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Wave-Wave Interactions in Quantum Plasmas

Amar Prasad Misra
Visva Bharati University, Bolpur, India

Wave-Wave interactions in quantum plasmas

Pattern dynamics and spatiotemporal chaos

A. P. Misra

Department of Mathematics, Visva-Bharati University, Santiniketan-731 235, India

In collaboration with P. K. Shukla

Institut für Theoretische Physik IV, Fakultät für Physik and Astronomie, Ruhr-Universität Bochum, D-44780, Germany

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Introduction: Quantum Zakharov equations (QZEs)

Nonlinear interaction of hf quantum Langmuir waves (QLWs) and lf quantum ion-acoustic waves (QIAWs) is governed by [PoP **12**, 012302 (2005)]

$$i\frac{\partial E}{\partial t}+\frac{\partial^2 E}{\partial x^2}-H^2\frac{\partial^4 E}{\partial x^4}=nE, \quad (1)$$

$$\frac{\partial^2 n}{\partial t^2}-\frac{\partial^2 n}{\partial x^2}+H^2\frac{\partial^4 n}{\partial x^4}=\frac{\partial^2 |E|^2}{\partial x^2}, \quad (2)$$

where $H=\hbar\omega_{pi}/k_B T_e$.

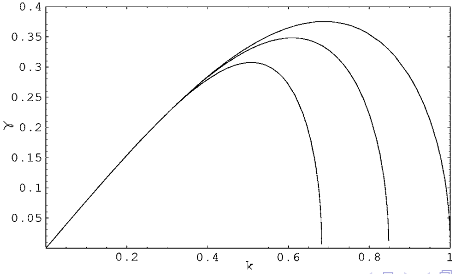
- When the electric field intensity is strong enough to reach the modified decay instability threshold, the interaction results to *weak turbulence.* LWs are then scattered off IAWs under $T_e >> T_i$.
- When the field intensity is so strong that the MI threshold is exceeded, the LWs are then essentially trapped by the density cavities of IAWs. The interaction is then said to be in *strong turbulence* regime.

Modulational instability (MI)

Assuming the perturbations $\propto \exp(ikx - i\omega t)$ from a spatially homogeneous field E_0,Growth rate of MI is [Marklund, PoP **12**, 082110 (2005); Misra & Shukla, PRE **79**, 056401 (2009)].

$$\gamma = \frac{1}{\sqrt{2}} \left[\bar{H} k^2 \sqrt{\bar{H}^2 + 8 \left| E_0 \right|^2 - 2\bar{H}^4 k^2 \left(2 - \bar{H}^2 k^2 \right)} - \bar{H}^2 k^2 \left(1 + \bar{H}^2 k^2 \right) \right]^{1/2}, \tag{3}$$

where $\bar{H} = 1 + H^2 k^2$. MI sets in for $k < \sqrt{2} E_0 / \left(1 + H^2 k^2 \right)$. From the right, $H = 0, 0.5, 1$ and $E_0 = \sqrt{0.5}$.



Temporal dynamics: Wave ansatz

$$E(x,t) = \sum_{m=-M/2}^{+M/2} E_m(t) e^{imkx} = \sum_{m=-M/2}^{+M/2} \rho_m(t) e^{i\theta_m(t)} e^{imkx}, \tag{4}$$

$$\nu(x,t) = \sum_{m=-M/2}^{+M/2} n_m(t) e^{imkx}, \tag{5}$$

where $M = [k^{-1}]$ represents the number of modes, $\rho_m = \rho_{-m}, \theta_m = \theta_{-m}, n_m = n_{-m}$. For three wave interactions, $M = 2$, $\rho_1 = \rho_{\pm 1}, n_1 = n_{\pm 1}$ etc., we can express the fields as [Misra et al, PoP **17**, 032307 (2010)]

$$E = E_0 + E_1 \cos(kx) \equiv \sqrt{N} \sin\left(\frac{a}{2}\right) \exp(i\theta_0) + \sqrt{2N} \cos\left(\frac{a}{2}\right) \exp(i\theta_1) \cos(kx), \tag{6}$$

$$n = N + n_1 \cos(kx), \tag{7}$$

where $N(\equiv n_0) = |E_{-1}|^2 + |E_0|^2 + |E_1|^2 =$ conserved plasmon no.

Three-wave model

Inserting Eqs. (6) and (7) into Eqs. (1) and (2), and following [Misra et al, PoP **17**, 032307 (2010)] we obtain the temporal system.

$$\dot{x}_1 = -\sqrt{2}\, x_3 \sin x_2\,, \tag{8}$$

$$\dot{x}_2 = k^2\left(1 + H^2 k^2\right) - \sqrt{2}\, x_3 \cos x_2\, \cot x_1\,, \tag{9}$$

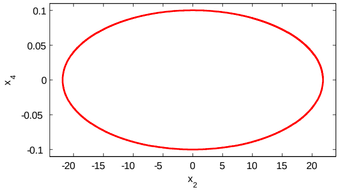
$$\dot{x}_3 = x_4\,, \tag{10}$$

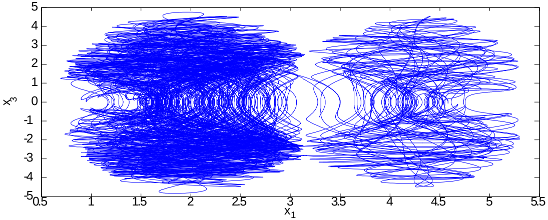
$$\dot{x}_4 = -k^2\left(1 + H^2 k^2\right) x_3 - \sqrt{2}\, N k^2 \cos x_2\, \sin\, x_1\,, \tag{11}$$

where $a = x_1, \varphi = x_2, n_1 = x_3, \dot{n}_1 = x_4$.

