The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2155-21

International Workshop on Cutting-Edge Plasma Physics

5 - 16 July 2010

Collisionless electrostatic shocks in striated electron temperatures in magnetized plasmas

Hans Pécseli

Oslo University
Norway

UNIVERSITETET
I OSLO

Collisionless electrostatic shocks in striated electron temperatures in magnetized plasmas

Patrick Guio and Hans Pécseli

References:

P. Guio, S. Børve, H. L. Pécseli, and J. Trulsen, *Low frequency waves in plasma with spatially varying electron temperature*, Ann. Geophys. **18**, 1613 (2001)

P. Guio and H. L. Pécseli, *Collisionless plasma shocks in striated electron temperatures*, Phys. Rev. Lett. **104**, 085002 (2010)

Linear Waves

Dispersion relation for low frequency electrostatic waves, with Boltzmann distributed electrons, assuming quasi-neutrality and magnetized cold ions, with $\Omega_{ci} < \Omega_{pi}$:

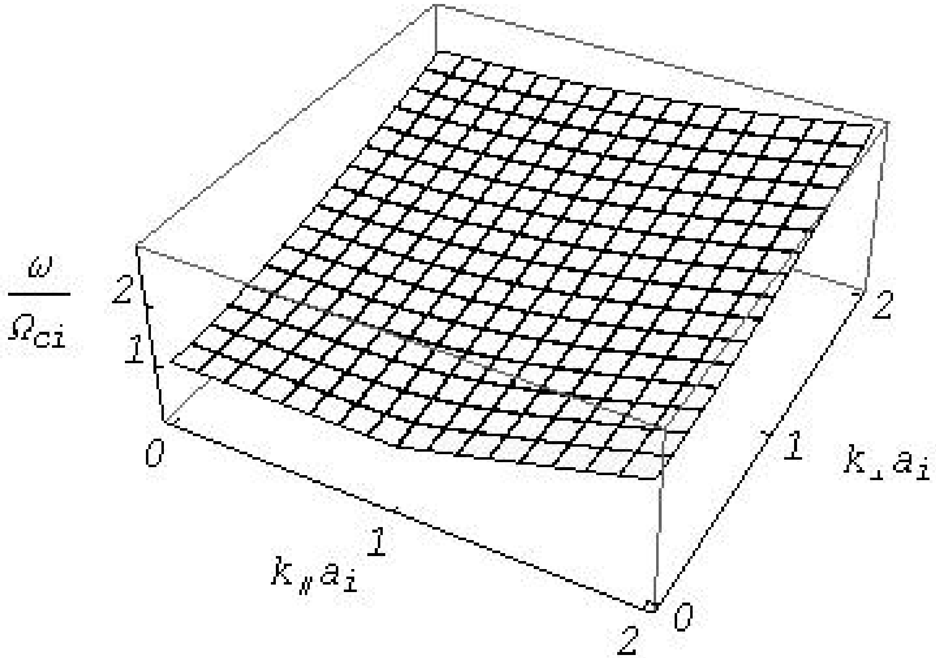
$$\omega^2\left(\omega^2 - \Omega_{ci}^2\right) - C_s^2\left(k_\perp^2\omega^2 + k_\parallel^2\left(\omega^2 - \Omega_{ci}^2\right)\right) = 0$$

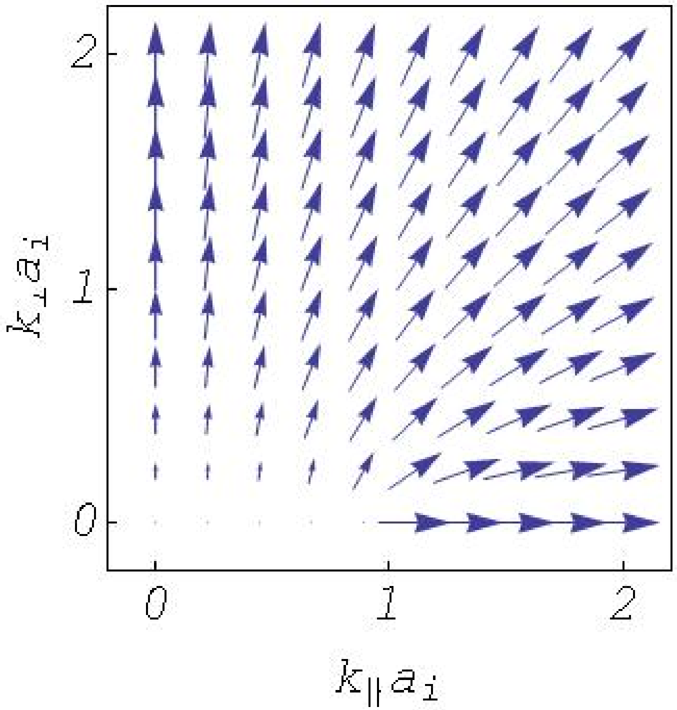
Limiting case a), ion cyclotron waves: $\omega^2 \approx \Omega_{ci}^2 + C_s^2 k_\perp^2$

