The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2155-34

International Workshop on Cutting-Edge Plasma Physics

5 - 16 July 2010

Properties of High Energy Density Plasmas

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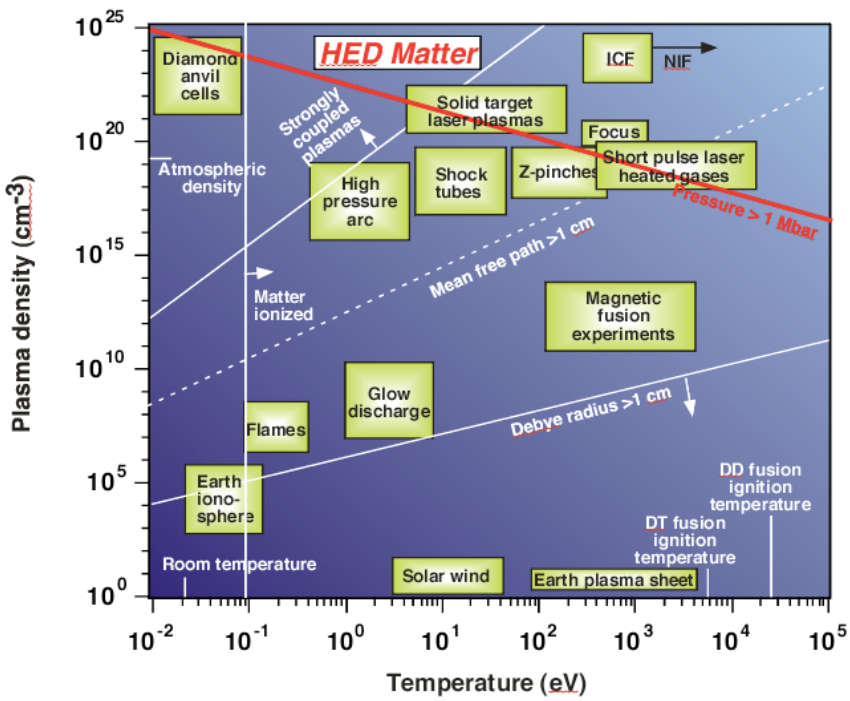
Summary notes for lectures delivered to the Trieste Summer School on Frontiers in Plasma Physics

Note: variables are summarized at the end of these notes. All units are CGS unless otherwise noted.

0. What is a High Energy Density (HED) Plasma?

High Energy Density Parameter	Value yielding equivalent to 1 Mbar – Energy density of 10^{11} J/m^3
Plasma density with T = 1 keV (10^7 K)	6 x 10^{20} cm^{-3}
Plasma temp at solid density (n≈10^{23} cm^{-3})	6 eV
Electromagnetic wave intensity	3 x 10^{15} W/cm^2
Electric field strength	1.5 x 10^{11} V/m
Magnetic field strength	500 T
Blackbody radiation temperature	400 eV
Current density for 100 MeV electrons	30 MA/cm^2
Laser intensity to yield 1 Mbar ablation pressure	4 x 10^{12} W/cm^2



I. Dense Plasma Kinetics

I.A Introduction and definition of the plasma parameter

Plasma regime considered in these lectures:
$n_e \geq 10^{19}$ cm^{-3}, kT_e = 10 eV – ~10 keV, p = ~ 10 kbar – 10 Gbar, highly ionized Z>1.

We are interested in understanding some of the fundamental properties of the HED plasmas
These include:
1) Equation of state, ie $p=f(n,T)$ and $E=g(n,T)$
2) Electrical conductivity, $\sigma(n, T)$
3) Thermal conductivity, $\kappa(n, T)$
4) Stopping power for fast particles dE/dx
5) Electron-electron and electron-ion relaxation times, τ_{ee}, τ_{ei}
6) Average charge state and charge state distribution, Z
7) Radiation absorption and emission properties,

We will discuss and develop simple models for (1), (2), (3), and (6) in lecture to illustrate the magnitude of effects in HED plasmas

Debye shielding:
The Debye length in a plasma that is *weakly coupled* is

$$\lambda_D = \sqrt{\frac{kT_e + kT_i}{4\pi e^2 Z n_o}}$$

The number of particles in a Debye sphere is called the plasma parameter:

$$\Lambda_p = \frac{1}{\sqrt{Zn_o}}\left(\frac{kT_e}{4\pi e^2}\right)^{3/2}$$

In an MFE plasma $\Lambda_p \approx 10^7$, in an HED plasma with $n_e \approx 10^{23}$ cm^{-3} and $kT_e \approx$ 100 eV, $\Lambda_p \approx 1.3$

I.B Collision frequency

Dense plasmas are very collisional, meaning they have a collision frequency comparable to the plasma frequency.

