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# Spin-ponderomotive force in plasmas

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# Spin-ponderomotive force in plasmas

## Spin-polarization & Spin-induced current

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**Fundamental interactions:**

- Gravitational (Space-time curvature without using the concept of force)
- Electromagnetic [Lorentz force: $\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$]
- Strong & Weak (Devoid of classical limit, in principle)

In modern language, the term EM forces is mainly used in a general sense.

# What is ponderomotive force (PF)?

## Why do we use PF?

- PF of EM waves is a key concept in plasma physics and plays a central role in intense laser plasma interactions.
- PF arises whenever a nonuniform oscillating electric field is present in a dielectric and can be seen as a slow time scale effect or the average effect due to some nonuniformity of the hf oscillations of the electric field. [$\mathbf{F}_p \sim -\left(q^2/4m\omega^2\right)\nabla E^2$]
- Unlike the Lorentz force, the PFs are inexact, nonlinear and rather cumbersome.
- So, what are the compelling reasons for introducing PFs instead of Lorentz force (LF)?

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- So, what are the compelling reasons for introducing PFs instead of Lorentz force (LF)?
- The answer is: To facilitate the solution of dynamical problems. The exact behavior of a dynamical system with the simple LF is very complex. By averaging over the period of oscillations, we obtain the more complex PFs, but the dynamics are simplified.

# Motivation

- Recent experimental progress in nano-scale plasmas [APL 91, 061108 (2007)], ultracold plasmas PRL 85, 4466 (2000), spintronics [Science 294, 1488 (2001)], and plasmonics [Sci. Am. 296, 56 (2007)].
- There is a possibility of observing quantum plasma effects through the electron spin in regimes otherwise thought to be classical [PRL 98, 025001 (2007)].
- In classical plasmas, the density fluctuation induced by PF of EM wave field leads to an electrostatic wake field as used in advanced particle accelerator schemes [Bingham, Nature 445, 721 (2007)].
- In other regimes, the back-reaction on the EM wave due to the density fluctuations leads to phenomena such as soliton formation, self-focusing or wave collapse. Such radiation pressure-like effects are widely used in high-intensity laser experiments and generalizations to include certain quantum plasma effects [Shukla et al, Phys.-Usp. 53, 51 (2010)].

- Microscopic approach [Tskhakaya, JPP **25**, 233 (1981)]
- Energy-momentum-stress tensor approach [Washimi & Karpman, Zhur. Eksp. Teor. Fiz. **71**, 1010 (1976)]

Equation of motion:

$$\frac{d\mathbf{v}}{dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}.\nabla\right)\mathbf{v} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right) \tag{1}$$

Assume $\mathbf{E} = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})\, E(z,t)\exp(ikz - i\omega t) +$ c.c., along $\mathbf{B} = B_0\hat{\mathbf{z}}$, $|(1/f)\,\partial f/\partial z| \ll k, |(1/f)\,\partial f/\partial t| \ll \omega$. Eq. (1) gives

$$\left(\frac{\partial}{\partial t} - i\omega\right)\mathbf{v} = \frac{q}{m}\mathbf{E} - \omega_c\mathbf{v}\times\hat{\mathbf{z}}, \tag{2}$$

Substituting the lowest order result [Eq. (3)] into the correction term involving the slow-time derivative, in Eq. (2)

$$v_{\pm} \equiv v_x \pm i v_y = \frac{iqE_{\pm}}{m\left(\omega \pm \omega_c\right)} \tag{3}$$

we get

$$v_{\pm} = \frac{q}{m} \frac{1}{\left(\omega \pm \omega_c\right)} \left[ iE_{\pm} + \frac{1}{\omega \pm \omega_c} \frac{\partial E_{\pm}}{\partial t} \right]. \tag{4}$$

