The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2155-31

International Workshop on Cutting-Edge Plasma Physics

5 - 16 July 2010

Asymmetry-driven structure formation in Pair Plasmas

Nana L. Shatashvili

Javakhishvili Tbilisi State University
Georgia

Asymmetry-driven structure formation in Pair Plasmas

Nana L. Shatashvili[2,3],
S. M. Mahajan[1] & V.I. Berezhiani[2,3]
(1) Institute for Fusion Studies, The University of Texas, Austin, USA
(2) Javakhishvili Tbilisi State University, Georgia
(3) Andronikashvili Institute of Physics, Georgia

Based On:

1. S.M. Mahajan & N.L. Shatashvili. *Phys. Plasmas*, 15 (10), 100701 (2008).
2. S.M. Mahajan, N.L. Shatashvili & V.I. Berezhiani. *Phys. Rev. E*, 80(6), 066404 (2009).
3. V.I. Berezhinai, S.M. Mahajan and N.L. Shatashvili. *Phys. Rev. A*, 81(5), 053812 (2010).
3. V.I. Berezhinai, S.M. Mahajan and N.L. Shatashvili. *J. Plasma Physics* 76(3&4), 467 (2010).

Outline

- **Pair Plasmas in astrophysical and laboratory conditions**
- **Electromagnetic structure formation in Pair Plasmas**
- **Classes of symmetry breaking in Pair Plasmas**
- **Model equations – *new* focusing-defocusing nonlinearity**
- **Formation of localized structures in Pair Plasmas with temperature asymmetry**
- **Initial mass asymmetry – difference with temperature asymmetry**
- **Stable optical vortex solitons in Pair Plasmas – formation & stability**
- **Stable localized EM pulses in asymmetric pair plasmas – light bullets,**
  spatiotemporal spinning solitons
- **Summary & Conclusions, *perspectives***

Pair plasmas in Astrophysical conditions

- **Pair plasmas = only positive- and negative-charged particles of equal mass**
  - special attention due to the astrophysical applications.
- **Early universe during the lepton era - ultra-relativistic electron-positron (e–p) pairs contribute** largely **to the matter contents of the Universe**.
- **Gamma-ray bursts** – *the most concentrated EM explosions in Universe* –
  believed to be related with enormous energy release ***in compact regions on short time-scales***.
- ***Energy release*** → **formation of a highly dense optically thick e–p plasma**
  that expands and cools down remaining **relativistic**.
- **Pair plasmas** exist in:
  active galactic nuclei, **relativistic jets**, **pulsar magnetospheres**.

Pair plasmas in Laboratory conditions

- **Many laboratory setups** in which **e-p pair plasmas are produced**.
- *Recent:* - **successful creation of "sufficiently" dense pair-ion (pi) plasmas** – first such plasma: equal-mass, positive & negative fullerene ions (C^{+}_{60} & C^{-}_{60}).
- **Fullerene plasma has a long enough life time - collective behavior can be experimentally investigated under controlled conditions**. **Frequencies** associated with collective modes (ω_p , ω_s , ω_A) - **low**.
- Group of Hatekayama & Oohara - considerable progress in production of *H^{+}–H^{-}* **PI plasmas**.
- **Many properties of pair plasmas are different from the ordinary electron-ion (e–i) plasma –** *a symmetric pair plasma, for instance, cannot sustain charge separation*
- **Controlled experiments - create a laboratory to simulate & understand a variety of phenomena taking place in astrophysical environments**.
- **A basic requirement for long-time-scale experiments**: the pair annihilation time scale is many orders of magnitude larger than ω_p^{-1}

Theoretical investigation of pair plasmas - 2 distinct tracks:

- **I -** emphasizes the special properties that stem from the **symmetric nature of the pair plasma** - a highly studied field both in astrophysical as well as laboratory contexts:
- **e-p plasmas**, problems like **solitary structure formation are studied quite extensively**.
- **Pair ion plasmas** - much of the linear as well as nonlinear studies **devoted to understanding & interpreting experimental results** – e.g dispersion curves of Oohara & Hatakeyama. PRL, **91**, 205005 (2003); Oohara, Date, & Hatakeyama. PRL, **95**, 175003 (2005)
- **II** - **phenomena that arise because the symmetry of the pair plasma is mildly broken through some mechanism** (creates some disparity between the fluids).
- **One result of symmetry breaking - creation of localized nonlinear structures**; **the particular properties of the structure depend upon the mode of symmetry breaking.**

Symmetry breaking between species

- **Natural to imagine that pair plasmas have to be symmetric:**

  q, n, T, m of positively & negatively charged particles are equal.

