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Modeling dispersive MHD turbulence

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Modeling dispersive MHD turbulence

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Outline

1. Brief review of solar wind and magnetosheath observations

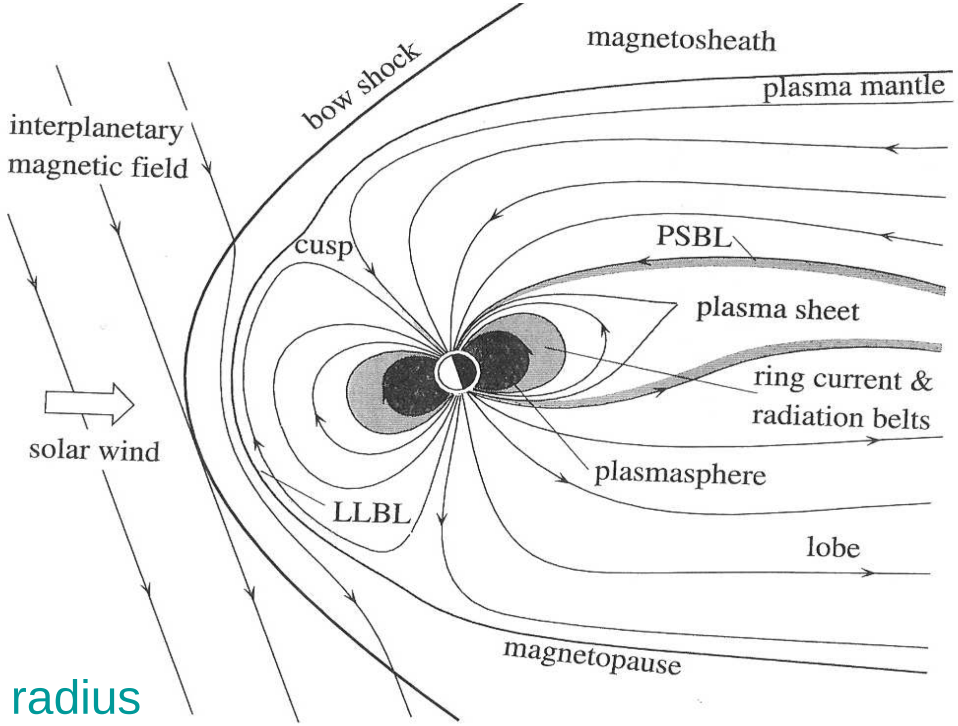
2. The DNLS equation as model of strong dispersive Alfvenic turbulence

3. Hall-MHD in one space dimension

4. Landau –fluid simulations (retain low-frequency kinetic effects)

5. Summary and conclusion

1. Solar wind and magnetosheath observations



Space plasmas such as the
solar wind or the Earth magnetosheath :

- Natural laboratories for accurate in situ measurements

- Turbulent magnetized plasmas with essentially no collisions.

- Cascades extend beyond the ion Larmor radius

- Small-scale coherent structures (filaments, shocklets, magnetosonic solitons, magnetic holes) with typical scales of a few ion Larmor radius.

- Dispersive and kinetic effects play a role.

Solar wind observations

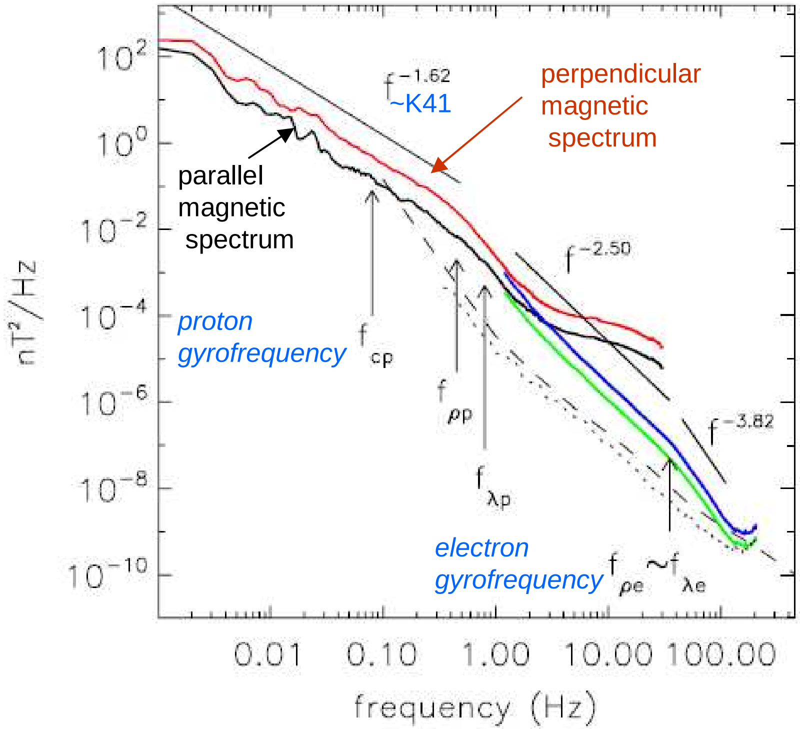
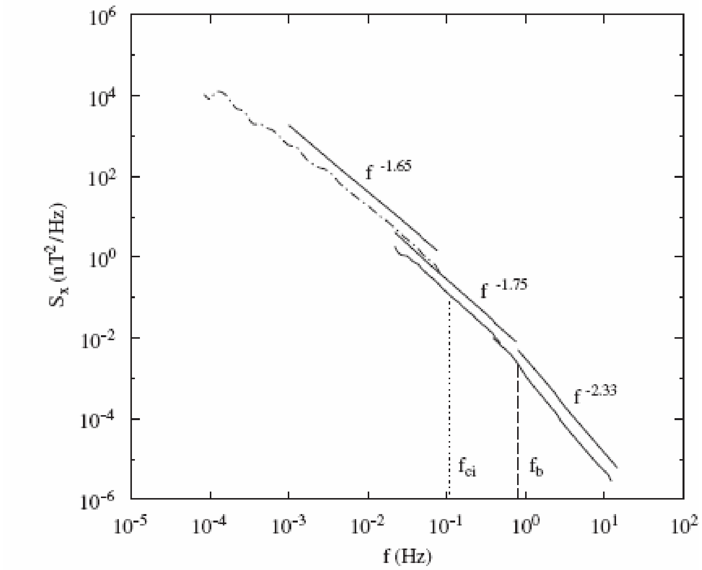


FIG. 2 (color online). The parallel (black) and perpendicular (red) magnetic spectra of FGM data ($f < 33$ Hz) and STAFF-SC data (respectively, light line; green online and dark line; blue online); $1.5 < f < 225$ Hz). The STAFF-SC noise level as measured in the laboratory and in-flight are plotted as dashed and dotted lines, respectively. The straight black lines are power law fits to the spectra. The arrows indicate characteristic frequencies defined in the text.

Sahraoui et al. PRL 102, 231102 (2009)



Energy spectra of B_x fluctuations measured by Cluster (full line) and by Helios 2 (dashed-dotted line)

Alexandrova et al. Planet. Space Sci. 55, 2224 (2007)

Excess of magnetic energy in the transverse components

Does the anisotropy persist at small scales?

Several power-law ranges:

Which cascades? Which waves? Which slopes?

At what scale does dissipation take place?
By which mechanism?
Important to estimate heating.

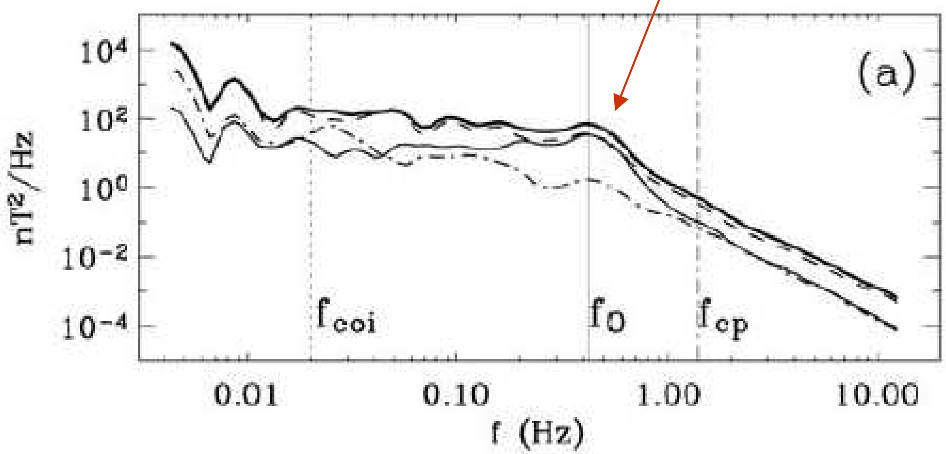
In the Earth magnetosheath

Important role of the temperature anisotropy:
Anisotropic ion cyclotron and mirror instabilities

Formation of coherent structures:
Magnetic filaments near the quasi-perpendicular shock.
Nonlinear mirror modes closer to the magnetopause

Alfvén vortices

The bump observed in the spectrum is associated with magnetic filaments (Alfvén vortices).



Drift kinetic Alfvén vortices also observed in the cusp
(*Sundkvist, Krasnoselskikh & Shukla, Nature **436**, 825, 2005*)

Alexandrova et al. JGR 111, A12208 (2006).

How to model turbulence in space plasmas?

3D Vlasov simulations are hardly possible on present-day computers.

Gyrokinetic simulations (*G. Howes PoP **15**, 055904 , 2008)* show the presence of cascades both in the physical and velocity spaces in the range $k_\perp \rho \geq 1$.

Applicability to space plasmas of the gyrokinetic theory (that concentrates on the quasi-transverse dynamics and average out the fast waves) is still to be validated.

Fluid models remain useful at large scales.

Simplest extension of usual MHD:

Hall-MHD: bifluid description where electron inertia is neglected.
Hall term is supplemented in the induction equation: ⟹ dispersive effects

Further extensions:

Landau fluids: retain low-frequency kinetic effects (Landau damping and FLR corrections)

Hall-MHD equations

$$\partial_t \rho + \nabla.(\rho \mathbf{u}) = 0$$

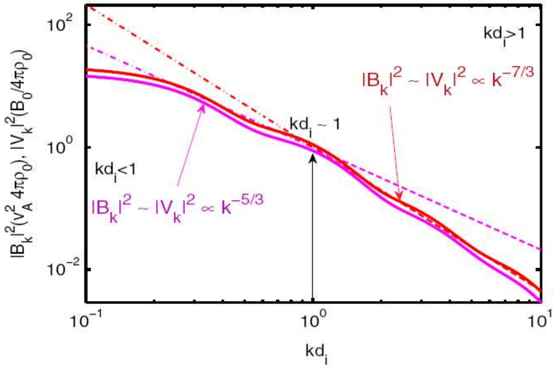
$$\rho(\partial_t \mathbf{u} + \mathbf{u}.\nabla \mathbf{u}) = -\frac{\beta}{\gamma}\nabla \rho^\gamma + (\nabla \times \mathbf{b}) \times \mathbf{b}$$

$$\partial_t \mathbf{b} - \nabla \times (\mathbf{u} \times \mathbf{b}) = \underbrace{-\frac{1}{R_i}\nabla \times \left(\frac{1}{\rho}(\nabla \times \mathbf{b}) \times \mathbf{b}\right)}_{\text{Hall term}}$$

$$\nabla.\mathbf{b} = 0$$

velocity unit: Alfvén speed
length unit : $R_i \times$ ion inertial length
time unit: $R_i \times$ ion gyroperiod
density unit: mean density
magnetic field unit: ambient field

In the presence of an ambient field, the Hall term induces dispersive effects.



Shaikh and Shukla, PRL 102, 045004 (2009)

Special attention paid to Alfvén waves
(that propagate on long distances without significant damping).

