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Quantum plasma fluid model for coherent stimulated emission of radiation by a dense relativistic electron beam

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Motivation

The essence of FEL

- Operates near resonant condition $\lambda_s \cong \lambda_w / 4\gamma_e^2$;
- e^- bunching inside potential well.

Description

- The beam electrons in the ponderomotive potential well are modelled by a discrete ensemble of initially uniform macro-electrons.

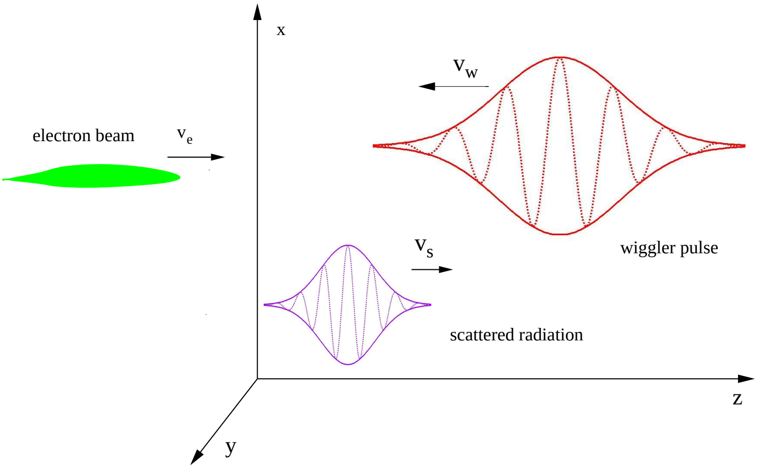
$\downarrow$ HOWEVER...

Motivation

For X-Ray

- Very high frequency such that $\frac{\hbar\omega_s}{\gamma_e m_e c^2} \geq 1$
- For very high frequency, this picture of macro-electrons in the potential well has to be reconsidered, since the longitudinal dimension of the potential is so small that, for any beam density, a potential well would be occupied at the maximum by one single electron.

Interaction scheme



Model

New model → Quantum backscattering

- FEL dynamics as a simple case of backscattering of intense optical wigglers by the energetic, but not ultra energetic, beam electrons, taking into account the photon recoil ($\hbar k_s/\gamma_e m_e c \geq 1$)

Previous models

- Mclever and Fedorov - Derived a set of general quantum equations starting from K. G. equation.;
- Smetanin - Also from K. G. Eq., showed that Q. FEL behaves as a two-level quantum oscilator;
- Preparata - By using the QFT.

Quantum FEL Scheme - Schrodinger-like equation

Energy conservation

$$E = \sqrt{p_e^2 c^2 + m_e^2 c^4} + V(\vec{r}, t)$$

- $E = \hbar w_e \longrightarrow e^-$ total relativistic energy;
- $\vec{p}_e = \hbar \vec{k}_e \longrightarrow e^-$ momentum;
- $V \longrightarrow$ ponderomotive potential
- For $V = 0$, E and $\vec{p}_e$ are constants. But, if $V \neq 0$ they will vary slowly in $\vec{r}, t$
- Eikonal approximation

$$E \longrightarrow \hbar \left(w_e + i \frac{\partial}{\partial t} \right), \qquad \vec{p}_e \longrightarrow \hbar (\vec{k}_e - i\nabla)$$

Quantum FEL Scheme - Schrodinger-like equation

- If ψ is the e^- wave function, then

$$\left|\frac{\partial}{\partial t}\psi\right| << w_e|\psi|, \qquad |\nabla\psi| << |\vec{k}_e|\psi$$

- From E expression, the evolution of the e^- wave packet is

Serbeto *et. al* Physics of Plasmas 15, 013110 2008

$$i\hbar\left(\frac{\partial}{\partial t} + \vec{v}_e\cdot\nabla\right)\psi + \frac{c^2}{2w_e}\left[\nabla^2 - \left(1 - \frac{m_e^2c^4}{\hbar^2 w_e^2}\right)(\hat{e}_{\parallel}\cdot\nabla)^2\right]\psi =$$

$$[V(\vec{r},t) - e\phi(\vec{r},t)]\psi$$

- ϕ is the plasma potential; $\vec{v}_e$ is the e^- velocity; $\hat{e}_{\parallel} = \vec{k}_e/k_e$.