Limiting case b), ion sound waves: $\omega^2 \approx C_s^2 k_\parallel^2$

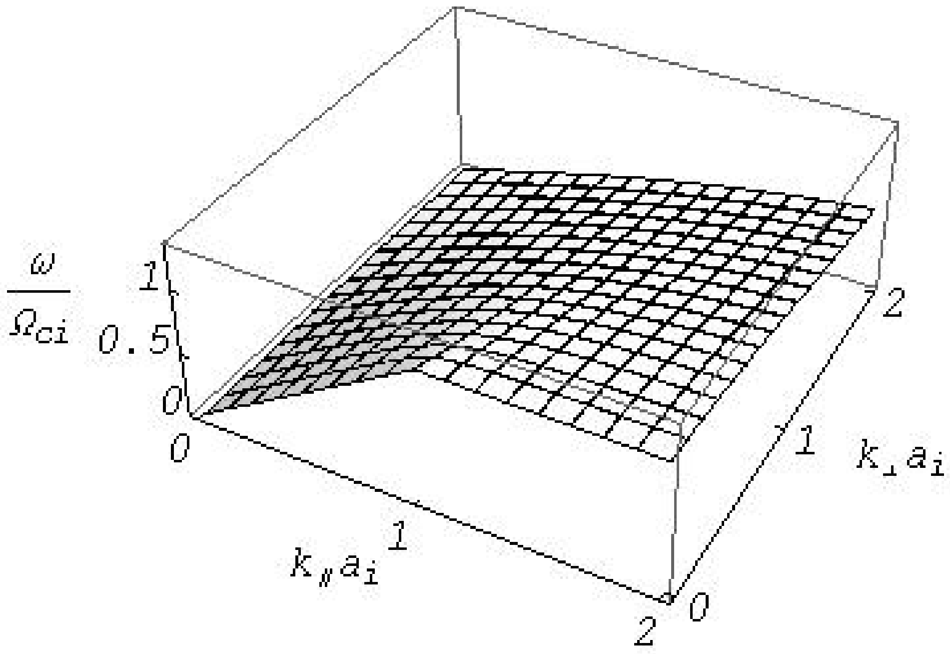
For $k_\perp^2 = 0$ *we have* $\left(\omega^2 - \Omega_{ci}^2\right)\left(\omega^2 - k_\parallel^2 C_s^2\right) = 0$, *where the ion sound branch crosses the electrostatic ion cyclotron resonance at* $k = \Omega_{ci} / C_s \equiv 1/a_i$, *where* a_i *is the effective ion Larmor radius*

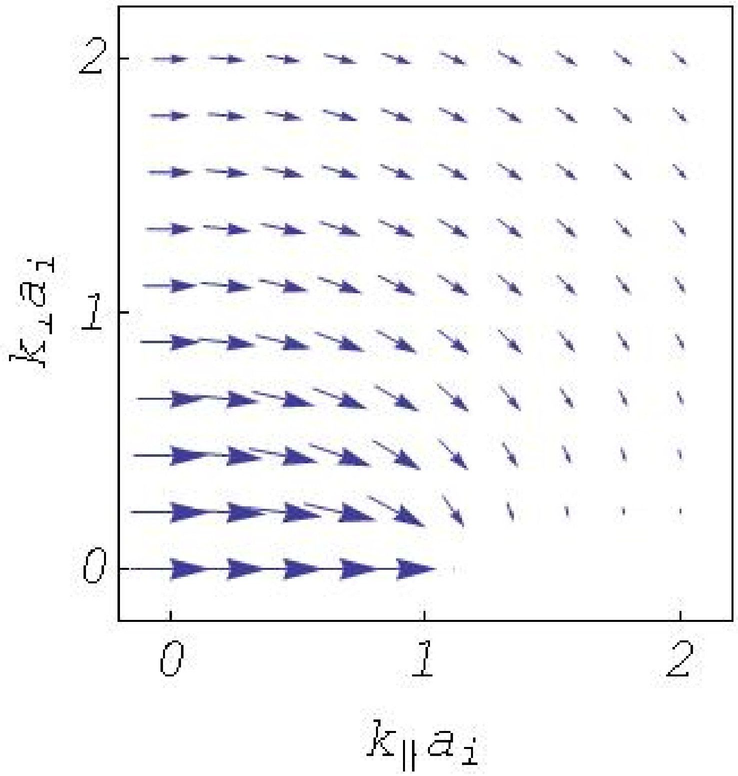
High frequency ion-wave, $\Omega_{ci} < \omega < \Omega_{pi}$





Low frequency ion wave, $\omega < \Omega_{ci} < \Omega_{pi}$





Plasma with electron temperature striations, $T_e(x)$ with $\mathbf{x} \perp \mathbf{B}$. General linearized fluid model for the electrostatic potential, with conditions as before:

$$\frac{\partial^4}{\partial t^4}\psi - \frac{T_e(x)}{M}\frac{\partial^2}{\partial t^2}\left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2}\right)\psi + \Omega_{ci}^2 \frac{\partial^2}{\partial t^2}\psi - \Omega_{ci}^2 \frac{T_e(x)}{M}\frac{\partial^2}{\partial z^2}\psi = 0$$