Focus on the Alfvén wave dynamics can be made using a long-wavelength reductive perturbative expansion (assume small amplitude weakly dispersive waves).

For waves propagating parallel or quasi-parallel to the ambient field:

Reductive perturbative expansion $\Longrightarrow$ derivative nonlinear Schrödinger equation

DNLS equation valid in any oblique direction in the large β limit (Ruderman, JPP 67, 271, 2002)

2. Dispersive Alfvenic turbulence

$$b = b_y + ib_z$$

The derivative nonlinear Schrödinger equation

$$\partial_t b + \alpha \partial_x (|b|^2 b) + i\delta \partial_{xx} b = 0$$

α depends on the beta of the plasma

Since DNLS equation is completely integrable by the inverse scattering technique, there is no resonant interactions and a regime of wave (or weak) turbulence is not possible.

In contrast, in the presence of driving and dissipation, a regime of strong dispersive turbulence characterized by the presence of structures can develop.

Driving:

Large-scale random forcing that is white noise in time in order to prescribe a constant rate of energy injection.

Dissipation:

Diffusivity $-\mu \partial_{xx} b$ (originating from viscosity and magnetic diffusivity in Hall-MHD equations: collisional plasma)

or Landau damping $\mathcal{H}(|b|^2) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{+\infty} \frac{|b|^2(x')}{x' - x} dx'$ (KDNLS, derived from Vlasov-Maxwell: non-collisional plasma)

lead to qualitatively similar results.

Randomly driven diffusive DNLS equation

$$\partial_t b + \partial_x(|b|^2 b) + i\delta\partial_{xx} b + \mu\partial_{xx} b = f$$

dispersion diffusion driving

No dispersion ($\delta = 0$) : Cohen-Kulsrud equation (Phys. Fluids 17, 2215, 1974):

Prototype of non strictly hyperbolic equation
(because of rotational invariance across the direction of propagation)

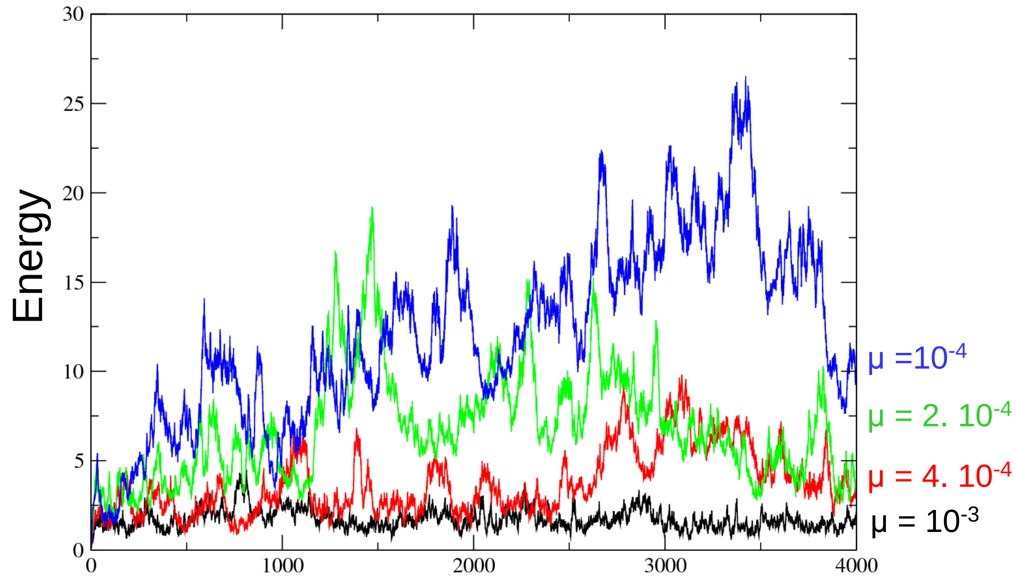
Model for intermediate shocks in MHD
(Kennel et al. Phys. Fluids B 2, 253 1990; 5 2877, 1993; Wu et al. PRL 68, 56, 1992)

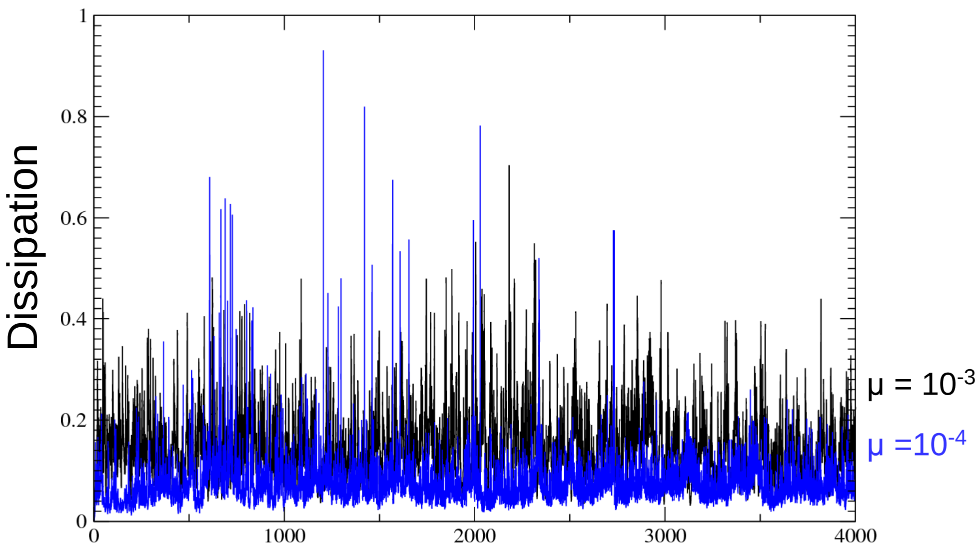
Question: Properties of the turbulent dynamics, depending on the dispersion and diffusion parameters.

Characteristics of the simulations:
Periodic domain of extension 16π
Driving at large scale (about $k=4\,\Delta k$) with a prescribed rate of energy injection ~ 0.012

Cohen-Kulsrud turbulence (no dispersion)

$$\partial_t b + \partial_x(|b|^2 b) = \mu \partial_{xx} b + f$$





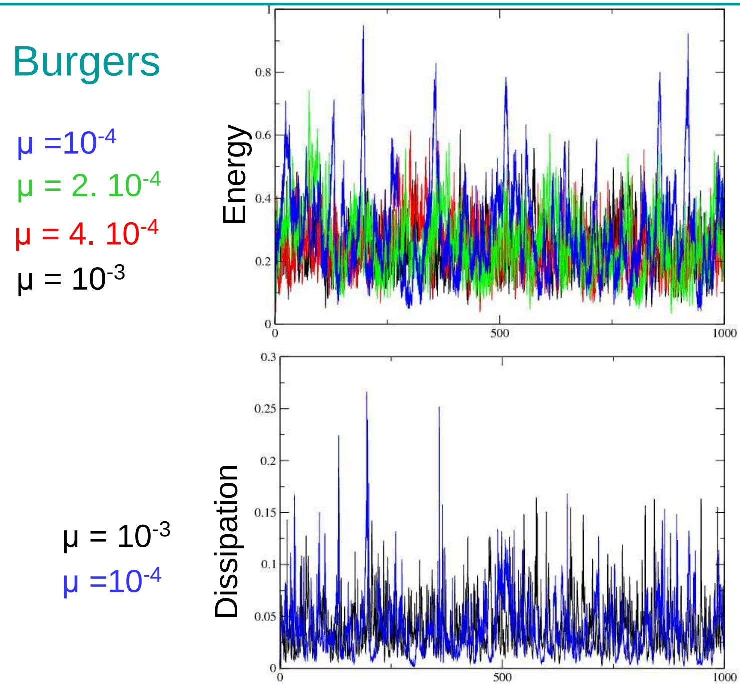
For fixed rate and scale of injection, the energy remains sensitive to the diffusion coefficient in the small dissipation limit: contrasts with the universality of Burgers and incompressible turbulence.

Small-scale dynamics affects the large-scale properties.

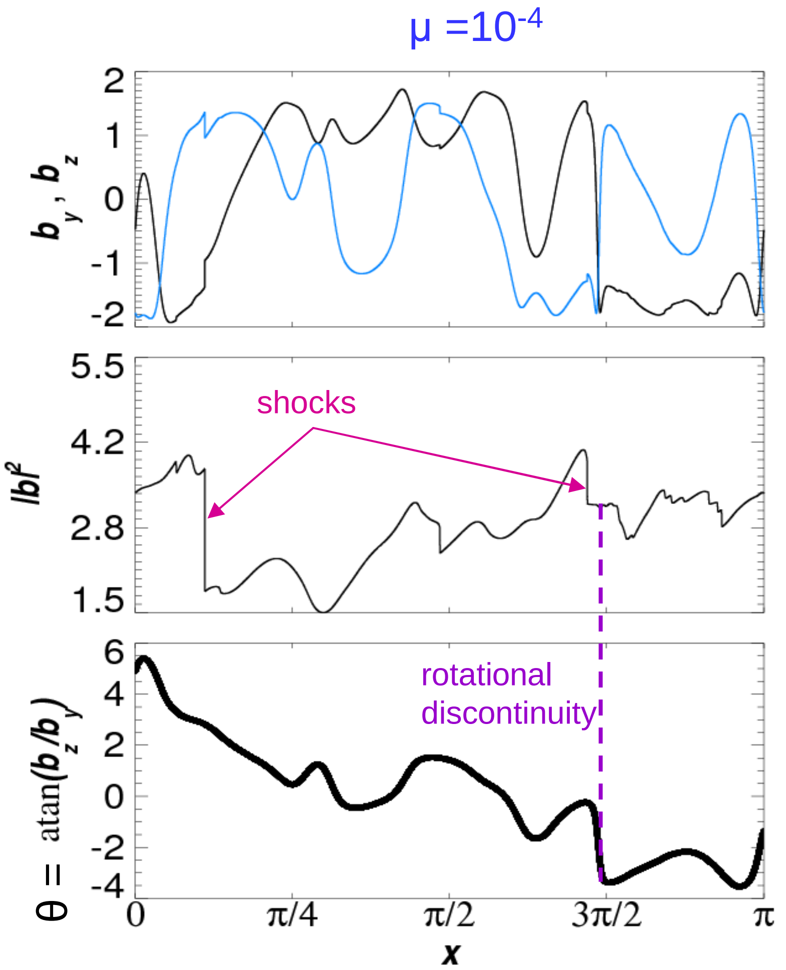
As the diffusion coefficient is reduced,

- slight decrease of the median dissipation
- higher and more intermittent peaks.