Quantum FEL Scheme - Schrodinger-like equation

- Transformation to the Stationary beam frame.

$$\vec{\zeta} = \vec{r} - \vec{v}_e t; \qquad t' = t; \qquad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \vec{v}_e \cdot \nabla$$

- Decomposing $\vec{\zeta}$ into parallel and perpendicular components to the e^- velocity ($\vec{\zeta} = \zeta_{\|}\hat{e}_{\|} + \zeta_{\perp}\hat{e}_{\perp}$), using $\hbar w_e = \gamma_e m_e c^2$, we get a Schrödinger-like equation

$$i\hbar\frac{\partial}{\partial t'}\psi + \frac{\hbar^2}{2m_e\gamma_e}\left\{\frac{1}{\gamma_e^2}\frac{\partial^2}{\partial\zeta_{\|}^2} + \frac{\partial^2}{\partial\zeta_{\perp}^2}\right\}\psi = (V - e\phi)\psi$$

- Longitudinal field

$$\nabla \cdot \vec{E} = \nabla \cdot \vec{E}_{\perp} + \nabla \cdot \vec{E}_{\|} = \nabla \cdot \vec{E}_{\perp} - \nabla^2\phi = 4\pi e|\psi|^2$$

Quantum FEL Scheme - Schrodinger-like equation

Gradient operator

$$\nabla = \left(\frac{\partial}{\partial \zeta_{\parallel}} \hat{e}_{\parallel}, \ \frac{\partial}{\partial \zeta_{\perp}} \hat{e}_{\perp} \right)$$

- ψ is the electron ensemble wave function (not single electron) in such a way that its density is $|\psi|^2 = n$.
- This normalization is valid since we are considering that the electrons in the ensemble interact only collectively through an electromagnetic potential,i.e., we are assuming a pure state ensemble of N-electrons (perfectly coherent particle sample) to represent the relativistic electron beam which interacts with an intense optical wiggler.

Quantum electron fluid

Quasi-classical approximation to the e^- wave function: Madelung transformation

$$\psi = A(\vec{\zeta}, t')e^{iS(\zeta,t')/\hbar};$$

L. D. Landau and E. M. Lifshitz, *"Quantum Mechanics", 3rd Ed.*

- S is the action of the particle;
- A is a slow-varying amplitude.
- Using this into the Schrodinger equation, we get

$$\frac{\partial}{\partial t}S + \frac{1}{2\gamma_e m_e}\left\{\frac{1}{\gamma_e^2}\left(\frac{\partial}{\partial \zeta_\parallel}S\right)^2 + \left(\frac{\partial}{\partial \zeta_\perp}S\right)^2\right\}$$

$$= -(V - e\phi) + \frac{\hbar^2}{2\gamma_e m_e}\left\{\frac{1}{A}\left(\frac{1}{\gamma_e^2}\frac{\partial^2}{\partial \zeta_\parallel^2} + \frac{\partial^2}{\partial \zeta_\perp^2}\right)A\right\}$$

Quantum electron fluid

- It reduces to the classical Hamilton-Jacob Eq. when $\hbar \longrightarrow 0$.
- The conservation eq. for the probability density, $A^2 = |\psi|^2$ is

$$\frac{\partial}{\partial t}A^2 + \frac{\partial}{\partial \zeta_{\|}}\left\{\frac{A^2}{m_e\gamma_e^3}\frac{\partial}{\partial \zeta_{\|}}S\right\} + \frac{\partial}{\partial \zeta_{\perp}}\left\{\frac{A^2}{m_e\gamma_e}\frac{\partial}{\partial \zeta_{\perp}}S\right\} = 0$$

Associations

$$A^2 = n, \quad \vec{p} = \nabla S, \quad \vec{v} = \left(\frac{p_{\|}}{m_e\gamma_e^3}\hat{e}_{\|}, \quad \frac{p_{\perp}}{m_e\gamma_e}\hat{e}_{\perp}\right);$$

Differential Operator D^2

$$D^2 = \left(\frac{1}{\gamma_e^2}\frac{\partial^2}{\partial \zeta_{\|}^2} + \frac{\partial^2}{\partial \zeta_{\perp}^2}\right)$$

Quantum electron fluid

- Electron momentum fluid equation:

$$\frac{\partial \vec{p}}{\partial t'} + \frac{1}{2}\nabla(\vec{v}\cdot\vec{p}) = -\nabla(V - e\phi) + \frac{\hbar^2}{2m_e\gamma_e}\nabla\left[\frac{1}{\sqrt{n}}D^2\sqrt{n}\right];$$

- Continuity:

$$\frac{\partial}{\partial t'}n + \nabla\cdot(n\vec{v}) = 0;$$

- Poisson: $\nabla^2\phi = 4\pi e(n - n_0);\quad -\nabla\cdot\vec{E}_\perp = 4\pi e n_0;$

n_0 $\rightarrow$ average beam density; n $\rightarrow$ density of an e^- fluid element;

$\vec{v}(\vec{p})$ $\rightarrow$ velocity(momentum) of this element in the moving frame.

