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Abstract

The magnetized plasma-wall transition (MPWT) layer occurring, e.g., near a tokamak divertor, typically consists of three dictinct sublayers: the Debye sheath (DS), the magnetic presheath (MPS), and the collisional presheath (CPS), with characteristic lengths λ_D (electron Debye length), ρ_i (ion gyro-radius), and λ (smallest relevant ion collision length), respectively. For analytical simplicity one usually assumes the ordering λ_D << ρ_i << λ, or, equivalently, $\varepsilon_{Dm} \equiv \frac{\lambda_D}{\rho_i} \to 0$ and $\varepsilon_{mc} \equiv \frac{\rho_i}{\lambda} \to 0$ ("asymptotic three- scale (A3S) limit"), in which the three sublayers are precisely defined. In the present work the equations and length scales governing the transition, or the "intermediate" regions between neighbouring sublayers (DS–MPS, MPS–CPS) in the A3S limit are derived, allowing one to avoid the singularities arising from the $\varepsilon_{Dm} \to 0$ and $\varepsilon_{mc} \to 0$ approximations. The MPS entrance and the related Bohm-Chodura condition is defined in a natural way. It is found that in the hydrodynamic $T_i = 0$ approximation the intermediate scales and equations have a universal form and a similar structure not only for the MPS–DS and CPS–MPS transitions, but also for the DS–CPS transition in the non-magnetized case.

1. Introduction

In the presence of an oblique magnetic field, the plasma-wall transition (PWT) layer can be divided into three regions (Fig. 1), namely: the Debye sheath (DS), the magnetic presheath (MPS) and the collisional presheath (CPS) with characteristic length scales λ_D (Debye length), ρ_i (ion gyroradius) and λ (relevant collisional or geometrical length), respectively [1]. For the limiting ordering $\lambda_D \ll \rho_i \ll \lambda$ ("asymptotic three-scale (A3S) limit"), i.e. for $\varepsilon_{Dm} \equiv \lambda_D/\rho_i \rightarrow 0$ and, $\varepsilon_{mc} \equiv \rho_i/\lambda \rightarrow 0$ (which must be clearly distinguished from the $\varepsilon_{Dm} = 0$ and $\varepsilon_{mc} = 0$ approximations, respectively) the DS can be characterized as collisionless and non-neutral ($n_i \neq n_e$, with n_i and n_e the ion and electron number densities, respectively), the MPS as collisionless and quasi-neutral ($n_i = n_e$), and the CPS as collisional and quasi-neutral [2]. In the classical PWT problem without magnetic field, monotonicity of the electric potential requires the fulfilment of the non-marginal Bohm condition at the DS-CPS interface. Chodura [2] was the first to investigate the quasineutral MPS in the case of an oblique magnetic field without any collisional effects, i.e. in the A3S limit.

The DS and MPS regions are separated by the "sheath edge", or "sheath entrance (SE)" which in the A3S limit is characterized by the "marginal Bohm criterion", $v_z = c_s \equiv [k(T_e + \gamma T_i)/m_i]^{1/2}$, where v_z is the z-component of the ion fluid velocity, c_s is the ion-sound velocity, k is the Boltzmann constant, γ is the local ion polytropic coefficient, T_e is the electron screening temperature and T_i is the ion temperature [3]. In this limit the sheath edge appears as a field singularity if viewed on the MPS scale, and as lying at infinity if viewed on the DS scale. The MPS and the CPS are separated by a similar but less known boundary surface, called the "MPS entrance (MPSE)". Below we will show *that quite in analogy with the DSE, the MPSE can be defined* in the *A3S limit as a surface were the electric field has a singularity if viewed on the CPS scale and lies at infinity if viewed on the MPS scale*. The condition satisfied at the A3S MPSE is the "Bohm-Chodura condition" $v_{||} = c_s$ [2, 4], where $v_{||}$ is the ion flow velocity component along the magnetic field. It easy to see that the dominant effect of the MPS is to deflect the ion orbits in such a way that the velocity component v_z can fulfil the Bohm condition at the DS entrance [1, 4]. In the absence of a magnetic field, or in the presence of a magnetic field perpendicular to the wall, the magnetic presheath does not exist as a distinct region at all.