Burgers



The energy accumulation when the diffusion coefficient is small is associated with the formation of large-scale structures.



$$b = |b|e^{i\theta}$$

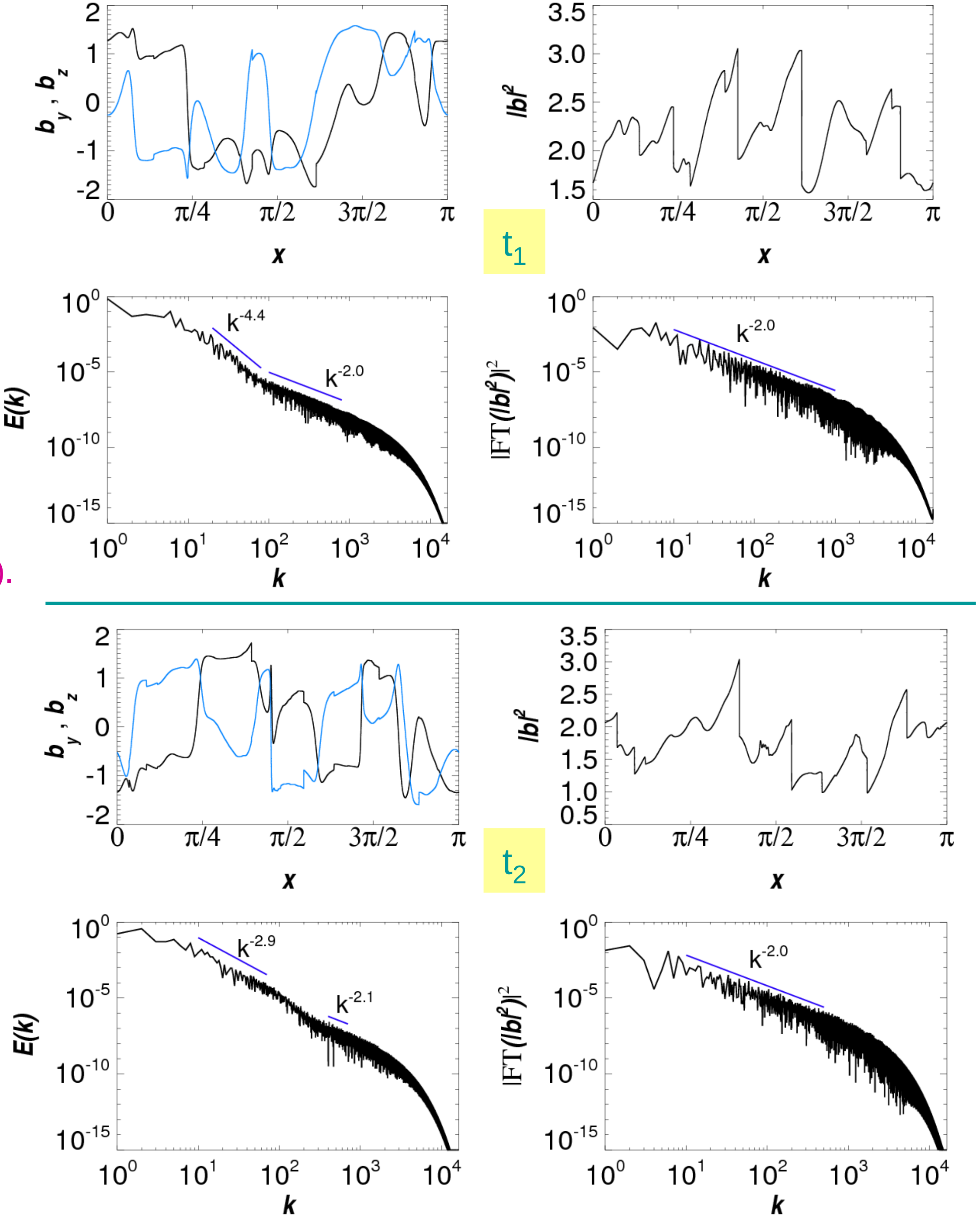
Singular structures include shocks and rotational discontinuities (change in the direction of the magnetic field without change in its magnitude)

$$b = |b|e^{i\theta}$$

$$\partial_t |b|^2 + 3|b|^2 \partial_x |b|^2 = \mu |b| [\partial_{xx}|b| - (\partial_x \theta)^2]$$
$$\partial_t \theta + |b|^2 \partial_x \theta = \mu [\partial_{xx}\theta + 2(\partial_x |b|)\partial_x \theta].$$

(in the absence of driving)

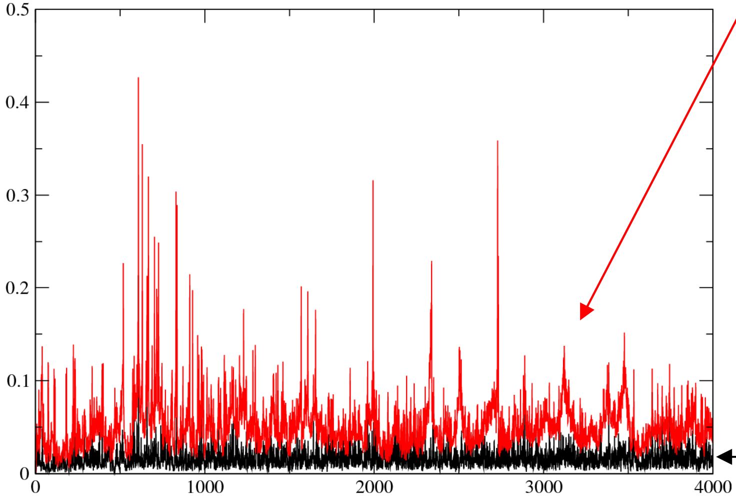
Energy spectrum: k^{-2} at small scales (shocks). Non universality at large and intermediate scales.



Dissipation:

$$2\mu\int\left(\frac{\partial b}{\partial x}\right)^2 dx=\mu\int\left(\frac{\partial |b|}{\partial x}\right)^2 dx+\mu\int|b|^2\left(\frac{\partial \theta}{\partial x}\right)^2 dx$$

shocks rotational discontinuities

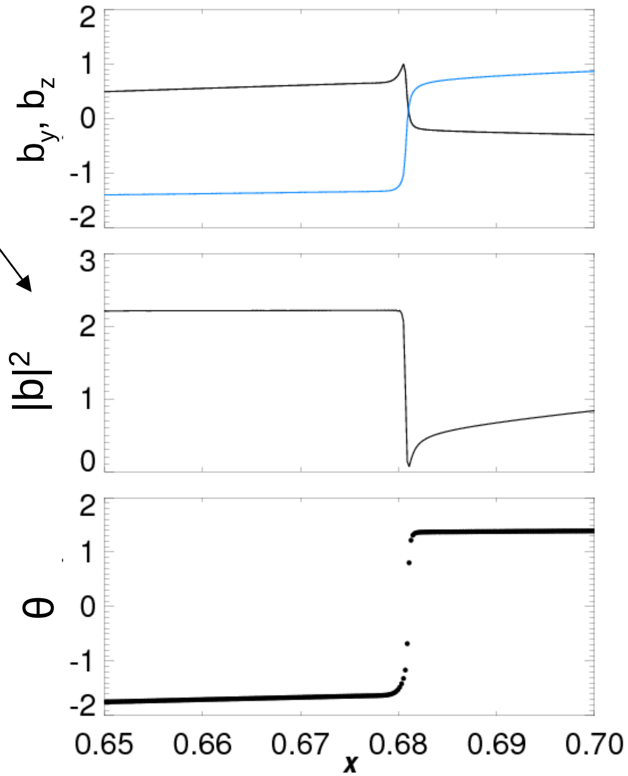


Contribution of the shocks

Dissipation originates from shocks and from rotational discontinuities (RD).

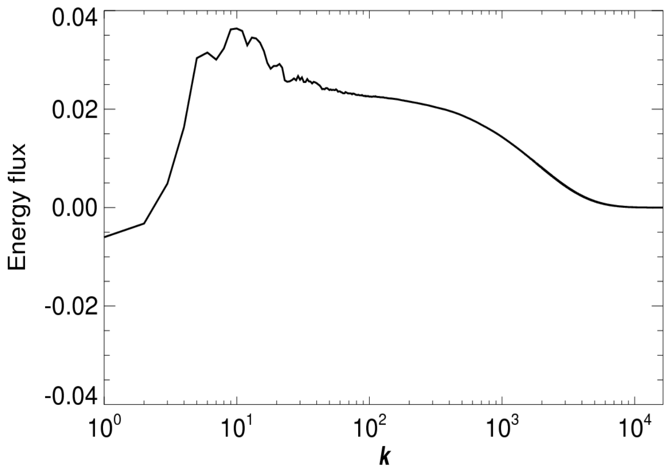
Both contributions remain comparable in the zero dissipation limit, but RD contribution always dominates.

Peaks of dissipation correspond to the coincidence of a shock (with overshooting) and of a rotational discontinuity.



Energy flux:

(averaged on 100 outputs distant of one time unit).



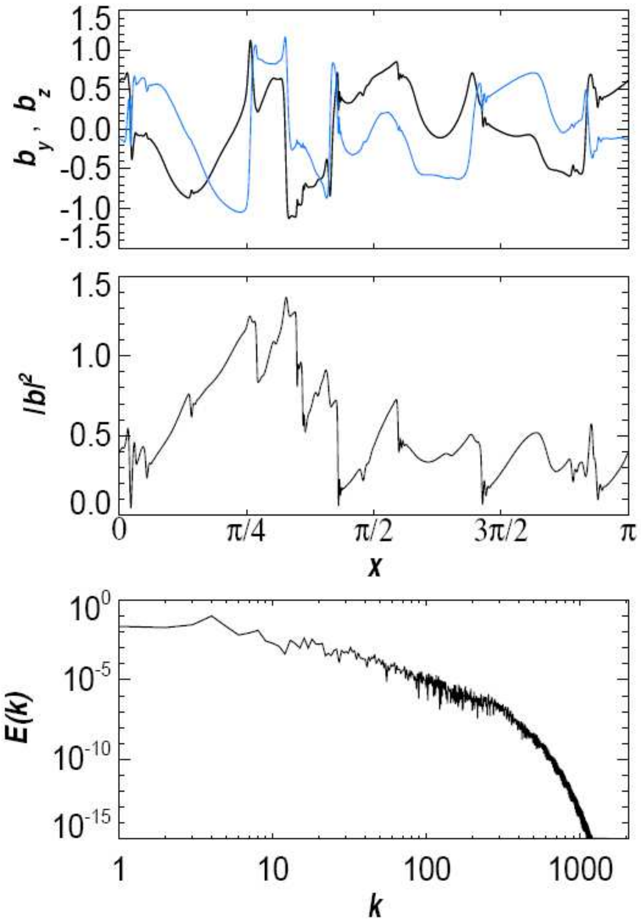
Dissipative DNLS equation:

Various kinds of structures depending on dissipation and dispersion

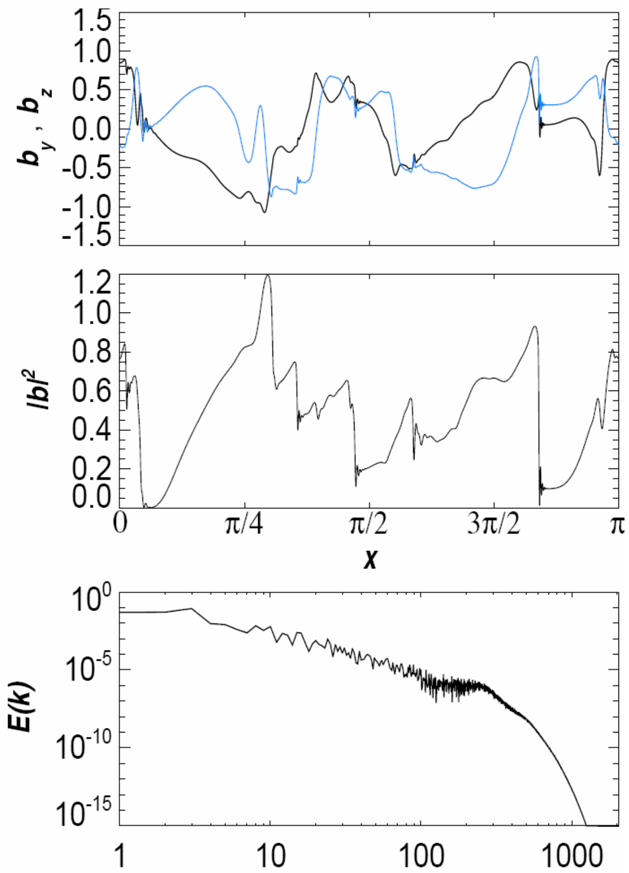
dispersion $\delta = 10^{-3}$
diffusion $\mu = 2.10^{-4}$
(same order of magnitude)

