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Theory and Simulation of Freak Waves Diversity of Padma`s Physics

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Diversity of Padma's Physics

International Advanced Workshop on the Frontiers of Plasma Physics
ICTP, Trieste, Italy, 5 – 9 July 2010

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Outline

A. WHAT ARE FREAK WAVES

B. NONLINEAR EFFECTS: MODULATIONAL INSTABILITY

C. KINETIC DESCRIPTION OF FREAK WAVES

The Great Wave
Katsushika Hokusai
(1832)

WHAT ARE FREAK WAVES

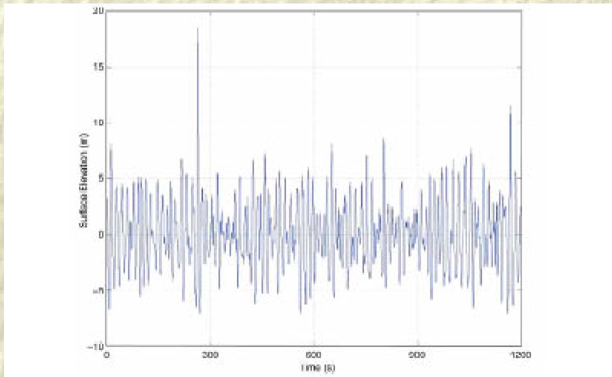
- ❏ In oceanography: waves that exceed 2 times the significant wave height

- ❏ Occur suddenly out of normal waves

- ❏ Known from oceanography: Hazard for offshore and oil industry

(a) Oil freighter Esso Languedoc, coast of Durban (1980)

(b) Norwegian tanker Wilstar, Agulhas current (1974)



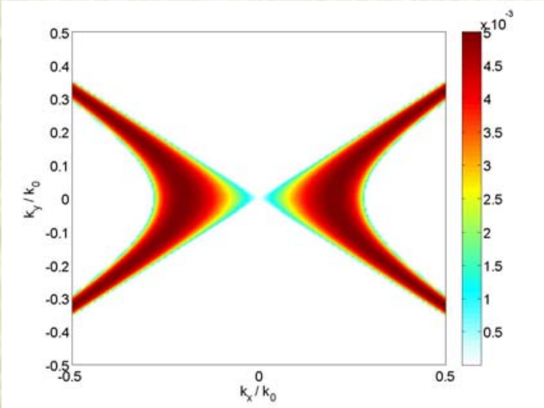
(c) Draupner Platform, the North Sea (New Year's Day 1995)

NONLINEAR SCHRÖDINGER EQUATION

$$i\left(\frac{\partial A}{\partial t}+\underbrace{\frac{\omega_0}{2k_0}}_{=v_{gr}}\frac{\partial A}{\partial x}\right)\underbrace{-\frac{\omega_0}{8k_0^2}}_{=D_x}\frac{\partial^2 A}{\partial x^2}+\underbrace{\frac{\omega_0}{4k_0^2}}_{=D_y}\frac{\partial^2 A}{\partial y^2}-\underbrace{\frac{\omega_0 k_0^2}{2}}_{=\xi}|A|^2A=0$$

Here $\eta=(1/2)A(\mathbf{r},t)\exp(-i\omega_0 t+ik_0x)+$ complex conjugate is the surface elevation.

- Modulationally unstable in x direction, stable in y direction! (Benjamin-Feir instability).



- Similar equations exist in plasma physics (e.g. nonlinear whistler and lower hybrid waves).

WIGNER TRANSFORM TO KINETIC MODEL

$$f(\mathbf{r},\mathbf{v},t) = \frac{1}{2(2\pi)^2}\int A^*(\mathbf{R}_+,t)A(\mathbf{R}_-,t)e^{i\boldsymbol{\lambda}\cdot(\mathbf{v}-v_{gr}\widehat{\mathbf{x}})}\,d^2\lambda, \qquad (1)$$

gives

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \frac{2i\xi}{(2\pi)^2}\int\int[I(\mathbf{R}_+,t) - I(\mathbf{R}_-,t)] \times\ f(\mathbf{r},\mathbf{v}',t)e^{i\boldsymbol{\lambda}\cdot(\mathbf{v}-\mathbf{v}')}d^2v'\,d^2\lambda = 0, \qquad (2)$$

where $I(\mathbf{r},t) = \int f(\mathbf{r},\mathbf{v},t)\,d^2v$ is the variance of the surface displacement (the wave intensity). Here $\mathbf{R}_\pm = \mathbf{r}\pm\bar{\bar{\mathbf{D}}}\cdot\boldsymbol{\lambda}$ and $\bar{\bar{\mathbf{D}}}\cdot\boldsymbol{\lambda} = D_x\lambda_x\widehat{\mathbf{x}} + D_y\lambda_y\widehat{\mathbf{y}}$