In the present paper we analyze the MPS-CPS and DS-MPS transition (intermediate) regions in the A3S limit. The equations bridging (or "matching") these regions and their characteristic lengths are determined. *We have found that the both of these transitions can be described by one and the same Painlevé equation,* quite similar to the DS– CPS transition of the unmagnetized PWT layer. Hence one can say that the Painlevé equation plays a universal role in the matching procedure in the hydrodynamic description. It is interesting to note, however, that in the unmagnetized PWT the Painlevé equation in fact represents the Poisson equation, while in our analysis of the MPS-CPS transition we have used the quasineutrality condition (instead of the Poisson equation) from very beginning of our analysis.

We have also found that the scale length l of the intermediate region between *any two neighbouring regions (denoted as 1 and 2 and having characteristic lengths* λ_1 *and* λ_2*,respectively, with* $\lambda_1 << \lambda_2$*) has the universal form* $l = \lambda_1^{4/5}\lambda_2^{1/5}$, earlier found also for the DS–CPS matching of the unmagnetized PWT layer [4].

2. Model and basic equation

The problem considered is one-dimensional, with the z axis perpendicular to the wall surface. The latter is placed at $z = 0$ and the plasma occupies the region $z < 0$. The electric potential $\Phi(z)$ decreases towards the wall monotonically. We assume a uniform magnetic field lying in the xz – plane and making a small angle α with the wall (see Fig. 2). The thermal motion of ions is neglected, $T_i \to 0$, whereas the electrons are the Boltzmann-distributed, $n_e = n_0 exp(e\Phi/kT_e)$. The ions are assumed to be produced by electron-impact ionization (accounted for by the source term $\nu_i n_e$ in the ion continuity equation, where ν_i is the ionization frequency), and to undergo charge-exchange collisions with the neutrals (with frequency ν_{cx}). To describe the MPWT layer we start from the ion fluid equations

$$\frac{d}{dz}(n_i v_z) = \nu_i n_e \,, \tag{1}$$

$$v_z \frac{dv_x}{dz} = \omega_c sin\alpha \cdot v_y - \left(\nu_{cx} + \nu_i \frac{n_e}{n_i}\right) v_x \,, \tag{2}$$

$$v_z \frac{dv_y}{dz} = \omega_c cos\alpha \cdot v_z - \omega_c sin\alpha \cdot v_x - \left(\nu_{cx} + \nu_i \frac{n_e}{n_i}\right) v_y \,, \tag{3}$$

$$v_z \frac{dv_z}{dz} = -\frac{Ze}{m_i}\frac{d\Phi}{dz} - \omega_c cos\alpha \cdot v_y - \left(\nu_{cx} + \nu_i \frac{n_e}{n_i}\right) v_z \,, \tag{4}$$

$$-\frac{d^2\Phi}{dz^2} = \frac{e}{\varepsilon_0}(Zn_i - n_e), \tag{5}$$

where $\omega_c = ZeB/m_i$ is the ion cyclotron frequency. In the dimensionless variables

$$u = \frac{v_z}{c_s}, \qquad \varphi = -\frac{e\Phi}{kT_e}, \qquad n = \frac{n_i}{n_0}\,, \qquad \rho = \frac{c_s}{\omega_c}, \tag{6}$$

the continuity, ion motion and Poisson equations (1) – (5) acquire the form

$$\frac{d}{dz}(nu) \ = \lambda_i^{-1} \cdot \exp(-\varphi), \tag{7}$$

$$\{D^2 + \ \rho^{-2}\}Du = \ \{D^2 + \ sin^2\alpha \cdot \rho^{-2}\}\frac{d\varphi}{dz}\,, \tag{8}$$

$$\lambda_D^2 \frac{d^2\varphi}{dz^2} = \ n - \ \exp(-\varphi), \tag{9}$$

where $\lambda_i = \ c_s/\nu_i$ is the "ionization length" and the three equations for the ion velocity components have been reduced to the single equation (8), with

$$D = u(d/dz) + \ \lambda^{-1}, \tag{10}$$

$$\lambda^{-1} = \ \lambda_{cx}^{-1} \ + \ \lambda_i^{-1} \exp(-\varphi)\,/n, \qquad \lambda_{cx} = \mathrm{c_s}/\nu_{cx} \ .$$