A simple model for electron scattering from immobile ions leads to a simple estimate of an electron momentum relaxation collision frequency:

$$\nu_{ei} = \frac{8\pi Z^2 e^4 n_i}{3^{3/2} m_e^{1/2} (kT_e)^{3/2}} \ln \Lambda_C$$

where the Coulomb logarithm is given by:

$$\Lambda_C = \frac{(kT)^{3/2}}{2Ze^2\sqrt{4\pi e^2 n_i}}$$

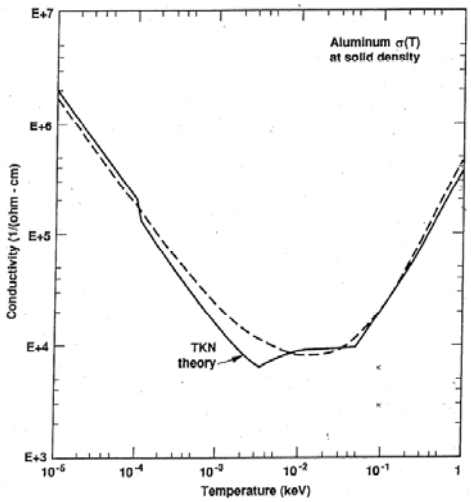
which is equal to the plasma parameter to within a factor of unity. In a strongly coupled plasma, it is common to replace the Debye length by the ion sphere radius in the upper limit of the Coulomb logarithm. Note that $\ln\Lambda_C<0$ for dense, cool plasmas, invalidating this common plasma picture.

I.C Plasma conductivity

A simple model for the DC conductivity of a dense plasma can be constructed using the electron momentum collision frequency, were the conductivity is defined by $E=J/\sigma$.

$$\sigma = \frac{e^2 n_e}{m_e \nu_{ie}} = \frac{3^{1/2}}{8\pi Z e^2 m_e^{1/2}} \frac{1}{\ln\Lambda_C} (kT_e)^{3/2}$$

This varies as $T_e^{3/2}$ in the ideal plasma regime and only weakly depends on density through the Coulomb logarithm.



Plot of conductivity in Al at solid density vs temperature. Reproduced from an article by RM More.

I.D Dense plasma non-ideal effects; strong coupling

A number of effects in dense plasmas become important in the 1-100 eV regime that cause conductivity deviation from the $(kT_e)^{3/2}$ behavior.

a) Coulomb logarithm approaches zero.

$$\sigma \sim \frac{1}{\ln\Lambda_C}$$

b) Atomic physics and ionization of high Z plasma constituents.

$$\sigma \sim \frac{1}{Z[kT_e]}$$

c) Electron degeneracy effects

$$kT_e \rightarrow kT_F = \frac{\hbar^2 \left(3\pi^2 n_e\right)^{2/3}}{2m_e}$$

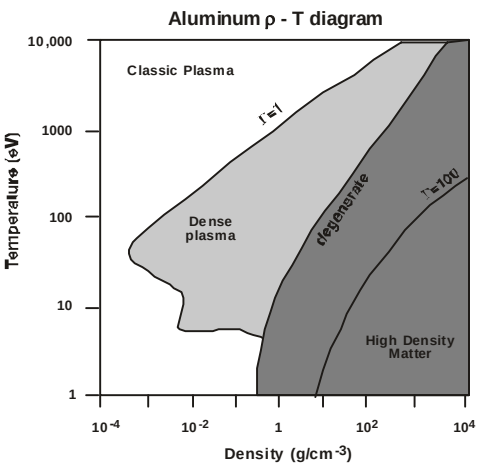
d) Two body collision approximation breaks down
A collision frequency in this dense regime can be estimated based on a collision frequency clamped by the time for electrons to travel the mean ion spacing (ignoring degeneracy effects):

$$\sigma_{cold} \approx \frac{Ze^2 n_i^{2/3}}{3^{1/6}(4\pi)^{1/3}} \frac{1}{(kT_e)^{1/2}}$$