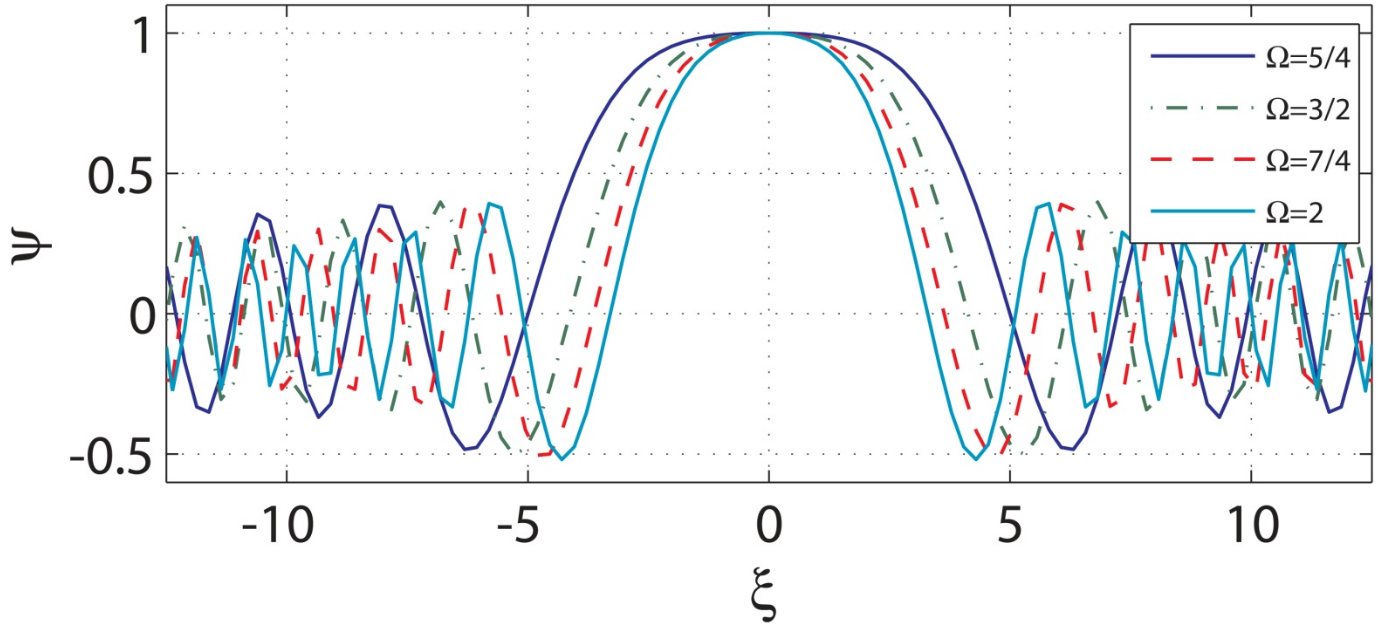
Fourier transform in time and z-coordinate; ω, k_z .
Normalized eigenvalue equation:

$$\frac{d^2}{d\xi^2}\hat{\psi} = (\Omega^2 - 1)\left(\frac{1}{\gamma^2} - \frac{T_0}{T_e(\xi)}\right)\hat{\psi}$$

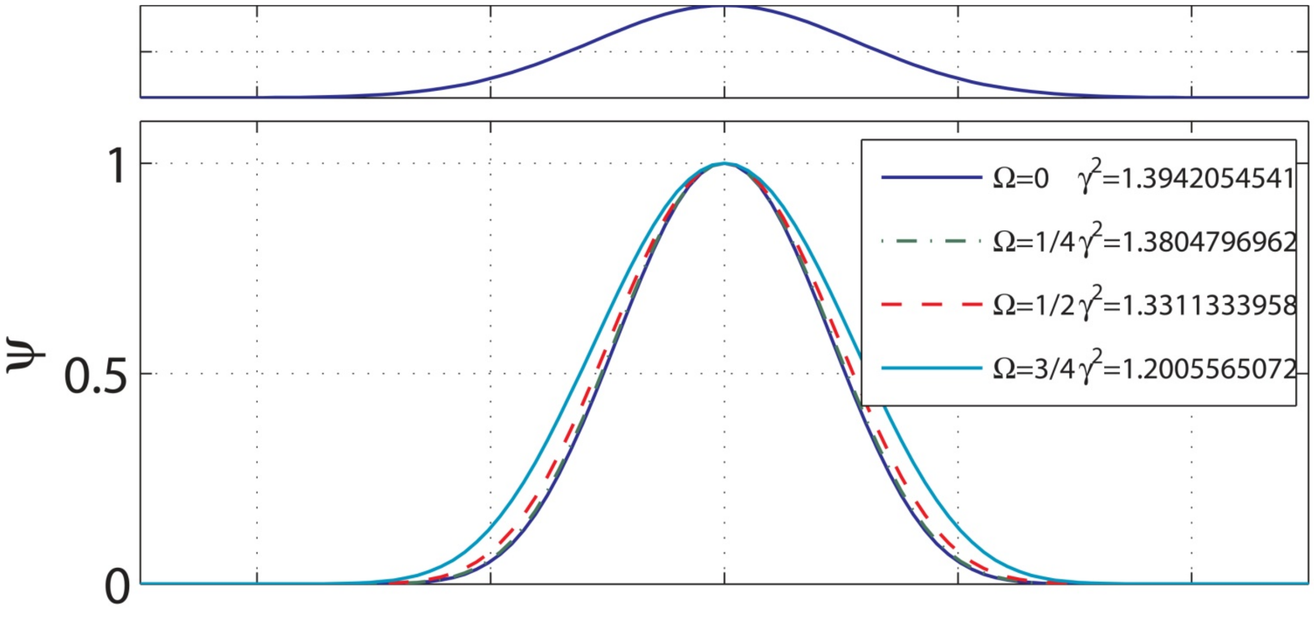
$$\gamma^2 \equiv \left(\frac{\omega}{k_z}\right)^2 \frac{M}{T_0}, \quad \xi \equiv x\frac{\Omega_{ci}}{C_s}, \quad \Omega \equiv \frac{\omega}{\Omega_{ci}},$$