The Wigner equation is the quantum analogue to the Vlasov equation for classical particles! (Prof. Tsintsadze's talk). See also J. E. Moyal, Math. Proc. Cambridge Phil. Soc. **45**, 99 (1949); T. Takabayasi, Prog. Theor. Phys. **11**, 341 (1954).

FOURIER TRANSFORM IN VELOCITY SPACE (ALBER'S MODEL)

Fourier-transform in velocity space

$$\widehat{f}(\mathbf{r}, \boldsymbol{\eta}, t) = 2 \int f(\mathbf{r}, \mathbf{v}, t) e^{i\boldsymbol{\eta}\cdot\mathbf{v}}\, d^2 v, \tag{3}$$

which transforms kinetic equation into

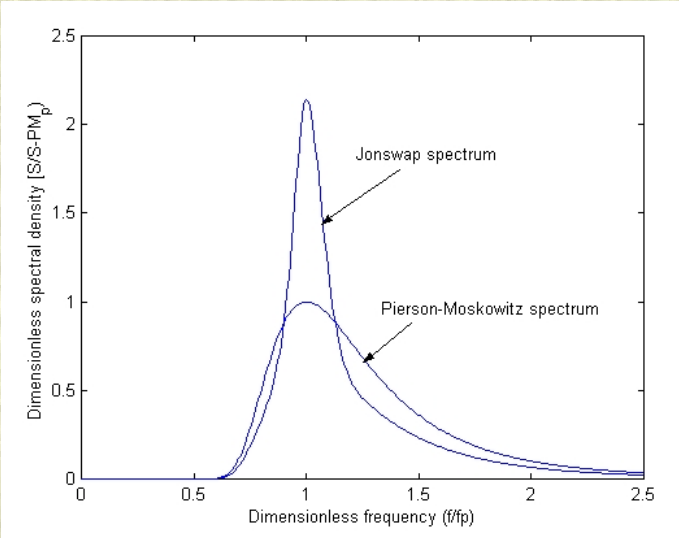
$$\frac{\partial \widehat{f}}{\partial t} - i\nabla_{\boldsymbol{\eta}} \cdot \nabla \widehat{f} + 2i\xi[I(\mathbf{r} + \bar{\bar{\mathbf{D}}} \cdot \boldsymbol{\eta}, t) - I(\mathbf{r} - \bar{\bar{\mathbf{D}}} \cdot \boldsymbol{\eta}, t)]\widehat{f}(\mathbf{r}, \boldsymbol{\eta}, t) = 0, \tag{4}$$

where $I = \widehat{f}(\mathbf{r}, \boldsymbol{\eta}, t)_{\boldsymbol{\eta}=\mathbf{0}}/2$. Also derived by Alber, Proc. R. Soc. Lond. A **363**, 252 (1978).

- f depends on x, y, v_x, v_y and t
- $\widehat{f}$ depends on x, y, η_x, η_y and t
- $\rightarrow$ Simulations in 4+1 dimensions!

