Joint ICTP-IAEA Course on Natural Circulation Phenomena and Passive Safety Systems in Advanced Water Cooled Reactors

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GOVERNING EQUATIONS IN TWO-PHASE FLUID NATURAL CIRCULATION FLOWS

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INTEGRAL SYSTEM EXPERIMENT SCALING METHODOLOGY

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Abstract

The objective of this lecture is to provide the participant with the methodology needed to design a single-phase or two-phase natural circulation Integral System Test Facility. By the conclusion of this lecture, the participant should be able to structure a Phenomena Identification Ranking Table (PIRT) for a Natural Circulation Based System and understand the method used to conduct a detailed scaling analysis to obtain the geometric dimensions and operating conditions for an integral system test facility.

1. INTRODUCTION

Integral system test (IST) facilities play a key role in the design, assessment and certification of innovative reactor designs. Data obtained using such facilities has been used to benchmark the best-estimate safety analysis computer codes used to evaluate nuclear plant safety. It has also been used to assess the effectiveness of safety system functions under simulated accident conditions. The purpose of this lecture is to describe a hierarchical scaling analysis method that can be used to design an integral system test facility in support of nuclear plant certification. This method has been successfully used to design the Advanced Plant Experiment at Oregon State University as part of the Westinghouse and U.S. Department of Energy programs conducted in support of AP600 and AP1000 design certification. It has also been used to design the Multi-Application Small Light Water Reactor (MASLWR) test facility currently being evaluated at OSU. Where appropriate, examples will be drawn from the scaling analyses for these test facilities.

1.1. Qualified data for plant certification

It is important to note that testing done in support of the certification of a nuclear power plant requires strict adherence to a quality assurance (QA) plan that has been developed, approved and implemented prior to the start of testing. This plan remains in force throughout the testing program and can vary depending on how the data will be used. The QA plan should address the control of every aspect of testing that can affect the quality of the data. Generally speaking, for reduced scaled thermal hydraulic test facilities, the data is the product. Typically, the data’s primary purpose will be to benchmark a computer code that will be used in safety analyses. Only qualified data is permitted in the certification process. Seek guidance from your laboratory or regulators regarding the QA program required for your specific test program.

The scaling analysis can be used to support the test program QA plan by identifying which features of the test facility (ie., geometric and operational features) must be controlled to assure that the important thermal hydraulic phenomena are accurately simulated in the facility. Examples of applicable QA criteria are found in Appendix B of Part 50 in Title 10 of the Code of Federal Regulations (10 CFR 50 Appendix B), and in NQA-1 issued by the American Society of Mechanical Engineers.

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1.2. Scaling analysis objectives

The general objective of a scaling analysis is to obtain the physical dimensions and operating conditions of a reduced scale test facility capable of simulating the important flow and heat transfer behavior of the system under investigation. To develop a properly scaled test facility, the following specific objectives must be met for each operational mode of interest.

- Identify the thermal hydraulic processes that should be modeled.
- Obtain the similarity criteria that should be preserved between the test facility and the full-scale prototype.
- Establish priorities for preserving the similarity criteria.
- Provide specifications for the test facility design
- Quantify biases due to scaling distortions.
- Identify the critical attributes of the test facility that must be preserved to meet Quality Assurance requirements.

Different similarity criteria are obtained for the different modes of system operation. These criteria depend on the geometry of the components, the scaling level required to address the transport phenomena of interest, and the initial and boundary conditions for each particular mode of operation. A method used to obtain similarity criteria for a test facility is presented in the following sections.

1.3. General scaling methodology

Meeting the scaling objectives of the previous section presents a formidable challenge. Therefore, to assure that these objectives are met in an organized and clearly traceable manner, a general scaling methodology (GSM) should be established. The model for this scaling methodology is largely drawn from the USNRC’s Severe Accident Scaling Methodology presented in NUREG/CR-5809 [1]. A flow diagram for a GSM is presented in Figure 1-1.

The first task outlined by the GSM is to specify the experimental objectives. The experimental objectives define the types of tests that will be performed to address specific design or certification needs. These objectives determine the general modes of operation that should be simulated in the test facility. There are practical limits with regard to what can be studied in a single facility. The expectations for the facility and its limitations should be identified early.

The second task outlined by the GSM is the development of a Phenomena Identification and Ranking Table (PIRT) [3]. The nature of scaling forbids exact similitude between a reduced scale test facility and a full-scale prototype. As a result, the design and operation of the test facility is based on simulating the thermal hydraulic processes most important to the system operational modes that will be explored. One very valuable function of a PIRT is to identify the different phases of a scenario and the most important thermal hydraulic phenomena within those phases that should be simulated in the test facility. Many of the thermal hydraulic phenomena of importance to LWR behavior during a Loss-of-Coolant Accident (LOCA) have already been identified in existing PIRTs [2]. Therefore it is of significant value to conduct a literature review to determine if a PIRT already exists for similar designs and transient conditions. Note that full-scale plant information is needed to develop a well-informed PIRT. In addition, the scaling analysis results will naturally identify the dominant thermal hydraulic phenomena for a specific process. Their importance to plant safety or operation however is determined by the PIRT. A description of the PIRT development process is presented in Section 2.
FIG. 1. General scaling methodology.

SPECIFY EXPERIMENTAL OBJECTIVES (1)

- Identify Range of Scenarios to be Examined in Test Facility

PHENOMENA IDENTIFICATION AND RANKING TABLE (PIRT) (2)

- Review Related PIRTs
- Define the Phases of each Scenario to be Examined
- Identify Important Thermal Hydraulic Phenomena for each Scenario

FULL-SCALE PLANT (PROTOTYPE) INFORMATION

- Plant Operating Conditions
- Physical Dimensions
- Isometric Drawings
- P&IDs
- Safety Logic and Setpoints
- Transient Code Analyses

DOCUMENT TEST FACILITY FINAL DESIGN AND OPERATION SPECIFICATIONS AND QA CRITICAL ATTRIBUTES (4)

If yes

DESIGN SPECIFICATIONS OPERATIONAL MODE #2 THROUGH N

PRIORITIZE SYSTEM DESIGN SPECIFICATIONS
The third step in the GSM is to perform a scaling analysis for each of the modes of operation specified by the experimental objectives and further defined by the PIRT. The method to be used herein is based on the Hierarchical Two-Tiered Scaling (H2TS) Methodology [1]. This method is particularly well suited for complex systems having interconnected components.

The scaling methodology is used to develop a set of similarity criteria for each mode of operation. Because it is impossible to identically satisfy all of the similarity criteria simultaneously, the set will include only those criteria that must be satisfied to simulate the most important phenomena identified by the PIRT. Having obtained the pertinent set of similarity criteria, a set of scale ratios can be derived to specify the physical dimensions of the test facility, its initial conditions and boundary conditions. To provide closure to the set of equations that define scale ratios, the designer must define a length scale and flow area or volume scale. In addition, the designer has the flexibility to select the working fluid, the pressure range, and structural materials. Some examples of the rationale for these selections are provided in a later section.

The scaling analysis also requires a numerical evaluation of the similarity criteria to determine if the scale model geometry selected, its boundary conditions, and its operating conditions introduce significant scaling distortions. Distortions are also evaluated relative to the other modes of operation. If the distortions are significant, another iteration is required. Perhaps the length scale, volume scale, or the working fluid selections need to be revised. An important outcome of this assessment process is the identification of the dominant phenomena.

Having completed the scaling analysis for a specific operational mode, the same process is repeated for all of the operational modes for the system. One may find that the scaling ratio requirements for one operational mode may partially conflict with the scaling ratio requirements of another operational mode. Therefore, the final step in the scaling analysis is to prioritize the system design specifications. A description of the Hierarchical Scaling Analysis Method is described in Section 3.

The fourth and final step of the GSM is to document all of the test facility design and operation specifications. All of the essential geometric features and operating parameters that must be controlled to assure an accurate simulation of the important thermal hydraulic phenomena are identified and designated as critical attributes for use in the quality assurance plan.