The wall is assumed to be completely absorbing. Therefore one may formulate an eigenvalue problem originating from the "plasma balance", i.e., the balance between ion creation due to ionization and ion absorption at the wall. Physically this eigenvalue represents, in fact, the ionization rate which is necessary to fulfil the plasma balance [5]. Here we do not consider this eigenvalue problem but are only interested in the analytic investigation of the time-independent MPWT sublayers and their analytic matching, deferring the eigenvalue problem to future investigations.

3. Analysis of the CPS and MPS regions and their matching

(3.1) CPS region in the $\boldsymbol{\varepsilon_{Dm}} = \mathbf{0}$ and $\boldsymbol{\varepsilon_{mc}} = \mathbf{0}$ approximation. The CPS and the MPS are both quasineutral and therefore $n = exp(-\varphi)$. In the CPS region the relation

$$\frac{d\varphi}{dz} = \ \frac{1}{u}\left(\frac{du}{dz} - \frac{1}{\lambda_i}\right) \tag{11}$$

is fulfilled. Introducing $\eta = z/\lambda$ as the position variable in "CPS normalization", and taking into account the smallness of the parameter $\varepsilon_{mc} = 0$ we find from (7) – (10)

$$\{sin^2\alpha - u^2\}\frac{d\,u}{d\,\eta} = u^2 + \frac{\lambda}{\lambda_i}\, sin^2\alpha \qquad (12)$$

From this equation it is obvious that at the point in the CPS region where $u = sin\alpha$, or when the ion velocity along the magnetic field line $u_{\infty||}$ ($u = u_{\infty||}\, sin\alpha$, see also Eq. (17) below) is equal to one,

$$u_{\infty||} = 1\,, \qquad (13)$$

the value du/dz and also the electric field $(E = d\varphi/dz)$ have a singularity. This point we define as the MPS entrance (MPSE). Relation (13) in fact represents the Bohm-Chodura criterion for the ion velocity in its marginal form. This new definition of the MPSE is quite analogous to the definition of the DS entrance in the unmagnetized PWT layer [2, 4]. Close to the MPSE from the CPS side, when Eq. (12) acquires the form

$$\frac{dw_c^2}{d\eta} = -sin\alpha\left(1 + \frac{\lambda}{\lambda_i}\right), \qquad (14)$$

where $w_c = u - sin\alpha$.

(3.2) MPS region.

(3.2.1) As mentioned above, the characteristic scale length of the MPS is of the order of the ion gyro-radius, so that for MPS analysis it is convenient to introduce the "MPS – normalized" position coordinate $\zeta = /\rho_i$. Keeping in mind the smallness of the parameters $\varepsilon_{Dm} = 0$ and, $\varepsilon_{mc} = 0$, the system (7) – (10) can be reduced to the single equation

$$\frac{d}{d\zeta}u\frac{d}{d\zeta}\frac{u^2-1}{u}\frac{du}{d\zeta} + \frac{du}{d\zeta} + Sin\alpha\frac{d}{d\zeta}\frac{1}{u} = 0. \qquad (15)$$

where the relations

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\zeta} = \frac{1}{u}\frac{du}{d\zeta} \qquad (16)$$

and $n = exp(-\varphi)$ have been used. Integrating Eq. (15) twice and using the boundary conditions,

$$\frac{\mathrm{du}}{\mathrm{d}\zeta} \to 0 \quad \text{and} \quad \mathrm{u} \to \mathrm{u}_\infty \left(= u_{\infty||} sin\alpha\right) \quad \text{at} \quad \zeta \to -\infty\,, \tag{17}$$

we find [4]

$$\frac{1-\mathrm{u}^2}{u}\frac{\mathrm{du}}{\mathrm{d}\zeta} = cos\alpha \cdot \sqrt{f(u)}\,, \tag{18}$$

where

$$f(u) = u_{\infty||}^2 + 2ln\left(u/u_{\infty||} sin\alpha\right) - u^2 - \left\{\frac{1}{cos\alpha}\left(u_{\infty||} + \frac{1}{u_{\infty||}}\right) - \frac{sin\alpha}{cos\alpha}\left(u + \frac{1}{u}\right)\right\}^2 . \tag{19}$$