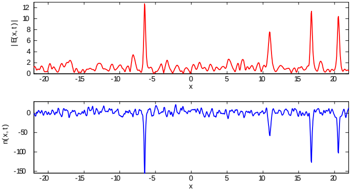
Three-wave interaction

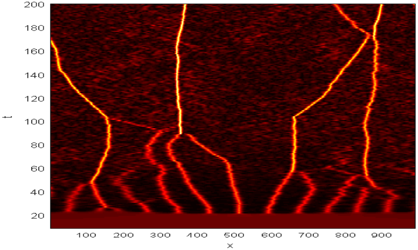
Periodic orbit & Chaotic attractor





$H = 0; k = 0.14, E_0 = 2, \; t = 200$

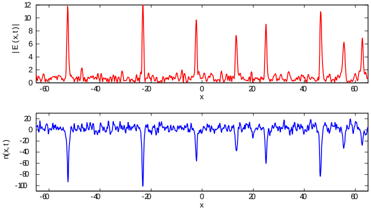


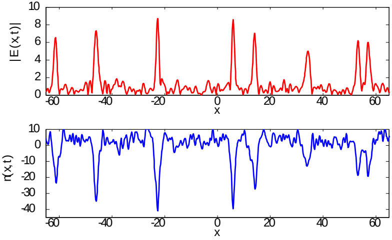


Spatiotemporal evolution (Contd.)

Quantum case

$H = 0.2$ (upper panel), 0.5 (lower panel); $k = 0.048, E_0 = 2,\ t = 200$

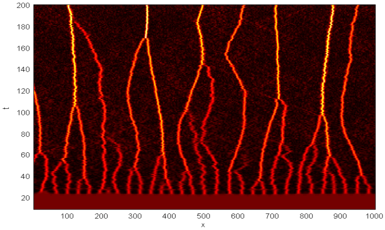


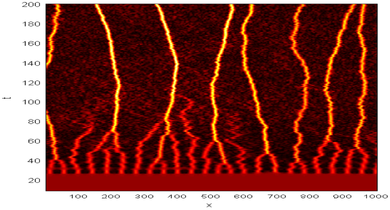


Spatiotemporal evolution (Contd.)

Quantum case

$H = 0.2$ (upper panel), 0.5 (lower panel); $k = 0.048, E_0 = 2, \; t = 200$





Temporal vs spatiotemporal evolution

- Three-wave model can be useful for the interaction of coupled LWs and QIAWs in the region $k_c/2 < k < k_c$ where the system exhibits stable oscillations.
- It can relatively be accurate in the region $k_0 \lesssim k < k_c/2$ where the system is in temporal chaos for a given E_0.
- For $k < k_0$ which indicates the excitation of many $(> [k_0^{-1}])$ unstable harmonic modes, the low-dimensional model fails to describe the dynamics of the coupled waves.
- Three-wave model exhibits periodic orbit in $k_c/2 < k < k_c$ and temporal chaos in $k_0 \lesssim k < k_c/2$.
- The space-time evolution reveals that the system is in TC for $k_1 < k \lesssim k_0$ and STC in $k_2 < k < k_1$.

Concluding remarks

- Envelope soliton is formed through the MI, which sets in if $k < \sqrt{2}E_0/\left(1 + H^2k^2\right)$, or $k < k_c$. The dynamics is subsonic in $k_c/2 < k < k_c$ where MI growth rate is small.
- H reduces the MI growth with cut-offs at lower wave numbers. The growth rate is max. at $H = 0$.
- The length scale of excitation of unstable harmonic modes is to be larger in quantum case compared to the classical one in order that a transition to STC emerge.
- Low-dimensional model predicts some basic features of the system as well as suggests an approximate region of k for the existence of regular and chaotic structures.
- Space-time evolution indicates that solitary patterns are first formed by the unstable harmonic modes. These modes are then excited by a master mode due to initial MI.

Concluding remarks (Contd.)

- As solitons experience oscillatory motions, they emit radiation due to irregular interaction with the QIA fields. Such radiations are broad-band in nature causing a growing number of modes to be involved in the stochastic dynamics.
- In the STC state, the system energy can flow faster the smaller is the wave number of modulation.
- Since our model is 1D in space, energy transfer is not a result of soliton collapse, rather it is purely generated by the nonintegrable nature as well as chaotic aspects of the system.
- The results could be important to turbulence in pulsar radiation where strong magnetic field plays an important role as well as in the next generation laser-solid density plasma interaction experiments.

Open issues

- Inclusion of IWN and deviation from quasineutrality [Misra, Shukla et al, PRE **81**, 046405 (2010)] might be important in the radiation of IAWs [Karpman, PoP **5**, 932 (1998)].
- More comprehensive investigation with 3D QZEs [Haas & Shukla, PRE **79**, 066402 (2009)] where arrest of Langmuir collapse might be possible by quantum effects [PRE **80**, 056405 (2009)].

Few words about P K Shukla

- Prudent and potential in all his nobel deed,
- Knowledgable and helpful to everyone in their need.
- Sober and Sombre in speeches and action,
- Helpful and encourages people with lot of motivation.
- Unparrel in talents and a personality so bright,
- Keen to know and accepts challenges with all his might.
- Love, best wishes and regards are all we can send,
- As we wish a very Happy Birthday to our dearest friend.