- **Observations + experiments indicate: asymmetry may appear at some stage of evolution**.

- **New results from a particle accelerator experiment** (Tevatron collider at Fermi National Accelerator Laboratory, Batavia) **suggest: matter wins the antimatter; an unbalanced ratio of matter to antimatter goes beyond imbalance predicted by St. Model –1% diff**.

- **Asymmetry could be engineered in experiments. Different species, not produced in identical conditions, e.g., could have different thermal speeds (temperatures).**

- **One could arrange experiments with different setups for different species:**

  there are fractions of heavier / lighter ions or

  there is a mixture of different mass / temperature species with opposite charges.

- This way **one could mimic the conditions pertinent to astrophysical pair plasmas**.

Examples of symmetry breaking

- **Most investigated example of broken symmetry - pair plasma contaminated by a small fraction of charged particles with different mass** (lighter/heavier than main species).
- **Symmetry breaking could also occur when the constituent elements of the two fluids have slightly different masses, or the fluids have slightly different temperatures**.
- **Symmetry breaking induces new properties - different from pure pair plasmas.**
- **Pair plasma, contaminated by the heavier immobile ions, can support 3D stable completely localized structures of EM radiation – "light-bullets",** *non-diffracting & nondisspersive EM pulses of pancake shape with large density bunching*.
- **Localized nonlinear structures of EM radiation were also found in a hot e-p relativistic plasma containing a small fraction of cold electron—ion component**.
- **Similar behavior could be expected in doped (or dust-contaminated) fullerene plasmas.**

Towards the model

- **Goal: to establish the existence of EM solitonic structures in pair plasmas .**

- **Plasma is underdense; EM pulse is longer then characteristic skin length of the plasma**.

- **e-i underdense plasma - formation of solitonic structures takes place only at $\omega \sim \omega_p$**

  in such plasma Raman instabilities dominate the process of soliton formation.

- **Pure symmetric pair plasma**, ponderomotive forces are same for different species –

  excitation of longitudinal waves by EM pulse & Raman instabilities can not develop.

- **Slightly asymmetric pair plasmas - ponderomotive forces acting on positively & negatively charged species are slightly different:**

  generation of weak ambipolar electrostatic potential ;

  effects related to Raman instabilities can be ignored to leading order

- **This potential plays a fundamental role in structure formation -**

  the binding "glue" that concentrates matter & radiation in a small region.

Model

The velocity distribution of particles is locally a relativistic Maxwellian.

The dynamics of the fluid of species α is contained in the equations:

α *represents negative and positive particles of any origin*

$$\frac{\partial}{\partial t}(G_\alpha \mathbf{p}_\alpha) + m_{0\alpha} c^2 \nabla (G_\alpha \gamma_\alpha) = e_\alpha \mathbf{E} + (\mathbf{u}_\alpha \times \mathbf{\Omega}_\alpha) \qquad (1)$$

whose curl converts the equation of motion to the vortex-dynamical form:

$$\frac{\partial \mathbf{\Omega}_\alpha}{\partial t} = \nabla \times (\mathbf{u}_\alpha \times \mathbf{\Omega}_\alpha) \qquad (2)$$

where: $\mathbf{p}_\alpha = \gamma_\alpha m_\alpha \mathbf{u}_\alpha$ is hydrodynamic momentum

$\mathbf{\Omega}_\alpha = (e_\alpha / c)\mathbf{B} + \nabla \times G_\alpha \mathbf{p}_\alpha$ is a generalized vorticity