- Shocks become dispersive
- Flattening of the spectrum at small scales

(small-scales oscillations)



t=50

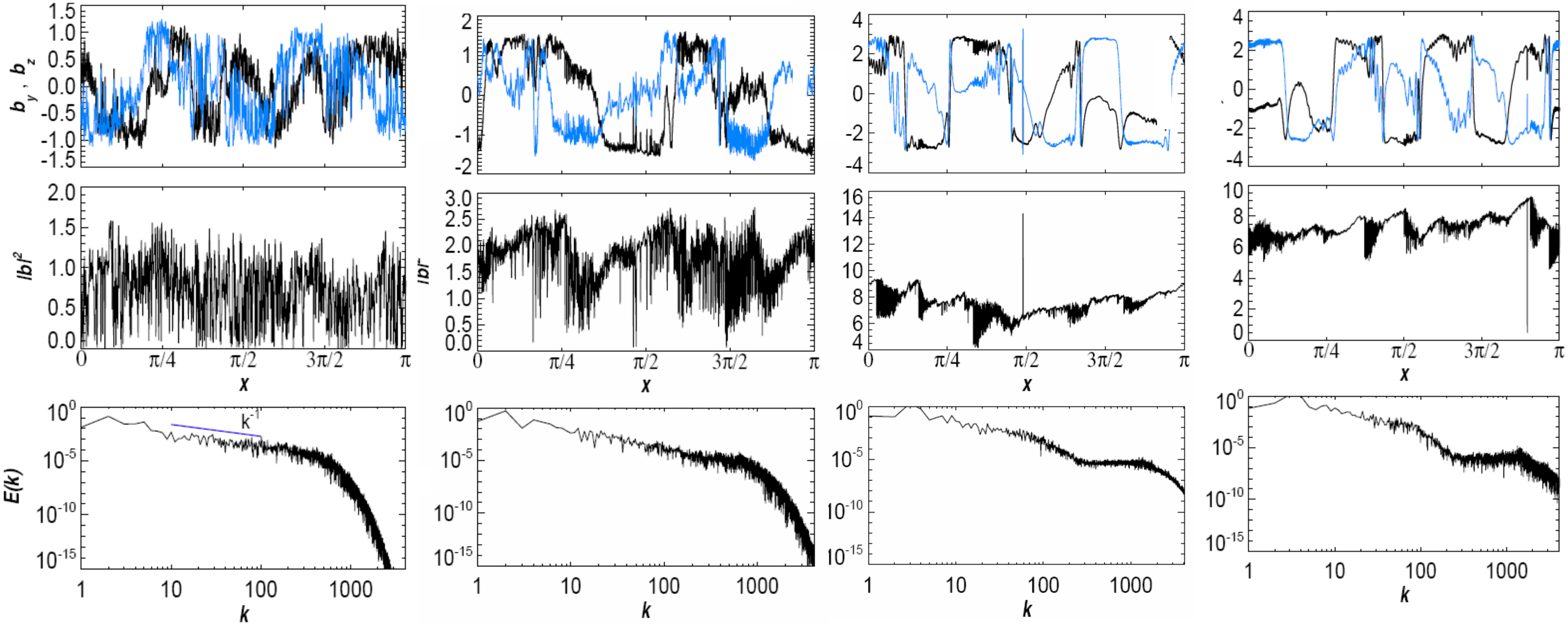


t=100

dispersion $\delta = 10^{-3}$
diffusion $\mu = 10^{-6}$

(smaller diffusion)

- Small-scale amplitude oscillations fill all the domain
- Beltramization of the large scales.
- Flattening of the small-scale spectrum
- Formation of intense bright and dark solitons



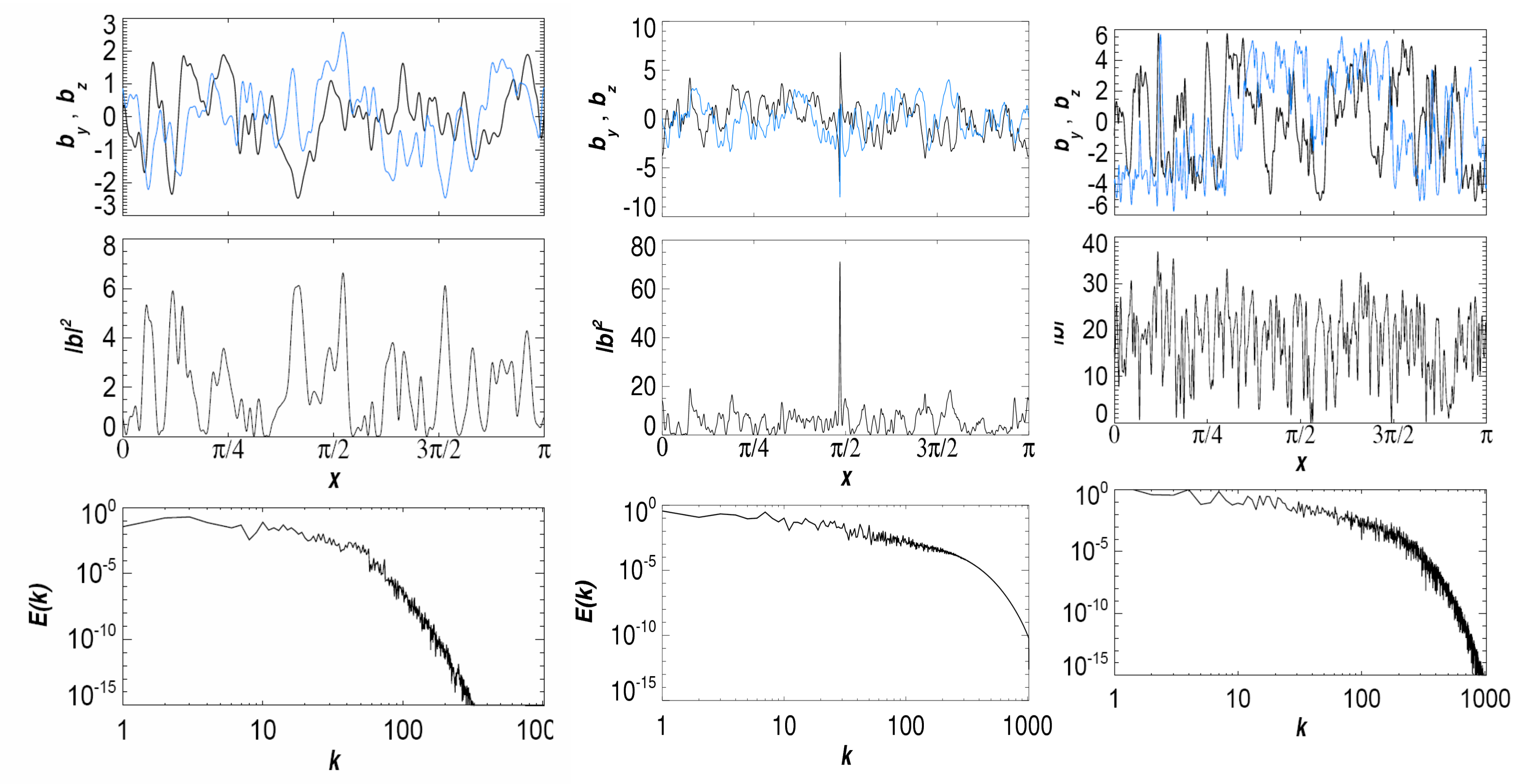
t=40 t=100 t=517 t=564.5

dispersion $\delta = 6.25\ 10^{-2}$
diffusion $\mu = 10^{-6}$

(stronger dispersion)

t

No shocks.
Early large-scales solitonic structures
evolve a quasi-collapse leading to small-scale fluctuations
No signficant beltramization



Early formed solitonic structures collapse into intense and persistent breathers
that then dissipate with strong radiation. Recurrent process.

Quasi-collapse of an oblique (i.e. with non-zero boundary conditions) bright DNLS soliton in the presence of a weak dissipation.

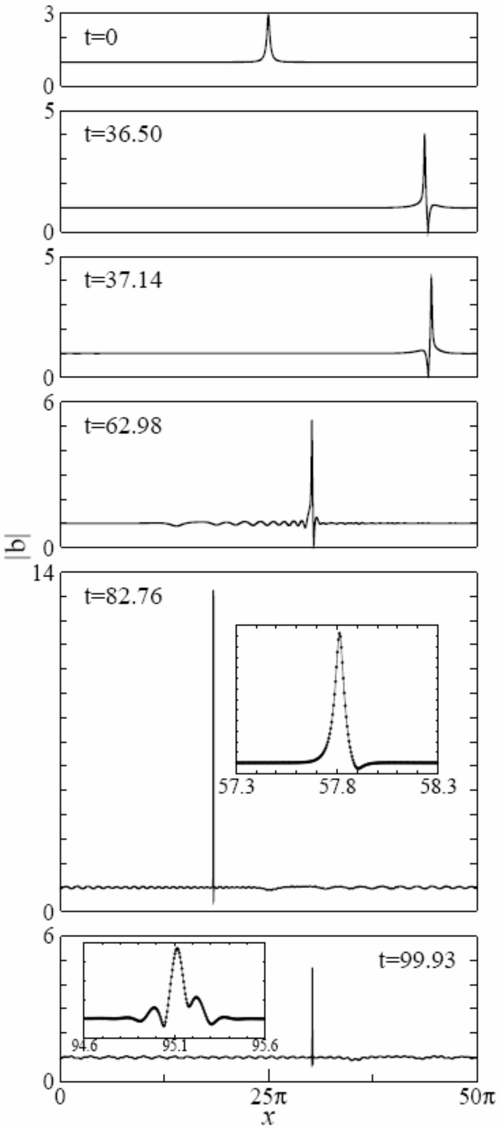
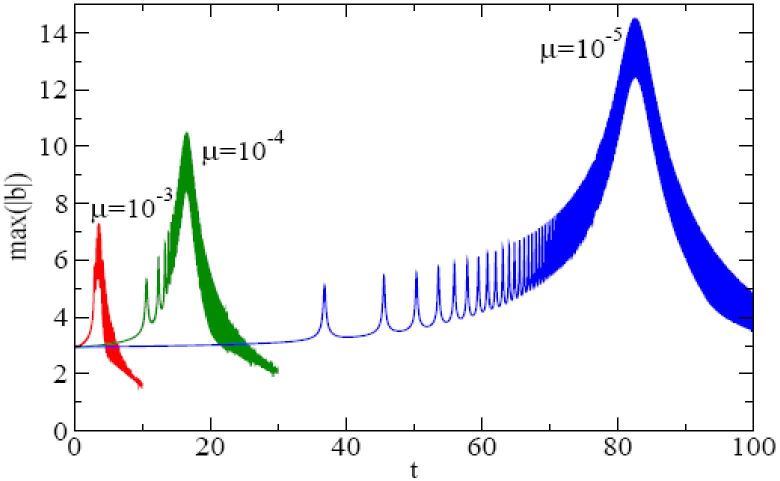
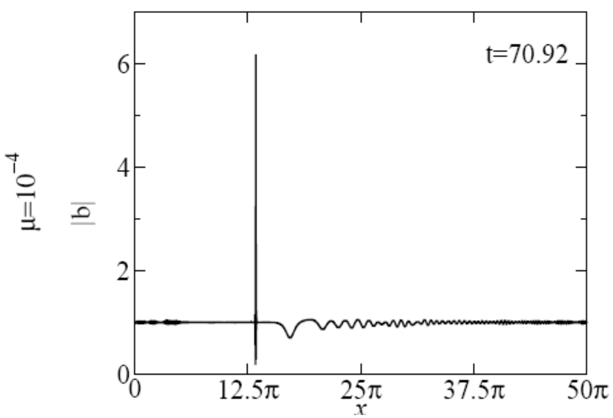


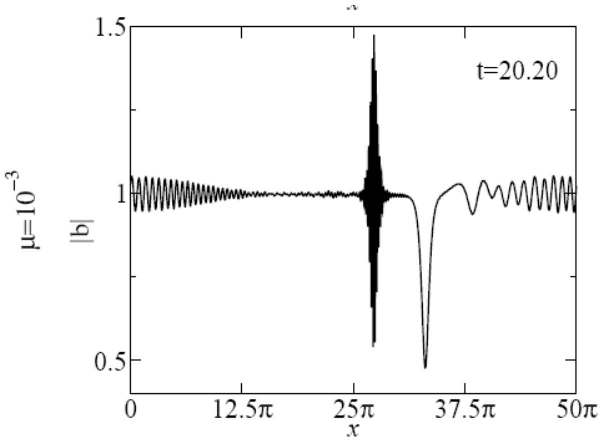
FIG. 1: Time evolution of an initial bright soliton with eigenvalue $\lambda = 0.975$, as prescribed by the diffusive DNLS equation with $\mu = 10^{-5}$. The inserts (where the mesh points are displayed) provide details of the solution near the soliton at the corresponding times.



