,00E-02		
0,4	3,21E-02		
0,45	5,94E-02		
0,5	6,97E-02		4,50E-09
0,55	8,26E-02		3,40E-08
0,6	9,49E-02	9,40E-04*	1,90E-03
0,65	0,1356	7,45E-03*	9,50E-03
0,7	0,1681	3,65E-02	3,90E-02
0,75	0,2093	1,06E-01	1,00E-01
0,8	0,2713	2,05E-01	1,98E-01
0,85	0,3095	3,91E-01	3,40E-01
0,9	0,3805	6,38E-01	4,40E-01
0,95	0,4641	1,00E+00	
1	0,6453		
1,05	0,9439		
1,1	0,9899		

Table 16 : Failure probabilities obtained by Monte-Carlo simulation (10000 or *100000 simulations) and FORM method performed on the non linear response surface model

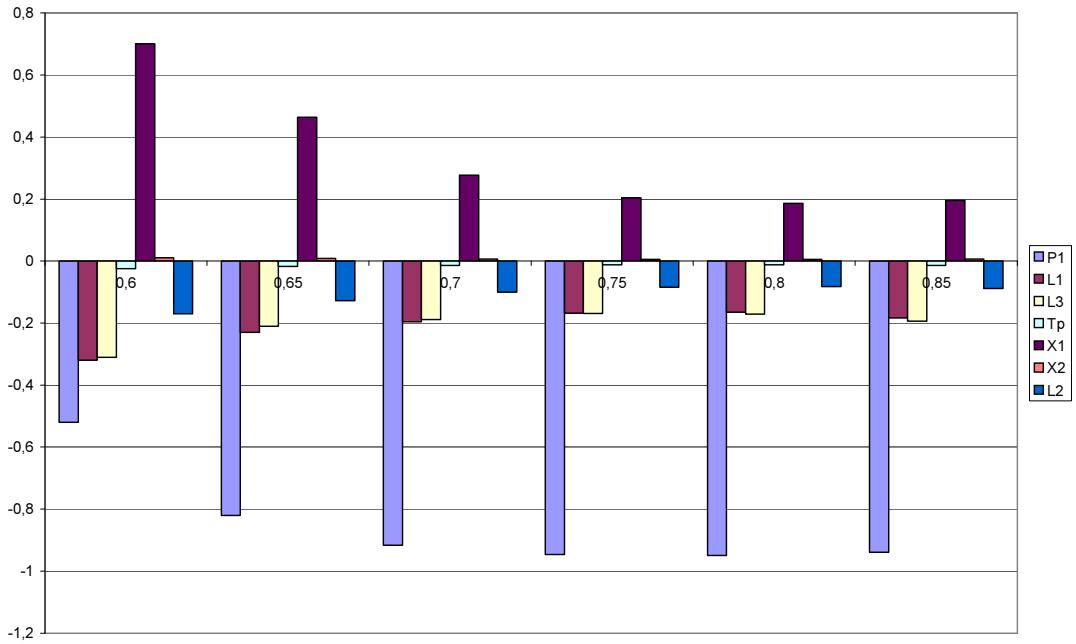
Figure 41 : Failure probabilities obtained by Monte-Carlo simulation and FORM method



The knowledge of the design point allows the determination the most influential variables on reliability. By supposing that there is a single design point, the vector directional cosine unit α is defined by $\alpha=u^*/\beta_{HL}$.
The components α_i are also called factors of sensitivity and the factors α_i^2 are interpreted like factors of importance associated to the variables U_i. A variable associated with one significant α_i is regarded as having a significant influence on the probability of failure.
Note: the area of maximum of probability of failure is often different from the area around the average. Consequently the most influential variables are often different for these two domains. In consequence, the factors of importance have a different interpretation that the sensitivity indices (Standardized regression Coefficients, Partial Correlation Coefficients…) calculated in global sensitivity analyses [1].
The figure 42 presents the evolution of the factors of sensitivity with respect to the criterion limit α. We can observe an increase in the importance of the variable P1 (vessel pressure) with α and a corresponding decrease of X1 (non-condensable fraction in the pressure vessel). A positive value for the α_i indicates that an increase of the corresponding variable X_i leads to an increase of the failure probability and a negative value that an increase of the corresponding variable leads to an decrease of the failure probability.