Similarly, using Faraday's law: $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$, we have

$$B_{\pm} = \pm \frac{ik}{\omega} E_{\pm} \pm \frac{1}{\omega} \frac{\partial E_{\pm}}{\partial z} \pm \frac{k}{\omega^2} \frac{\partial E_{\pm}}{\partial t}. \tag{5}$$

$$F_C \equiv \langle \frac{q}{m}\mathbf{v} \times \mathbf{B} \rangle_z = \frac{iq}{2m}\left(v_+ B_+^* - v_+^* B_+\right) \text{ for RCP,}$$
$$= \frac{iq}{2m}\left(v_-^* B_- - v_- B_-^*\right) \text{ for LCP.} \quad (6)$$

Substitution of Eqs. (4) and (5) into Eq. (6) yields [Karpman & Washimi, JPP (1977)]

$$F_C = -\frac{e^2}{2m^2\omega\left(\omega \pm \omega_c\right)}\left[\frac{\partial}{\partial z} \pm \frac{k\omega_c}{\omega\left(\omega \pm \omega_c\right)}\frac{\partial}{\partial t}\right]|E|^2. \quad (7)$$

The spin-evolution equation:

$$\frac{d\mathbf{S}}{dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}.\nabla\right)\mathbf{S} = -\frac{2\mu}{\hbar}\left(\mathbf{B}\times\mathbf{S}\right). \tag{8}$$

Linearized spin evolution equation:

$$\left(\partial_t - i\omega\right)\mathbf{S} = -\left(2\mu/\hbar\right)\left(B_0\hat{\mathbf{z}}\times\mathbf{S} + S_0\mathbf{B}\times\hat{\mathbf{z}}\right), \tag{9}$$

where $|S_0| \equiv \hbar/2$, $\mu = -g\mu_B/2, \mu_B = e\hbar/2m$. As before, in the lowest order,

$$S_\pm \equiv S_x \pm iS_y = \mp\frac{2\mu S_0}{\hbar\left(\omega\pm\omega_g\right)}B_\pm, \tag{10}$$

where $\omega_g = (g/2)\omega_c$. Then finally, the expression for the perturbed spin becomes

$$S_\pm = \frac{2\mu S_0}{\hbar\left(\omega\pm\omega_g\right)}\left[\mp B_\pm \pm \frac{i}{\left(\omega\pm\omega_g\right)}\frac{\partial B_\pm}{\partial t}\right], \tag{11}$$

$$
\begin{aligned}
F_S \equiv \frac{2\mu}{m\hbar}\langle \mathbf{S}.\nabla \mathbf{B}\rangle_z &= \frac{2\mu}{m\hbar}\langle S^a \nabla B_a\rangle_z \\
&= \frac{2\mu}{m\hbar}\left( S_+ \frac{\partial B_+^*}{\partial z} + S_+^* \frac{\partial B_+}{\partial z}\right) \quad \text{for RCP}, \\
&= \frac{2\mu}{m\hbar}\left( S_- \frac{\partial B_-^*}{\partial z} + S_-^* \frac{\partial B_-}{\partial z}\right) \quad \text{for LCP}. \qquad (12)
\end{aligned}
$$

Substitution of Eq. (11) into Eq. (12) yields

$$
F_S = \mp \frac{4\mu^2}{m\hbar^2} \frac{S_0}{(\omega \pm \omega_g)} \left[ \frac{\partial}{\partial z} - \frac{k}{(\omega \pm \omega_g)} \frac{\partial}{\partial t} \right] |B|^2. \qquad (13)
$$

# Classical and spin-induced PF: Few comments

$$F_C = -\frac{e^2}{2m^2\omega(\omega \pm \omega_c)}\left[\frac{\partial}{\partial z} \pm \frac{k\omega_c}{\omega(\omega \pm \omega_c)}\frac{\partial}{\partial t}\right]|E|^2, \tag{14}$$

$$F_S = \mp\frac{4\mu^2}{m\hbar^2}\frac{S_0}{(\omega \pm \omega_g)}\left[\frac{\partial}{\partial z} - \frac{k}{(\omega \pm \omega_g)}\frac{\partial}{\partial t}\right]|B|^2. \tag{15}$$