$$T_0 = \frac{1}{2}(\max T_e(x) + \min T_e(x))$$

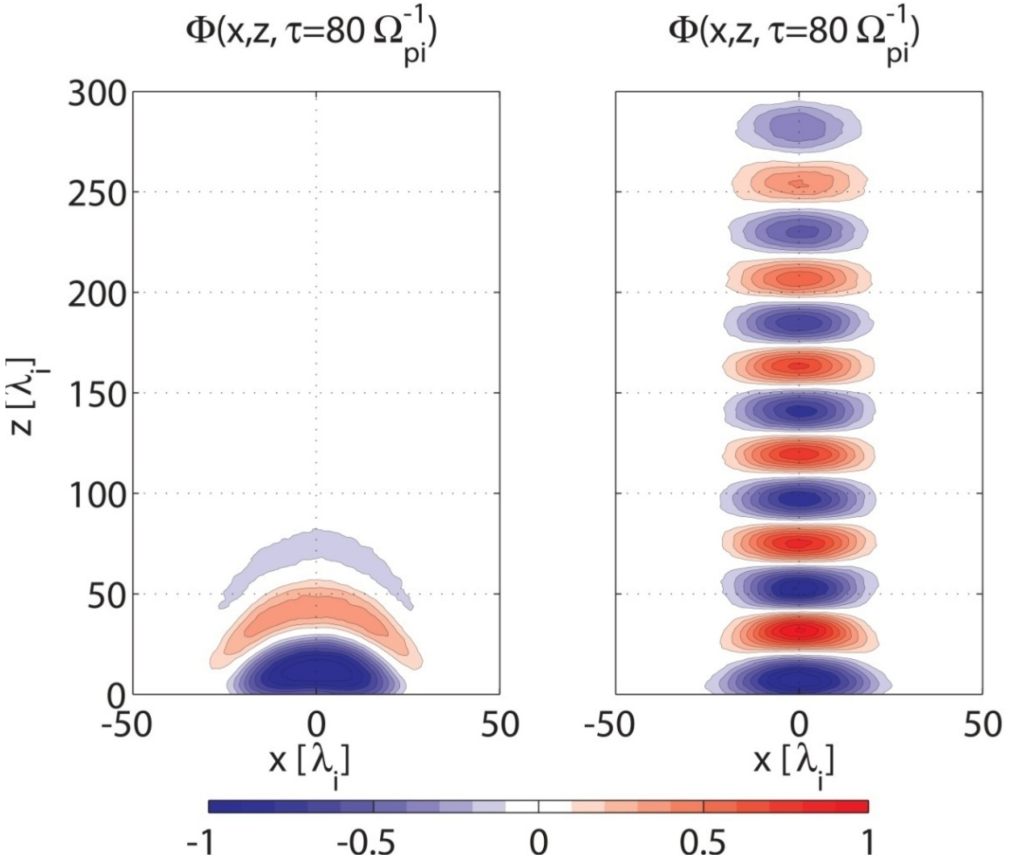
High frequency, $\omega > \Omega_{ci}$, i.e. $\Omega^2 > 1$, ”leaking” eigenmodes



Electron temperature striation (top), and lowest order waveguide-mode (trapped) eigenfunctions, $\omega < \Omega_{ci}$, i.e. $\Omega^2 < 1$. There may also be higher order eigenfunctions, with one or more zero-crossings, depending on $T(x)$.



Particle-in-cell (PIC) simulation of the propagation of linear ion waves in an electron temperature duct *(T_e = 25 T_i),* for two magnetic field intensities:



Summary:

For $\omega > \Omega_{ci}$ we have a continuoum of eigenvalues γ with corresponding global eigenfunctions.

For $\omega < \Omega_{ci}$ we have a discrete set of eigenvalues $\min\sqrt{(T_e/T_0)} < \gamma < \max\sqrt{(T_e/T_0)}$ with corresponding localized eigenfunctions.

For $\omega < \Omega_{ci}$ we also have a continuoum of eigenvalues $\gamma < \min\sqrt{(T_e/T_0)}$ with corresponding global eigenfunctions.

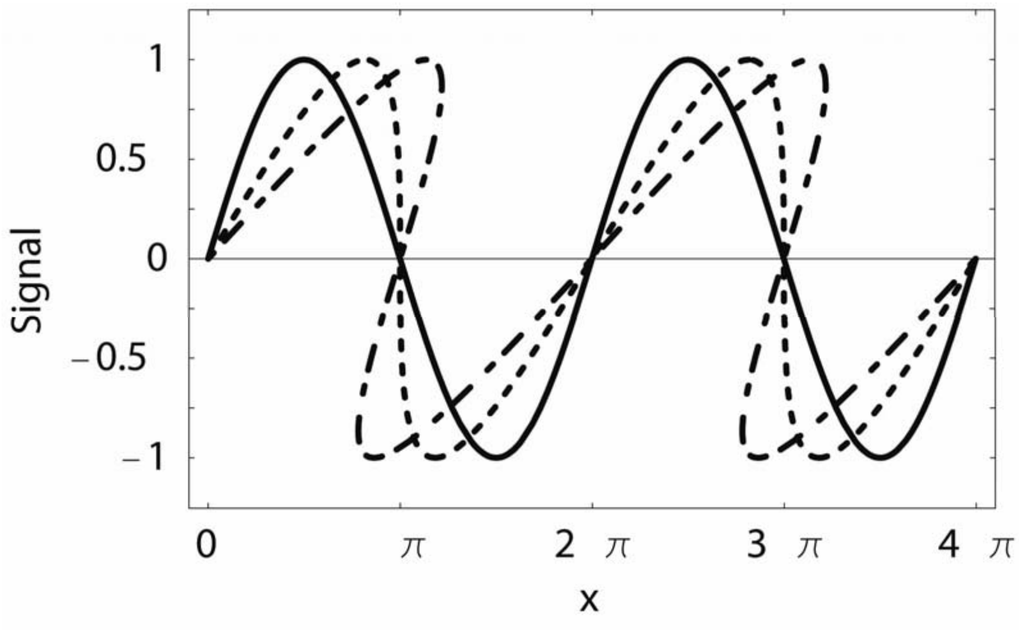
Nonlinear Waves

These waveguide modes have many interesting nonlinear wave properties, such as wave particle interactions, wave-decay, etc.