Strong Coupling: Dense, cool plasmas are usually strongly coupled, meaning that the interparticle potential energy is comparable to the thermal kinetic energy. The strong coupling parameter is the ratio of the potential energy to the kinetic energy of the plasma particles:

$$\Gamma = \frac{Z^2 e^2 \left(\frac{4}{3}\pi n_i\right)^{1/3}}{kT}$$

Plasmas with $\Gamma > 1$ are said to strongly coupled; liquids usually have $\Gamma > 100$.



This is a density-temperature plot of an aluminum plasma showing the regime of strong coupling.

II. Dense Plasma Thermodynamics

II.A Dense plasma equation of state

The equation of state of a plasma can usually be broken into three contributions, the ions, the electrons and radiation. So:
$p(n,T)=p_i+p_e+p_r$; $E(n,T)=E_i+E_e+E_r$

1) *Ions*
In HED plasmas, the ion EOS can almost always be described as an ideal gas EOS:

$$p_i = n_i kT_i \text{ and } E_i = \tfrac{3}{2} n_i kT_i$$

which are pretty accurate until the plasma ions become very strongly coupled, then a Coulomb correction must be included.

2) *Electrons*
At temperatures well above the Fermi temperature, the electrons can also be described by an ideal gas, or *polytropic* EOS in which the adiabatic index is γ=5/3. In dense plasmas an additional correction needs to be made due to the Coulomb forces between the particles (discussed in II.D). The pressure associated with the electrons in the "Ion Sphere Model" is:

$$p_e = Zn_i kT_e - \tfrac{3}{10} Z^2 e^2 n_i^{4/3}$$

The internal energy of the electrons has three components, a thermal component, a correction for the Coulomb forces and internal energy that goes into the ionization of the ions in the plasma. (We will consider the internal energy in ionization in section II.D). Considering only their thermal component and the Coulomb correction.

$$E_e = \tfrac{3}{2} Zn_i kT_e - \tfrac{9}{10} \frac{Z^2 e^2}{R_0} n_i$$

where $R_0 = 3^{1/3}(4\pi n_i)^{-1/3}$ is the mean ion sphere spacing. As the electron temperature decreases and approaches the Fermi temperature, degeneracy effects must be considered (discussed in the next section).

3) *Radiation*
At very high temperatures in optically thick plasmas, radiation is strongly coupled to the plasma and it dominates the pressure. In this regime:

$$p_r = \frac{\pi^2}{\hbar^3 c^3} (kT_r)^4$$

Because the radiation field has six degrees if freedom (3D+2 polarizations), it acts like a polytropic gas with γ=4/3. Therefore

$$E_r = 3p_r$$

It is possible to estimate the regime in which radiation dominates the EOS by finding when p_r=p_e. This occurs at high temperature:

$$kT > \frac{\hbar c n_e^{1/3}}{\pi^{2/3}}$$

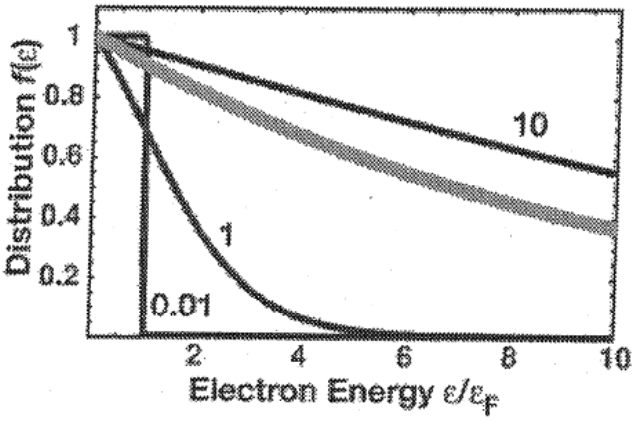
This implies that radiation dominates in a solid density plasma at temperature > ~500 eV.