- Eq. (15) is applicable to arbitrary EM wave propagation parallel to $\mathbf{B}_0$.
- The overall structure of the SPF is similar to its classical counterpart (14).
- The frequency resonances occur at $\omega_g = (g/2)\omega_c$.
- The dependence on $S_0$ means that spin-up and spin-down populations drift in opposite directions relative to $\mathbf{B}_0$.
- For $\omega << \omega_c$, typically the time-derivative part is negligible, whereas it is crucial for the classical contribution.
- For $\omega >> \omega_c$, $|F_S| / |F_C| \equiv \frac{\hbar k}{mv_p}\left(1 + \frac{v_g}{v_p}\right) \sim \frac{\hbar k}{mc}$ for $v_g, v_p \sim c$.

# Fluid model

From Spin MHD model [Brodin & Marklund, NJP **9**, 277 (2007)]

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0, \tag{16}$$

$$m\left(\frac{\partial}{\partial t} + \mathbf{v}_\alpha \cdot \nabla\right)\mathbf{v}_\alpha = q_\alpha\left(\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}\right) - \frac{\nabla P_\alpha}{n_\alpha} + \frac{2\mu}{\hbar} S_\alpha^a \nabla B_a, \tag{17}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_\alpha \cdot \nabla\right)\mathbf{S}_\alpha = -\frac{2\mu}{\hbar}\mathbf{B} \times \mathbf{S}_\alpha, \tag{18}$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0}\sum_\alpha q_\alpha n_\alpha, \tag{19}$$

where $\mu \equiv -g\mu_B/2$, $\mu_B \equiv e\hbar/2m$, $g \approx 2.0023192$.

# Equations for low-frequency response

Define $N_{1,2} = n_u \pm n_d$ and $V_{1,2} = (v_u \pm v_d)/2$ and $n_{0u} = n_{0d} \equiv n_0/2$. Neglecting the thermal effects $v_{\rm th}^2 \ll v_g^2$,

$$\partial_t N_{1,2} = -n_0 \partial_z V_{1,2}, \tag{20}$$

$$\frac{\partial V_1}{\partial t} = \frac{q}{m} E_l - \frac{q^2}{2m^2\omega(\omega \pm \Omega)} \left[ \frac{\partial |E|^2}{\partial z} \pm \frac{k\Omega}{\omega(\omega \pm \Omega)} \frac{\partial |E|^2}{\partial t} \right] \tag{21}$$

$$\frac{\partial V_2}{\partial t} = \mp \frac{4\mu^2 k^2 S_0}{m\hbar^2\omega^2(\omega \pm \omega_g)} \left[ \frac{\partial |E|^2}{\partial z} - \frac{k}{(\omega \pm \omega_g)} \frac{\partial |E|^2}{\partial t} \right], \tag{22}$$

$$\frac{\partial E_l}{\partial z} = \frac{q}{\varepsilon_0}(N_1 - n_0). \tag{23}$$

# Wave equations

In the comoving frame $\xi = z - v_g t$,

$$v_g^2 \frac{\partial^2 N_1}{\partial \xi^2} + \omega_p^2 N_1 = \frac{\varepsilon_0 \omega_p^2}{2m\omega(\omega \pm \Omega)} \left[1 \mp \frac{k v_g \Omega}{\omega(\omega \pm \Omega)}\right] \frac{\partial^2 |E|^2}{\partial \xi^2}, \tag{24}$$

$$v_g^2 \frac{\partial^2 N_2}{\partial \xi^2} = \pm \frac{\varepsilon_0 \omega_p^2 k^2 S_0}{2m^2\omega^2(\omega \pm \omega_g)} \left[1 + \frac{k v_g}{(\omega \pm \omega_g)}\right] \frac{\partial^2 |E|^2}{\partial \xi^2}. \tag{25}$$

Some Notes:

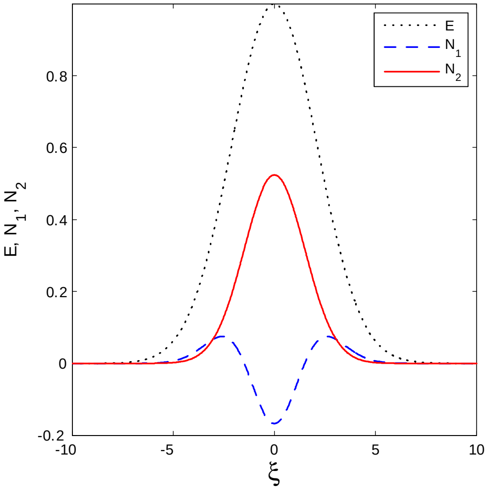
- $N_2 \propto |E|^2$, whereas $N_1$ is non-locally related to $|E|^2$ due to the possible excitation of a plasma oscillation wake-field with a characteristic wavelength $\lambda_p \equiv v_g/\omega_p$.
- Ion dynamics is neglected with pulse lengths fulfilling $L_p \lesssim c/\omega_{pi}$. That is, we consider an EM pulse interacting with a plasma metal where $\omega_p/2\pi \simeq 10^{16}\,\mathrm{s}^{-1}$.
- We consider $\omega_p \lesssim kc$ and use the estimate $\partial^2|E|^2/\partial\xi^2 \sim |E|^2/L_p^2$, where $L_p \gg k^{-1}$ is the length of the hf pulse.

In the limit $\omega_c \to 0$, (when e.g., the laser frequency >>the cyclotron frequency) $[J_{\mathrm{M}\pm} = (kg\mu_B/2\hbar)\sum_\alpha n_\alpha \mathbf{S}_{\alpha\pm}]$

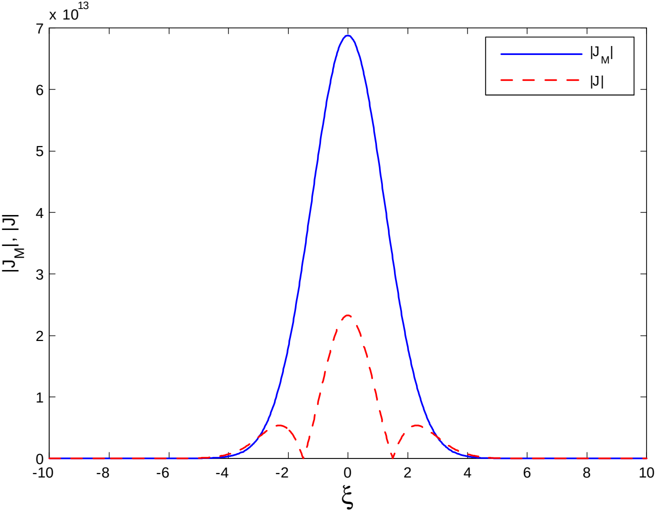
$|N_2/N_1| \sim \left(\hbar\omega_p/mc^2\right)\left(kL_p\right)^2\left(c/v_p\right)^2\left(\omega\omega_p/k^2v_g^2\right)\left(1+v_g/v_p\right)$,
$\Gamma \equiv |J_{\mathrm{M}\pm}/J_\pm| \approx (\hbar\omega/mv_p^2)|N_2/N_1|$.
For $v_g, v_p \sim c$, we have $|N_2/N_1| \sim \left(\hbar\omega_p/mc^2\right)\left(kL_p\right)^2$,
$\Gamma \sim \left(\hbar\omega_p/mc^2\right)^2\left(kL_p\right)^2$.

- Spin polarization: UV-Laser of wavelength, $\lambda = 80\,\mathrm{nm}$ and pulse length, $L_p = 15\,\mu\mathrm{m}$ gives moderate spin-polarization ($N_2/N_1 \approx 3$ at the center, **Fig. 1**). A longer pulse length or a shorter wavelength will give a higher degree of spin-polarization. For an intense pulse resulting in $N_2 \gg N_1$ we obtain a strongly spin-polarized plasma.
- Back-reaction on the EM-pulse: XFEL with $\lambda = 1\,\mathrm{nm}$, pulse length $L_p = 30\,\mu\mathrm{m}$, and a metallic plasma density with $\omega_p/2\pi = 10^{16}\ \mathrm{s}^{-1}$ gives $\Gamma \approx 3$ (**Fig. 2**). These parameters are relevant for the XFEL at DESY (http://xfel.desy.de). For $\lambda = 0.1\,\mathrm{nm}$, $\Gamma \approx 200$.