Figure 2 presents the general scaling methodology implemented for the MASLWR test facility. The scaling analysis was divided into four modes of operation:

- Natural Circulation Scaling Analysis
- Sump Recirculation Scaling Analysis
- Reactor Coolant System Depressurization Scaling Analysis
- Containment Pressurization Scaling Analysis

Other nuclear power plant designs undergo different operating modes during transients. Therefore, familiarity with plant operations is essential to obtaining a properly scaled test facility. The Phenomena Identification and Ranking Table (PIRT) helps the designer define the operating modes for a particular reactor type and transient.
Specify Experiment Objectives

SBLOCA PIRT

Perform Scaling Analysis

Natural Circulation Scaling Analysis

Sump Recirculation Scaling Analysis

Reactor Coolant System Depressurization Scaling Analysis

Containment Pressurization Scaling Analysis

Develop \( \Pi \) Groups and Similarity Criteria

Develop \( \Pi \) Groups and Similarity Criteria

Develop \( \Pi \) Groups and Similarity Criteria

Develop \( \Pi \) Groups and Similarity Criteria

Significant Scaling Distortions in Important \( \Pi \) Groups?

Significant Scaling Distortions in Important \( \Pi \) Groups?

Significant Scaling Distortions in Important \( \Pi \) Groups?

Significant Scaling Distortions in Important \( \Pi \) Groups?

System Design Specifications

System Design Specifications

System Design Specifications

System Design Specifications

Evaluate Key T/H PIRT Processes to Prioritize System Design Specification

OSU Test Facility Design Specifications and Q/A Critical Attributes

FIG. 2. General scaling methodology for the MASLWR test facility.
2. PHENOMENA IDENTIFICATION AND RANKING TABLES (PIRT)

The primary function of a Phenomena Identification and Ranking Table (PIRT) is to rank, relative to a well-defined figure of merit, the importance of systems, components, processes and phenomena in driving a particular plant response. An detailed description of this technique is described in Wilson and Boyack, 1998 [3]. A PIRT can be used to provide guidance in establishing the requirements in:

- Separate effects and integral effects test (SET, IET) programs, where the objective is to insure that the important phenomena are considered in the scaling, design, instrumentation layout and operation of a test facility.
- Code development and improvement programs, where the objective is to insure that the code adequately predicts the important phenomena; and
- Code uncertainty quantification programs, where the objective is to identify and determine the importance of the various sources of code uncertainty.

The role of the PIRT in guiding an integral system scaling analysis is the focus of this section. In this regard, the goal of the PIRT is to identify the most important thermal hydraulic phenomena for a specific scenario (e.g., Single-Phase Natural Circulation Flow, SBLOCA, etc) for the full-scale plant being investigated. Figure 3 briefly outlines the method used to develop a PIRT for a specific plant and scenario.

![Diagram of PIRT development process]

**FIG 3. Process used to develop a “Phenomena Identification and Ranking Table” (PIRT).**
As shown in the GSM outlined in Figure 1, the development of a PIRT requires having already identified the types of scenarios to be investigated in an experimental facility and having obtained a significant amount of data on the full-scale plant. Typically, the PIRT developers consist of an expert panel capable of deducing how the full-scale plant might behave for the scenario being considered. They must have access to the plant operating conditions, drawings, physical dimensions, safety logic and setpoints. Quite often code calculations are used to gain insights into plant behavior during a scenario of interest.

2.1. Evaluation criteria and ranking scale

Having established an expert team with adequate knowledge of the full-scale plant, the first step is to define the basis and method for determining the importance of a phenomenon or component. This requires establishing both a set of evaluation criteria, usually related to the safety of the plant, and a ranking system. An evaluation criterion defines the area of impact the phenomenon has relative to the plant operation and the ranking system defines the extent of impact. The following is an example of the evaluation criterion and ranking method used for the MASLWR SBLOCA PIRT.

**Evaluation Criterion:** How does this particular phenomenon in this particular component impact the fuel’s Peak Cladding Temperature (PCT) during this phase of the scenario?

**Ranking Scale:** High (H), Medium (M), Low (L), Plausible (P), Inactive or Not Applicable (I)

- A High Ranking means that the phenomenon significantly impacts the PCT during a specific phase of the scenario.
- A Medium Ranking means the phenomenon has a moderate impact on the PCT during a specific phase of the scenario.
- A Low Ranking means that the phenomenon has little impact on the PCT during a specific phase of the scenario.
- A Plausible Ranking means that the phenomenon has not been previously assessed in other designs or its impact on PCT is not well understood or modeled by computer codes. For purposes of test facility scaling, these phenomena were considered highly ranked.
- An Inactive or Not Applicable Ranking means that the phenomenon cannot physically impact the PCT during a specific phase of the scenario.

The evaluation is quite specific so as to limit the response to a relatively narrow window. The high, medium low scale is easiest to apply, however, a scale of 1-5 is often used because it permits a greater differentiation in ranking. The value of (1) assigned to the lowest of the low in importance, (2) low importance, (3) Moderate importance (4) high importance and (5) the highest of the high in importance.

2.2. PIRT framework

The layout of the PIRT, that is the structure of the table itself, is essential to the ranking process. First the scenario of interest must be divided into well-defined logical phases. Next the plant must be divided into the key systems of the plant. These key systems are further subdivided into components. All of the phenomena than could occur within a component during a phase of the transient are then listed. This process of dividing the scenario into phases and the plant into systems, components and identifying plausible phenomena requires significant effort and lays the framework for the PIRT. In addition to having access to full-scale plant data and analyses for the scenario in question, the use of existing PIRTs developed for similar scenarios in similar plants would be of some assistance.
Three phases of a SBLOCA in MASLWR were identified and defined as follows:

- **Phase 1 - Blowdown** begins with the opening of the break and ends with the ADS initiation.
- **Phase 2 - ADS Operation** begins with the opening of the ADS valve and ends when the containment and reactor system pressures are equalized.
- **Phase 3 - Long Term Cooling** begins with the equalization of the containment and reactor system pressures and ends when stable cooling is established.

The breakdown of the MASLWR design into systems, components and phenomena within components is shown in Table 1.

### 2.3. Expert ranking methods and PIRT results

There are various techniques used by expert panels to come to consensus regarding the rank for a specific phenomenon. A formal ranking technique, known as the Analytical Hierarchy Process (AHP), is described in Saaty, 1982 [4]. The method requires a pair-wise comparison of phenomena. That is, the expert only considers two phenomena at a time to determine which one is more important. After this information is gathered, the AHP method is applied to obtain the relative importance of each phenomenon. The simplest ranking method is a group discussion on each phenomenon followed by a vote of the experts. If the vote is not unanimous with regard to a rank, additional discussion may be encouraged to resolve the difference, the chairman may offer a compromise rank, or the default may be to apply the highest rank. It is very important that the basis for an assigned rank be justified and documented. The use of a tape recorder is recommended during the deliberation process. It will prove very valuable when developing the ranking rationale document. Table 1 presents the SBLOCA PIRT for the MASLWR design.

### 2.4. High-ranked phenomena

For the test facility designer, the ultimate result of a PIRT is to obtain a list and ranking of all of the thermal hydraulic phenomena pertinent to the full-scale plant for the specific scenarios that will be examined in the test facility. As a minimum, it is desired that all of the phenomena that have been highly ranked (i.e., ranked “H” or “4-5” on their respective scales) be properly simulated in the test facility. The method for doing so is addressed in Chapter 3.
<table>
<thead>
<tr>
<th>System</th>
<th>Component</th>
<th>Process/Phenomenon</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vent Valves</td>
<td>Valves</td>
<td>Mass Flow (Choked/Nonchoked)</td>
<td>1 2 3</td>
</tr>
<tr>
<td>Piping</td>
<td>Line Flow Resistance</td>
<td>M  M  M</td>
<td></td>
</tr>
<tr>
<td>ADS</td>
<td>Valves</td>
<td>Mass Flow (Choked/Nonchoked)</td>
<td>1 2 3</td>
</tr>
<tr>
<td>Piping</td>
<td>Line Flow Resistance</td>
<td>I  M  M</td>
<td></td>
</tr>
<tr>
<td>Sparger</td>
<td>Condensation</td>
<td>I  H  M</td>
<td></td>
</tr>
<tr>
<td>Sparger</td>
<td>Energy Release</td>
<td>I  H  M</td>
<td></td>
</tr>
<tr>
<td>ADS</td>
<td>Piping</td>
<td>Mass Flow (Choked/Nonchoked)</td>
<td></td>
</tr>
<tr>
<td>Containment Shell</td>
<td>Internal</td>
<td>Thermal Stratification</td>
<td>I  I  L</td>
</tr>
<tr>
<td>Passive Safety Recirculation</td>
<td>ADS Heat-up of Sump</td>
<td>I  L  M</td>
<td></td>
</tr>
<tr>
<td>Passive Safety Recirculation</td>
<td>Condensation (Surface of Pool)</td>
<td>L  I  I</td>
<td></td>
</tr>
<tr>
<td>Passive Safety Recirculation</td>
<td>Thermal Stratification</td>
<td>L  L  L</td>
<td></td>
</tr>
<tr>
<td>Passive Safety Recirculation</td>
<td>Recirculation (Flow Resistance)</td>
<td>I  I  H</td>
<td></td>
</tr>
<tr>
<td>Passive Safety Recirculation</td>
<td>Resupply from Containment</td>
<td>L  L  H</td>
<td></td>
</tr>
<tr>
<td>Primary Coolant System</td>
<td>Hot Leg Riser</td>
<td>Flow Resistance (wall/control rod tubes)</td>
<td>M  M  M</td>
</tr>
<tr>
<td>SG Tube Annulus</td>
<td>SG Tube Condensation</td>
<td>H  H  H</td>
<td></td>
</tr>
<tr>
<td>Reactor System</td>
<td>Vessel - Control Rods</td>
<td>Flow Interfacial Drag</td>
<td>L  H  L</td>
</tr>
<tr>
<td>Reactor System</td>
<td>Vessel - Core Subchannels</td>
<td>Mass Flow</td>
<td>H  H  H</td>
</tr>
<tr>
<td>Reactor System</td>
<td>Vessel - Downcomer</td>
<td>Flow Interfacial Drag</td>
<td>L  H  L</td>
</tr>
</tbody>
</table>