Maximum soliton amplitude versus time for various diffusion coefficients.

(Sanchez-Arriaga et al., PRE, in press)

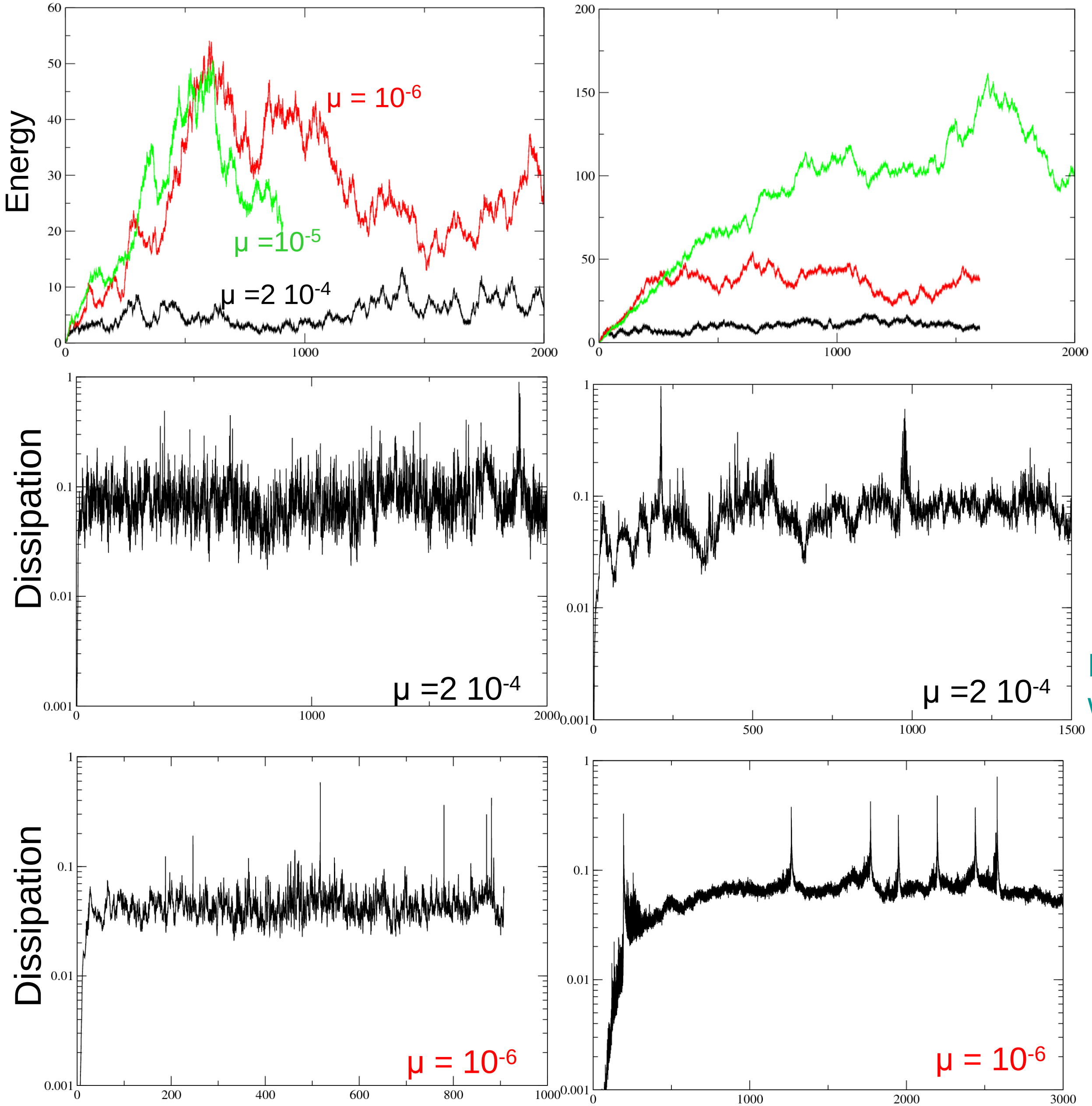




Formation of magnetic holes (Hamilton et al., JGR 2009) and of wavepackets.

Energetic quantities

$\delta = 10^{-3}$ $\delta = 6.2510^{-2}$

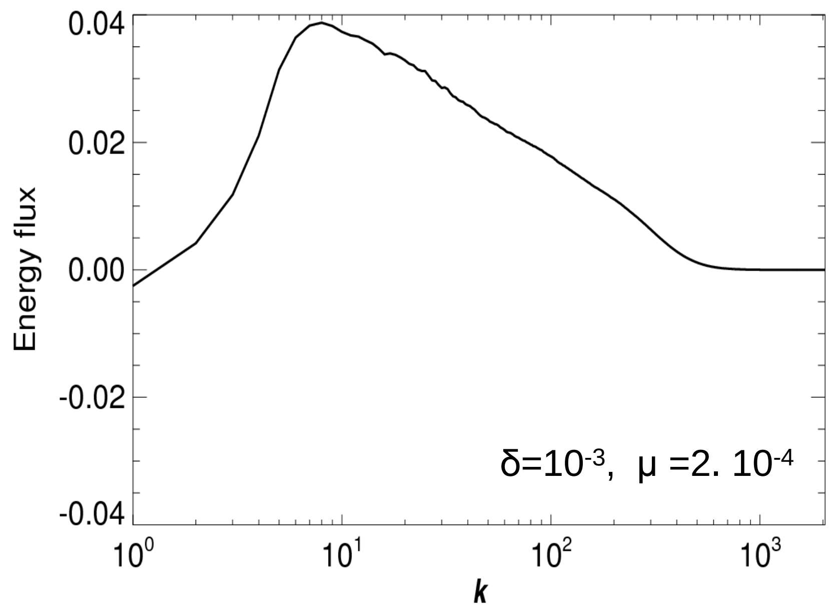


Energy saturates at a level that increases as the diffusion is reduced and/or the dispersion increased.

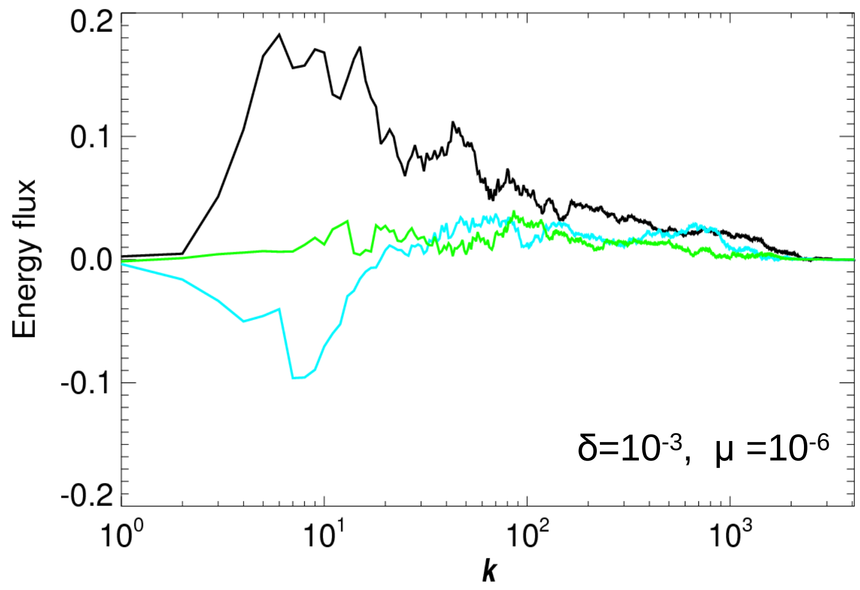
Dissipation peaks are associated with the dissipation of intense solitons.

Energy fluxes

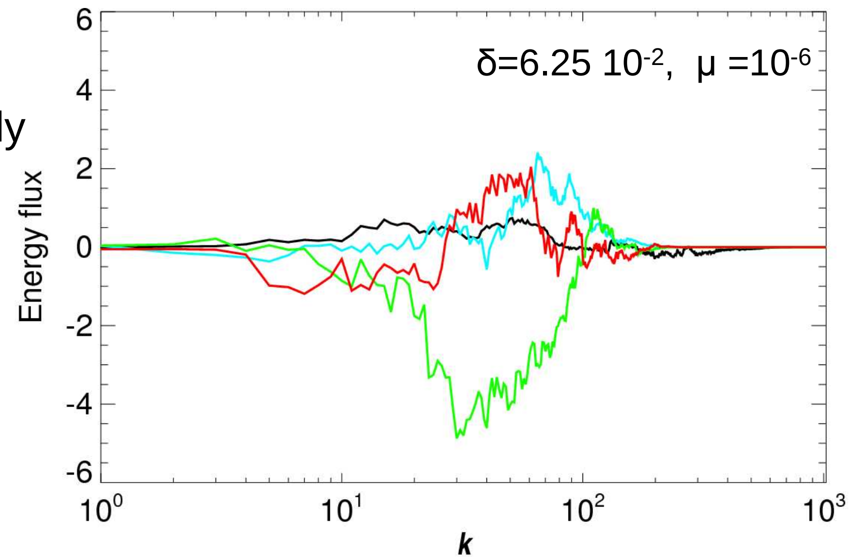
(averaged on 100 outputs distant by one time unit)



(various times)



In the presence of dispersion (even small) and a very small diffusivity, the energy flux strongly fluctuates both in time and from scale to scale.
No usual inertial range, but rather energy transfer through intermittent bursts.



Fluctuations of the energy flux are larger when the dispersion is stronger.

3. Hall-MHD

DNLS equation isolates Alfvén wave dynamics by averaging over the interactions with magnetosonic waves.

Valid in the limit of a long-wavelength weakly nonlinear asymptotics on a long but nevertheless limited time interval.

A more global picture of the plasma dynamics is obtained by retuning to the Hall-MHD equations

Hall-MHD is a rigorous limit of collisionless kinetic theory for:

$T_i << T_e$
$\omega << \Omega_i$
$k_{||} v_{thi} << \omega << k_{||} v_{the}$

Irose et al. , Phys. Lett. A 330, 474 (2004)
Ito et al., PoP 11, 5643 (2004)
Howes, NPG 16, 219 (2009)

It correctly reproduces whistlers and KAW's for small to moderate β.

It contains waves that are usually damped in a collisionless plasma and whose influence in the turbulent dynamics has to be evaluated.

One-dimensional simulations:

Random driving (white noise in time) on the **transverse components** of velocity *(kinetic driving)* or magnetic field *(magnetic driving)*.