II.B Degeneracy effects in the electron EOS

When $kT_e \to kT_F$ the degeneracy of the electrons becomes important in determining the pressure and internal energy. The energy distribution of the electrons approaches a Fermi degenerate distribution. For very cold plasma, electrons fill continuum levels up to the Fermi energy E_F

$$E_F = kT_F = \frac{\hbar^2}{2m_e} \left(3\pi^2 n_e\right)^{2/3}$$

At solid density with $n_e \approx 10^{23}$ cm^{-3}, $E_F = 8$ eV.



Plot of electron energy distributions for various values of T_e/T_F. The gray line shows a Maxwellian for $T_e/T_F = 10$. Reproduced from Drake's book on HED physics.

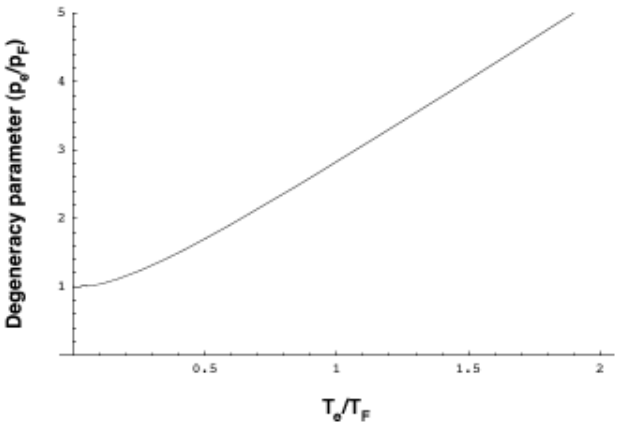
When $kT_e << kT_F$ we can say that

$$p_e = \frac{\left(3\pi^2\right)^{2/3} \hbar^2 n_e^{\ 5/3}}{5m_e}$$

and

$$E_e = \frac{3}{5} E_F$$

The *degeneracy parameter* is the ratio of the electron pressure to its ideal, T=0, Fermi degenerate pressure. It is shown below as a function of temperature.



Plot of the degeneracy parameter as a function of temperature in units of Fermi temperature

II.C LTE and Saha equilibrium

We now desire a model to determine how highly ionized constituent ions are in a dense plasma. This will allow us to determine the average Z.

It is convenient to approximate the ionization potential of ions with the hydrogenic ionization potential, namely $I_p = Z^2 I_H$. (where I_H is 13.6 eV)

Local thermodynamic Equilibrium (LTE): To assess the ionization states using equilibrium statistical mechanics we require the ion populations to be dominated by collisional processes over radiative processes. This condition can be roughly determined by comparing collision de-excitation rates with radiative rates to a quantum level p:

$$n_e[cm^{-3}] > 5 \times 10^{15} Z^6 \sqrt{kT_e[eV]} p^{-9/2}$$

This implies that higher lying states come into equilibrium at much lower densities than states near the ground state.

Saha equations: The density of individual ionization states can be found from Boltzman statistics leading to a series of equilibrium equations relating the densities of charge states to each other and to the electron density:

$$\frac{n_e n_Z}{n_{Z-1}} = 2\frac{W_Z(T)}{W_{Z-1}(T)}\left(\frac{m_e kT_e}{2\pi\hbar^2}\right)^{3/2} \exp\left[-\frac{I_p^{(Z-1)}}{kT_e}\right]$$

These equations are completed by the conservation of charge condition: $n_e = \sum_{j=1}^{Z_{nuc}} n_j$

It can be shown that, in a high Z plasma, these equations lead to a distribution of ionization states that is strongly peaked about an average Z. Using the Saha equation and this strong peaking, we can estimate this average Z if we assume hydrogen-like ionization potentials for all ions. This average Z is

$$\overline{Z} \cong \sqrt{\frac{kT_e}{I_H}} \sqrt{\ln\left[\frac{2}{n_e}\left(\frac{m_e kT_e}{2\pi\hbar^2}\right)^{3/2}\right]}$$

Because of the logarithmic dependence on density, we can say that for most high Z, HED plasmas, the average ionization is $\overline{Z} \approx 0.55\sqrt{kT_e[in\ eV]}$