Note that confidence intervals of the reliability results can be obtained by bootstrap techniques [Zio, 2008].

Figure 42: ***Evolution of the factors of sensitivity of the input variables with respect to the criterion limit α.***



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ANNEXE 1: PASSIVE SYSTEM CLASSIFICATION

Following the IAEA definition [IAEA, 1991], a passive system is either a system which is composed of passive components and structures or a system which uses active components in a very limited way to initiate subsequent passive operation.

Some broad categories of passivity can be drawn for qualitative evaluation and classification. The following categories can be considered as passive:

Category A

This category is characterized by :
- no signal inputs of "intelligence", no external power sources or forces,
- no moving mechanical parts,
- no moving working fluid.

Examples of safety features included in this category are:
- physical barriers against the release of fission products, such as nuclear fuel cladding and pressure boundary systems,
- hardened building structures for the protection of a plant against seismic and other external events,
- core cooling systems relying only on heat radiation and/or conduction
- static components of safety related passive systems (e.g. , tubes, pressurizers, accumulators) as well as structural parts (e.g., supports, shields).

Category B

This category is characterized by :
- no signal inputs of "intelligence", no external power sources or forces,
- no moving mechanical parts, but
- moving working fluids.

The fluid movement is only due to thermal-hydraulic conditions occuring when the safety function is activated.

Examples of safety features included in this category are:
- reactor shutdown/emergency cooling systems based on injection of borated water from an external water pool,
- reactor emergency cooling systems based on air or water natural circulation in heat exchangers immersed in water pools (inside the containment),
- containment cooling systems based on natural circulation of air flowing around the containment walls.

Category C

This category is characterized by :
- no signal inputs of "intelligence", no external power sources or forces,
- moving mechanical parts, whether or not moving working fluids are also present.

The fluid motion is characterized as in category B; mechanical movements are due to imbalances within the system (e.g., static pressure in check and relief valves, hydrostatic pressure in accumulators) and forces directly exerted by the process.

Examples of safety features included in this category are:

- emergency injection systems consisting of accumulators or storage tanks and discharge lines equipped with check valves.
- mechanical actuator, such as check valves and spring-loaded relief valves.

Category D

This category addresses the intermediary zone between active and passive where the execution of the safety function is made through passive methods as described in the previous categories except that internal intelligence is not available to initiate the process. In these cases an external signal is permitted to trigger the passive process.

Examples of safety features included in this category are:

- emergency core cooling systems, based on gravity-driven flow of water, activated by valves which break open on demand.

APPENDIX 2: CRITERIA ADOPTED IN THE SELECTION OF THE DESIGN AND CRITICAL PARAMETERS? THEIR RANGES AND CORRESPONDING PROBABILITY DISTRIBUTIONS.

	Design Parameter	Unit	Nominal Value	Range	Discrete Initial Values & Probabilities					
P_1	RPV pressure	MPa	7	0.2-9	0.2 *0.05*	1 *0.1*	3 *0.15*	7 *0.5*	9 *0.2*	value *pdf*
L_1	RPV collapsed level	m	8.7	5-12	5 *0.05*	7 *0.1*	8.7 *0.5*	10 *0.2*	12 *0.15*	
L_3	POOL level	m	4.3	2-5	2 *0.1*		4.3 *0.8*		5 *0.1*	
$T_p(0)$	POOL initial temperature	K	303	280-368	280 *0.1*		303 *0.8*		368 *0.1*	
-	System geometry: layout	-	-	Not assigned			- 1.0			