JONSWAP SPECTRUM

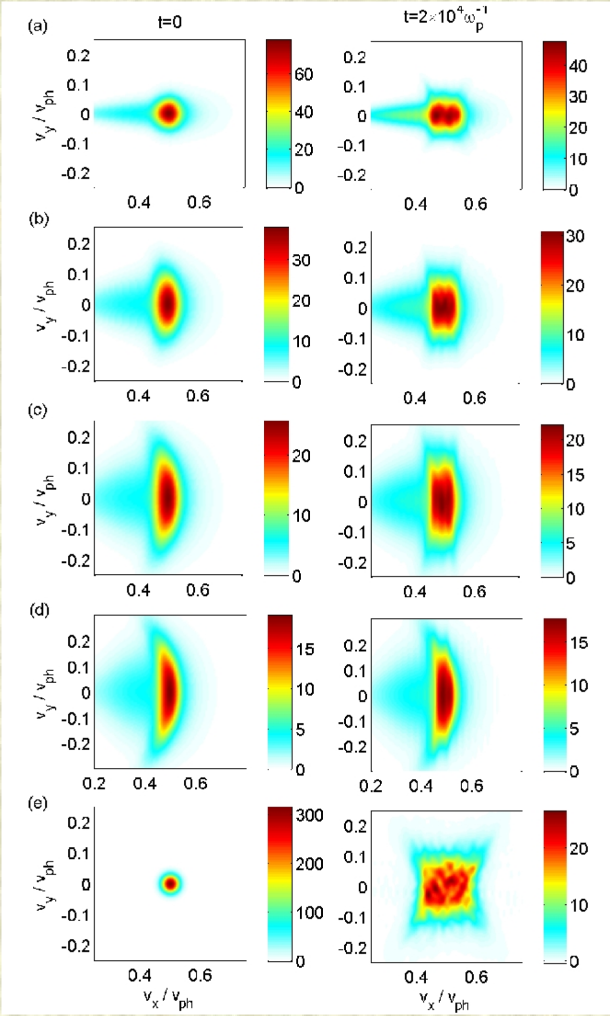
$$S(\omega) = \frac{\alpha_P g^2}{\omega^5} \exp\left(-\frac{5}{4}\frac{\omega_p^4}{\omega^4}\right) \gamma^{\exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]} G(\theta), \qquad (5)$$



Directional waves $G(\theta) = G_0[1+\cos(\theta)]^{N/2}/2^{N/2}, \qquad (6)$

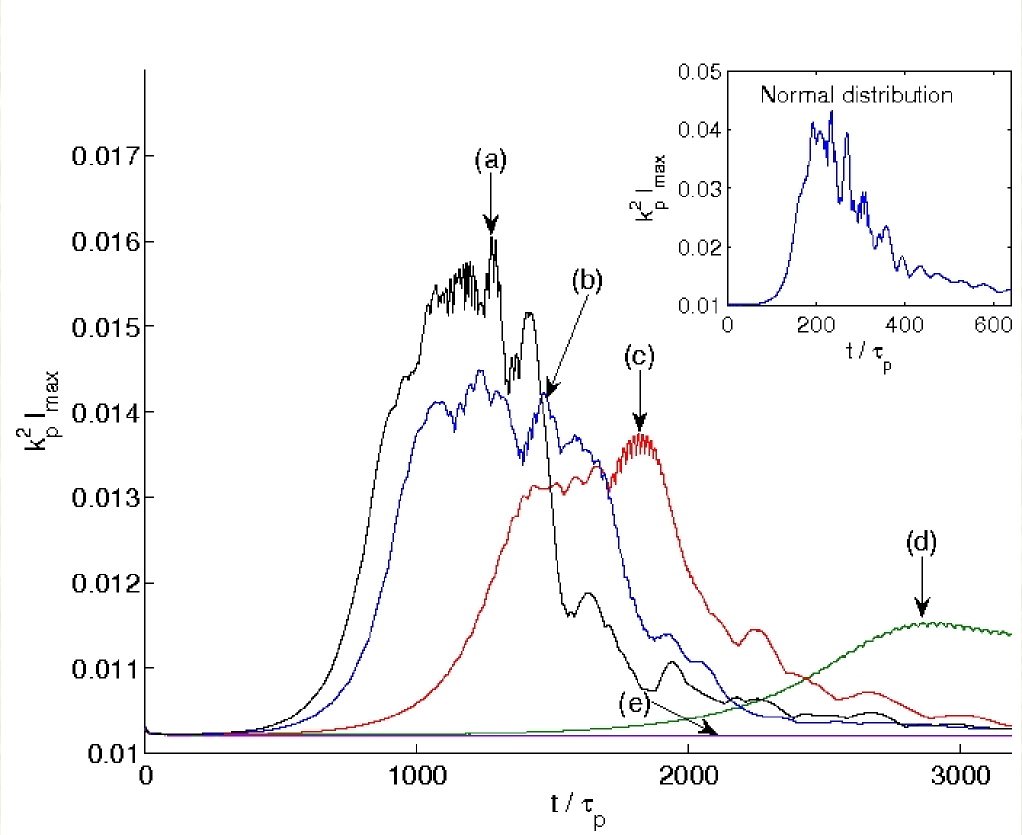
Eliasson and Shukla, Phys. Rev. Lett. **105**, 014501 (2010)

VELOCITY DISTRIBUTION OF WAVES



Directional spectra with $N = 840$, 200, 90, 50 + narrow Gaussian.