$\gamma_\alpha = (1 - \mathbf{u}_\alpha^2 / c^2)^{-1/2} = (1 + \mathbf{p}_\alpha^2 / m_{0\alpha}^2 c^2)^{1/2}$ $\quad m_\alpha G_\alpha(z_\alpha) = m_\alpha K_3(z_\alpha) / K_2(z_\alpha)$

$m_\alpha G_\alpha(z_\alpha) = m_\alpha K_3(z_\alpha)/K_2(z_\alpha)$ is the **thermally enhanced "effective mass",**

K_ν are the modified Bessel functions $[z_\alpha = m_\alpha c^2/T_\alpha]$,

For **non-relativistic temperatures**: $(T_\alpha \ll m_\alpha c^2)$ $G_\alpha = 1{+}5T\alpha/2m_\alpha c^2$

For **relativistic temperatures**: $(T_\alpha \gg m_\alpha c^2)$ $G_\alpha = 4T_\alpha/m_\alpha c^2 \gg 1$

The **relativistic thermal pressure in rest frame** $P_\alpha = (n_\alpha/\gamma_\alpha)T_\alpha$

Where n_α is the **density in laboratory frame** & $\gamma_\alpha \nabla P_\alpha = m_\alpha c^2 n_\alpha \nabla G_\alpha.$

The system of Eqs. (1)-(2) is augmented by the **equation of state**:

$$\frac{n_\alpha z_\alpha}{\gamma_\alpha K_2(z_\alpha)} \exp[-\mathrm{z}_\alpha \mathrm{K}_2(\mathrm{z}_\alpha)] = \mathrm{const}_\alpha \tag{3}$$

which yields the usual results:

$n_\alpha^{-} T_\alpha^{3/2} = const$ in non-relativistic limit for mono-atomic gas, $n_\alpha^{-} T_\alpha^{3} = const$ ultra-relativistic case for photons.

(2) = > **If generalized vorticity is initially zero** $(\boldsymbol{\Omega}_\alpha = 0)$ **everywhere in space,**

it will remain zero for all subsequent times.

***We assume*: before the EM radiation is "switched on"** the **generalized vorticity of system is 0**.

For both species we have the continuity equation:

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0. \tag{4}$$

In terms of the vector (***A***) and electrostatic ($\boldsymbol{\varphi}$) potentials

the basic equations take the form (Coulomb gauge $\nabla \cdot \mathbf{A} = 0$):

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - c^2 \triangle \mathbf{A} + c \frac{\partial}{\partial t}(\nabla \varphi) - 4\pi c \mathbf{J} = 0 \tag{5}$$

$$\triangle \varphi = -4\pi \rho \tag{6}$$

where for **the charge and current densities** we have respectively:

$$\rho = \sum_\alpha e_\alpha n_\alpha \qquad \mathbf{J} = \sum_\alpha e_\alpha n_\alpha \mathbf{u}_\alpha \tag{7}$$

The equilibrium state is characterized by charge neutrality: $n_0^+ = n_0^-$

Subscript α hereafter will indicate negative (α = -) and positive (α = +) ions.

In ***dimensionless variables***:

$$\mathbf{p}^\pm = \frac{\mathbf{p}^\pm}{m^- c}\,, \quad n^\pm = \frac{n^\pm}{n_0^\pm}\,, \quad T^\pm = \frac{T^\pm}{m^- c^2}, \quad \mathbf{A} = \frac{|e|\mathbf{A}}{m^- c^2}\,, \quad \phi = \frac{|e|\varphi}{m^- c^2}\,, \quad \mathbf{r} = \frac{\omega_-}{c}\mathbf{r}\,, \quad t = \omega_- t\,,$$

We get the ***set of equations***:

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \triangle \mathbf{A} + \frac{\partial}{\partial t}(\nabla \phi) + \left[\frac{n^- \mathbf{\Pi}^-}{\Gamma^-} - \frac{n^+ \mathbf{\Pi}^+}{\Gamma^+}\right] = 0, \tag{8}$$

$$\triangle \phi = n^- - n^+ \tag{9}$$

$$\frac{\partial}{\partial t}\Pi^\pm + \nabla \Gamma^\pm = \mp \frac{\partial \mathbf{A}}{\partial t} \mp \nabla \phi \tag{10}$$

$$\frac{\partial n^\pm}{\partial t} + \nabla \cdot \left(n^\pm \frac{\mathbf{\Pi}^\pm}{\Gamma^\pm}\right) = 0 \tag{11}$$

Where $\omega_- = (4\pi e^2 n_0^- / m^-)^{1/2}$ is the ***Langmuir frequency of negative species***,

and it was convinient to introduce **temperature dependent momentum** $\mathbf{\Pi}^\pm = G^\pm \mathbf{p}^\pm$

and **relativistic factor**: $\Gamma^\pm = G^\pm \gamma^\pm = \sqrt{(G^\pm)^2 + (\mathbf{\Pi}^\pm)^2}$.

The **equations of state for species then read:**

$$\frac{n^\pm}{\Gamma^\pm f(T^\pm)} = \frac{1}{G_\infty^\pm f(T_\infty^\pm)} \tag{12}$$

with: $f(T^\pm) = [T^\pm K_2(1/T^\pm)/G^\pm] \exp[\mathrm{G}^\pm/\mathrm{T}^\pm]$ and $T_\infty^\pm$ the equilibrium temperature.

We study the **propagation** (along the z axis) **of a circularly polarized EM wave:**

$$\mathbf{A}_\perp = \frac{1}{2}(\mathbf{x} + i\mathbf{y})A(\mathbf{r}_\perp, z, t)exp(ik_o z - i\omega_o t) + c.c., \tag{13}$$

Where A (z,t) is a slowly varying function of z and t .

Assumption: longitudinal extent of pulse is << than its transverse dimensions $(\partial A/\partial z \gg \nabla_\perp A)$

The **gauge condition gives** $A_z = (i/k_0)(\nabla_\perp \cdot \mathbf{A}_\perp);\ |A_z| \ll |A_\perp|$.

Transverse component of (10) gives: $$\mathbf{\Pi}_{\perp}^{\pm} = \mp\mathbf{A}_{\perp} \tag{14}$$

particle hydrodynamic moments are assumed to be zero at infinity where the field vanishes.

Note: for longitudinal motion the equations of motion can be treated one-dimensionally.

Longitudinal dynamics is described by (11) & the *z*-component of Eq. of motion (10)

Note: due to the circular polarization of EM wave

$$\gamma^{\pm} = [1+|A|^2/(G^{\pm})^2+(\Pi_z^{\pm})^2/(G^{\pm})^2]^{1/2}$$

Introducing the following variables:

$$\xi = z - v_g t \qquad \tau = t$$

Assuming $v_g \partial/\partial\xi \gg \partial/\partial\tau$.

straightforward algebra gives the **integral of motion**:

$$G^{\pm}\left[1+\frac{|A|^2}{(G^{\pm})^2}+\frac{(\Pi_z^{\pm})^2}{(G^{\pm})^2}\right]^{1/2} - v_g\Pi_z \pm \phi = G_{\infty}^{\pm}(T_{\infty}^{\pm}) \tag{15}$$

We deal with **transparent plasmas**, i.e, $\omega_0 >> 1$, & $v_g \approx 1$ (found both in astrophys. & Lab. Cond.)