# Graphical representation (Fig. 1)



# Graphical representation (Fig. 2)



# Validity of the fluid model: Some remarks

- Division of electrons in spin-up and spin-down populations. This is suggested by the Stern-Gerlach experiment where particles passing through (an inhomogeneous magnetic field) the Stern-Gerlach apparatus are deflected either up or down by a specific amount, i.e., spin angular momentum is quantized and can take only discrete values. This division of electrons is common in semiconductors [Science **294**, 1488 (2001), *ibid* **312**, 1883 (2006)].
- The theoretical basis for the division is two-fold:

1. In the unperturbed plasma (before the pulse arrival), there are two discrete spin states (up & down) relative to the magnetic field with $n_{0u} \approx n_{0d} \equiv n_0/2$. In thermodynamic equilibrium, the latter is valid whenever $\mu_B B_0 << k_B T_e$.
2. Spin coupling of the electrons to the wave magnetic field is of opposite signs depending on the unperturbed spin states.

# Some remarks (Contd.)

- The discrete initial states prevent the two spins from mixing. However, spin-flips due to e.g., collisions can, in principle, mix the up and down states too fast. For significant spin flips to occur, we must have $\nu_{sp} > v_g/L_p$. In the specific example, $v_g/L_p \gtrsim 10^{14}$Hz, and hence neglected the spin-flips [*Quantum Statistics of Nonideal plasmas*, Springer, 2005], and the identity of the spin states can be retained for a sufficient time.
- Since the external magnetic field is considered constant and thus provides no magnetic dipole force, it is not responsible for the spin states, rather the separation is due to the inhomogeneous wave magnetic field. So, our results will be valid still for $B_0 = 0$. The importance of the external magnetic field is that it affects the transverse wave modes and determines the spin-precession frequency.

# Some remarks (Contd.)

- We have omitted quantum effects like particle dispersion (Bohm potential), Fermi pressure, and considered only the spin effects. For the specific example considered here, these effects are shown to be negligible as $V_F$ is compared to $v_g$ ($\sim c$). Furthermore, the particle dispersion only affects length-scales that of the order of the Compton length, which is considerably shorter than the pulse length considered in our example.
- Whether nonrelativistic treatment is valid. Due to weak field expansion of the wave amplitude, the electrons are not trapped in the potential well of the electrostatic oscillations. Hence the velocity of the electrons is much smaller than $v_p$ or $v_g$ of the electrostatic oscillations that are induced, and is essentially limited by the electron quiver velocity which is much smaller than $c$. Moreover, the relativistic nonlinearities are cubic, and our main focus in this work is with a quadratic one associated with the PF.

# Concluding Remarks

- Generalization of the classical ponderomotive force to include the electron spin effect in a magnetized plasma.
- SPF is applicable to arbitrary EM waves propagating along an external magnetic field. In the limit of $\omega >> \omega_c$, the spin contribution to the ponderomotive force has opposite directions for RCP and LCP waves. Thus, an experiment on spin-polarization along these lines must consider CP rather than linearly polarized light.
- SPF can induce a strong spin-polarization in a plasma even if the initial up and down states of electrons are equally populated. A longer pulse length ($>$ UV-laser) or shorter wavelength can give rise a higher degree of spin-polarization.
- In an unmagnetized plasma, the nonlinear back-reaction from the spin induced current can be even larger than the classical one when the EM pulse has sufficiently short-wavelength.

# Open Issues

- Generalizations to, e.g., arbitrary direction of propagation.
- Inclusion of relativistic effects may play a role for the full dynamical evolution.
- SPF can well be applied to nonlinear lf propagation of Whistler waves, Alfvén waves etc.
- Instead of producing a spin-polarized plasma, another use of PF may be 'Isotope Separation' [Weibel, PRL **44**, 377 (1980)].