II.D Thermodynamics of ionizing plasmas

We can now determine the electron pressure and internal energy of a high Z HED plasma that is ionized according to the Saha equation. To do this we assume the "Ion Sphere Model" where the plasma is assumed to be composed of non-interacting ions of charge Z surrounded by a neutralizing, uniform density cloud of electrons with radius equal to the mean ion spacing. The pressure is the pressure due to the thermal energy corrected by the attractive free energy associated with the Coulomb forces between particles:

$$p_e = Zn_i kT_e - \tfrac{3}{10} Z^2 e^2 n_i^{4/3}$$

The internal energy in electrons now must be corrected to account for the energy needed for ionization of the constituent ions:

$$E_e = \tfrac{3}{2} Z n_i k T_e - \tfrac{9}{10} \frac{Z^2 e^2}{R_0} n_i + \sum_j^{Z_{nuc}} \left[n_j \left(\sum_s^j I_p^{(s)} \right) \right]$$

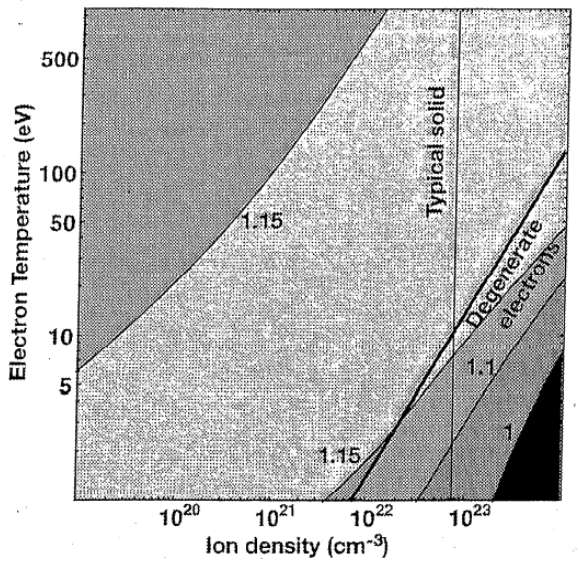
(In both equations, Z really refers to $\overline{Z}$.) The ionization energy term can be estimated assuming hydrogen-like ionization potentials for all the ions and summing them up to the average ionization state, Z:

$$E_{ionization} \approx n_i \sum_{j=1}^{Z} j^2 I_H = n_i \frac{I_H}{6} Z(1+Z)(1+2Z) \approx \frac{1}{3} Z^3 I_H n_i$$

This ionization energy has an effect on the effective adiabatic index of the plasma. Defining the effective adiabatic index by the relation

$$E_e = p_e \big/ \left(\gamma_{eff} - 1 \right)$$

we find that this simple model for the pressure and internal energy indicate that γ drops from the ideal gas value of 5/3 to ~1.3 in a dense ionizing plasma. Below is a contour plot of the estimated γ_{eff} in a high Z ionizing plasma as a function of temperature and density (reproduced from the book of P. Drake).



Contour plot of γ_{eff} as a function of temperature and density.

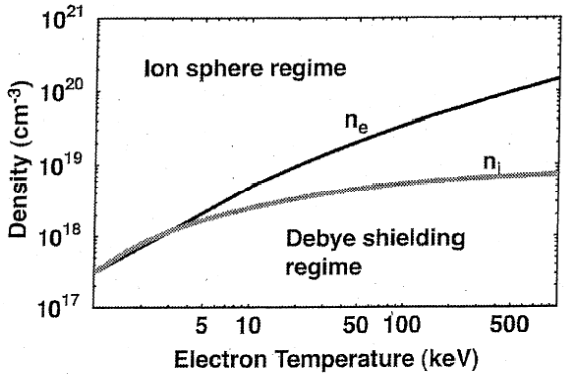
II.E Atomic physics in dense plasmas; continuum lowering

The fields felt by an atom or ion in a dense plasma affect the atomic structure. This manifests itself in two principal ways

1) Radiative lines are broadened
2) The ionization potential is lowered

Continuum lowering: The second effect is a consequence of the fact that neighboring charged particles deform the confining potential of an ion immersed in a plasma. This lowers its effective ionization potential. This effect can be thought of in two limits, the Debye shielding model and the ion sphere

model. The Debye shielding model is appropriate in tenuous plasma, where the Debye length is greater than the interparticle main spacing. In HED plasmas, because there are so few particles in a Debye sphere, we must resort to the ion sphere model. The regimes of validity for these two models are illustrated in the plot below (reproduced from the text book of P. Drake).