Then with $G_\infty^\pm \equiv G_\infty^\pm(T_\infty^\pm)$ straightforward algebra leads to:

$$n^\pm = \frac{\gamma^\pm}{\gamma^\pm - p_z} \qquad \gamma^\pm - p_z = \frac{G_\infty^\pm}{G^\pm}\left[1 \mp \frac{\phi}{G_\infty^\pm}\right]^{-1} \qquad \frac{n^\pm}{\gamma^\pm} = \frac{G^\pm}{G_\infty^\pm}\left[1 \mp \frac{\phi}{G_\infty^\pm}\right]^{-1} \tag{16}$$

And

$$p_z^\pm = \frac{G^\pm}{2G_\infty^\pm}\left[1 + \frac{|\mathbf{A}|^2}{(G^\pm)^2} - \frac{G_\infty^\pm}{(G^\pm)^2}\left[1 \mp \frac{\phi}{G_\infty^\pm}\right]^2\right]\left[1 \mp \frac{\phi}{G_\infty^\pm}\right]^{-1} \tag{17}$$

$$\gamma^\pm = \frac{G^\pm}{2G_\infty^\pm}\left[1 + \frac{|\mathbf{A}|^2}{(G^\pm)^2} + \frac{G_\infty^\pm}{(G^\pm)^2}\left[1 \mp \frac{\phi}{G_\infty^\pm}\right]^2\right]\left[1 \mp \frac{\phi}{G_\infty^\pm}\right]^{-1} \tag{18}$$

which allow us to write **densities fully in terms of potentials** φ **and** A :

$$n^\pm = \frac{G^\pm}{2G_\infty^\pm}\left[\left(1 + \frac{|\mathbf{A}|^2}{(G^\pm)^2}\right)\left(1 \mp \frac{\phi}{G_\infty^\pm}\right)^{-2} + 1\right] \tag{19}$$

Formation of localized structures in pair plasmas - temperature asymmetry

- **Pure pair plasma:** $T^- = T^+$ - the radiation pressure gives equal longitudinal momenta to the negative & positive ions (effective masses are equal $G^- = G^+ = G$)
  no charge separation ($n^- = n^+$ & $\varphi = 0$)

- Situation changes by introducing a **small fraction of heavy ions**; with the **"symmetry breaking" between hot electrons and positrons, it becomes possible to generate a finite φ**

- **Existence of electrostatic potential is possible due to the small fraction of different temperature electrons; pair plasmas respond similarly.**

- **Assumption:** "asymmetry" through temperature difference $T^- \neq T^+$ (both experimentally & observationally justified) → different "effective masses" even though the real masses are equal ($m^- = m^+$).

As a first step we assume, that temperatures are only slightly different (β ~ 1)

$$\frac{G_\infty^-}{G_\infty^+} = \beta \neq 1 \tag{20}$$

(16) =>

$$\frac{n^+}{\gamma^+} = \frac{G^+}{G^+_\infty}\left[1 - \frac{\phi}{G^+_\infty}\right]^{-1} = \frac{g^+}{1-\beta\hat{\phi}} \qquad \frac{n^-}{\gamma^-} = \frac{G^-}{G^-_\infty}\left[1 + \frac{\phi}{G^-_\infty}\right]^{-1} = \frac{g^-}{1+\hat{\phi}}, \qquad (21)$$

With $g^\pm = (G^\pm/G^\pm_\infty)$ and $\hat{\phi} = \phi/G^-_\infty$, $\hat{A} = A/G^-_\infty$.

(19) =>

$$n^+ = \frac{1}{2}\left[1 + \frac{(g^+)^2 + \beta^2|\hat{A}|^2}{(1-\beta\hat{\phi})^2}\right] \qquad n^- = \frac{1}{2}\left[1 + \frac{(g^-)^2 + |\hat{A}|^2}{(1+\hat{\phi})^2}\right] \qquad (22)$$

Below we omit $\hat{}$ & it will be assumed that φ & A are normalized to G^-_∞.

The expression for the total current $\mathbf{J}^\pm = n^\pm \mathbf{\Pi}^\pm/\Gamma^\pm$ is easily derived to be:

$$\mathbf{J}_\perp = -\left[\frac{1}{1-\beta\,\phi} + \frac{1}{1+\phi}\right]\mathbf{A} \qquad (23)$$