The function $f(u)$ satisfies the following relations

$$f(u_\infty) = 0\ , \tag{20}$$

$$\frac{df}{du}\Big|_{u_\infty} = f'(u_\infty) = 0\,, \tag{21}$$

$$\frac{d^2f}{du^2}\Big|_{u_\infty} = f''(u_\infty) = \frac{2}{u_\infty^4} tg^2\alpha(1-u_\infty^2)\left(u_{\infty||}^2 - 1\right). \tag{22}$$

$$\frac{d^3f}{du^2}\Big|_{u_\infty} = f'''(u_\infty) = \frac{4}{u_\infty^3}\left(1 + \frac{3}{cos^2\alpha}\frac{1-\mathrm{u}_{\infty||}^2}{\mathrm{u}_{\infty||}^2}\right) \tag{23}$$

(3.2.2). First we analyze the region close to the MPS–DS interface from the MPS side. From Eqs. (16) and (18) it follows that at $u \to 1$ the electric field runs into a singularity. The corresponding point defines the DS entrance (DSE). The equation describing the region close to the DSE from the MPS side, $u - 1 = w_{mD}$ ($w_{mD} \ll 1$), then acquires the form

$$\frac{dw_{mD}^2}{d\zeta} = -cos\alpha\sqrt{f(1)}\,. \tag{24}$$

The formal similarity with Eq. (14) is obvious.

(3.2.3). Now we investigate the region close to the CPS–MPS interface from the MPS side. For small velocity shift $u - u_{||}\, sin\alpha = w_{mc}$, which takes place at $\zeta \ \rightarrow -\infty$, from (18) – (22) we obtain the equation

$$\frac{d^2 w_{mc}}{d\zeta^2} = \frac{u_{\infty||}^2 - 1}{u_{\infty||}^2 (1 - u_\infty^2)} w_{mc}. \tag{25}$$

Hence because $u_\infty^2 < 1$, for the monotonic decay of the potential at $\zeta \ \rightarrow -\infty$ (according to Eq. (16) $\varphi - \varphi_m \ \propto w_{mc}$, where φ_m is the potential at the MPSE) the Bohm – Chodura criterion, must be fulfilled in the non-marginal form: Eq. (25) demands supersonic flow $u_{\infty||}^2 > 1$ along the magnetic field lines. The same result was derived by Chodura from the different arguments based on plasma dispersion relation [2, 4].

(3.3) Matching of the MPS and CPS regions. As it is shown above, the equations (14) and (25) (and therefore their solutions), describing separately the CPS and the MPS sublayers at the vanishing parameter $\varepsilon_{mc} = 0$, do not match smoothly: at the MPS entrance from the CPS side the electric field runs into a singularity indicating that the subsequent interface is infinitely thin on the CPS scale $\eta = z/\lambda$; while from the MPS side (it means on the MPS scale $\zeta = z/\rho_i$) the MPS entrance is shifted in the infinity $\zeta \rightarrow -\infty$, where the electric field is zero $\left(\frac{d\varphi}{d\zeta} \rightarrow 0\right)$. As we see the behaviour of plasma characteristics at the different sides of the CPS – MPS interface is quite similar to the one observed at CPS-DS matching in the unmagnetized PWT layer [4, 5]. Therefore we can follow the procedure used there. The matching of the CPS and the MPS sublayers for small but arbitrarily finite ε_{mc} ($\varepsilon_{mc} \rightarrow 0$) requires some common basis for both sublayers to be described simultaneously. For this purpose we try to construct an equation for a space region termed as “intermediate region”, which exists between the two regions and where the properties of both regions co-exist. In principle, the equation for the intermediate region should consider the collision effects of the CPS (which are absent in the MPS) as well as take into account the ion gyro- motion in the magnetic field (which is neglected in the CPS). Hence both regions must be treated on same appropriate scale for better comparison of their involvements. So we introduce the normalized coordinate $\chi = z/l_{mc}$ for the intermediate region, with the "intermadiate" scale length l_{mc} being from the supposed range $\rho_i \ll l_{mc} \ll \lambda$. Under these restrictions we obtain from (7) – (10) an equation which bridges the CPS and the MPS sublayers:

$$\frac{d}{d\chi}\left\{\frac{\rho_i^2}{l_{mc}^2} u \frac{d}{d\chi}\frac{u^2-1}{u}\frac{du}{d\chi} + \left(u + \frac{sin^2\alpha}{u}\right)\right\} =$$

$$= -\frac{l_{mc}}{\lambda}\left\{1 + \frac{\lambda}{\lambda_i}\frac{sin^2\alpha}{u^2}\right\}. \qquad (26)$$

In fact, assuming formally $l_{mc} \simeq \lambda$ we obtain Eq. (12). If $l_{mc} \simeq \rho_i$ then (26) reduces to Eq. (15). In addition to the normalized length scale, we introduce in the intermediate region the normalized velocity w according to the relation $u - sin\alpha = \beta w$. By appropriately choosing the characteristic scale l_{mc} and the coefficient β we can make the contribution of the collision effect (the rhs of Eq. (26)) to have the same order as the contribution of the ion gyro-motion (the first term in the lhs of Eq. (26)). Assuming the contributions of every term in (26) to be equal we find

$$l_{mc} = \rho_i^{4/5}\lambda^{1/5} \qquad \text{and} \qquad \beta = (\rho_i/\lambda)^{2/5}\,. \qquad (27)$$

We see that the powers in (27) are the same as those obtained for the problem of the CPS–DS matching in the unmagnetized PWT layer [4]. The intermediate-region equation (26) can be represented in the form

$$\frac{d^2w}{d\chi^2} = \frac{1}{sin\alpha \cdot cos^2\alpha} w^2 + \left(1 + \frac{\lambda}{\lambda_i}\right)\frac{1}{cos^2\alpha}(\chi - \chi_m), \qquad (28)$$

where χ_m is the value corresponding to the MPS entrance.

4. Analysis of the DS region and its matching with MPS

When traveling from the unperturbed plasma to the wall, the ions are first accelerated along the magnetic field fines in the CPS. Then, in the MPS they are progressively reoriented towards the wall. And finally, in the DS, they are strongly accelerated in the direction normal to the wall. The DS entrance from the MPS side in the $\varepsilon_{Dc} = 0$ approximation has already been defined above (see section (**3.2.2.**)). On the MPS scale it is a singular point of the electric field. In the approximation $\varepsilon_{Dm} \equiv (\lambda_D/\rho_i) = 0$ and $\varepsilon_{Dc} \equiv (\lambda_D/\lambda) = 0$ the behaviour of ions in the DS is described by the equations

$$\frac{d}{d\xi} nu = 0, \tag{29}$$

$$u\frac{du}{d\xi} = \frac{d\varphi}{d\xi}, \tag{30}$$

$$\frac{d^2\varphi}{d\xi^2} = n - exp\big(-(\varphi - \varphi_s)\big), \tag{31}$$

which follow from the system (7) – (10). In (29) – (31) the dimensionless coordinate $\xi = z/\lambda_D$ is introduced and also the Poisson equation rather than the quasineutrality condition is used. In the sheath variable ξ the DSE is shifted to the infinity ($\xi \to -\infty$), where $\varphi \to \varphi_s$, and $u \to u_s$ (φ_s and u_s are the potential and the ion velocity at the DSE) [4, 5]. In the region close to the DSE from the DS side Eqs. (29) and (30) give

$$n = 1 - \frac{\varphi - \varphi_s}{u_s} + \frac{3}{2}\frac{(\varphi - \varphi_s)^2}{u_s^4}, \tag{32}$$

and for the potential we obtain the equation

$$\frac{d^2(\varphi - \varphi_s)}{d\xi^2} = \frac{u_s^2 - 1}{u_s^2}(\varphi - \varphi_s)\,. \tag{33}$$