Validity regimes for the two approaches to finding the ionization potential lowering in a plasma

In the Ion Sphere Model, we calculate the continuum lowing by finding the difference between the Coulomb energy of the *Z-1* charged ion and the ionized *Z* charged ion. We assume that electrons are uniformly distributed in the a sphere of radius R_0.

$$\Delta I_p^{(Z)} = \frac{9}{5}\left(\frac{4\pi}{3}\right)^{1/3} Ze^2 n_i^{1/3}$$

Pressure ionization: At high pressure, continuum lowering can be so large that it ionizes the ions directly. When this becomes important can be found by equating the ionization potential lowering with a hydrogenic ionization potential. This suggests that at a temperature of 10 eV, pressure ionization is important at density ~ 2 x 10^{23} cm^{-3}.

III. Energy Transport in Dense Plasmas

Heat flow in HED plasmas can often be treated as diffusive heat flow where

$$\frac{\partial T}{\partial t} = \frac{\kappa(T)}{\rho c_V}\nabla^2 T$$

III.A Electron thermal heat conduction in plasmas

A simple model for heat flow by electron conduction can be derived using our simple formula for electron collisional relaxation.

$$\kappa_e = \frac{9\sqrt{3}}{16\pi}\frac{k(kT_e)^{5/2}}{m_e^{1/2} Ze^4 \ln\Lambda_C}$$

Note that this varies as the 5/2 power with electron temperature. Also note that more sophisticated models give the same dependence but with a larger coefficient.

III.B Nonlocal electron thermal conduction and flux limiters

In a large thermal gradient, as is common in short pulse laser solid target experiments, the diffusive approximation breaks down. The heat flow in the plasma in that case can be approximated by saying that it cannot exceed the heat flow of freely flowing electrons across a plane in the plasma. Put simply:

$\mathbf{S} = -\kappa_e \nabla T_e$ for $\mathbf{S} < f\,\mathbf{S}_{\max}$

$\mathbf{S} = -f\,\mathbf{S}_{\max}$ otherwise

where f is the flux limiter with value usually chosen around 0.1 and the maximum free streaming heat flux is

$$\mathbf{S}_{\max} = \frac{3^{3/2}}{2} n_e (kT_e)^{3/2}$$

In large electron temperature gradients the heat flow is often nonlocal. A good phenomenological model for this is to say that

$$S(x) = \int_{-\infty}^{\infty} S_{SH}(x') G(x,x') dx'$$

where $S_{SH}(x) = -\kappa_e \nabla T_e$ and the nonlocal transport function can be a variety of empirically derived functions. A common nonlocal transport model uses

$$G(x,x') = \frac{1}{2a\lambda_e} \exp\left[-\frac{|x-x'|}{a\lambda_e} \right]$$

which convolves the local diffusive heat flux with an exponential function with width roughly that of the electron mean free path. a is an empirically derived constant, which is usually taken to be of the order of ~ 10.

III.C Radiative heat flow and Rossland mean free path

At high temperatures, energy transport is dominated by photons. If the plasma is optically thick, we treat the radiation field as a black body and we can use the diffusive heat flow approximation. By averaging over all photon energies, we can derive a formula for the diffusive heat flux:

$$\mathbf{S}_R = -\frac{16\sigma_{SB}\lambda_R T^3}{3} \nabla T$$

Here it is usually assumed that the radiation and the electrons are in equilibrium so T represents both the electron and the effective radiation temperature. The Rossland mean free path, λ_R, is a photon mean free path that is appropriately averaged over a function found by calculating the heat flux from photons of specific frequencies and averaging over a Plankian spectrum:

$$\lambda_R = \int_0^{\infty} \frac{15}{4\pi^4} \left(\frac{h\nu}{kT} \right)^4 \frac{h}{kT} \frac{e^{h\nu/kT}}{\left(e^{h\nu/kT} - 1 \right)^3} \lambda_\nu d\nu$$

The Rossland mean free path depends on the absorption properties of the ions in the plasma. The three principal photon absorption mechanisms dominated this mean free path are:

1) Free-free transition -> inverse bremsstrahlung
2) Bound-free transitions -> photo-ionization
3) Bound-bound -> line absorption