		Choice justification(+)	Parameter range	Range justification	Probability values justification(*)
DESIGN PARAMETERS	P_1 **Reactor Pressure** **(I.C.)**	The vessel pressure determines the operating pressure of the passive system, hence the heat transfer capability of the Isolation Condenser.	0.2 - 9 MPa (nominal: 7 Mpa)	0.2 MPa = minimum pressure value below which GDCS actuates. 9 MPa = maximum pressure value above which SRV actuate. Both SRV and GDCS largely affect the IC performance. Eventual IC actuation in combination with SRV and GDCS may constitute an additional design constraint not addressed in the present study.	7 MPa (nominal operation pressure) is the most probable value expected for the operation of the IC: half the total distribution probability has been assigned to this value.
	L_1 **Reactor Liquid (collapsed) Level** **(I.C.)**	The vessel level characterizes the total coolant mass hence the thermal capacity or thermal inertia for the system.	5 – 12 m (nominal: 8.7 m)	5 m = limit value for core uncovery. 12 m = elevation of the connection between the RPV and the steam line. Level value below 5 m implies unsafe conditions for the core and un-successful mission for the IC. Level value above 12m is challenging the safe operation of the SRV. Both of these conditions imply inadequacy of the overall system design.	See above
	L_3 **Pool Level** **(I.C.)**	The pool level characterizes the thermal capacity of the heat sink in the long run and may affect the heat transfer coefficient across the the IC tube bundle.	2 - 5 m (nominal: 4.3 m)	2 m = minimum pool level, resulting in heat exchanger uncovery. 5 m = maximum pool level, resulting in overfilling. Level value below 2 m implies restoration by tools not part of the present study and above 5 m implies malfunction of the water control system.	A significant confidence for the parameter to be kept at nominal value (resulting in a probability equal 0.8) is assumed. This is justified by the assumed operation of a system (the water level control) not considered in the present study.
	T_p **Pool Temperature** **(I.C.)**	The liquid temperature of the pool is Initial Condition affects the heat transfer capability of the heat sink.	280 - 368 K (nominal: 303 K)	280 K = minimum water temperature to avoid freezing. 368 K = maximum allowable water temperature at ambient pressure to avoid boiling. Initial temperature below 280 K is assumed non-realistic and temperature above 368 may imply undetected circulation across the IC and failure the water control system.	see above

(+) The (initial) value of each of the considered quantities may correspond to the end status of a system transient, when the IC operation is required.
(*) Probability values are peaked to the nominal value, and decrease gradually towards the minimum and maximum allowed values. No finalised study has been carried out to propose the relevant probability distributions. A 'fault-tree-analogous' approach can be used to this aim.

	Critical Parameter	Discrete Values & Probabilities							
x_1	RPV non-condensable fraction	0.	0.01	0.1	0.2	0.5	0.8	1.	value
		0.719	*0.12*	*0.07*	*0.05*	*0.03*	*0.01*	*0.001*	*pdf*
x_2	Non-condensable fraction at the Inlet of IC piping	0.	0.01	0.1	0.2	0.5	0.8	1.	
		0.71	*0.12*	*0.07*	*0.05*	*0.03*	*0.01*	*0.01*	
θ	Inclination of the IC piping on the suction (deg)	0.		1.		5.		10.	
		0.5		*0.4*		*0.08*		*0.02*	
C_2	Heat Losses piping – IC Suction (kW)	0.		5.		20.		100.	
		0.10		*0.7999*		*0.10*		*0.0001*	
$L_2(0)$	Initial condition liquid level - IC tubes, inner side (%)	0.			50.			100.	
		0.1			*0.1*			*0.8*	
UL	Undetected leakage (m^2)	0.		1.E-5		5.E-5		10.E-5	
		0.8899		*0.1*		*0.01*		*0.0001*	
POV	Partially opened valve in the IC discharge line (%)	1.		10.		50.		100.	
		0.001		*0.01*		*0.1*		*0.889*	