- $\frac{\hbar^2}{2m_e\gamma_e}\nabla\left[\frac{1}{\sqrt{n}}D^2\sqrt{n}\right]$ accumulates all quantum effects. [R. P. Feynman, R. B. Leighton and M. Sands, *"Feynman Lectures on Physics"*, *2nd Edition*, *Vol.3*, *Chapter 21*.]

Quantum electron fluid

- FEL is a 1-D phenomenon. Transverse effects are neglected
- Electron momentum fluid equation:

$$\frac{\partial}{\partial t'}p + v\frac{\partial}{\partial \zeta}p = -\frac{\partial}{\partial \zeta}(V - e\phi) + \frac{\hbar^2}{2m_e\gamma_e^3}\frac{\partial}{\partial \zeta}\left(\frac{1}{\sqrt{n}}\frac{\partial^2}{\partial \zeta^2}\sqrt{n}\right)$$

- Continuity

$$\frac{\partial}{\partial t'}n + \frac{\partial}{\partial \zeta}(nv) = 0$$

- Poisson

$$\frac{\partial^2}{\partial \zeta^2}\phi = 4\pi e(n - n_0)$$

- p, v and ζ now stand for the longitudinal components of $\vec{p}$, $\vec{v}$ and $\vec{\zeta}$.

Radiation and wiggler fields

- Assuming the circularly-polarized fields, and using the wave equation, we get the Slow-varying amplitude equations:
- Optical wiggler

$$\left[\frac{\partial}{\partial t'} - (v_w + v_e)\frac{\partial}{\partial \zeta}\right] a_w = \frac{w_p^2}{2\gamma_e \omega_w} a_s \delta n e^{-i(k_l\zeta - [w_l - k_l v_e]t')}$$

- Radiation field

$$\left[\frac{\partial}{\partial t'} + (v_s - v_e)\frac{\partial}{\partial \zeta}\right] a_s = -\frac{w_p^2}{2\gamma_e \omega_s} a_w \delta n e^{i(k_l\zeta - [w_l - k_l v_e]t')}$$

- $v_{s(w)} = c^2 k_{s(w)}/\omega_{s(w)}$ is the radiation(optical wiggler) group velocity.

Plasma equation

- 1-D e^- fluid perturbed equations;

$$\frac{\partial}{\partial t'}\delta p = -\frac{\partial}{\partial \zeta}V + e\frac{\partial}{\partial \zeta}\phi + \frac{\hbar^2}{4m_e\gamma_e^3}\frac{\partial^3}{\partial \zeta^3}\delta n$$

$$\frac{\partial}{\partial t'}\delta n + \frac{1}{m\gamma_e^3}\frac{\partial}{\partial \zeta}\delta p = 0$$

$$\frac{\partial^2}{\partial \zeta^2}\phi = (4\pi e n_0)\delta n$$

- A simple manipulation lead us to Eq. which represents the space-chage oscillation excited by a ponderomotive force through V

$$\left[\frac{\partial^2}{\partial t'^2} + \left(\frac{\hbar}{2m_e\gamma_e^3}\right)^2\frac{\partial^4}{\partial \zeta^4} + \frac{w_p^2}{\gamma_e^3}\right]\delta n = \frac{1}{m\gamma_e^3}\frac{\partial^2}{\partial \zeta^2}V$$

Ponderomotive potential/Matching conditions

- From our fields and Ponderomotive Force definition

$$V = \frac{mc^2}{2\gamma_e}\left(ia_w a_s^* e^{i(k_l\zeta - \Omega_l t)} + cc\right)$$

- Matching conditions:
- Energy: $\hbar w_l = \hbar w_s - \hbar w_w;$
- Momentum: $\hbar k_l = \hbar k_s + \hbar k_w;$
- $\Omega_l = w_l - k_l v_e$