**NOTES**

| Phase 1 - Blowdown | H - Significantly Impacts PCT | I - Inactive during the transient Phase |
| Phase 2 - ADS Operation | M - Moderately Impacts PCT | P - Plausible |
| Phase 3 - Long Term Cooling | L - Little Impact on PCT | |

Table 1. Small break loca PIRT for the MASLWR design (continued)
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<table>
<thead>
<tr>
<th>System</th>
<th>Component</th>
<th>Process/Phenomenon</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor System (continued)</td>
<td>Vessel - Fuel Rods</td>
<td>Fuel Heat Transfer</td>
<td>H</td>
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<tr>
<td></td>
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<td>H</td>
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<table>
<thead>
<tr>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1 - Blowdown</td>
</tr>
<tr>
<td>Phase 2 - ADS Operation</td>
</tr>
<tr>
<td>Phase 3 - Long Term Cooling</td>
</tr>
</tbody>
</table>

3.  HIERARCHICAL TWO-TIERED SCALING (H2TS) METHODOLOGY

This section describes the Hierarchical Two-Tiered Scaling (H2TS) method. This method has been successfully used to develop the similarity criteria necessary to scale the APEX-600 and APEX-1000 systems for LOCA transients. The H2TS method was developed by the USNRC and is fully described in Appendix D of NUREG/CR-5809 [1].

Figure 4 is taken from NUREG/CR-5809. It presents the four basic elements of the H2TS analysis method. The first element consists of subdividing the plant into a hierarchy of systems. Each system is subdivided into interacting subsystems which are further subdivided into interacting modules which are further subdivided into interacting constituents (materials) which are further subdivided into interacting phases (liquid, vapor or solid). Each phase can be characterized by one or more geometrical configurations and each geometrical configuration can be described by three field equations (mass, energy and momentum conservation equations). Each field equation can incorporate several processes. Figure 5 shows the general approach used for a system breakdown and Figure 6 presents an example based on the MASWLR design.

After identifying and subdividing the system of interest, the next step is to identify the scaling level at which the similarity criteria should be developed. This is determined by examining the phenomena being considered. For example, if the phenomenon being considered involves mass, momentum or energy transport between materials such as water and solid particles, then the scaling analysis would be performed at the constituent level. If the phenomenon of interest involves mass, momentum or energy transport between vapor and liquid, then the scaling analysis would be performed at the phase level. Therefore identifying the scaling level depends on the phenomenon being addressed.
Thermal hydraulic phenomena involving integral reactor coolant system interactions, such as primary system depressurization or loop natural circulation, would be examined at the “system” level. Thermal hydraulic phenomena, such as steam generator heat transfer, would be examined at the “subsystem” level. Specific interactions between the steam-liquid mixture and the stainless steel structure would be examined at the “constituent” level.

The H2TS method requires performing a “Top-Down” (system) scaling analysis. The top-down scaling analysis examines the synergistic effects on the system caused by complex interactions between the constituents deemed important by the PIRT. Its purpose is to use the conservation equations at a given scaling level to obtain characteristic time ratios and similarity criteria. It also identified the important processes to be addressed in the bottom-up scaling analysis.

The H2TS method also required performing a “Bottom-Up” (process) scaling analysis. This analysis provides similarity criteria for specific processes such as flow pattern transitions and flow dependent heat transfer. The focus of the bottom-up scaling analysis is to develop similarity criteria to scale individual processes of importance to system behavior as identified by the PIRT.

FIG. 4. Flow diagram for the hierarchical, two-tiered scaling analysis. [1].
FIG. 5. System breakdown into components, geometries, fields and processes.

FIG. 6. MASLWR system breakdown into components, geometries, fields and processes.
3.1. Time ratios

The basic objective of the H2TS method is to develop sets of characteristic time ratios for the transfer processes of interest. This is done by writing the control volume balance equations for each constituent “k” as follows:

\[
\frac{dV_k \psi_k}{dt} = \Delta [Q_k \psi_k] + \sum (j_{kn} A_{kn}) + S_k
\]

where

\[
\Delta [Q_k \psi_k] = \left[ Q_k \psi_{k, in} - Q_k \psi_{k, out} \right]
\]

In equation (1) the \( \psi_k \) term represents the conserved property; \( \psi_k = \rho, \rho u \) or \( \rho \varepsilon \) (mass, momentum or energy per unit volume), \( V_k \) is the control volume, \( Q_k \) is the volumetric flow rate, \( j_{kn} \) is the flux of property \( \psi_k \) transferred from constituent “k” to “n” across the transfer area \( A_{kn} \). Hence, \( \Delta [Q_k \psi_k] \) represents the usual mass, momentum, or energy convection terms, and \( \sum j_{kn} A_{kn} \) represents transport process terms such as condensation and \( S_k \) represents the distributed sources, such as decay power or body forces acting internal to the control volume. Equation (1) can be put in dimensionless form by specifying the following dimensionless groups in terms of the constant initial and boundary conditions:

\[
V_k^+ = \frac{V_k}{V_{k,0}}, \psi_k^+ = \frac{\psi_k}{\psi_{k,0}}, Q_k^+ = \frac{Q_k}{Q_{k,0}}, j_{kn}^+ = \frac{j_{kn}}{j_{kn,0}}, A_{kn}^+ = \frac{A_{kn}}{A_{kn,0}}, S_k^+ = \frac{S_k}{S_{k,0}}
\]

Substituting these groups into equation (1) yields:

\[
V_{k,0} \psi_{k,0} \frac{dV_k^+ \psi_k^+}{dt} = Q_{k,0} \psi_{k,0} \Delta [Q_k^+ \psi_k^+] + \sum (j_{kn,0} A_{kn,0}) j_{kn}^+ A_{kn}^+ + S_{k,0}^+ S_k^+
\]

Dividing both sides of this equation by \( Q_{k,0} \psi_{k,0} \) yields:

\[
\tau_k \frac{dV_k^+ \psi_k^+}{dt} = \Delta \left[ Q_k^+ \psi_k^+ \right] + \sum \Pi_{kn} j_{kn}^+ A_{kn}^+ + \Pi_{sk} S_k^+
\]

where the residence time of constituent “k” is

\[
\tau_k = \frac{V_{k,0}}{Q_{k,0}}
\]

and the characteristic time ratio for a transfer process between constituents “k” and “n” is given by:

\[
\Pi_{kn} = \frac{j_{kn,0} A_{kn,0}}{Q_{k,0} \psi_{k,0}}
\]

The characteristic time ratio for the distributed source term within the control volume is given by:

\[
\Pi_{sk} = \frac{S_{k,0}}{Q_{k,0} \psi_{k,0}}
\]

Because each transfer process has a characteristic time ratio, it is possible to rank the importance of each process by comparing the time ratios. If a specific transfer process is to have the same effect in the prototype and the model, then the characteristic time ratios must be preserved.
3.2. Similarity of trajectories in dimensionless phase space