		Choice justification(+)	Parameter range	Range justification	Probability values justification(*)
CRITICAL PARAMETERS	X_1 **Non-condensables fraction in RPV** **(I.C.)**	The gas presence in the RPV affects the heat transfer capability of the Isolation Condenser, with a significant degradation of the heat transfer coefficient. This situation may occur following various transients from events like pin failure , clad overheating (hydrogen production,), failure in the actuation of sub-systems connected with the primary loop. The total mass of non-condensables in the RPV depends on the steam volume, hence it is connected with the design parameter L_1 .	0 – 1 fraction (nominal: 0)	0 = absence of non-condensables; 1 = presence of 100% of non-condensables in the steam/gas region of the RPV	(**) Probability values decrease as X_1 increase; the lower limit corresponds to the nominal value hence to the Max.prob.value. The presence of non-condensables has been judged generally more unlikely than for the design parameter cases, thus a major probability has been assigned to the lower limit condition ($P_{(X1=0)}$ = 0.719). The upper limit condition has been considered very unlikely to occur, thus a very low value has been assigned to it. Due to a significant sensitivity of the heat transfer coefficient to the non-condensable fraction, 7 discrete values have been considered for the investigation of the parameter effects.
	X_2 **Non-condensables fraction at Isolation Condenser tube bundle inlet** **(I.C.)**	The gas presence in the IC line affects the heat transfer capability of the Isolation Condenser in the early stage in the transient, with a significant degradation of the heat transfer coefficient. This situation could occur for the concentration of gases in the upper zone of the draining pipe line, in the passive system loop. The presence of non-condensables could be caused e.g. by an incorrect maintenance of the passive system piping or by gas accumulation during the nominal operational life of the system.	0 – 1 fraction (nominal: 0)	0 = absence of non-condensables; 1 = presence of 100% of non-condensables at the inlet header of the Isolation Condenser tube bundle	Same comment as in (**), except for the upper limit probability, considered more likely to occur if compared with the complete filling of the RPV with non-condensables.

	θ **Piping inclination at IC inlet** **(B.C.)**	This Boundary Condition could affect the natural circulation flow rate. It corresponds to a wrongly welded pipe, with a downward inclination that could disturb the mixture flowing towards the IC inlet header. The consequences could be an alteration of the flow regime or a concentration of the gases, if present, in that zone of the circuit.	0 – 10 deg (nominal: 0 deg)	0 deg = corresponds to the design layout, i.e. horizontal piping; 10 deg = this limited value for the inclination angle has been assumed (10 deg) in order to consider it as not easily apparent and recognizable at a preliminary or sight check.	Probability values decrease as θ increase; the lower limit corresponds to the nominal value hence to the Max.prob.value. The occurrence of the nominal condition or even a small pipe inclination have been considered as the most probable conditions, hence most of the probability distribution (90%) lies within these values.
	C_2 **Thermal losses at Passive System piping - draining side** **(B.C.))**	This Boundary Condition could affect both the natural circulation flow rate and the heat transfer in the IC. tube bundle. A steam condensation for the mixture flowing upward towards the I.C., increases the system depressurization and decreases the driving density difference between the hot and cold legs.	0 - 100 kW (nominal: 5 kW)	0 kW = corresponds to the ideal case of no heat dissipation; 100 kW = corresponds to the worst condition for the heat losses of the fluid through the piping surface	A complete loss of thermal isolation for the piping has been judged very unlikely, hence a very low probability (P=1.e-4) has been assigned to the thermal losses upper limit. The nominal or pivot value is considered as the most realistic (P=0.8).
	L_2 **Fluid Level into IC tube bundle** **(I.C.)**	This Initial Condition affects the natural circulation flow rate in the early period of the transient, because of its direct influence on the driving force.	0 - 100 % (nominal: 100%)	0 % = the lower limit pertains to the worst situation, i.e. the complete lack of liquid into the heat exchanger tube bundle; 100 % = pertains to the nominal condition, i.e. the complete filling of the HX tubes	The largest confidence has been assigned to the nominal condition (P=0.8), i.e. the steam coming from the RPV condenses into the bundle and fill it completely. Marginal and equal probabilities are considered for the other two possible values. A reasonable cause to consider such an abnormal situation is the presence of non-condensables in the piping. Therefore a **conditional probability** holds for L_2 : a non-nominal value for the liquid level in the IC tube bundle is accepted only in the case a corresponding non-condensable fraction X_2 is present in the bundle inlet, otherwise the liquid fills the bundle completely.