Free-electron laser instability

- Normalized set of FEL equations:

$$\left[\frac{\partial}{\partial \tau} - (\beta_w + \beta_e)\frac{\partial}{\partial \chi}\right] a_w = \frac{\omega_p}{2\gamma_e \omega_w} \bar{a}_s \tilde{n}$$

$$\left[\frac{\partial}{\partial \tau} - (\beta_e - \beta_s)\frac{\partial}{\partial \chi} - i\frac{\delta}{\omega_p}\right] \bar{a}_s = -\frac{\omega_p}{2\gamma_e \omega_s} a_w \tilde{n}^*$$

$$\left[\frac{\partial}{\partial \tau} + \beta_q\frac{\partial}{\partial \chi} + \frac{\nu}{\omega_p}\right] \tilde{n} = \frac{c^2 k_l^2}{4\gamma_e^4 \Omega_l \omega_p} a_w \bar{a}_s$$

where the damping (ν) and the mismatching (δ) have been introduced. Here, $\tau = \omega_p t'$, $\chi = (\omega_p/c)\zeta$, and $\Omega_l^2 - 1/4(\hbar/\gamma_e^3 m_e)^2 k_l^4 - \omega_p^2/\gamma_e^3 = 0$

Free-electron laser instability

- $\beta_q = \hbar k_l/\gamma_e m_e c \geq 1$ is the quantum parameter. β_e and $\beta_s(\beta_w)$ are the normalized electron beam and radiation(wiggler) group velocities.

Steady-state FEL equations: Universal normalization

$$\frac{\partial \hat{a}_w}{\partial \chi} = -\hat{a}_s \hat{n} cos\phi$$

$$\frac{\partial \hat{a}_s}{\partial \chi} = S \hat{a}_w \hat{n} cos\phi$$

$$\frac{\partial \hat{n}}{\partial \chi} + \hat{\nu}\hat{n} = \hat{a}_w \hat{a}_s cos\phi$$

$$\frac{\partial \phi}{\partial \chi} = -\hat{\delta} - \left(\frac{\hat{a}_w \hat{a}_s}{\hat{n}} + S\frac{\hat{a}_w \hat{n}}{\hat{a}_s} - \frac{\hat{a}_s \hat{n}}{\hat{a}_w} \right) sin\phi$$

where

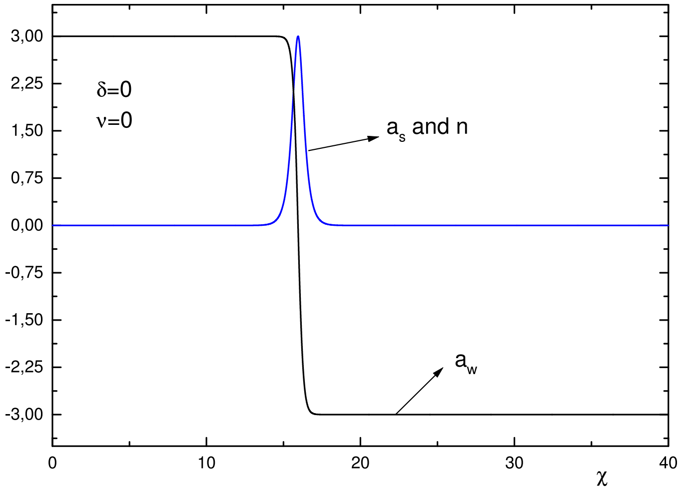
$$\hat{n} = n\sqrt{|C_1|C_2}; \quad \hat{a}_s = a_s\sqrt{C_0 C_2}; \quad \hat{a}_w = a_w\sqrt{C_0|C_1|}; \quad S = \frac{|C_1|}{C_1}$$

$$C_0 = \frac{c^2}{4\gamma_e^4}\frac{k_l^2}{\Omega_l w_p}\frac{1}{\beta_q} = \frac{ck_l}{2w_p}\left(\frac{mc\gamma_e}{\hbar k_l}\right)^2,$$

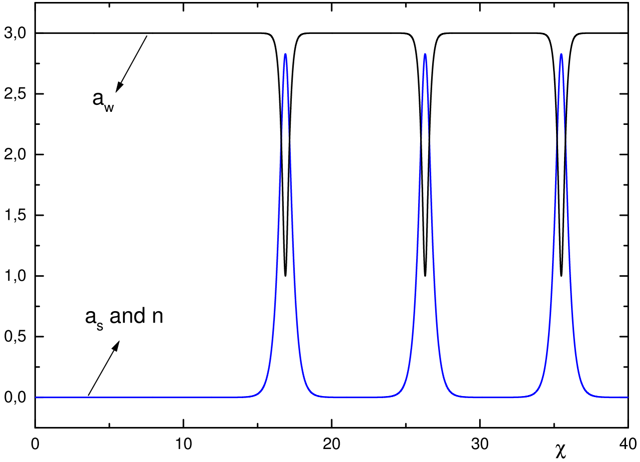
$$C_1 = \frac{w_p}{2w_s\gamma_e}\frac{1}{(\beta_e - \beta_s)}, \quad C_2 = \frac{w_p}{2w_w\gamma_e}\frac{1}{(\beta_w + \beta_e)}$$

$$\hat{\nu} = \frac{\nu}{\omega_p\beta_q}, \quad \hat{\delta} = \frac{\delta}{\omega_p(\beta_e - \beta_s)}$$