One of the goals of a test program is to operate the test facility such that the various trends that evolve in the facility for a given scenario would be the same for a similar scenario in the full-scale prototype when the results are plotted in dimensionless phase space. For example, given a dimensionless depressurization rate equation of the form:

\[
\frac{dP^+}{dt^+} = -\sum \Pi_i S_i^+ 
\]

plotting a scenario’s pressure history as \(P^+ = P/P_o\) versus \(t^+ = t/t_{RCS}\) would yield overlaying curves for the two facilities. This condition can be achieved by satisfying the following requirements:

1. The scenarios are initiated from the same initial condition in dimensionless phase space. In this case \(P^+\) at \(t^+ = 0\), is 1.
2. The rate of change, (i.e., slope), is preserved in dimensionless phase space. This imposes the following scaling criterion:

\[
\left( \frac{dP^+}{dt^+} \right)_R = 1 
\]  

(10)

Satisfying the requirement given by equation (10) implies preserving all of the dimensionless \(\Pi\) groups on the right hand side of the balance equation. If the two requirements listed above are satisfied, then the following is true:

\[
\left( \frac{P}{P_o} \right)_m = \left( \frac{P}{P_o} \right)_p
\]

(11)

This means that the dimensionless pressure at any point along the scenario trajectory will be the same in model and in the prototype.

3.3. Process ranking using characteristic time ratios (\(\Pi\) Groups)

Let us define \(M[ (\Pi_{i,j}),(\Pi_{i+1,j}),...,(\Pi_{N_i,N_j})]\) as the set of time ratios that characterize all of the individual processes that occur during the evolution of a transient. The subscripts \(i, j, N_i, N_j\) identify the specific process, the hierarchical level, the total number of specific processes and the total number of hierarchical levels respectively. Because of differences in geometrical scale and fluid properties, it is impossible to exactly duplicate the “time ratio set” for the full-scale prototype, \(M_p\), in a reduced scale model. That is, exact similitude for all processes cannot be preserved; therefore:

\[
M_p \neq M_m 
\]

(12)

The subscript, \(p\), refers to the full-scale prototype and the subscript, \(m\), refers to the reduced scale model. It is possible to design a reduced scale test facility that preserves the similitude of a subset of time ratios \(T [\Pi_{i,j}]\) that characterize the processes of greatest importance to the transient. This optimizes the model design to investigate the important processes while distorting the less important processes. To determine which processes govern the overall evolution of a transient, numerical estimates of the characteristic time ratios for the prototype and the model must be obtained for each hierarchical level of interest. Physically, each characteristic time ratio, \(\Pi_i\), is composed of a specific frequency, \(\omega_i\), which is an attribute of the specific process, and the residence time constant, \(\tau_{cv}\), for the control volume. That is:

\[
\Pi_i = \omega_i \tau_{cv} 
\]

(13)
The specific frequency defines the mass, momentum or energy transfer rate for a particular process. The residence time defines the total time available for the transfer process to occur within the control volume. A numerical value of:

$$\Pi_i < 1$$  \hspace{1cm} (14)

means that only a small amount of the conserved property would be transferred in the limited time available for the specific process to evolve. As a result, the specific process would not be important to the overall transient. Numerical values of:

$$\Pi_i \geq 1$$  \hspace{1cm} (15)

means that the specific process evolves at a high enough rate to permit significant amounts of the conserved property to be transferred during the time period, $\tau_{cv}$. Such processes would be important to the overall transient behavior.

### 3.4. Similarity criteria and scaling ratio development

The scaling analysis results in a set of characteristic time ratios (dimensionless $\Pi$ groups) that characterize the various processes in the mode of operation being examined. Because it is impossible to identically satisfy all of the similarity criteria simultaneously, the set only includes those criteria that must be satisfied to scale the most important phenomena identified by the PIRT. Similarity Criteria are developed by requiring that the characteristic time ratios for a subset of specific processes in the prototype (usually those of greatest importance) match in the model at each hierarchical level. That is,

$$T \{ \Pi_i \}_{m} = T \{ \Pi_i \}_{p}$$  \hspace{1cm} (16)

Specifically, for each process, there will be a similarity criterion given by:

$$\frac{\Pi_m}{\Pi_p} = 1$$  \hspace{1cm} (17)

Each similarity criterion can be satisfied by adjusting the physical geometry, fluid properties, and operating conditions of the model; thus optimizing the model design for the specific process of interest. In the process of satisfying the similarity criteria, the scale ratios for the components and operating conditions are obtained.

### 3.5. Evaluation of scale distortion

The scaling criteria were evaluated to determine if the scale model geometry, boundary conditions or operating conditions would introduce significant scaling distortions. Distortions were also evaluated relative to other modes of operation.

The effect of a distortion in the model for a specific process can be quantified as follows:

$$DF = \frac{[\Pi_i]_p - [\Pi_i]_m}{[\Pi_i]_p}$$  \hspace{1cm} (18)

The distortion factor, $DF$, physically represents the fractional difference in the amount of conserved property transferred through the evolution of a specific process in the prototype to the amount of conserved property transferred through the same process in the model during their respective residence times. A distortion factor of zero would indicate that the model ideally simulates the specific process.
A distortion factor of +0.05 would indicate that the specific process in the model transfers 5 percent less of the conserved property (on a scaled basis) than the same process in the prototype. The distortion factor can also be written as:

\[ \text{DF} = 1 - \left[ \omega_{\text{R}} \right] \left[ \tau_{\text{cv}} \right] \]  

or

\[ \text{DF} = 1 - \left[ \Pi_{\text{R}} \right] \]  

The degree to which a specific transfer process could impact a particular transient can be determined by comparing the maximum characteristic time ratio for each of the transfer processes that arise during the transient. The following section provides some examples for the MASLWR design.

4. SINGLE-PHASE NATURAL CIRCULATION SCALING ANALYSIS

This section briefly outlines the results of the H2TS single-phase natural circulation scaling analysis method as applied to the MASLWR test facility. Figure 7 provides a flow diagram that describes the scaling analysis process for this operational mode. First, a top-down scaling analysis was performed. This included an analysis at the system level (integrated loop behavior) for normal operating conditions. The objective of the top-down scaling analysis was to scale the primary loop mass flow rates and core and steam generator heat exchange rates. Following the top-down scaling analysis, a bottom-up/process scaling analysis was performed to develop similarity criteria to scale specific thermal hydraulic phenomena such as the hydraulic resistance and the core decay power.
FIG 7. Scaling analysis flow diagram for single-phase natural circulation.
4.1. Top-down scaling analysis for single-phase natural circulation

The loop being considered consists of the core, which serves as a heat source, the riser, the annular downcomer region between the riser and the reactor vessel, and the helical steam generator coil that serves as the heat sink. A simple sketch is presented in Figure 8. As shown in this figure, the primary loop is divided into a hot fluid side having an average temperature $T_H$ and a cold fluid side having an average temperature $T_C$.

![Diagram of single-phase natural circulation flow within MASLWR](image)

**FIG. 8.** Hot and cold regions of single-phase natural circulation flow within MASLWR.

Mass, momentum and energy control volume balance equations can be written for each component. For purposes of the single-phase natural circulation scaling analysis, the following assumptions were made:

1. The flow was one-dimensional along the loop axis, therefore fluid properties were uniform at every cross-section.
2. The Boussinesq approximation was applicable.
3. The fluid was incompressible.
4. $T_c$ is constant
5. Form losses, primarily in the core and steam generator regions, dominate the loop resistance.

By implementing the Boussinesq approximation, all of the fluid densities in the loop were assumed equal to an average fluid density except for those that comprise the buoyancy term. $T_M$ is a mixed mean temperature for the system.
The fact that the components of the loop remain liquid filled during the natural circulation mode of operation coupled with the third assumption eliminates the time dependence in the component mass conservation equation. Applying these assumptions to the component balance equations and integrating the momentum and energy equations over the entire loop, yielded the momentum and energy balance equations for fluid transport around the loop as shown in Table 2.

The loop momentum and energy balance equations were made dimensionless by normalizing each term relative to its steady-state initial or boundary condition. These terms are also shown in Table 2.