Using: **the quasi-neutrality condition** $n^+ = n^-$ (characteristic length-scale of wave L >>1)

for a transparent plasma, heating of both fluids is very weak (implying $g^\pm \sim 1$)

we derive: $\phi \sim (1-\beta)\,\psi(|A|^2)$ where $\psi(|A|^2) \leq 1$

Then the nonlinear term of (8) is found to be:

$$\mathrm{NL} = \frac{1}{1-\beta\,\phi} + \frac{1}{1+\phi} - 2 \simeq -\phi\,[(1-\beta) - 2\,\beta\,\phi] \qquad (24)$$

Where we do not neglect ***Φ*** w.r.t $(1-\beta) \ll 1$

To complete Maxwell's equations, **we have to relate *Φ* (< 1) with its source - temperature difference between species!**

Super-relativistic temperature pair plasmas

For $T^{\pm} \gg 1$ we get $G^{\pm} = 4T^{\pm}(\gg 1)$, $g^{\pm} = T^{\pm}/T^{\pm}_{\infty}$

Eq. (12) reads

$$\frac{n^{\pm}}{\gamma^{\pm}} = \left(\frac{T^{\pm}}{T^{\pm}_{\infty}}\right)^3 = g^{\pm 3} \tag{25}$$

Which yields

$$g^{+} = \frac{1}{(1-\beta\phi)^{1/2}}, \qquad g^{-} = \frac{1}{(1+\phi)^{1/2}} \tag{26}$$

and

$$\phi = \frac{(1-\beta)}{2}\,\frac{\kappa|A|^2}{(1+\beta\,\kappa|A|^2)} \quad \text{with} \quad \kappa \equiv \frac{2}{3} \tag{27}$$

thus, $\phi \sim (1-\beta)$ when $|A|^2 \gg 1$ and $\phi \leq (1-\beta)$ when $|A|^2 \leq 1$.

(27) => **Heating / cooling of both fluids is weak** - $g^{+} \sim (1 + 0.5\beta\,\phi)$; $g^{-} \sim (1 - 0.5\,\phi)$

$$n^{\pm} \sim 1 + \frac{\beta}{2}|A|^2. \tag{28}$$

Non-relativistic temperature pair plasmas

For $T^{\pm}, T^{\pm}_{\infty} \ll 1$ and $G^{\pm} = 1 + 5T^{\pm}/2$

Relevant relations are:

$$\frac{n^{\pm}}{\gamma^{\pm}} = \left(\frac{T^{\pm}}{T^{\pm}_{\infty}}\right)^{3/2}, \quad g^{\pm} \simeq 1 + \frac{5}{2}(T^{\pm} - T^{\pm}_{\infty}), \tag{28}$$

$$\beta = \frac{G^{-}_{\infty}}{G^{+}_{\infty}} \simeq 1 + \frac{5}{2}(T^{-}_{\infty} - T^{+}_{\infty}). \tag{29}$$

$$g_{\pm} = 1 + H_{\pm}, \qquad H_{+} = \frac{5}{2}\beta T^{+}_{\infty}\phi \qquad \text{and} \qquad H_{-} = -\frac{5}{2}T^{-}_{\infty}\phi\ . \tag{30}$$

And we find: $g_{+} + g_{-} \simeq 2 + (\beta - 1)\phi\ ,\qquad g_{+} - g_{-} \simeq \frac{5}{2}\phi T^{-}_{\infty}(1+\beta),$

From which:

$$g_{+}^{2} - g_{-}^{2} \simeq \phi T^{-}_{\infty}(1+\beta) \tag{31}$$

And **again** (**with all the similar consequences as in ultra- relativistic case**):

$$\phi = \frac{(1-\beta)}{2}\frac{\kappa|A|^{2}}{(1+\beta\kappa|A|^{2})}, \quad \text{with} \quad \kappa \equiv \frac{1}{2} \tag{32}$$

Localized structures

From (27) & (32) =>

$$J_\perp \simeq -(2-\phi\left[(1-\beta)-2\beta\,\phi\right])\,A = -\left[2-\frac{(1-\beta)^2}{2}\frac{\kappa\,|A|^{\,2}}{(1+\beta\,\kappa\,|A|^{\,2})^{\,2}}\right]A \qquad (33)$$

With $\kappa = 2/3$ ($\kappa = 1/2$) for relativistic (non-relativistic) temperatures.