Hence the Bohm condition must be fulfilled in the non-marginal form, $u_s^2 > 1$, Eq. (33) formally coincides with Eq. (25), describing the ion velocity at the CPS–MPS interface from the MPS side. Moreover the situation is quite analogous to the fracture on the shape of the electric potential at the CPS-MPS interface: on the MPS scale the electric field in the $\varepsilon_{Dm} = 0$ and $\varepsilon_{Dc} = 0$ approximation again runs into a singularity, while on the DS scale the MPS–DS interface is shifted to infinity, $(\xi \to -\infty)$, where the electric field according to Eq. (33) tends to zero [6]. Therefore, for matching such distinctly different sublayers as the MPS and the DS (the MPS is quasi-neutral, while in the DS there is space charge and the influence of the magnetic field is negligible), we can repeat the procedure used for matching the CPS with the MPS given above in section **(3.3)**.

To bridge the MPS and the DS we again assume the existence of an intermediate region between them. Both MPS and DS are collisionless in their bulks thanks to the smallness of parameters ε_{Dm} and ε_{mc} : $\varepsilon_{Dm} = \frac{\lambda_D}{\rho_i} \to 0$ and $\varepsilon_{mc} = \rho_i/\lambda \to 0$. Introducing the normalized coordinate, $= z/l_{mD}$, quite analogous to [4] we can construct the equation, which describes the properties of both sublayers.

$$\frac{\lambda_D^2}{l_{mD}^2}\frac{\partial^2 w}{\partial \tau^2} = w^2 + \frac{l_{mD}}{\rho_i} cos\alpha\sqrt{f(1)}(\tau - \tau_s)\,. \qquad (34)$$

In deriving this equation the relations $u = 1 + w \ (w \ll 1)$ and $\varphi - \varphi_0 \backsimeq w$ have been are used, τ_s is the DS entrance position in the limit $\varepsilon_{Dm} = 0$ and $\varepsilon_{mc} = 0$. For $l_{mD} \backsimeq \lambda_D \ (\lambda_D \ll \rho)$ Eq. (34) reduces to the equation describing the DS in the region close to its entrance (see Eqs. (32) and (33) with $u_s = 1,$ when in the Poisson equation (33) the third term in the rhs of (32) is taken into account). For the case when $l_{mD} \backsimeq \rho_i$, Eq. (34) gives a result which correctly describes the MPS at the DS entrance (cf., Eq. (24)). By appropriately choosing the characteristic scale length l_{mD} we can make the contribution of the magnetic field to have the same order as that of charge separation. For the renormalized potential w_{mD} $(w = sw_{mD})$ we obtain from (34)

$$\frac{\partial^2 w}{\partial \tau^2} = w^2 + cos\alpha\sqrt{f(1)}(\tau - \tau_s)\,. \qquad (35)$$

The intermediate scale-length has the form

$$l_{mD} = \lambda_D^{4/5} \cdot {\rho_i}^{1/5} \quad \text{and} \quad s = (\lambda_D/\rho_i)^{2/5}\,. \qquad (36)$$

The similarity with (27) is obvious. The intermediate equations (28) and (35) both represent the Painlev*é* equation. By introducing appropriate new variables one can represent these equations in the form

$$\frac{\partial^2 w}{\partial x^2} = w^2 + x\,. \qquad (37)$$

Analytic and numerical investigation of the Painlev*é* equation can be found in [4, 5, 7]. In Fig. 3, the solid line shows the solution of Eq. (37), which obviously It is obvious bridges the solutions of the neighbouring regions.