Table 3 presents the dimensionless momentum and energy balance equations for the loop. The characteristic time constant and the characteristic ratios (Π groups) arising in the equations are also defined in the table. Lastly, the steady-state solution for the governing equation is presented. The steady-state solution is obtained by setting the time dependent term to zero and setting all of the “+” superscripted terms to unity. This results in a very simple equation. That is, the Richardson Number, Π_Ri equals the Loop Resistance Number Π_Fl. Thus, substituting equation (35) into (40) yields the well-known equation for the fluid velocity at the inlet to the core.

4.2. Bottom-up scaling analysis for single-phase natural circulation

The bottom-up scaling analysis provides the information required to evaluate the dimensionless groups obtained in the Top-Down Scaling Analysis. These are local transport models and correlations. For a single-phase natural circulation loop, models are needed for:

- Core Decay Power,
- Friction and Form Loss Coefficient Models
- Core Heat Transfer Models

For the MASLWR test facility design, core decay power was modeled using ANSI/ANS-5.1.24, 1979 with a G-Factor included. Friction and form losses were modeled using the Moody Friction factor correlations and standard tables for loss coefficients for fittings, elbows and valves. Orifices were used to obtain the desired pressure drop around the loop.

4.3. Similarity criteria and scale ratios for a single-phase fluid natural circulation loop

The similarity criteria for a single-phase fluid natural circulation loop is obtained by setting the model to prototype ratios of each dimensionless group in Table 3 to unity. Expressions for the time, length, and velocity scale ratios are obtained using the ratio of the characteristic time constant and the steady state solution for the fluid velocity at the core inlet. The results are presented in Table 3.

The flow area scale ratio shown in equation (45) was set to unity to preserve the kinematic behavior within the loop components. To obtain a closed set of design parameters, the designer must “select” values for the time, length, flow area and the loop resistance scale ratios. In addition, the designer must also select the working fluid. Hence the designer has significant flexibility for the loop design.

**Exercise**: Assuming fluid property similitude (i.e., same working fluid), find the core power scale ratio given, \( \tau_R = 0.5 \) (i.e., isochronicity); \( l_R = 0.25 \); \( a_R = 1:50 \); \((\Pi_Fl)_R = 1\)
Table 2. Governing equations and normalizing ratios for single-phase natural circulation

**Governing Balance Equations**

*Loop Momentum Balance Equation:*

$$
\sum_{i=1}^{N} \left( \frac{l_i}{a_i} \right) \frac{d\dot{m}_i}{dt} = \beta g \rho(T_H - T_C) I_{th} - \frac{\dot{m}^2}{\rho_a c_p} \sum_{i=1}^{N} \left[ \frac{1}{2} \left( \frac{\frac{fl}{d_h} + K}{a} \right) \left( \frac{a_c}{a_i} \right)^2 \right]
$$

*Loop Energy Balance Equation:*

$$
C_{sys} \rho(T_H - T_C) = \dot{m} \rho(T_H - T_C) - \dot{q}_{SG} - \dot{q}_{loss}
$$

**Initial and Boundary Conditions**

$$
t^+ = \frac{\dot{f}}{\tau_{loop}}
$$

$$
\dot{m}^+ = \frac{\dot{m}}{\dot{m}_o}
$$

$$
\left\{ \sum_{i=1}^{N} \left[ \frac{1}{2} \left( \frac{\frac{fl}{d_h} + K}{a} \right) \left( \frac{a_c}{a_i} \right)^2 \right] \right\}^+ = \frac{\sum_{i=1}^{N} \left[ \frac{1}{2} \left( \frac{\frac{fl}{d_h} + K}{a} \right) \left( \frac{a_c}{a_i} \right)^2 \right]}{\sum_{i=1}^{N} \left[ \frac{1}{2} \left( \frac{\frac{fl}{d_h} + K}{a} \right) \left( \frac{a_c}{a_i} \right)^2 \right]_{o}}
$$

$$
(T_M - T_C)^+ = \frac{(T_M - T_C)}{(T_M - T_C)_o}
$$

$$
(T_H - T_C)^+ = \frac{(T_H - T_C)}{(T_H - T_C)_o}
$$

$$
\dot{q}_{SG}^+ = \frac{\dot{q}_{SG}}{\dot{q}_{SG,o}}
$$

$$
\dot{q}_{loss}^+ = \frac{\dot{q}_{loss}}{\dot{q}_{loss,o}}
$$
### Table 3. Dimensionless equations and \( \Pi \) groups for single-phase natural circulation

#### Dimensionless Balance Equations

**Loop Momentum Balance Equation:**

\[
\Pi_L \frac{d\bar{h}^+}{dt^+} = \Pi_{Ri} \left( T_H - T_C \right)^+ - \Pi_{Fl} \left( \bar{m}^+ \right)^2 \left\{ \sum_{i=1}^{N} \left[ \frac{1}{2} \left( \frac{f_l}{d_i} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right] \right\}^+ \tag{30}
\]

**Loop Energy Balance Equation:**

\[
\frac{1}{\gamma} \frac{d(\bar{T}_M - T_C)^+}{dt^+} = \Pi_{\gamma} \bar{m}^+ \left( T_H - T_C \right)^+ - \Pi_{SG} \dot{q}_{SG}^+ - \Pi_{loss} \dot{q}_{loss}^+ \tag{31}
\]

#### Characteristic Time Constant

\[
\tau_{loop} = \sum_{i=1}^{N} \frac{l_i}{u_i} = \sum_{i=1}^{N} \tau_i = \frac{M_{sys}}{\bar{m}_a} = \frac{M_{sys}}{\rho_i u_c a_c} \tag{32}
\]

#### Dimensionless Groups

**Loop Reference Length Number:**

\[
\Pi_L = \sum_{i=1}^{N} \frac{l_i}{l_{ref}} \frac{a_c}{a_i} \tag{33}
\]

where:

\[
l_{ref} = \frac{M_{sys}}{\rho_l a_c} \]

**Loop Richardson Number:**

\[
\Pi_{Ri} = \frac{\beta g \left( T_H - T_C \right)_o L_{th}}{u_{co}^2} \tag{34}
\]

or

\[
\Pi_{Ri} = \frac{\beta g \dot{q}_{co} L_{th}}{\rho_i a_c C_{pl} u_{co}^3} \tag{35}
\]

**Loop Resistance Number:**

\[
\Pi_{Fl} = \sum_{i=1}^{N} \left\{ \frac{1}{2} \left( \frac{f_l}{d_i} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right\} \tag{32}
\]

**Ratio of Specific Heats:**

\[
\gamma = \frac{C_{pl}}{C_{vl}} \tag{36}
\]

**Loop Energy Ratio:**

\[
\Pi_{\gamma} = \frac{T_H - T_C}{T_M - T_C} \tag{37}
\]

**Steam Generator Heat Transport Number:**

\[
\Pi_{SG} = \frac{\dot{q}_{SG,o}}{\rho_i u_{co} a_c C_{pl} \left( T_M - T_C \right)_o} \tag{38}
\]

**Loop Heat Loss Number:**

\[
\Pi_{Loss} = \frac{\dot{q}_{loss,o}}{\rho_i u_{co} a_c C_{pl} \left( T_M - T_C \right)_o} \tag{39}
\]

**Steady-State Solution**

\[
\Pi_{Ri} = \Pi_{Fl} \tag{40}
\]

**Core Inlet Velocity:**

\[
u_{co} = \left( \frac{\beta g \dot{q}_{co} L_{th} \dot{g}_{co}}{\rho_i a_c C_{pl} \Pi_{Fl}} \right)^{\frac{1}{3}} \tag{41}
\]
Table 4. Scale ratios for a single-phase natural circulation loop

**Time Scale Ratio:**

\[
\tau_{\text{loop},R} = \left( \frac{\rho_l C_{pl}}{\beta} \right)_{R}^{\frac{1}{2}} \left( \frac{a_c \Pi_{fl} l^2}{\dot{q}_{co}} \right)_{R}^{\frac{1}{2}}
\]  
(42)

**Fluid Velocity Scale Ratio:**

\[
u_R = \left( \frac{\beta}{\rho_l C_{pl}} \right)_{R}^{\frac{1}{2}} \left( \frac{\dot{q}_{co} l}{a_c \Pi_{fl}} \right)_{R}^{\frac{1}{2}}
\]  
(43)

**Loop Length Scale Ratio:**

\[l_R = (l_{th})_R
\]  
(44)

**Flow Area Scale Ratio (Kinematic Similarity):**

\[
\left( \frac{a_c}{a} \right)_R = 1
\]  
(45)

**Loop Energy Scale Ratio:**

\[
\left[ \frac{(T_H - T_C)_o}{(T_M - T_C)_o} \right]_R = 1
\]  
(46)

**Steam Generator Power Scale Ratio:**

\[
\left( \frac{\dot{q}_{SG}}{\dot{q}_{co}} \right)_R = 1
\]  
(47)

**Heat Loss Scale Ratio:**

\[
\left( \frac{\dot{q}_{loss,o}}{\dot{q}_{co}} \right)_R = 1
\]  
(48)
5. TWO-PHASE NATURAL CIRCULATION SCALING ANALYSIS

This section presents a scaling analysis for two-phase natural circulation. Figure 9 provides a flow diagram which describes the scaling analysis process for this operational mode. First a top-down integral system scaling analysis was performed. The primary objective of the top-down scaling analysis was to scale the primary loop mass flow rates. Following the top-down scaling analysis, a bottom-up scaling analysis was performed to scale the loop resistance for two-phase flow conditions and to assess the degree of similarity between flow pattern transitions in the model and prototype.