Questions:

- Influence of the dispersion strength (by changing the size of the domain and thus the ratio between the scale of energy injection and the ion inertial length).
- Influence of the direction of propagation.
- Influence of the type of driving (kinetic or magnetic).
- Spectral transfer versus coupling between various MHD modes.

Laveder et al., Planet. Space Sci., in press

Parallel propagation, Large-scale kinetic driving

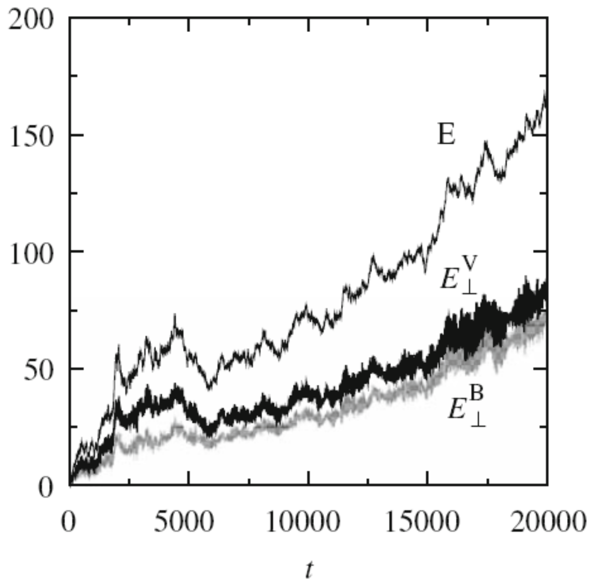
(weakly dispersive regime)

$L=16\pi\, l_i$

$K_{inj}\, l_i=1/2$

$\beta=2$

$\gamma=5/3$



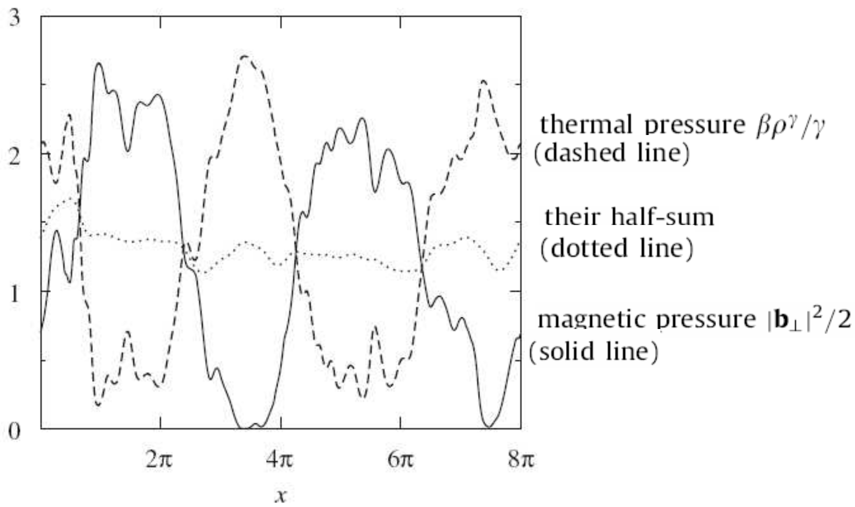
Energy does not saturate:

significant inverse transfer

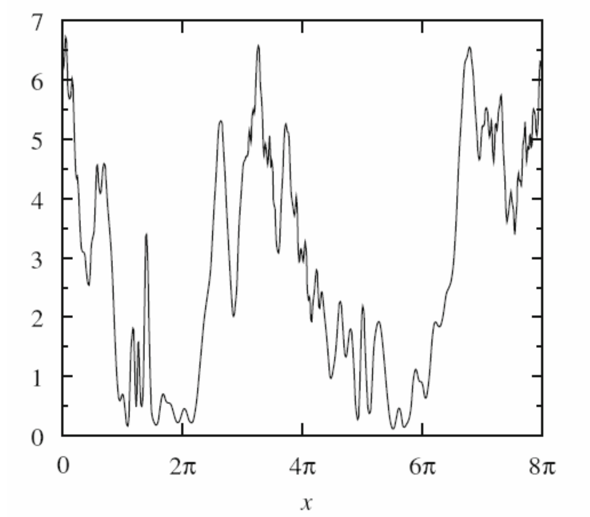
(time unit: inverse ion gyrofrequency)

Etablishment of a pressure-balanced state

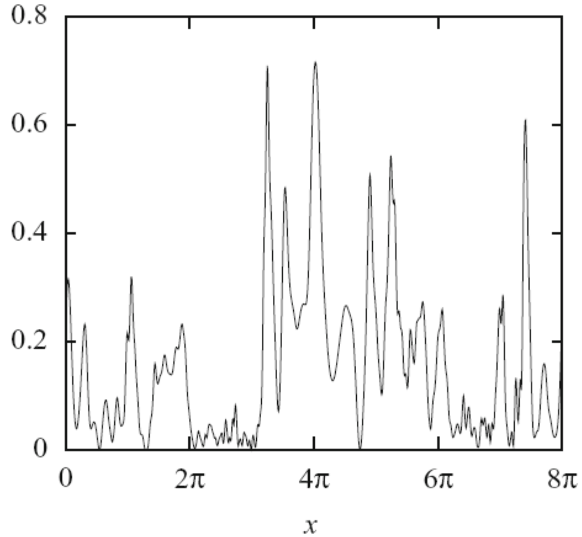
(not limited to largest scales: persists when largest scales are filtered out).



Transverse velocity $|\mathbf{v}_\perp|^2$



After filtering the n=1 mode



Solitonic waves

that survive collisions

Large-scale kinetic driving (continued)

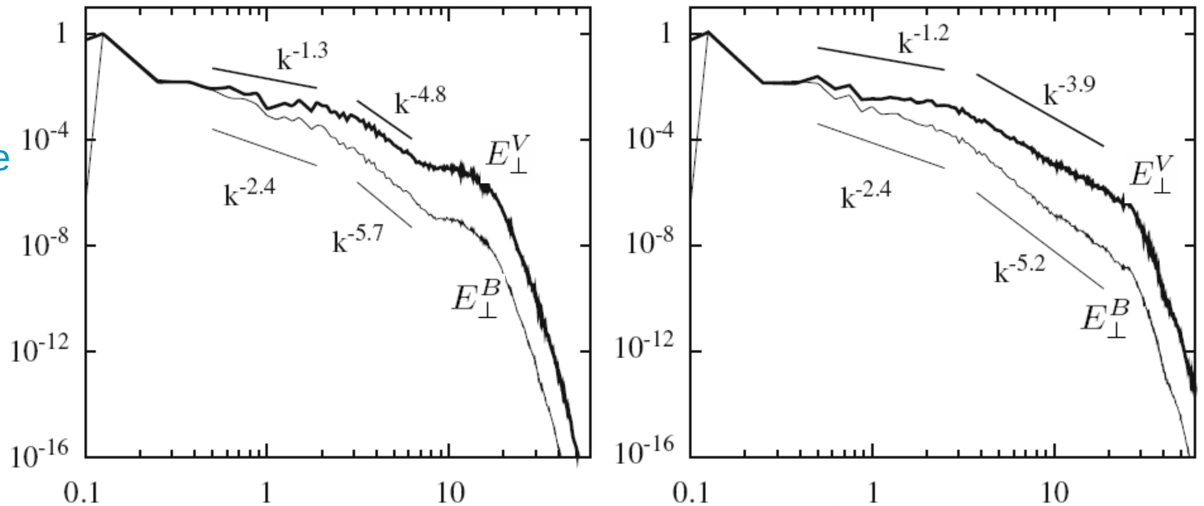
Energy spectra: Distinct power laws at large and small scales:

Transition near the ion inertial scale (consistent with solar wind observations: Leamon et al. '98, Golstein and Robert '99, Alexandrova et al. '06, Sahraoui et al. '09)

Non universal small-scale exponents

(influence of small-scale cusp-like structures and wave-packets)

Qualitatively similar to weakly dispersive DNLS spectrum.



Kinetic $|\hat{\mathbf{v}}_y|^2+|\hat{\mathbf{v}}_z|^2$ (thick line) and magnetic $|\hat{\mathbf{b}}_y|^2+|\hat{\mathbf{b}}_z|^2$ (thin line) spectra averaged over the time interval $t=18\,200-18\,250$ (left)
$t=19\,500-19\,550$ (right)

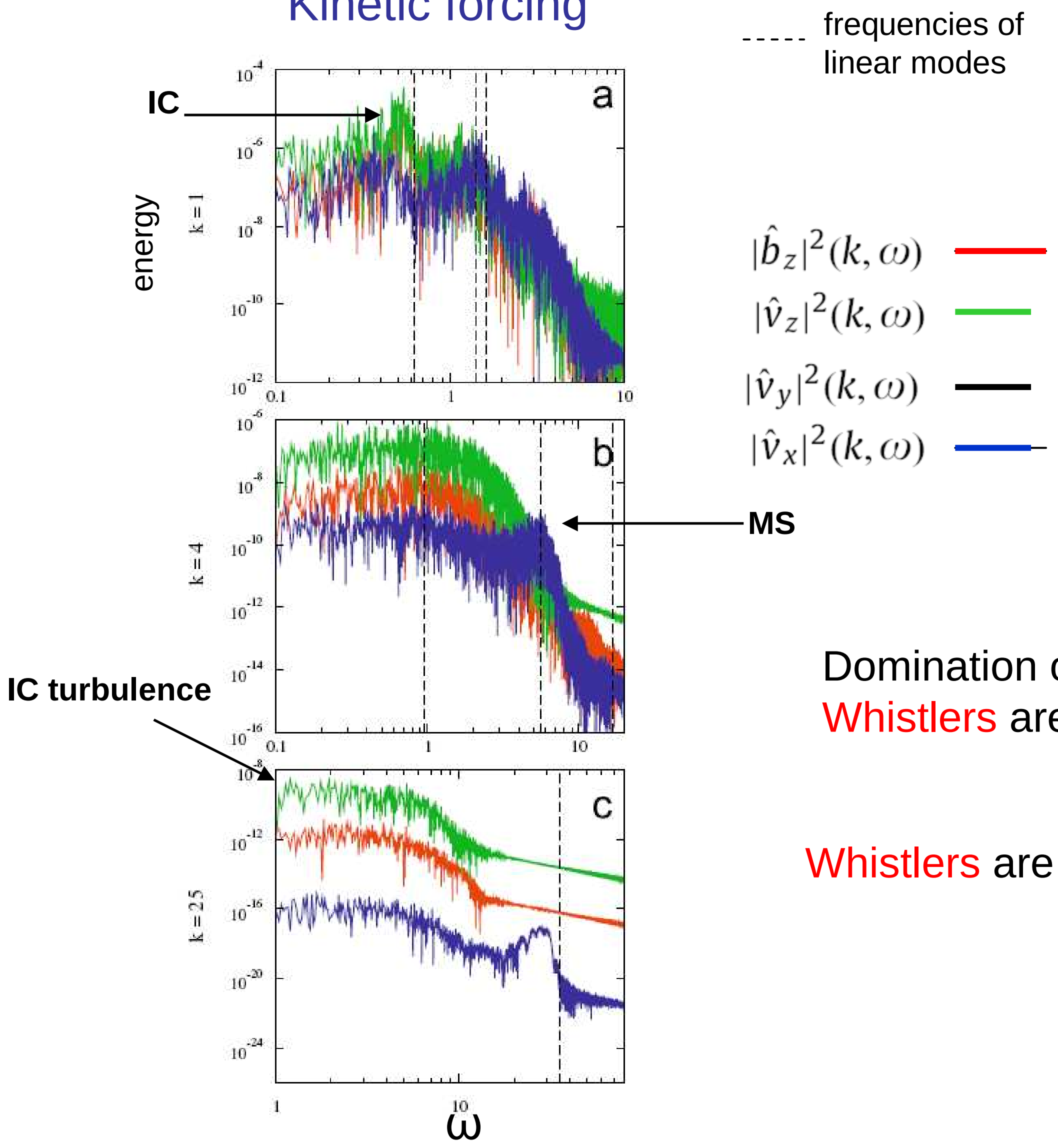
Transverse kinetic spectrum shallower than the magnetic one