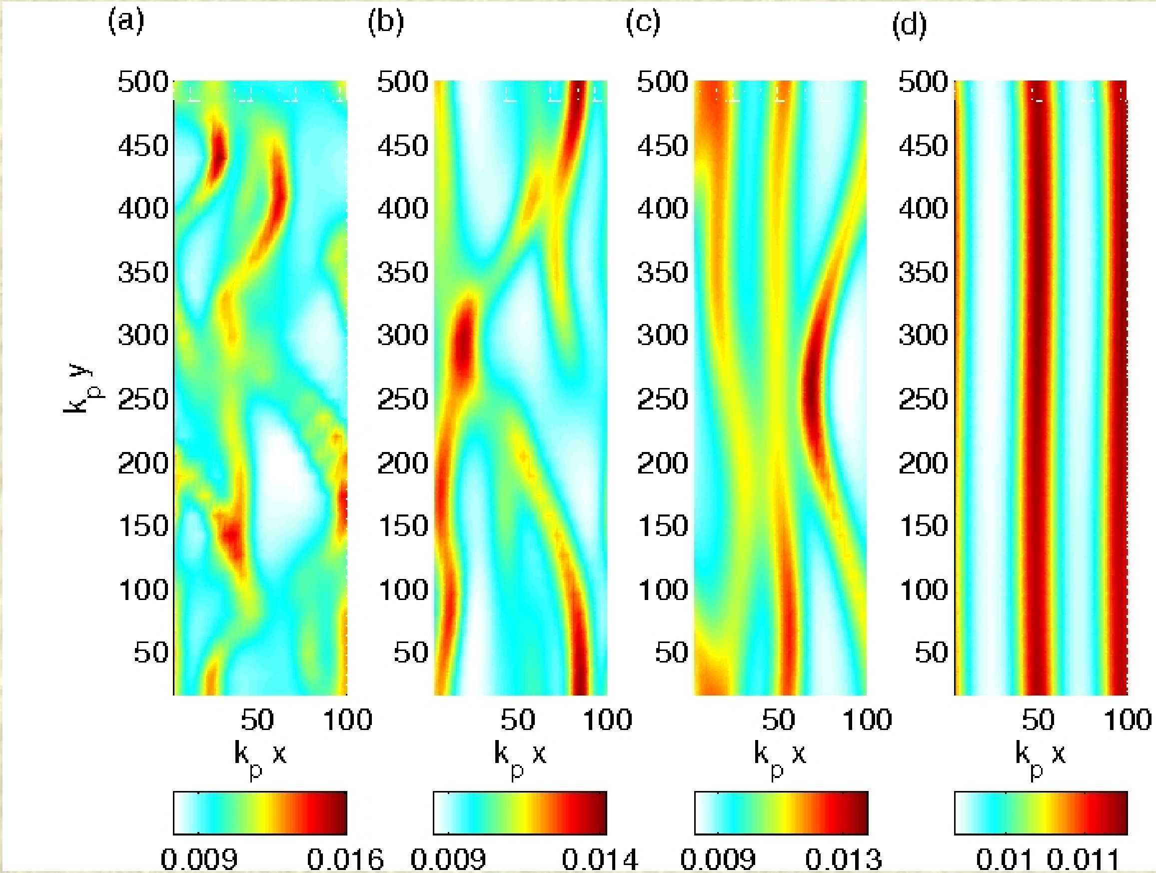
MAXIMUM WAVE AMPLITUDE



Higher maximum amplitudes for directional spectra!