Here we amphasize only one of these properties; collisionless shock formation.

Simple nonlinear waves: $\frac{\partial}{\partial t}u + (u + C_s)\frac{\partial}{\partial x}u = 0$

Wave breaking in the frame moving with C_s:



Burger's equation, with kinematic viscosity *v*:

$$\frac{\partial}{\partial t}u + u\frac{\partial}{\partial x}u = \nu\frac{\partial^2}{\partial x^2}u$$

Shock solution; a balance between harmonic generation by the nonlinearity and dissipation of short scales.

Assume a monotonically varying, step-like, perturbation propagating with velocity *U*,

$$(u - U)\frac{d}{dx}u = \nu\frac{d^2}{dx^2}u$$

There exist a position $x = x_0$ where $d^2u/dx^2 = 0$ and $du/dx \neq 0$. Then $U = u(x_0)$.

Solution obtained for instance by Cole-Hopf transformation:

$$h(x-Ut) = \frac{A}{1+\exp\left((x-Ut)\frac{U}{\nu}\right)}$$

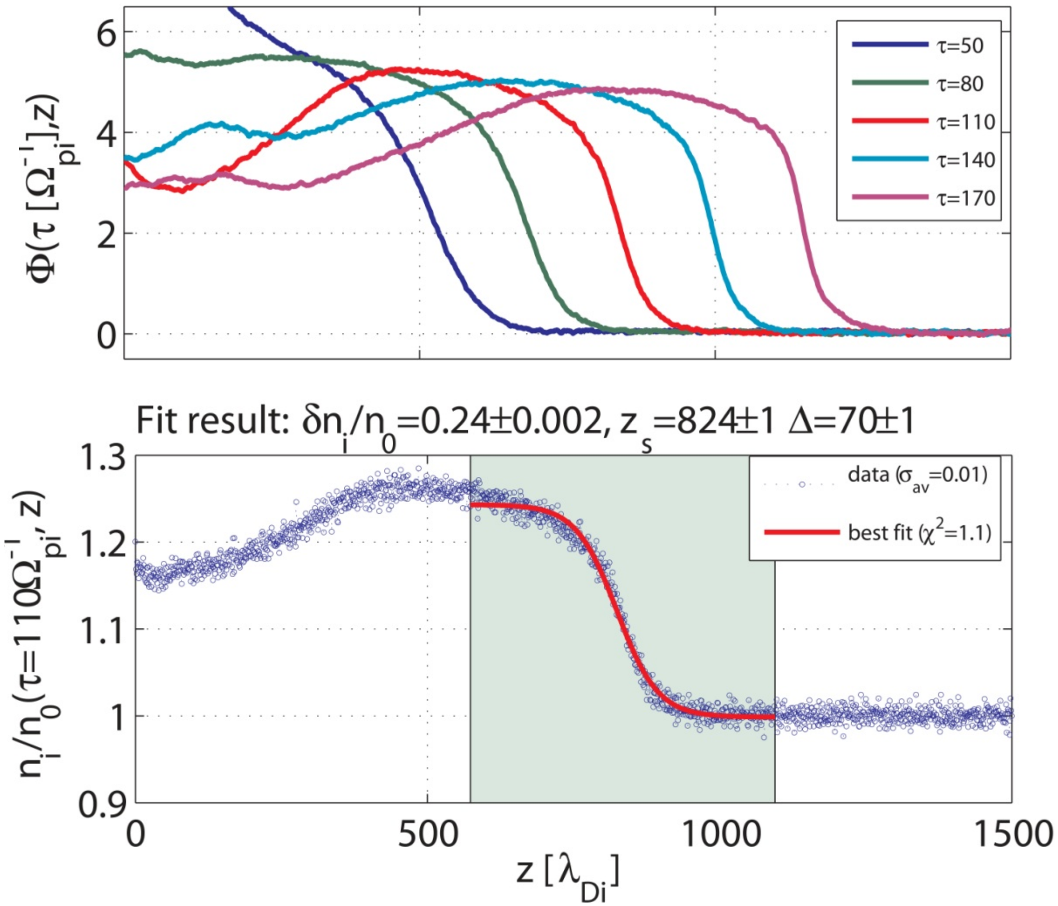
with $A = 2U$. The shock width, in particular, is: $\nu/U = ½\ (\nu/A)$
The shock velocity is half of the peak velocity-perturbation.