Dominance of kinetic on magnetic modes suggests an ion-cyclotron turbulence

Parallel propagation , weakly dispersive regime (continued):

Wave identification
by frequency analysis of
individual Fourier modes

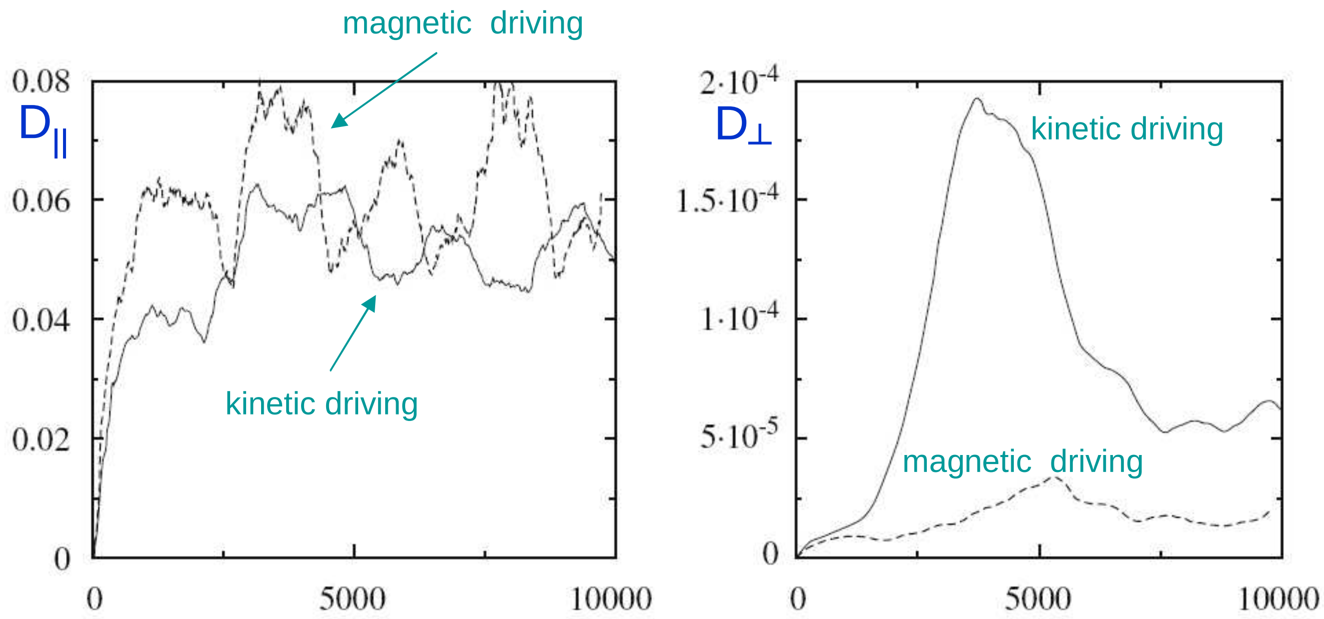
Kinetic forcing



Domination of ion-cyclotron waves.
Whistlers are subdominant.

Whistlers are more important with magnetic driving.

Energy dissipation affects dominantly the parallel field components



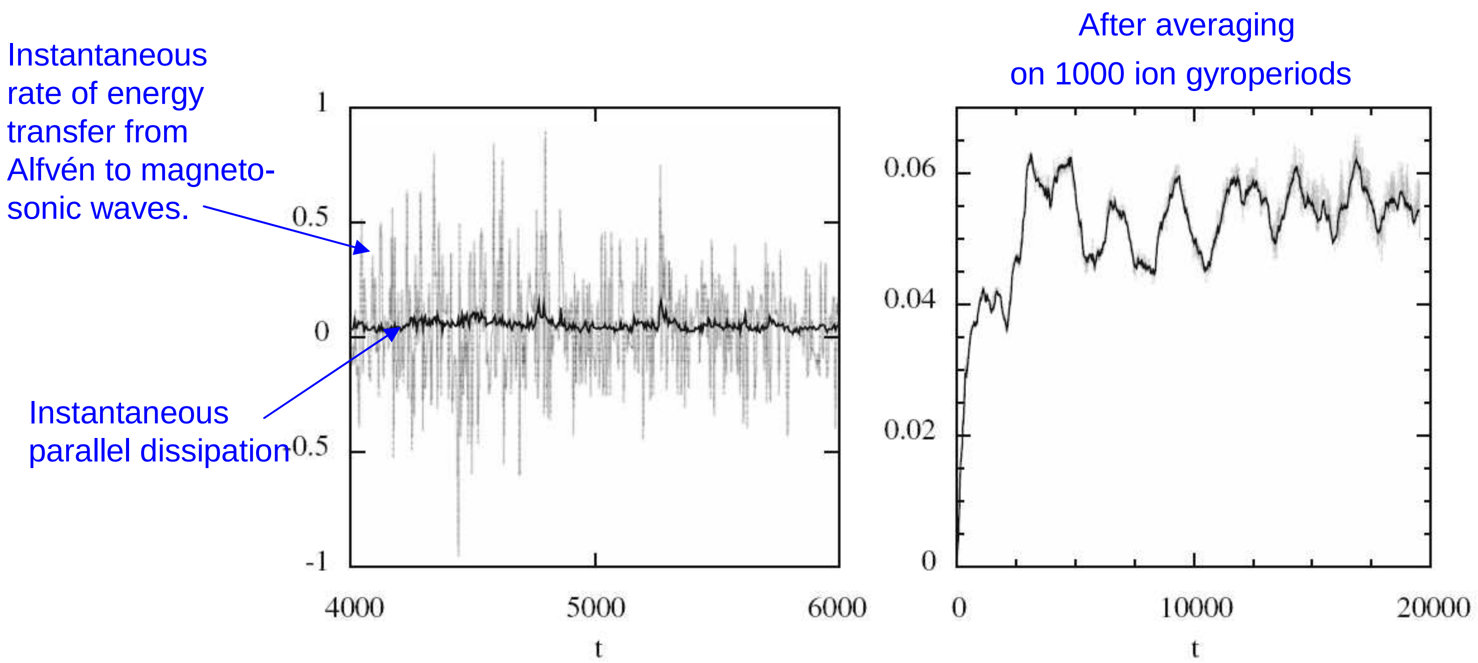
Left: time evolution of the parallel dissipation $D_{\|}$ for the kinetic (solid line) and magnetic (dashed line) drivings
Right: same for the perpendicular dissipation $D_\perp$

Dominant dissipation of the parallel components

In the weakly dispersive regime, injected energy is transferred to sonic waves and dissipated through a cascade of acoustic waves.

Sonic wave turbulence is the dominant phenomenon, although small scales also form on the transverse field components but on a longer time scale.

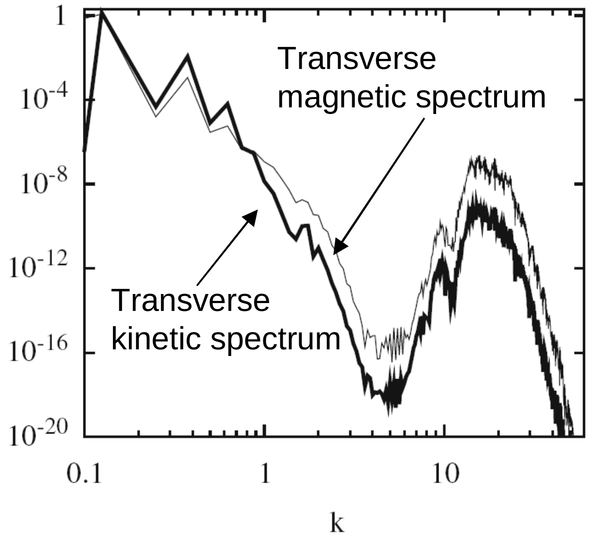
Strongly intermittent energy transfer from Alfvén to magnetosonic waves



(kinetic driving)

Forcing no longer persistent but monitored to maintain a constant total energy:

In the case of a large-scale kinetic forcing (weakly dispersive regime), a spectral hole forms at intermediate scales, showing the absence of Alfvén wave cascade.



kinetic $|\hat{\mathbf{v}}_y|^2+|\hat{\mathbf{v}}_z|^2$ (thick line) and magnetic $|\hat{\mathbf{b}}_y|^2+|\hat{\mathbf{b}}_z|^2$ (thin line) spectra

Differently, in a more dispersive regime, an Alfvén wave cascade is possible.

(qualitative agreement with weak-turbulence analysis on Vlasov equation: Yoon & Fang 2008).

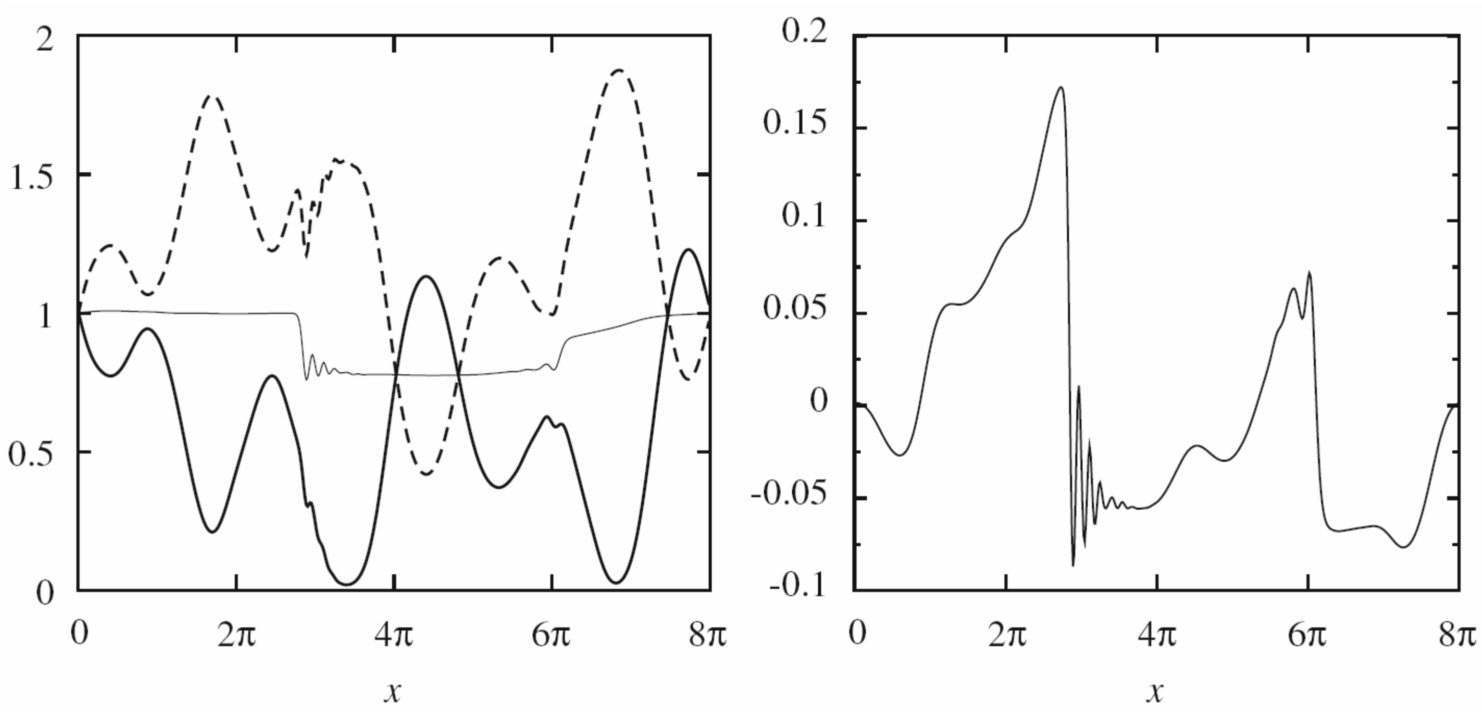
No spectral gap in the case of monitored driving.

