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Parametric Instabilities in Plasmas

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Outline

A. TEST CHARGE POTENTIALS

B. PARAMETRIC INSTABILITIES IN PLASMAS

C. NONLINEAR ELECTROMAGNETIC WAVES

D. ULTRACOLD QUANTUM PLASMAS

TEST CHARGE POTENTIALS

Slow test charge in a collisional plasma

$$\phi = \frac{q_t}{4\pi\varepsilon_0 r} e^{-r/\lambda_D} + \frac{q_t}{4\pi\varepsilon_0 r} \frac{4\cos\theta}{\sqrt{2\pi}} \left(\frac{\lambda_D}{r}\right)^2 \frac{v_0}{v_t} \left(1 - \frac{\widetilde{\nu} r}{v_t}\right)$$

Stenflo, Yu & Shukla, Phys. Fluids **16**, 450 (1973)

Effects of external fields

$$\phi = \frac{q_t}{4\pi\varepsilon_0 r} \frac{q^2 E_0^2}{m^2 \omega_0^2 (\omega_p^2 - \omega_0^2)} \frac{3\cos^2\theta - 1}{2r^2}$$

Z. Physik B **20**, 105 (1975)

PARAMETRIC PROCESSES IN PLASMAS

EM dispersion relation:

$$\omega_0^2 = \omega_{pe}^2 + k_0^2 c^2$$

Matching conditions:

$$\omega_\pm = \omega \pm \omega_0$$

$$\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0$$

Coupled equations:

$$c^2 \mathbf{k}_\pm \times (\mathbf{k}_\pm \times \mathbf{E}_\pm) + (\omega_\pm^2 - \omega_{pe}^2)\mathbf{E}_\pm \approx \omega_{pe}^2 \frac{\delta n}{n_0} \mathbf{E}_{0\pm}$$

$$\varepsilon(\omega, \mathbf{k})\delta n = -\frac{k^2 \varepsilon_0}{m_e \omega_0^2} \chi_e (1 + \chi_i)(\mathbf{E}_{0+} \cdot \mathbf{E}_- + \mathbf{E}_{0-} \cdot \mathbf{E}_+)$$

where

$$\varepsilon(\omega, \mathbf{k}) = 1 + \chi_e(\omega, \mathbf{k}) + \chi_i(\omega, \mathbf{k})$$

PARAMETRIC PROCESSES IN PLASMAS (continued)

If $\omega_0 \gg \omega_{pe}$:

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} = \frac{k^2|\mathbf{k}_+ \times \mathbf{v}_0|^2}{k_+^2(k_+^2c^2 - \omega_+^2 + \omega_{pe}^2)} + \frac{k^2|\mathbf{k}_- \times \mathbf{v}_0|^2}{k_-^2(k_-^2c^2 - \omega_-^2 + \omega_{pe}^2)}$$

where

$$\mathbf{v}_0 = \frac{q_e\mathbf{E}_0}{m_e\omega_0}$$

If $kv_{ti} < \omega \ll kv_{te}$:

$$\chi_e \approx \frac{1}{k^2\lambda_D^2}(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{kv_{te}})$$

$$\chi_i \approx -\frac{\omega_{pi}^2}{\omega^2} + i\frac{\sqrt{\pi/2}}{k^2\lambda_D^2}\frac{T_e}{T_i}\frac{\omega}{kv_{ti}}\exp\left(-\frac{\omega^2}{2k^2v_{ti}^2}\right)$$

PARAMETRIC PROCESSES IN PLASMAS (continued)

If $\omega_0 \gg \omega_{ce}$:

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} = k^2 \sum_{+,-} \left[\frac{|\mathbf{k}_\pm \times \mathbf{v}_0|^2}{k_\pm^2 (k_\pm^2 c^2 - \omega_\pm^2 + \omega_{pe}^2 - i\nu_e \omega_{pe}^2/\omega_\pm)} - \frac{|\mathbf{k}_\pm \cdot \mathbf{v}_0|^2}{k_\pm^2 \omega_\pm^2 \varepsilon(\omega_\pm, \mathbf{k}_\pm)} \right]$$

where

$$\chi_e = \frac{\omega_{pe}^2}{k^2 v_{te}^2} \left\{ 1 - \sum_{n=-\infty}^{\infty} I_n(b) e^{-b} \left[\omega - \omega^* \left(1 - \frac{n\omega}{b\omega_{ce}} \right) \right] \int_{-\infty}^{\infty} \frac{F_z \, dv_z}{\omega - k_z v_z - n\omega_{ce}} \right\}$$

and

$$b = \frac{k_\perp^2 v_{te}^2}{\omega_{ce}^2}, \quad \omega^* = -\frac{k_y \kappa v_{te}^2}{\omega_{ce}^2}, \quad \kappa = -\frac{1}{n_0} \frac{\partial n_0}{\partial x}$$

Yu, Spatschek & Shukla, Z. Naturforsch. **29a**, 1736 (1974).