![Scaling analysis flow diagram for two-phase natural circulation.](image-url)
5.1. Top-down scaling analysis for two-phase natural circulation

Figure 10 depicts the loop geometry considered for this analysis. The loop is divided into two regions; a two-phase region with a fluid density $\rho_{TP}$ and a single-phase region with a fluid density $\rho_l$. The simplifying assumptions are as follows:

Integral Analysis Assumption:

- Constant core inlet enthalpy
- Uniform fluid properties at every cross-section,
- Homogeneous flow in the two phase region,
- Chemical Equilibrium – no chemical reactions,
- Thermal Equilibrium – both phases at the same temperature,
- The sum of convective accelerations due to vaporization and condensation are negligible,
- Viscous effects included in determination of form losses only.
- Form losses, primarily in the core and steam generator regions, dominate the loop resistance.

The assumptions listed above were applied to the mass, momentum, and energy equations for each component in the loop to obtain the conservation equations. The equations were then integrated over their respective single-phase and two-phase regions to obtain the loop balance equations. These equations are listed in Table 5. Also listed in Table 5 are the normalized initial conditions that were substituted into the balance equations to make them dimensionless. The set of dimensionless balance equations is presented in Table 6. It includes the set of dimensionless groups (i.e., characteristic time ratios) that characterize two-phase natural circulation in the loop. The ratio of each group was used to form the similarity criteria needed for test facility scaling.

Unlike single-phase natural circulation, a simple analytical expression for the velocity at the core inlet cannot be readily obtained from the steady-state solution. This is due to the fact that the two-phase mixture density is dependent on core flow rate. The resulting steady-state expression for the velocity is a cubic equation as described in Table 7. The coefficients for this equation include the terms for the subcooled liquid phase.

The problem becomes much more manageable when considering saturated conditions at the core inlet. Examining this special case (i.e. zero subcooling) results in the simpler dimensionless equation (78) presented in Table 7. It is a cubic equation expressed in terms of the Zuber Number and dimensionless coefficients. This equation can be used to obtain the desired scale ratios for two-phase natural circulation as shown in the next section.
FIG. 10. Regions of single-phase and two-phase natural circulation within MASLWR.
Table 5. Governing equations and normalizing ratios for two-phase natural circulation

### Governing Balance Equations

#### Loop Momentum Balance Equation:
\[
\sum_{i=1}^{N} \left( \frac{l_i}{a_i} \right) \frac{d\dot{m}}{dt} = g(\rho_i - \rho_{TP})L_{th} - \dot{m}^2 \rho_i \frac{\sum_{SP} \left[ \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right]}{\rho_i \sum_{TP} \left[ \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right]}
\]

(49)

#### Loop Energy Balance Equation:
\[
M_{sys} \frac{d(e_M - e_l)}{dt} = \dot{m}(h_{TP} - h_l) - \dot{q}_{SG} - \dot{q}_{loss}
\]

(50)

#### Equilibrium Vapor Quality at Core Exit:
\[
x_e = \frac{h_{TP} - h_l}{h_{fg}}
\]

(51)

#### Homogeneous Two-Phase Fluid Mixture Density:
\[
\rho_{TP} = \frac{\rho_f}{1 + x_e \left( \frac{\rho_f - \rho_{fg}}{\rho_{fg}} \right)}
\]

(52)

### Initial and Boundary Conditions

\[
t^* = \frac{t}{\tau_{loop}}
\]

(53)

\[
\dot{m}^* = \frac{\dot{m}}{\dot{m}_o}
\]

(54)

\[
\rho_i - \rho_{TP} = \frac{(\rho_i - \rho_{TP})}{(\rho_i - \rho_{TP})_o}
\]

(55)

\[
e_M - e_l = \frac{(e_M - e_l)}{(e_M - e_l)_o}
\]

(56)

\[
\left( \frac{h_{TP} - h_l}{h_{TP} - h_l}_o \right) = \frac{(h_{TP} - h_l)}{(h_{TP} - h_l)_o}
\]

(57)

\[
\dot{q}_{SG}^* = \frac{\dot{q}_{SG}}{\dot{q}_{SG, o}}
\]

(58)

\[
\dot{q}_{loss}^* = \frac{\dot{q}_{loss}}{\dot{q}_{loss, o}}
\]

(59)

\[
\left( \frac{\rho_f}{\rho_{TP}} \right)_o = \frac{(\rho_f / \rho_{TP})}{(\rho_f / \rho_{TP})_o}
\]

(60)

\[
\left[ \sum_{SP} \left[ \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right] \right]^* = \frac{\sum_{SP} \left[ \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right]}{\sum_{TP} \left[ \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right]}
\]

(61)

\[
\left[ \sum_{TP} \left[ \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right] \right]^* = \frac{\sum_{TP} \left[ \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right]}{\sum_{TP} \left[ \frac{1}{2} \left( \frac{fl}{d_h} + K \right) \left( \frac{a_c}{a_i} \right)^2 \right]}
\]

(62)
Table 6. Dimensionless equations and $\Pi$ groups for two-phase natural circulation

**Dimensionless Balance Equations**

**Loop Momentum Balance Equation:**

$$\Pi_l \frac{dm^*}{dt^*} = \Pi_{\rho_{\text{ref}}} (\rho_l - \rho_{\text{TP}})^* + \left(\frac{\rho_l}{\rho_{\text{TP}}}\right)^* \sum_{i=1}^{N} \left(\frac{1}{2} \left( \frac{fl}{dh} + K \right) \left( \frac{a_i}{a_l} \right)^2 \right)$$

$$- \left(\frac{\rho_l}{\rho_{\text{TP}}}\right)^* \left( \sum_{i=1}^{N} \left( \frac{1}{2} \left( \frac{fl}{dh} + K \right) \left( \frac{a_i}{a_l} \right)^2 \right) \right)$$

(62)

**Loop Energy Balance Equation:**

$$\frac{d(e_M - e_l)^*}{dt^*} = \Pi_{\rho_{\text{ref}}} \left( h_{\text{TP}} - h_l \right)^* - \Pi_{\text{SG}} \left( \dot{q}_{\text{SG}}^* \right)^* - \Pi_{\text{loss}} \left( \dot{q}_{\text{loss}}^* \right)^*$$

(63)

**Characteristic Time Constant**

$$\tau_{\text{loop}} = \sum_{i=1}^{N} \frac{l_i}{u_i} \sum_{i=1}^{N} \tau_i = \frac{M_{\text{sys}}}{\dot{m}_o} = \frac{M_{\text{sys}}}{\rho_l u_{\text{co}} a_c}$$

(64)

**Dimensionless Groups**

**Loop Reference Length Number:**

$$\Pi_l = \sum_{i=1}^{N} \frac{l_i}{l_{\text{ref}}} \frac{a_i}{a_l}$$

(65)

where:

$$l_{\text{ref}} = \frac{M_{\text{sys}}}{\rho_l a_c}$$

**Loop Richardson Number:**

$$\Pi_{\text{Re}, \rho} = \frac{g \left( \rho_l - \rho_{\text{TP}} \right) a_c L_{\text{th}}}{\rho_l u_{\text{co}}^2}$$

(66)

**Loop Liquid-Phase Resistance Number:**

$$\Pi_{\text{f}2} = \sum_{i=1}^{N} \left( \frac{1}{2} \left( \frac{fl}{dh} + K \right) \left( \frac{a_i}{a_l} \right)^2 \right)$$