Postulate in Fourier space:

$$\frac{\partial}{\partial t}\hat{u}_{\|} + \frac{i}{2}\alpha\, k_{\|}\left[\hat{u}_{\|} \otimes \hat{u}_{\|}\right] = -\frac{1}{\tau(k_{\|})}\hat{u}_{\|}\, H\left(|\, k_{\|}\,| - \frac{\Omega_{ci}}{C_s}\right)$$

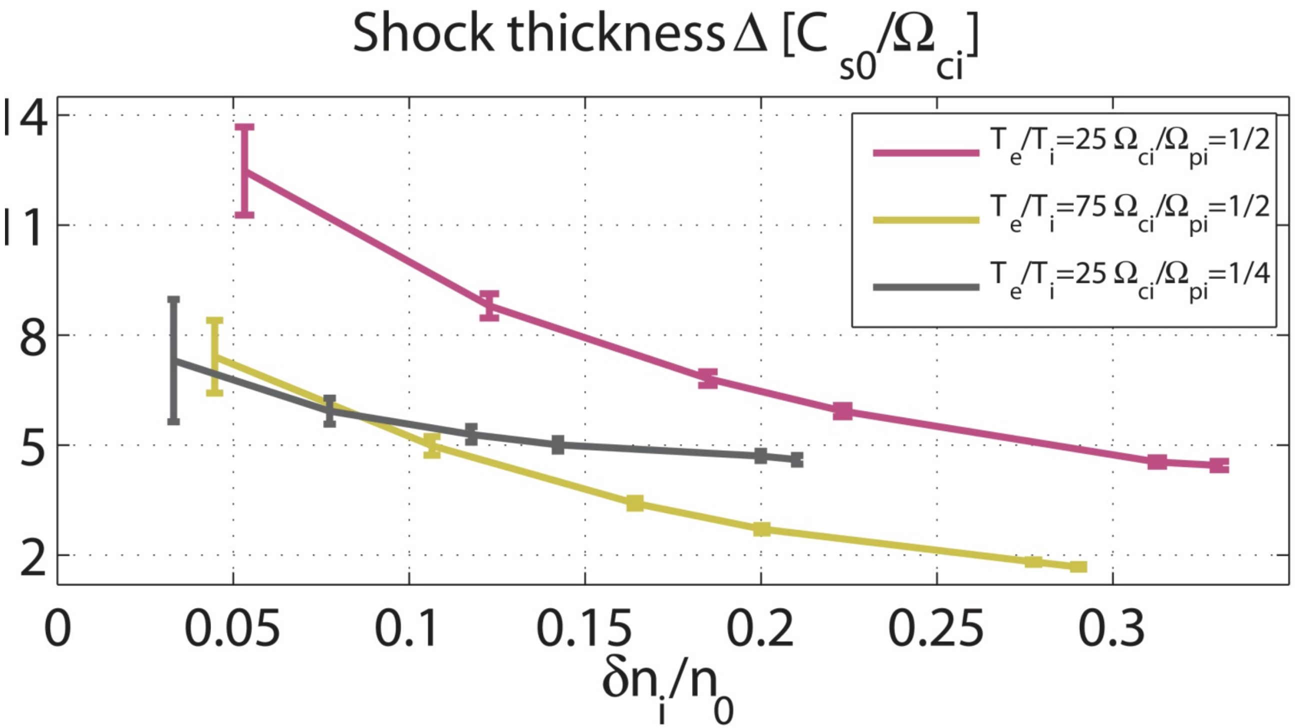
where $\tau(k_{\|})$ is the time it takes a ”leaking” mode to escape from the waveguide. We introduced *H(x)* as Heaviside’s step function, while α originates from an expansion in eigenmodes.

Shock formation, propagation, and shock fitting in PIC-simulation (no quasi-neutrality assumed):