In MFE plasmas and highly ionized astrophysical plasmas, the first usually dominates. In high Z HED plasmas, however, the other two absorption mechanisms are most important. In HED plasmas that are dominated by one ion species (say an ion stripped to the K-shell) we can roughly estimate the Rossland MFP by considering only photo absorption from the ground state. Numerical integration yields the approximate formula:

$$\lambda_R \approx 3\frac{h^4c^3}{Z^2e^6n_0I_p}(kT)^3$$

We see that in this case the radiative heat conductivity $\sim T^6$ and is very nonlinear with temperature. Using a formula similar to this for the Rossland MFP, we can estimate the relative conductivities of radiation and electrons:

$$\frac{\kappa_R}{\kappa_e} \sim 5\times 10^{18}\frac{(kT[eV])^{5/2}}{n_e[cm^{-3}]}$$

which suggests that in a solid density plasma, radiative heat flow dominates when $kT_e > \sim 500$ eV.

III.D Thermal waves

When an impulsive input of energy is made into a localized region of a plasma, thermal gradients lead to a thermal wave. Such a wave will usually have a thermal profile that is steep at the front due to the nonlinear nature of heat conduction in a plasma. If the thermal conductivity varies with temperature nonlinearly with the power law $\kappa = aT^n$ a thermal wave will travel outward, in planar geometry as

$$x_f(t) = \alpha_0 a^{1/n+2} U^{n/n+2} t^{1/n+2}$$

which indicates that an electron conduction driven thermal wave will travel outward as $t^{2/9}$ and a radiation driven wave will travel as $t^{1/8}$. When a thermal wave is driven by a constant heat input (such as by a constant long laser pulse) the wave travels as:

$$x_f(t) = \varepsilon_0^{n/n+2} a^{1/n+2} t^{n+1/n+2}$$

Which indicates that such a wave travels almost linearly from the energy deposition point with time.

Variables used in these notes

n_e	Electron density [cm^{-3}]
k	Boltzman constant [erg/deg]
T_e	Electron temperature [deg]
T_i	Ion temperature [deg]
p_e, p_i p_r	Pressure, electron, ion, radiation [$dyne/cm^2$]
Z	Charge state
E_e, E_i, E_r	Internal energy per unit volume for electrons, ions, radiation [erg/cm^3]
σ	Conductivity
κ	Thermal conductivity [erg-deg/cm-s]
n_0	Average particle density [cm^{-3}]
λ_D	Debye length [cm]
Λ_p	Plasma paramter
e	Electron charge [esu]
ν_{ei}	Electron relaxation collision frequency [s^{-1}]
Λ_C	Coulomb logarithm
m_e	Electron mass
T_F	Fermi temperature [deg]
E_F	Fermi energy [erg]
h	Planck constant [erg-s]
σ_{cold}	Conductivity in the ion sphere limit
Γ	Strong coupling parameter
R_0	Mean ion spacing in the ion sphere model [cm]
c	Speed of light [cm/s]
I_H	Ionization potential of hydrogen [erg]
n_Z	Ion density of the Zth charge ion species [cm^{-3}]
W_Z	Partition function of the Zth charge state
$I_p^{(Z)}$	Ionization potential of the Zth charge state [erg]
Z_{nuc}	Charge of the nuclear core of a plasma species
$E_{ionization}$	Internal energy associated with ionization [erg/cm^3]
G	Nonlocal heat transport convolution function
$\Delta I_p^{(Z)}$	Continuum lowering of the ionization potential [erg]
κ_e	Electron thermal heat conductivity
c_V	Specific heat [erg/deg]
S, S_R	Heat flux [erg/cm^2-s]
S_{max}	Maximum heat flux possible by electron free streaming [erg/cm^2-s]
f	Flux limiter
γ	Adiabatic index
λ_R	Rossland mean free path [cm]
σ_{SB}	Stefan Boltzman constant [$erg/s\text{-}cm^2\text{-}deg^4$]
ν	Photon frequency [s^{-1}]
λ_ν	Mean freem path of a photon of frequency ν [cm]
x_f	Thermal wave front position [cm]
α_0	Thermal wave constant
a	Thermal conductivity cofactor
U	Energy deposited per unit
ε_0	Energy deposited per time per unit area