	UL **Undetected Leakage in the Passive System piping** **(B.C.)**	This Boundary Condition directly affects both the flow rate and the global thermal capacity of the primary fluid. Extension of the activity might imply consideration of different positions for the 'undetected leakage'	0 - 10e-5 m^2 (nominal: 0 m^2)	0 m^2 = no leakage 10.e-5 m^2 = maximum area of undetected leakage	Probability values decrease as *UL* increase; the lower limit corresponds to the nominal value hence to the Max.prob.value. The keeping of a leakage undetected for the mission time of the system has been judged very unlikely to occur, thus a major probability has been assigned to the lower limit condition ($P_{(UL=0)}$ = 0.8899) and a very low value has been assigned to the upper limit (P= 1e-4).
	POV **Uncomplete Valve opening** **(B.C.)**	This boundary Condition represents the presence of an abnormal form loss in the pressure drops of the passive system, hence a direct degradation of the natural circulation.	1 - 100 % (nominal: 100%)	1 % = minimum percentage of valve .open; 100 % = complete opening of the valve. The considered range of variation may cover a number of uncertainties connected with the definition (in the code input deck) of the pressure loss coefficient. An extension of the activity might imply consideration of different positions for 'incomplete valve opening'.	Probability values increase as *POV* increase; the upper limit corresponds to the nominal value hence to the Max.prob.value. The keeping of a valve partially open has been judged unlikely to occur, thus a major probability has been assigned to the upper limit condition (P= 0.889) and the lowest value has been assigned to the lower limit (P= 1e-3).

(+) Same comment as in the previous table.
(*) Same comment as in the previous table.

APPENDIX 3: FMEA ANALYSIS

Table A3.1: FMEA Table for the Isolation Condenser System

Component	Failure Mode	Causes	Prev. Actions on Causes	Consequences	Corr/Prev. Act. On Consequences	Comment	Severity Rank
System piping	Rupture	Material defects and aging; Corrosion; Abnormal operat. conditions; Vibrations; Local. Stresses; Impact of heavy loads (missile)	Adequate welding process quality; Water chemistry control; InService inspect; Design against missile generation	LOCA in the Drywell; Instantaneous loss of natural circulation; Emptying of the circuit; Loss of heat removal capability; Loss of reactor coolant inventory	Isolate the breached loop; Safety relief valves actuation; Automatic reactor depressurization; Gravity Driven Cooling System actuation;	Includes both steam lineand drain line Critical Parameter: Undetected Leakage	V
	Leak	Material defects and aging; Corrosion; Abnormal operat. conditions; Vibrations; Local. stresses	Adequate welding process quality; Water chemistry control; InService inspect.	Small LOCA in the Drywell; Slow emptying of the circuit and natural circulation arrest for long periods of operation; Reduced heat removal capability	Leak monitoring; Isolate the breached loop; Safety relief valves actuation	Critical Parameter: Undetected Leakage	III
Tube Bundle of the heat exchangers of the IC	Single pipe rupture	Wearing due to vibration and corrosion	Preventive maintenance; Water chemistry control; Leak monitoring	Release of primary water to the pool; Slow emptying of the circuit and natural circulation arrest for long periods of operation; Reduced heat removal capability	Flow monitoring; Isolate the breached loop; Safety relief valves actuation	Critical Parameter: Undetected Leakage	IV
	Multiple pipe rupture	Wearing due to corrosion, vibration and pressure transient	Preventive maintenance; Water chemistry control; Leak monitoring	Release of primary water to the pool; Natural circulation stop; Emptying of the	**Isolate the breached loop;** Safety relief valves actuation; Automatic reactor	Critical Parameter: Undetected Leakage	V