Quasi-transverse propagation
80°

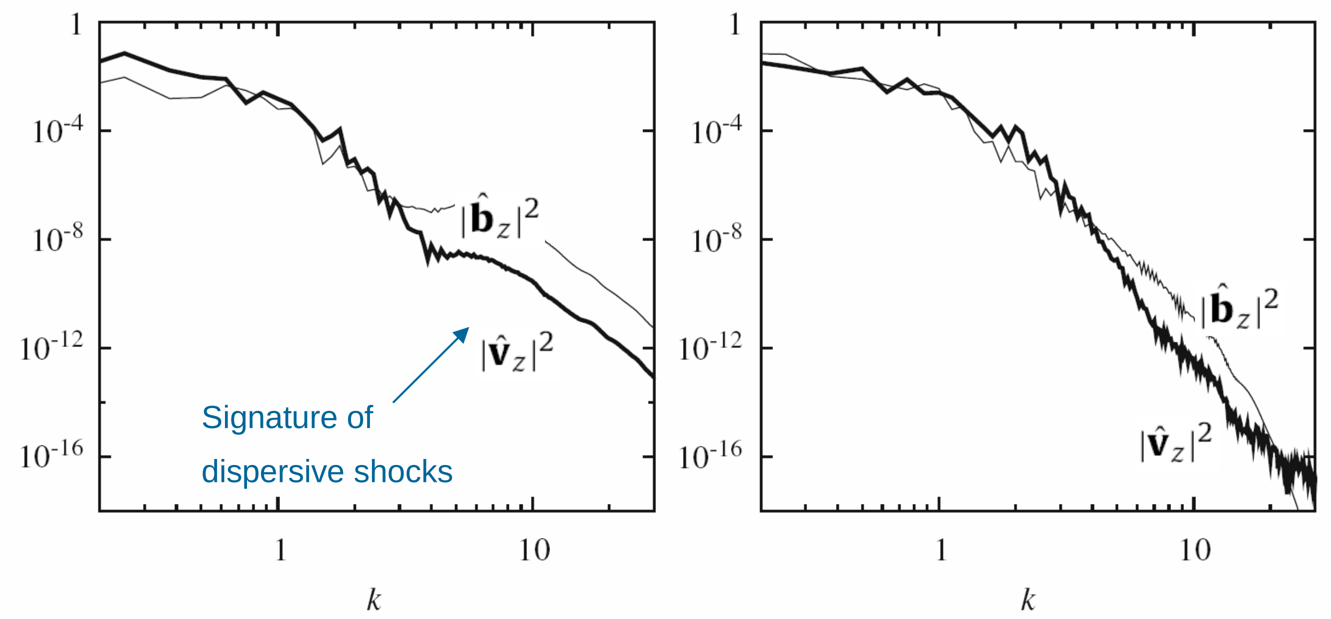
Large-scale kinetic driving
(weakly dispersive regime)

Early inverse cascade that saturates

Pressure-balanced state perturbed by dispersive shocks



Quasi transverse propagation, weakly dispersive regime (continued):



Locally averaged transverse kinetic and magnetic spectra near t=2500 and t=10500

Strong fluctuations of the energy spectra. Whistler modes are dominant.

Contrast with

Strongly dispersive regime (not shown): Kinetic Alfvén wave are significant

Quasi-transverse dynamics amenable of a more refined modeling, including small-scale low-frequency kinetic effects **(Landau fluids).**

4. Beyond the Hall MHD description: Landau fluid models

Hall-MHD cannot capture important dynamical properties of space plasmas related to their collisionless character.

Fully kinetic description requires enormous computational ressources

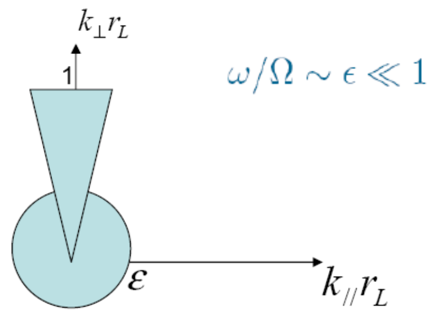
Landau fluid models: extend MHD by retaining low-frequency kinetic effects (Landau damping and finite Larmor radius (FLR) corrections), together with exact equations for the (anisotropic) pressures and heat fluxes.

First introduced in the context of large-scale MHD (Snyder et al. PoP 1997).
Then extended to retain Hall effect and FLR corrections in order to describe quasi-transverse ionic scales obeying the gyrokinetic scaling (Passot & Sulem, PoP 2007).

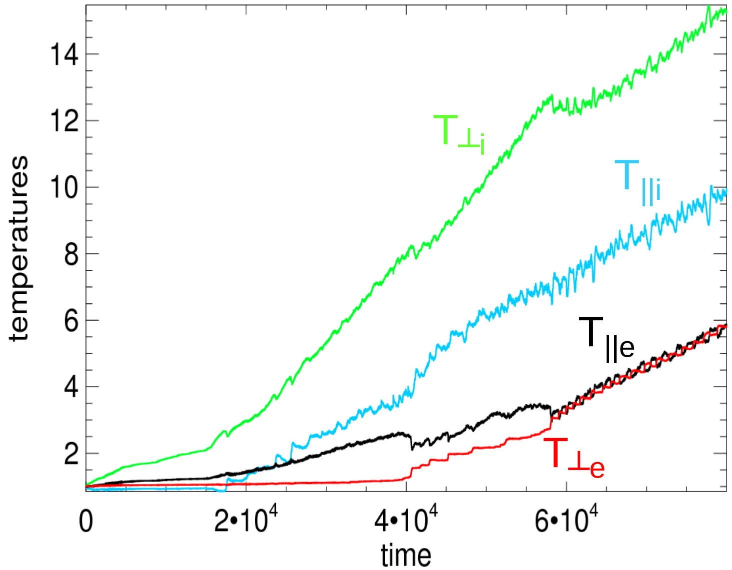
Closure relation at the level of the 4th order cumulants (retains Landau damping)

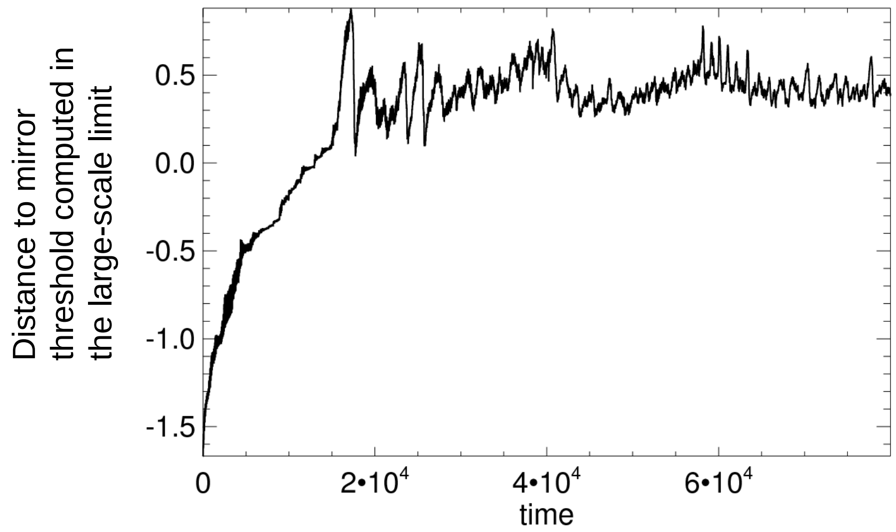
FLR corrections to all retained moments

Consistent with the linear kinetic theory in the low-frequency limit

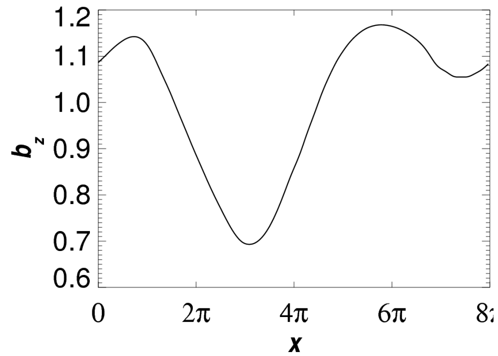


In contrast with gyrokinetic theory, fast magnetosonic modes are not averaged out and their large-scale dynamics is accurately reproduced.

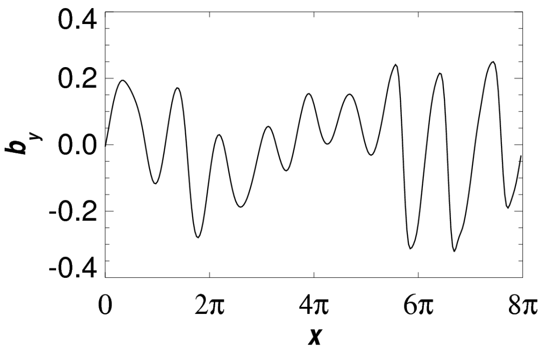




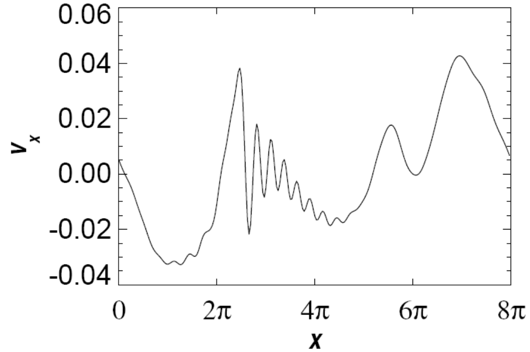
Mirror structure



Intermittent formation
of kinetic Alfvén waves



whistlers



Turbulence driven at large scale
in a way that maintains the kinetic
energy close to a prescribed value.

Initially $\beta_i = 0.6$.

Development of temperature anisotropy
is regulated by mirror instabilities
that maintain the system close to the
instability threshold.

Analysis of solar wind data: proton
temperature anisotropy is constrained
by oblique instabilities (mirror and firehose):
Hellinger et al. GRL 33, L099101 (2006).
Bale et al. PRL 103, 21101 (2009).

Could be due to a Hall instability
within magnetic field gradient
associated with mirror structures
(Stasiewicz et al. JGR 2001) .

5. Conclusion

- At least in 1D, dispersive MHD turbulence displays non universal properties.

- Influence of the structures (both at large and small scales) on the dynamics

- Strongly intermittent transfer both between the scales and between the modes.

- The existence of a cascade of parallel Alfvén waves requires a sufficiently dispersive regime.

- When retaining dynamical equations for gyrotropic pressures and heat fluxes, together with Landau damping and FLR corrections *(Landau fluid models)*,
  turbulence at $\beta=O(1)$ generates temperature anisotropies
  (dominant perpendicular temperature of the ions and parallel temperature of the electrons),
  leading to ion mirror instabilities that maintain the system close to the instability threshold.
  Presence of whistlers.
  Intermittent generation of kinetic Alfvén waves that **could** be due to a Hall instability in the inhomogeneities associated with mirror structures (work in progress).

Further developments:

- Detailed analysis of the dynamics driven by the temperature anisotropies.

- Extension to 3 dimensions, after simplification of the model.