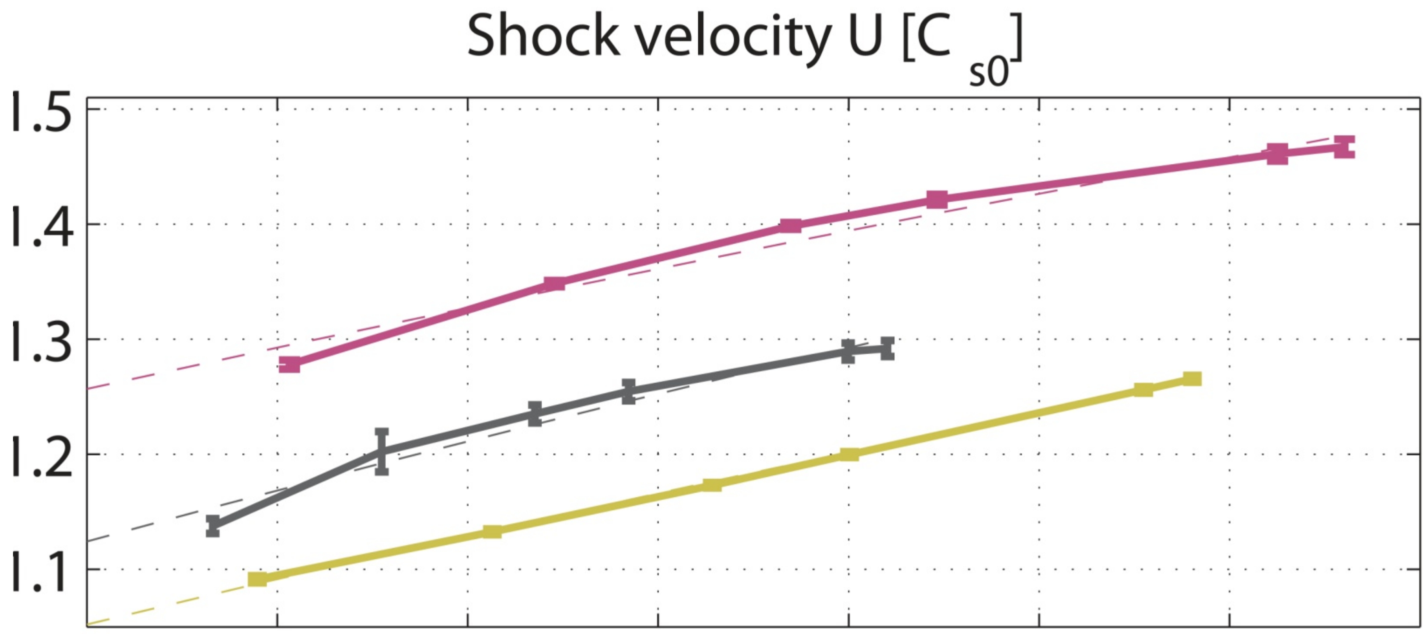


UNIVERSITETET
I OSLO

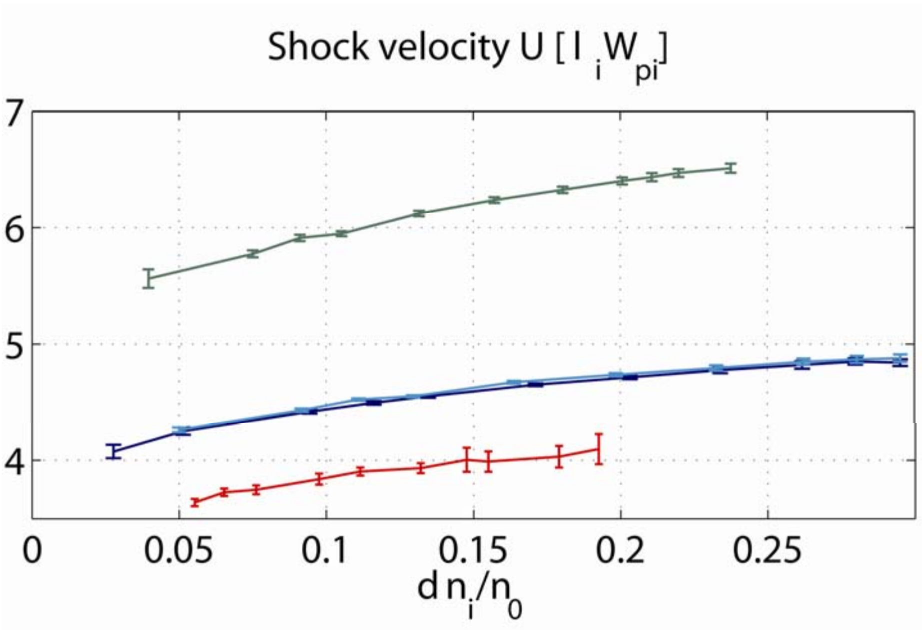
Shock thickness for varying parameters:

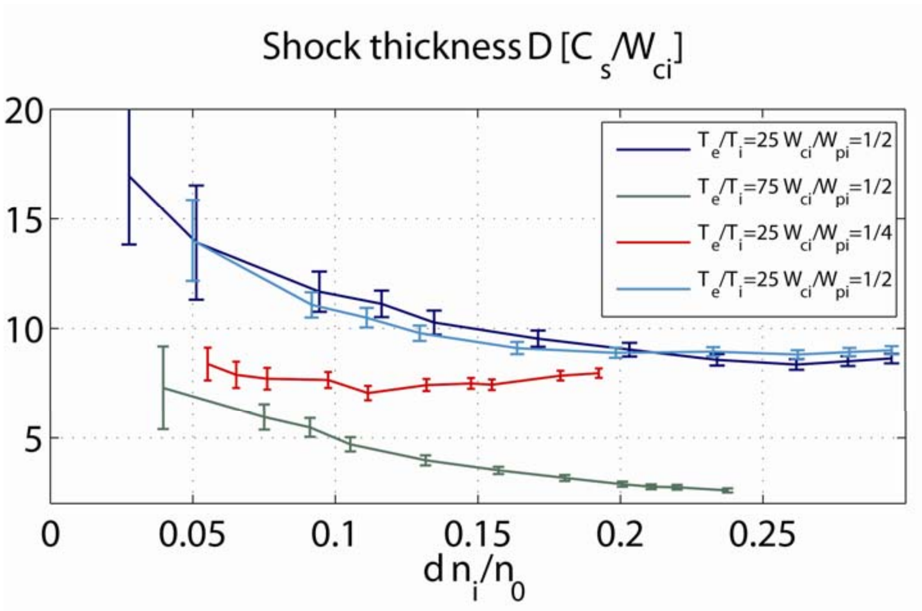


Shock velocity for varying parameters, $T_e/T_i = 25, 75$ and $\Omega_{ci} / \Omega_{pi} = ½, ¼$



Supplementary data, with different fitting-model:





The processes are time reversible: you can ”run the movie backwards” at any time you choose, and everything remains physically acceptable, but the observed phenomenon has nevertheless all the basic features of a classical collisional, or dissipative, shock. This observation remains true also in a fluid model of the problem, although here it is illustrated by a PIC-particle simulation!

Assume $\tau = D/u_g$ where D is the width of the striation.
Model equation for propagating shock-like structure:

$$(u - U)\frac{d}{dx}u = -\int_{\Omega_{ci}/C_s}^{\infty} \frac{u_g(k)}{D}\,\hat{u}(k)\sin(k\,x)\,dk.$$

Within this model we have $U = u(x=0)$.

Note that the group velocity u_g will in general have D as a parameter!

To lowest order we have $u \approx C_s\,\delta n/n_0 \approx C_s\,e\varphi/T_e$.