				circuit; Loss of reactor coolant inventory; Loss of heat removal capability	depressurization; Gravity Driven Cooling System actuation		
	Single pipe plugging	Crud in the cooling loop; Foreign object in the cooling loop	Water chemistry control; Yearly test of pipes flow; Preventive maint.	No consequences		Critical Parameter: HX Plugged Pipes	II
	Multiple pipe plugging	Violent pressure and vibration transient detaching large amount of crud from pipes walls.	Water chemistry control; **Use of suitable materials for cooling loop pipes;** Preventive maintenance	Natural circulation stop; Loss of heat removal capability; Reactor pressure and temperature increase	Safety relief valves actuation	Critical Parameter: HX Plugged Pipes	III
Drain valve on the return condensate line	Valve fails to open	Control circuit failure; Loss of electric power to motor; Electric motor failure	Redundancy of control devices; Signal to the operator; InService inspect.	Non triggering of Isolation Condenser if bypass valve does not operate; Loss of heat removal capability; Reactor pressure and temperature increase	Reactor pressure and temperature control; Safety relief valves actuation; Realignment by the operator; Corrective maintenance	Critical Parameter: Partially Open Valve	III
	Inadvertent valve closing	Spurious signal; **Control circuit failure;** Human error	Redundancy of control devices; Signal to the operator; Procedured actions	Natural circulation stop in case bypass valve does not operate; Loss of heat removal capability; Reactor pressure and temperature increase	Reactor pressure and temperature control; Safety relief valves actuation; Realignment by the operator; Corrective maintenance	Critical Parameter: Partially Open Valve	IV
Natural Circulation	Envelope failure	Material defects and aging; Corrosion; Abnormal operat. conditions; Vibrations;	Adequate welding process quality; Water chemistry control; InService inspect;	LOCA in the Drywell; Instantaneous loss of natural circulation; Emptying of the	Isolate the breached loop; Safety relief valves actuation; Automatic reactor depressurization;	Includes both steam line and drain line Critical Parameter: Undetected	V

		Local. Stresses; Impact of heavy loads (missile)	Design against missile generation	circuit; Loss of heat removal capability; Loss of reactor coolant inventory	Gravity Driven Cooling System actuation	Leakage	
	Cracking	Material defects and aging; Corrosion; Abnormal operat. conditions; Vibrations; Local. stresses	Adequate welding process quality; Water chemistry control; InService inspect.	Small LOCA in the Drywell; Slow emptying of the circuit and natural circulation arrest for long periods of operation; Reduced heat removal capability	Leak monitoring; Isolate the breached loop; Safety relief valves actuation	Critical Parameter: Undetected Leakage	IV
	Modification of surface characteristics	Oxidation; Aerosol deposits	Water chemistry control	Reduction in heat exchange efficiency; Reduced heat removal capability	Flow monitoring	Critical Parameter: Oxide Layer	II
	Thermal stratification	Temperature dishomogeneity; Density variations; Onset of local thermalhydraulic phenomena	Process control (pressure, flow, temperature)	Reduction of heat convection; Natural circulation blockage; Loss of heat removal capability; Reactor pressure and temp.	Flow monitoring; Reactor pressure and temperature control; Safety relief valves actuation	Critical Parameter: Piping Layout, Heat Loss	IV
	Non condensable build-up	Onset of chemical phenomena; radiolysis products; impurities	Water chemistry control (PH, O2, H2)	Reduction in heat exchange efficiency; Reduction of heat convection; Natural circulation blockage; Loss of heat removal capability; Reactor pressure and temp. increase	Flow monitoring; Reactor pressure and temperature control; Purging through vent lines Safety relief valves actuation	Critical Parameter: Non-Condensables Fraction	II
	Heat dissipation	Thermal insulation degradation; Inaccurate material assembly	InService inspect.	Reduction of heat convection; Natural circulation impairment	Flow monitoring	Critical Parameter: Heat Loss	III