(67)

**Loop Two-Phase Resistance Number:**

$$\Pi_{\text{f}2} = \sum_{i=1}^{N} \left( \frac{1}{2} \left( \frac{fl}{dh} + K \right) \left( \frac{a_i}{a_l} \right)^2 \right)$$

(68)

**Loop Energy Ratio:**

$$\Pi_e = \frac{\dot{q}_{\text{co}}}{\dot{m}_o (e_M - e_l)}$$

(69)

**Steam Generator Heat Transport Number:**

$$\Pi_{\text{SG}} = \frac{\dot{q}_{\text{SG}, o}}{\dot{m}_o (e_M - e_l)}$$

(70)

**Heat Loss Number:**

$$\Pi_{\text{loss}} = \frac{\dot{q}_{\text{loss}, o}}{\dot{m}_o (e_M - e_l)}$$

(71)

**Density Ratio:**

$$\Pi_{\rho} = \left( \frac{\rho_l}{\rho_{\text{TP}}} \right)$$

(72)
Table 7. Steady state solution for two-phase natural circulation flow rate

**Governing Equation:**

\[ \Pi_{w,\rho} = \Pi_{f_{1}} + \Pi_{f_{2}} \Pi_{\rho} \]  

(73)

**Liquid Velocity at Core Entrance:**

\[ u_{co}^3 + \phi_{a}u_{co}^2 + \phi_{u_{co}} - \phi_{c} = 0 \]  

(74)

where:

\[ \phi_{a} = \frac{\dot{q}_{co} \Delta \rho}{\rho_f a_c} \left( \frac{\Pi_{f_{1}} + \Pi_{f_{2}} \rho_g h_{fg} - \Delta \rho \Pi_{f_{2}} h_{sub}}{\Pi_{f_{1}} \rho_{fg}^2 h_{fg}^2 - (\Pi_{f_{1}} + \Pi_{f_{2}}) \rho_{fg} h_{fg} \Delta \rho + \Pi_{f_{2}} (\Delta \rho)^2 h_{sub}^2} \right) \]  

(75)

\[ \phi_{b} = \frac{\Delta \rho}{\rho_f a_c} \left( \frac{\Pi_{f_{1}} \dot{q}_{co}^2 + L_{th} \rho_f a_c h_{sub} \rho_{fg} h_{fg}}{\Pi_{f_{1}} \rho_{fg}^2 h_{fg}^2 - (\Pi_{f_{1}} + \Pi_{f_{2}}) \rho_{fg} h_{fg} \Delta \rho + \Pi_{f_{2}} (\Delta \rho)^2 h_{sub}^2} \right) \]  

(76)

\[ \phi_{c} = \frac{1}{\rho_f a_c} \left( \frac{L_{th} \dot{q}_{co} \Delta \rho \rho_{fg} h_{fg}}{\Pi_{f_{1}} \rho_{fg}^2 h_{fg}^2 - (\Pi_{f_{1}} + \Pi_{f_{2}}) \rho_{fg} h_{fg} \Delta \rho + \Pi_{f_{2}} (\Delta \rho)^2 h_{sub}^2} \right) \]  

(77)

where:

\[ \begin{aligned} 
\Delta \rho &= (\rho_f - \rho_g) \\
\psi_{a} &= \left( \frac{\Delta \rho}{\rho_f} \right) \left( \frac{\Pi_{f_{1}} + \Pi_{f_{2}}}{\Pi_{f}} \right) \\
\psi_{b} &= \left( \frac{\Delta \rho}{\rho_f} \right)^2 \left( \frac{\Pi_{f_{2}}}{\Pi_{f}} \right) \\
\psi_{c} &= \left( \frac{\Delta \rho}{\rho_f} \right) \left( \frac{g L_{th} \rho_g^2 h_{fg}^2 a_c^2}{\dot{q}_{co}} \right) \left( \frac{1}{\Pi_{f}} \right) 
\end{aligned} \]  

(78)

(Saturated Conditions at Inlet):

\[ \Pi_{z} + \psi_{a} \Pi_{z} + \psi_{b} \Pi_{z} - \psi_{c} = 0 \]  

(79)

where:

\[ \Pi_{z} = \frac{1}{N_{Zuber}} = \frac{\rho_g \dot{m}_{co} h_{fg}}{\rho_f \dot{q}_{co}} \]  

(80)

\[ \psi_{a} = \left( \frac{\Delta \rho}{\rho_f} \right) \left( \frac{\Pi_{f_{1}} + \Pi_{f_{2}}}{\Pi_{f}} \right) \]  

\[ \psi_{b} = \left( \frac{\Delta \rho}{\rho_f} \right)^2 \left( \frac{\Pi_{f_{2}}}{\Pi_{f}} \right) \]  

\[ \psi_{c} = \left( \frac{\Delta \rho}{\rho_f} \right) \left( \frac{g L_{th} \rho_g^2 h_{fg}^2 a_c^2}{\dot{q}_{co}} \right) \left( \frac{1}{\Pi_{f}} \right) \]  

(81)

(82)
5.1.1. Scale Ratios for Saturated Two-Phase Natural Circulation

To simulate the same fluid velocity behavior in the model as in the full-scale prototype, the coefficients of equation (78) must be scaled properly. It is noted that the form of equation (78) is a special class of catastrophe function known as a constraint catastrophe. A general method of scaling single-state variable catastrophe functions was developed by Reyes [5]. That is, it is required that the following condition be satisfied:

$$\beta^3 \left( \Pi_{zp}^1 + \psi_{zp} \Pi_{zp}^2 + \psi_{zp} \Pi_{zp} - \psi_{zp} \right) = \Pi_{zm}^1 + \psi_{zm} \Pi_{zm}^2 + \psi_{zm} \Pi_{zm} - \psi_{zm} \quad (83)$$

This condition is satisfied by scaling each of the coefficients as follows:

$$\beta = \left( \Pi_z \right)_R \quad (84)$$
$$\beta = \left( \psi_a \right)_R \quad (85)$$
$$\beta^2 = \left( \psi_b \right)_R \quad (86)$$
$$\beta^3 = \left( \psi_c \right)_R \quad (87)$$

Evaluating each of the ratios results in the scale ratios presented in Table 8.

<table>
<thead>
<tr>
<th>Table 8. Scale ratios for a saturated two-phase natural circulation loop</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Scale Ratio:</strong> ( \tau_{loop,R} = \frac{l_R}{(L_{sh})_{R}^{1/2}} )</td>
</tr>
<tr>
<td><strong>Fluid Velocity Scale Ratio:</strong> ( u_R = \left( L_{sh} \right)_R^{1/2} )</td>
</tr>
<tr>
<td><strong>Power Scale Ratio:</strong> ( \left( \dot{q}<em>{co} \right)<em>R = a</em>{c,R} \left( L</em>{sh} \right)<em>R^{1/2} \left( \frac{\rho_f \rho</em>{fg}}{\Delta \rho} \right) )</td>
</tr>
<tr>
<td><strong>Assuming:</strong> ( \frac{a}{a} ) ( \left( a_{c,R} \right) = 1 )</td>
</tr>
<tr>
<td><strong>Steam Generator Power Scale Ratio:</strong> ( \left( \frac{\dot{q}<em>{SG,co}}{\dot{q}</em>{co,R}} \right) = 1 )</td>
</tr>
</tbody>
</table>
5.2. Bottom-up scaling analysis for two-phase natural circulation flow

Several important local phenomena need to be scaled to evaluate the scale ratios given in Table 8. The two-phase friction and form loss coefficients are needed to evaluate equation (94). Heat transfer models for the steam generator and interconnecting piping heat loss are needed to evaluate equations (92) and (93). Core heat transfer may be of interest for some studies. Local transport models are provided as part of the lectures in this series given by Saha, et al. (2004).

Transitions in two-phase flow patterns may impact integral system behavior. Therefore an assessment must be made for each component in the system to determine which two-phase flow patterns may be delayed or entirely missed in a reduced scale test facility. In their paper, Schwartzbeck and Kocamustafaogullari [6] catalogued the applicable flow pattern transition criteria for horizontal and vertical flows. These have been summarized in Table 9.