NONLINEAR ELECTROMAGNETIC WAVES

$$i\partial_t E + iv_g\partial_z E + \frac{v_g'}{2}\partial_z^2 E - \Delta E = 0$$

$$\partial_t^2 N - c_s^2\partial_z^2(N + N^2) - c_s^2\lambda_D^2\partial_z^4 N =$$

$$= \frac{1}{m_i}\partial_z \sum_\sigma \frac{q^2}{m\omega(\omega+\omega_c)}\left(\partial_z + \frac{k\omega_c}{\omega(\omega+\omega_c)}\partial_t\right)|E|^2$$

where the wave frequency

$$\omega^2 = k^2c^2 + \sum_\sigma \frac{\omega\omega_p^2/\gamma}{\omega+\omega_c/\gamma}$$

and nonlinear frequency shift

$$\Delta = \frac{v_g}{2kc^2}\sum_\sigma \frac{\omega\omega_p^2}{\omega+\omega_c}\left(N - \frac{kv_z\omega_c}{\omega(\omega+\omega_c)} - \frac{\omega}{c^2}\frac{q^2}{m^2}\frac{|E^2|}{(\omega+\omega_c)^3}\right)$$

Shukla & Stenflo, Phys. Rev. A **30**, 2110 (1984); Phys Fluids **28**, 1576 (1985)

NONLINEAR ELECTROMAGNETIC WAVES (continued)

$$(\partial_t^2 + \nu_h \partial_t + \omega_{pe}^2)\mathbf{E} - v_{te}^2 \nabla\nabla \cdot \mathbf{E} + c^2 \nabla \times \nabla \times \mathbf{E} = -\omega_{pe}^2 N \mathbf{E}$$

$$(\partial_t^2 + \nu_l \partial_t - c_s^2 \nabla^2) N = \frac{q_e^2}{m_e m_i \omega_0} \nabla^2 (|\mathbf{E}|^2 + \widetilde{T})$$

$$(\partial_t + \frac{1}{\tau} - \nabla \cdot H \cdot \nabla)\widetilde{T} = \frac{4\nu_e}{3} |\mathbf{E}|^2$$

Stenflo & Shukla, J. Plasma Phys. **64**, 353 (2000)

ULTRACOLD QUANTUM PLASMAS

$$(\partial_t^2 - c^2\nabla^2 + \omega_{pe}^2)\mathbf{E} + \omega_{pe}^2 \frac{\delta n}{n_0}\mathbf{E} = 0$$

$$\left(\partial_t^2 + \frac{\hbar^2}{4m_e m_i}\nabla^4 - \frac{m_e}{m_i}v_{te}^2\nabla^2\right)\delta n = \frac{n_0 e^2}{2m_e m_i \omega_0^2}\nabla^2|\mathbf{E}|^2$$

Shukla & Stenflo, J. Plasma Phys. **13**, 044505 (2006)

PARAMETRIC INSTABILITIES IN QUANTUM PLASMAS

$$\omega^2 - \frac{\hbar^2 k^4}{4m_e m_i} = \frac{\omega_{pe}^2 e^2 k^2}{2m_e m_i c^2}|\mathbf{A}_0|^2\left(\frac{1}{D_+} + \frac{1}{D_-}\right)$$

$$|\mathbf{A}_0|^2 = \frac{c^2}{\omega_0^2}|\mathbf{E}_0|^2, \qquad D_\pm = \omega_\pm^2 - k_\pm^2 c^2 - \omega_{pe}^2$$

Growth rate:

$$\gamma_B = \frac{\omega_{pe}}{2}\frac{e|\mathbf{A}_0|}{m_e c}\left(\frac{m_e}{m_i}\right)^{1/4}\left(\frac{m_e}{\hbar\omega_0}\right)^{1/2}, \qquad \text{if } D_- \approx 0$$

$$\omega \ll \Omega_B \equiv \frac{\hbar k^2}{2\sqrt{m_e m_i}}$$

$$\omega = \mathbf{k}\cdot\mathbf{v}_g \pm \left[\delta^2 - \frac{\delta\omega_{pe}^2 e^2 k^2 |\mathbf{A}_0|^2}{2\omega_0 \Omega_B^2 m_e m_i c^2}\right]^{1/2}$$

$$\mathbf{v}_g = \frac{c^2\mathbf{k}_0}{\omega_0}, \qquad \delta = \frac{k_0^2 c^2}{2\omega_0}$$

Thank you for listening!