Table A3.2: HAZOP Table for the Isolation Condenser System

PARAMETER: Pressure

Guide Word	Deviation	Possible Causes	Consequences	Safeguards/Interlocks	Actions Required
More of	High Pressure	Non-condensable build-up; Surface modifications (crud, oxidation); HX tube plugging; HX tube rupture Partial valve opening	Natural circulation degradation and reduced heat transfer capability; T increase	Safety relief valve actuation; Vent line valve actuation	Corrective maintenance; Operator action
Less of[1]	Low Pressure	N/A			
No/None	No Pressure	N/A			

PARAMETER: Temperature

Guide Word	Deviation	Possible Causes	Consequences	Safeguards/Interlocks	Actions Required
More of	High Temperature	Non-condensable build-up; Surface modifications (crud, oxidation); HX tube plugging; HX tube rupture Partial valve opening	Natural circulation degradation and reduced heat transfer capability; P increase	Safety relief valve actuation; Vent line valve actuation	Corrective maintenance; Operator action
Less of[1]	Low temperature	N/A			
No/None	No Temperature	N/A			

PARAMETER: Flow-rate

Guide Word	Deviation	Possible Causes	Consequences	Safeguards/Interlocks	Actions Required
More of[1]	High Flow	N/A			
Less of	Low Flow	Modifications of surface characteristics (crud deposition, oxidation); Non-condensable build-up; Thermal stratification; Pipe partial plugging; Pipe leak; HX single pipe plugging; rupture Drain valve partial opening	Natural circulation degradation and reduced heat transfer capability; T and P increase	Safety relief valve actuation; Vent line valve actuation	Corrective maintenance; Operator action

No/None	No Flow	Non-condensable build-up; Thermal stratification; Pipe plugging; Pipe rupture; HX Multiple pipe plugging; HX Multiple pipe rupture; Drain valve closed;	Natural circulation stop and loss of heat transfer capability; T and P increase	Safety relief valve actuation; Vent line valve actuation; Automatic Depressurization System actuation; Gravity Driven Cooling System actuation	Corrective maintenance; Operator action

PARAMETER: Drain valve opening

Guide Word	Deviation	Possible Causes	Consequences	Safeguards/Interlocks	Actions Required
More of	N/A				
Less of	Reduced Opening	Partial blockage	Natural circulation degradation and reduced heat transfer capability; T and P increase	Safety relief valve actuation;	Corrective maintenance; Operator action
No/None	No Opening	Loss of electrical power; Circuit control failure; Electrical motor failure; Valve stuck	Natural circulation stop and loss of heat transfer capability; T and P increase	Safety relief valve actuation; Vent line valve actuation; Automatic Depress. System actuation; Gravity Driven Cooling System actuation	Corrective maintenance; Operator action

PARAMETER: Exchanged heat flux

Guide Word	Deviation	Possible Causes	Consequences	Safeguards/Interlocks	Actions Required
More of[1]	High flux	N/A			
Less of	Low Flux	Non-condensable build-up; Surface modifications (crud, oxidat.) HX single tube plugging; HX single tube rupture	Natural circulation degradation and reduced heat transfer capability; T and P increase	Safety relief valve actuation; Vent line valve actuation	Corrective maintenance; Operator action
No/None	No Flux	Non-condensable build-up; HX multiple tube plugging; HX multiple tube rupture	Natural circulation stop and loss of heat transfer capability; T and P increase	Safety relief valve actuation; Vent line valve actuation; Automatic Depress. System actuation; Gravity Driven Cooling System actuation	Corrective maintenance; Operator action

[1] This deviation is not evaluated, even if it implies an overcooling of the system that could potentially induce to thermal stresses on core structures and reactor components, like the heat exchange.