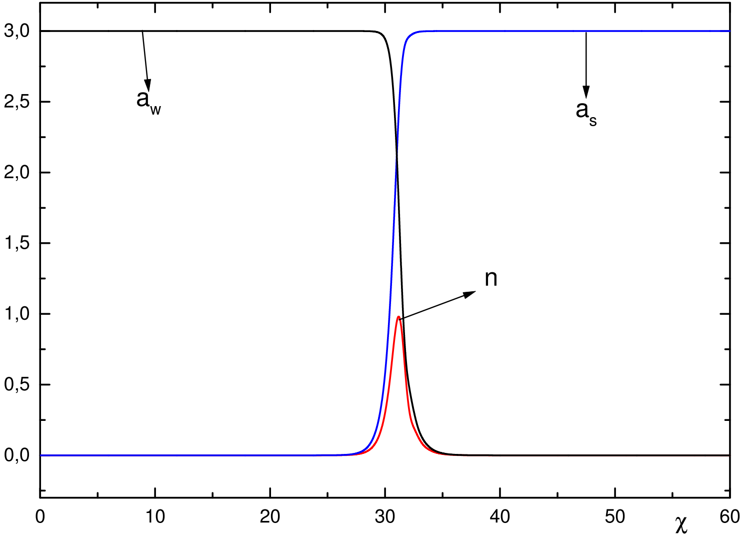
- These equations can be solved by numerical methods or by the scheme of Galeev *et. al Reviews of Plasma Physics, (Plenum, New York), Vol.7, Chapter 1 (1979)* soliton

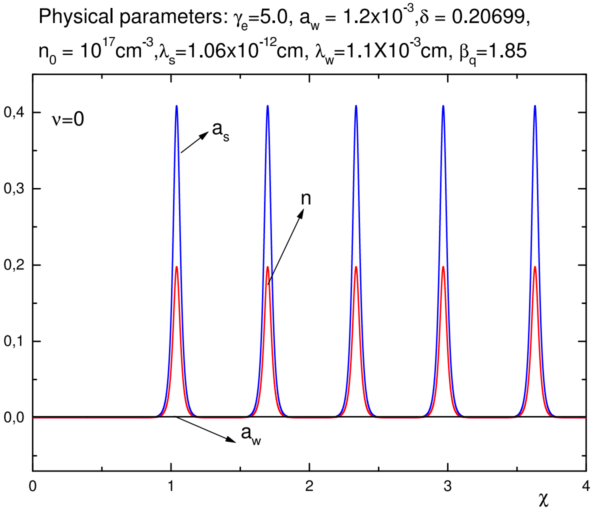

3,00
2,25
1,50
0,75
0,00
-0,75
-1,50
-2,25
-3,00
0
10
20
30
40
χ
δ=0
ν=0
a_s and n
a_w

repeated explosion ($\delta = 2$, $\nu = 0$)



soliton + shock (ν=4, δ=2)




Physical parameters: γe=5.0, aw = 1.2x10-3, δ = 0.20699,
n0 = 10^17 cm^-3, λs=1.06x10^-12 cm, λw=1.1X10^-3 cm, βq=1.85
ν=0
as
n
aw
0,4
0,3
0,2
0,1
0,0
0
1
2
3
4
χ

Conclusions

- We have presented a quantum plasma fluid model to describe the stimulated emission of coherent radiation by a relativistic electron beam interacting with an intense electromagnetic wave as a three-wave interaction process. This stimulated emission is treated as a Compton scattering process, **without the usual nonlinear electron bunching in the ponderomotive potential;**
- S. charge is negligible for dense and relativistic e^- beam, since the quantum term $1/4(\hbar/\gamma_e^3 m_e)^2 k_l^4 >> \omega_p^2/\gamma_e^3$;
- This fluid model is simpler and more realistic to describe and simulate quantum FEL than previous quantum model based on the Wigner distribution description. (*Bonifacio, 2005, Nucl. Instr. and Meth. A; Europhys. Lett.; and Serbeto, 2008, Phys. Plasmas*)

That's all folks.

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- [5] A. Hasegawa, *"Plasma Instabilities and Nonlinear Effects" Chapter 4*
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