<table>
<thead>
<tr>
<th>Horizontal Pipe Flow</th>
<th>Vertical Pipe Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stratified-Smooth to Stratified-Wavy:</strong></td>
<td><strong>Bubbly to Slug Flow:</strong></td>
</tr>
<tr>
<td>( (l_R)^{\alpha} ) = ( \alpha_R \left(1 - \frac{\alpha}{1 - x}\right)^{\alpha} \left(\frac{\Delta \rho^3 \nu}{\rho_f^2 \rho_g}\right)^{\frac{\nu}{2}} )</td>
<td>( \alpha_R = \left(\frac{\rho_f}{\Delta \rho}\right)^{\frac{1}{4}} )</td>
</tr>
<tr>
<td><strong>Stratified to Intermittent or Annular-Dispersed Liquid:</strong></td>
<td><strong>Slug/Churn to Annular Flow:</strong></td>
</tr>
<tr>
<td>( u_R = D_k^{\frac{1}{2}} \left(\frac{\Delta \rho}{\rho_f}\right)^{\frac{\nu}{2}} )</td>
<td>( u_R = \left(\frac{\sigma \Delta \rho}{\rho_g^2}\right)^{\frac{1}{4}} )</td>
</tr>
<tr>
<td><strong>Intermittent or Dispersed Bubbly to Annular-Dispersed:</strong></td>
<td></td>
</tr>
<tr>
<td>( \alpha_R = \left(\frac{\rho_f}{\Delta \rho}\right)^{\frac{1}{2}} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 9. Scale ratios for a saturated two-phase flow pattern transitions
REFERENCES


NOMENCLATURE

Acronyms

ADS – Automatic Depressurization System
AP600 – Advanced Passive 600 MW(e)
H2TS – Hierarchical Two-Tiered Scaling
MASLWR – Multi-Application Small Light Water Reactor
PIRT – Phenomena Identification and Ranking Table
PCT – Peak Cladding Temperature
SBLOCA- Small Break Loss-of-Coolant-Accident

Symbols

a – flow area
d_h – hydraulic diameter
e – specific internal energy
f – friction factor
g – gravity constant
h – enthalpy
j – flux term
K- Loss Coefficient
L_m – Distance between heat source and heat sink thermal centers
\dot{q} - power or heat transport rate
Q – Volumetric flow rate
S – Source term
T – Temperature
u – velocity
V – Volume
x – vapor quality

Greek Symbols

Π - Characteristic Time Ratio (Dimensionless Parameter)
ρ - density
τ- residence time constant
\psi_k = ρ, ρu or ρε (mass, momentum or energy per unit volume)
ACKNOWLEDGEMENT

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APPENDIX A
DESCRIPTION OF THE MASLWR DESIGN

The Multi-Application Small Light Water Reactor (MASLWR) is a 150 MW(t) modular nuclear reactor that uses natural circulation for primary loop cooling. Figure A.1 shows a single power generation module for the MASLWR design. MASLWR implements an integrated reactor vessel with an internal helical coil steam generator. The reactor vessel is enclosed in a high-pressure steel containment vessel that is partially filled with water to serve as a suppression pool. The containment vessel in turn resides in a large exterior cooling pool that acts as the ultimate heat sink.

![MASLWR exterior cooling pool and turbine-generator set](image)

**FIG. A.1.** Schematic of the MASLWR exterior cooling pool and turbine-generator set.

### A.1 MASLWR primary loop design

Because MASLWR uses natural circulation for primary loop flow, reactor coolant pumps are not needed. In this regard, its primary flow loop is quite simple as shown in Figure A.2. The long vertical tube in the center of the reactor vessel is called the riser and functions like a chimney to enhance the driving head of the natural circulation flow. Starting from the bottom of the riser, fluid enters the core which is located in a shroud connected to the riser entrance. Upon traveling through the core, the fluid is heated and thus ascends through the riser due to its buoyancy. Hot fluid in the surrounding annulus, outside the riser is cooled by convective heat transfer to a helical coil steam generator. The fluid inside the tubes is at a lower pressure, hence boiling occurs inside the tubes to generate superheated steam. The steam produced within the tube side of this coil travels on to the turbine generator set where it is used to produce electrical power. The cooled primary fluid in the annulus is negatively buoyant and descends to the bottom of the vessel and the inlet of the core thereby completing its loop.
FIG. A.2. Schematic of the MASLWR reactor cooling system and containment.
Table A.1 lists the steady-state operating conditions for MASLWR. The design provides a 53°C temperature rise from the core inlet to the core outlet. In addition, it is designed to provide superheated steam at the helical coil outlet to eliminate the need for separators and driers. The secondary side pressure was selected so that off-the-shelf low pressure steam turbines could be implemented.

Table A.1 MASLWR steady-state operating conditions

<table>
<thead>
<tr>
<th>Primary Side</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor Power (MW)</td>
<td>150.00</td>
</tr>
<tr>
<td>Primary Pressure (MPa)</td>
<td>7.60</td>
</tr>
<tr>
<td>Primary Mass Flow Rate (kg/s)</td>
<td>597.00</td>
</tr>
<tr>
<td>Reactor Inlet Temperature (K)</td>
<td>491.80</td>
</tr>
<tr>
<td>Reactor Outlet Temperature (K)</td>
<td>544.30</td>
</tr>
<tr>
<td>Saturation Temperature (K)</td>
<td>565.00</td>
</tr>
<tr>
<td>Reactor Outlet Void Fraction</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Secondary Side</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Steam Pressure (MPa)</td>
<td>1.50</td>
</tr>
<tr>
<td>Steam Outlet Quality</td>
<td>1.00</td>
</tr>
<tr>
<td>Steam Temperature (K)</td>
<td>481.40</td>
</tr>
<tr>
<td>Saturation Temperature (K)</td>
<td>471.60</td>
</tr>
<tr>
<td>Feedwater Temperature (K)</td>
<td>310.00</td>
</tr>
<tr>
<td>Feedwater Flowrate (kg/s)</td>
<td>56.10</td>
</tr>
</tbody>
</table>

A.2 MASLWR passive safety system SBLOCA operations

This section briefly describes the evolution of a Small Break Loss-of-Coolant Accident in MASLWR. Because MASLWR is an integrated reactor system, there are very few plausible primary break scenarios. In the event of a small break, the MASLWR passive safety systems would respond to the accident. The passive safety systems consist of the following components:

- Two, independent, small diameter, steam vent valves (SVV)
- Two, independent, small diameter, Automatic Depressurization System (ADS) valves
- Two, independent, small diameter, Sump Recirculation Valves (SV)
- A high-pressure containment vessel with an internal pressure suppression pool, and
- An external cooling pool that serves as the ultimate heat sink for the high-pressure containment and reactor decay heat.

Let us postulate the inadvertent opening of an ADS valve. Figure A.3 provides a schematic of the postulated pressure trend for illustration purposes. As shown in the figure, the transient begins with a relatively short blowdown period that consists of a subcooled blowdown into the suppression pool within the stainless steel containment. The suppression pool consists of the annular space bounded by the exterior surface of the reactor vessel and inner surface of the containment walls. It is partially filled with water. This water region is integral to the long term removal of decay heat following system depressurization (blowdown). The rapid rise in containment pressure results in a Safety Injection signal which automatically opens the steam vent valves, the ADS valves and the sump recirculation valves. The ADS blowdown period serves to further reduce the reactor vessel pressure well below the saturation pressure corresponding to the hot leg temperature. A major advantage to the small volume, high-pressure containment is that the blowdown quickly results in equalizing the containment and reactor vessel pressures, effectively terminating the blowdown. As the pressures become equalized, a natural circulation flow path is established in which the sump fluid enters through the sump recirculation valves, descends through the downcomer region in the lower portion of the reactor vessel, rises through the core and riser, and finally exits through the upper vent valve into the containment as a saturated vapor. From the vent valve, the fluid returns to the sump via condensation on the containment walls and/or water surface, thus completing its circuit.
Finally, to ensure the long term removal of heat from the containment and thus to moderate the containment pressure, the containment itself is submerged in an outer pool of water which is open to the atmosphere. Within this pool, thermal energy is transferred from the outer containment wall to the atmosphere via natural convection and circulation of the water. The pool is formed in the space between the outer containment wall and the inner wall of the concrete structure in which the containment is placed.

In conclusion, a SBLOCA in MASLWR can be divided into three phase, or modes of operation, a blowdown phase, an ADS operation phase and a long term cooling phase. A more detailed description of the MASLWR design will be made available as part of the documentation for the IAEA Small to Medium Reactor (SMR) program.