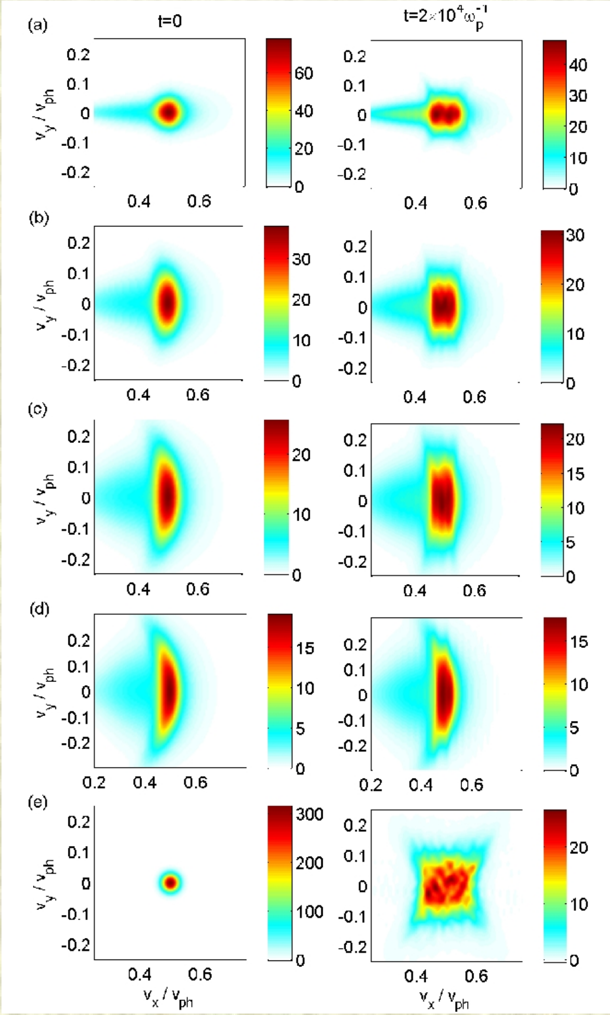
Eliasson and Shukla, Phys. Rev. Lett. **105**, 014501 (2010)

SPACE DISTRIBUTION DISTRIBUTION OF WAVES



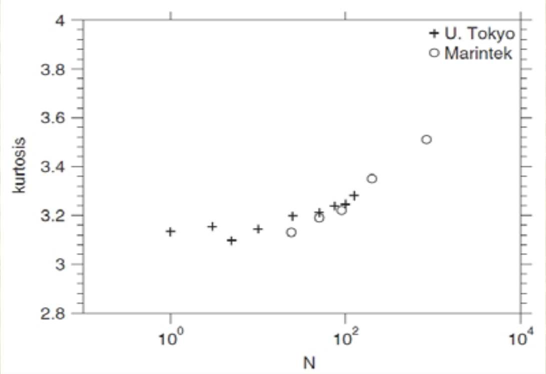
Wave energy focused into narrow bands in space

VELOCITY DISTRIBUTION OF WAVES

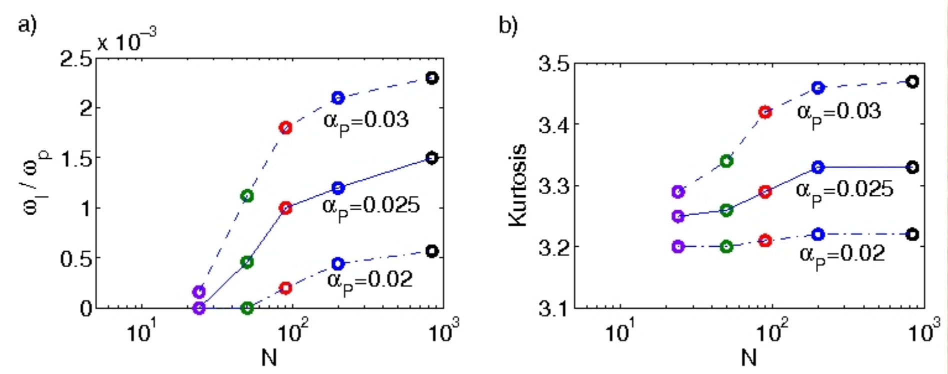


Broadening of wave spectra leads to stabilization of the system.

GROWTH RATE AND KURTOSIS



(d) Experiment



(e) Simulation

Kurtosis

$$\kappa = \frac{\langle \eta^4 \rangle}{\langle \eta^2 \rangle^2} \tag{7}$$

$\kappa = 3$ for Gaussian process. $\kappa > 3$ indicates extreme events.

DO FREAK WAVES REALLY EXIST ?

SUMMARY & FUTURE PERSPECTIVES

- Have presented a kinetic model for nonlinear water waves.
- Simulation and experiments show instability for narrow-band directional spectra.
- Prediction of probability of freak waves from underlying energy distribution.
- Refinement and comparison with more exact models.
- Model and methods applicable to nonlinear quantum plasmas, wave-kinetic models of laser-plasma interactions, etc.
- Padma’s physics not confined to plasmas!

HAPPY BIRTHDAY!