As a test-function use $\widehat{u}(k) = Ae^{-\frac{1}{2}(k\Delta)^2}/k$, i.e. $U = \alpha A$, with $\alpha = const$.

$$\frac{d}{dx}\left((u-U)\frac{d}{dx}u\right) \approx -A\int_{\Omega_{ci}/C_s}^{\infty} \frac{u_g(k)}{D} e^{-\frac{1}{2}(k\Delta)^2} \cos(k\,x)\,dk.$$

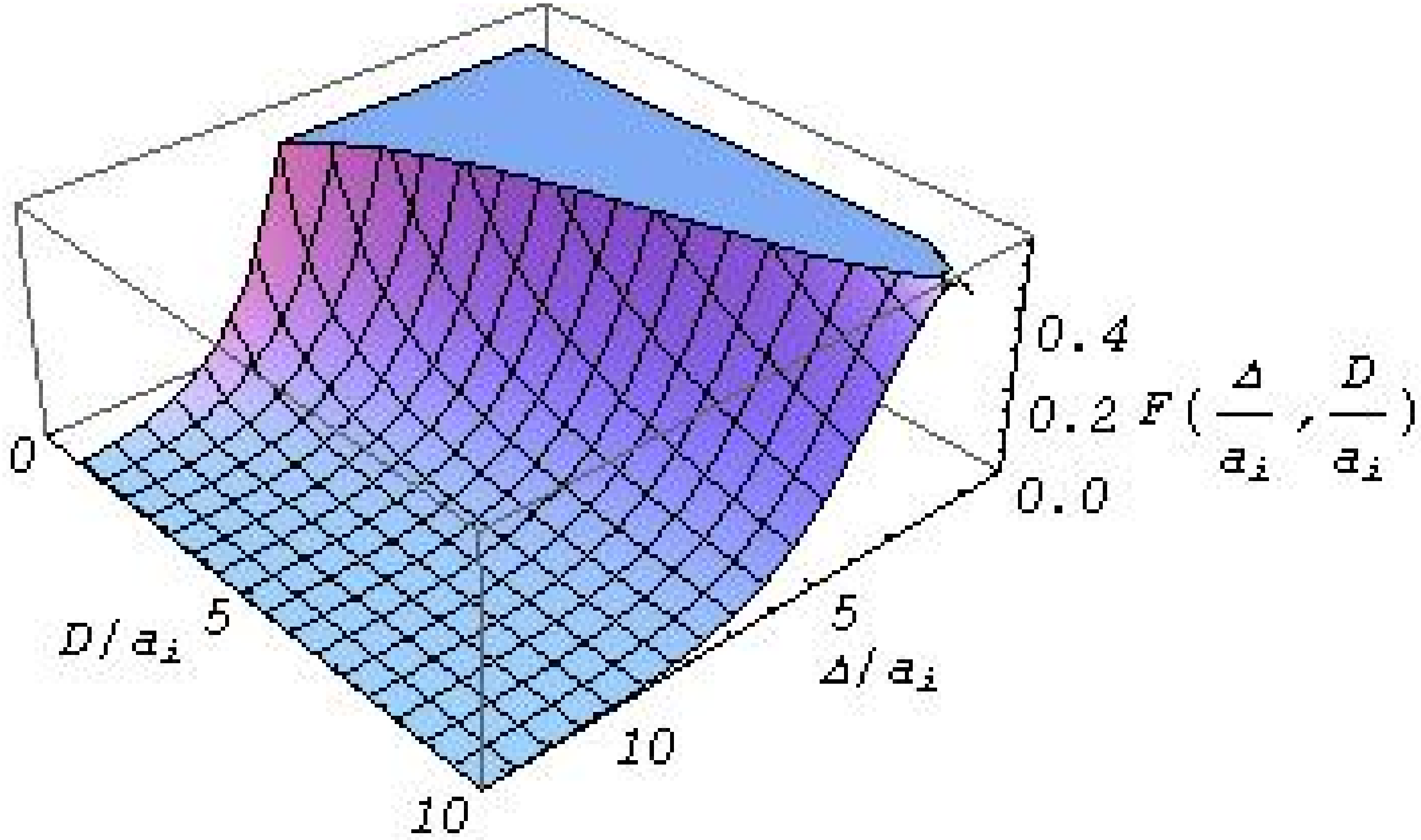
From the model equation we have at $x = 0$

$$\left(u'(x=0)\right)^2 \approx \left(\frac{A}{\Delta}\right)^2 \approx \frac{A}{D\Delta} \int_{\Delta\Omega_{ci}/C_s}^{\infty} u_g\left(\gamma \frac{C_s}{\Delta\Omega_{ci}}\right) e^{-\frac{1}{2}\gamma^2} d\gamma.$$

$$i.e. \quad \frac{A}{C_s} \approx \frac{\Delta}{D} F\left(\frac{\Delta\Omega_{ci}}{C_s}, \frac{D\Omega_{ci}}{C_s}\right), \quad recalling$$

that D *enters* u_g *as a parameter*

Numerical integration



Summary:

1) The existence of trappen waveguide modes in striated electron temperatures was demonstrated in a linear fluid model for magnetized plasmas.
2) By simple physical arguments we gave reasons for the existence of shock-like nonlinear waveforms. The dissipation mechanism is purely time-reversible.
3) By numerical PIC-plasma simulations we demonstrated the existence of such shock-solutions, and gave empirical relations for the amplitude dependence of some basic parameters.
4) A simple fluid model explains parts of these amplitude variations.

Thank you for your attention

UNIVERSITETET
I OSLO

Dear Padma: Happy Birthday!