5. Summary and conclusion

In the unmagnetized PWT layer the Painlev*é* equation, which governs the intermediate region between the CPS and the DS, in fact represents Poisson's equation [4], In our analysis of the CPS –MPS transition, on the other hand, we have used the quasi-neutrality condition (instead of Poisson's equation) from the very beginning, again obtaining the Painlev*é* equation for the CPS-MPS intermediate region.

The intermediate scale of the DS-CPS transition *without magnetic field* was found in the form $l_r = (\lambda_D^4 \cdot \lambda)^{1/5}$ [4]. According to our results (27) and (36), the dependence of the characteristic scale-lengths on the sublayers' parameters for the magnetized PWT is quite similar. These findings indicate that in the hydrodynamic, $T_i = 0$, approximation the intermediate scale between two sublayers 1 and 2 with characteristic lengths λ_1 and λ_2, $\lambda_1 < \lambda_2$, has the general form $l_r = \lambda_1^{4/5} \cdot \lambda_2^{1/5}$.

Apparently both the CPS-MPS and the MPS-DS transitions are described by the Painlevé equation (see (28) and (35)), quite similar to the case of the *unmagnetized* PWT layer [1]. Hence one can say that at least for the cold-ion hydrodynamic approximation the Painlevé equation plays somewhat a universal role in the matching procedure.

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Figure captions

Fig.1 Structure of the MPWT.

Fig.2 MPWT geometry.

Fig.3 Matching potential $w(x)$ on the intermediate scale (see Eq. (37)). The point $x = 0$ corresponds to the DSE (or MPSE). The curve $w_l(x)$ describes the potential in the DS region close to the DSE (or in the MPS region close to MPSE). The curve $w_r(x)$ corresponds to the MPS region close to the DSE (or to the CPS region close to the MPSE) .

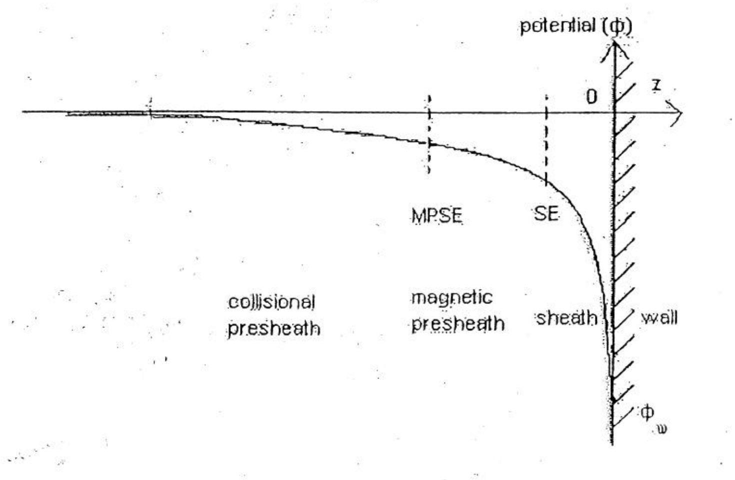

Fig.1

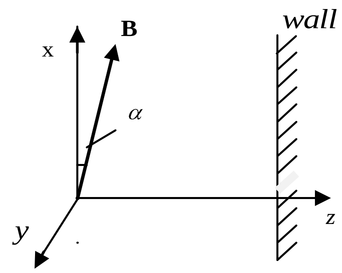

Fig.2

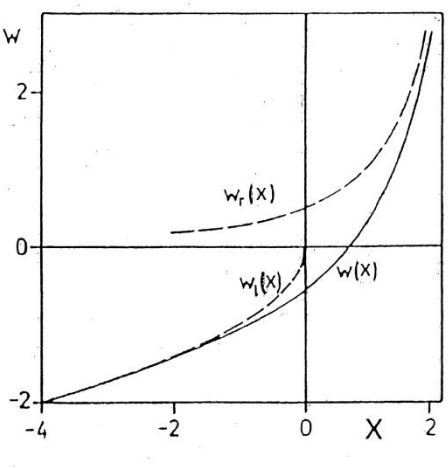

Fig.3