Final equations reads:

$$2i\omega_0\frac{\partial A}{\partial\tau}+\frac{1+\beta}{\omega_0^2}\,\frac{\partial^2 A}{\partial\xi^2}+\nabla_\perp^2\,A+\frac{(1-\beta)^2}{2}\,\frac{\kappa\,|A|^{\,2}}{(1+\beta\,\kappa\,|A|^{\,2})^2}\,A=0, \qquad (34)$$

with $\epsilon^2 \equiv \frac{1}{4}(1-\beta)^2 \ll 1,$

Where we have redefined mass as → $m_- \rightarrow m_- G_\infty^-$

Dispersion relation: $\omega_0^2 = \quad k_0^2 + (1+\beta)$ [$\omega_0^2 = k_0^2\,c^2 + (1+\beta)\,\omega_-^2$)]

Despite $\partial A/\partial\xi \gg \nabla_\perp A$ **II & III terms can be comparable** (transparent plasma $(\omega_0^2 \gg 2)$

With self-evident renormalization Eq. (34) is written as:

$$i\frac{\partial A}{\partial \tau} + \frac{\partial^2 A}{\partial \xi^2} + \nabla_\perp^2 A + F(|A|^2) \cdot A = 0, \tag{35}$$

With the **nonlinearity function**:

$$F(|A|^2) = \frac{|A|^2}{(1+|A|^2)^2} \ . \tag{36}$$

Eq. (35) is a NLS with saturating nonlinearity $F(|A|^2)$ **of new type** – has an unusual form –

In ultra-relativistic case $|A|^2 \gg 1$ $F(|A|^2)$ $\rightarrow 0$

From refractive index $\delta n_{nl} = F(I)$ ($I = |A|^2$ being intensity)

Plasma is self-focusing $d(\delta n_{nl})/dI > 0$ provided $I < 1$

Plasma is de-focusing $d(\delta n_{nl})/dI < 0$ for $I > 1$

- ***For a localized intense EM pulse with peak intensity*** $I_m > 1$ ***medium reacts differently to different parts of EM pulse!***

Modulational instability

Eq. (35) is satisfied by the **plane wave solution**:

$$A = A_0 \exp(i\tau F(|A_0|^2)) + c.c..$$

- Standard stability analysis shows that linear modulation with frequency Ω & wave number **K obeys dispersion relation:**

$$\Omega^2 = \mathbf{K}^2[\mathbf{K}^2 - 2A_0^2(1 - A_0^2)/(1 + A_0^2)^3]$$

Which exhibits **a purely growing mode** if $A_0 < 1$ &

$$K < K_{cr} = \sqrt{2A_0^2(1 - A_0^2)/(1 + A_0^2)^3}$$

For ultra-relativistic case $(A_0 > 1)$ **there is no modulational instability.**

- **One can expect that the modulation instabilityof moderately intense field** $(A_0 < 1)$ **in the nonlinear stage will lead to the break up of the field into soliton-like pulses**

with a characteristic length corresponding to the optimum scale of instability $(\sim \sqrt{2}/K_{cr})$

Stable solitonic soultions

For **stationary solitons**, we look for solutions that are **”spherical” symmetric**: $A = A(r)\exp(i\lambda)$

λ - a constant measuring the **nonlinear frequency shift**.

In terms of the radial variable $r = (\mathbf{r}_\perp^2 + \xi^2)^{1/2}$ ***Eq. (35) reduces to ODE that can not be analytically solved.***

It is helpful to rewrite it as the equation describing **a ”particle” moving with friction in the potential:**

$$\frac{d}{dr}\left[\left(\frac{dA}{dr}\right)^2 + V(A)\right] = -\frac{2(D-1)}{r^{D-1}}\left(\frac{dA}{dr}\right)^2 \tag{37}$$

wih $V(A) = -\lambda A^2 + \ln(1+A^2) - A^2/(1+A^2).$ Here $D(=1,2,3)$ is a dimension.

Stationary solitary solutions

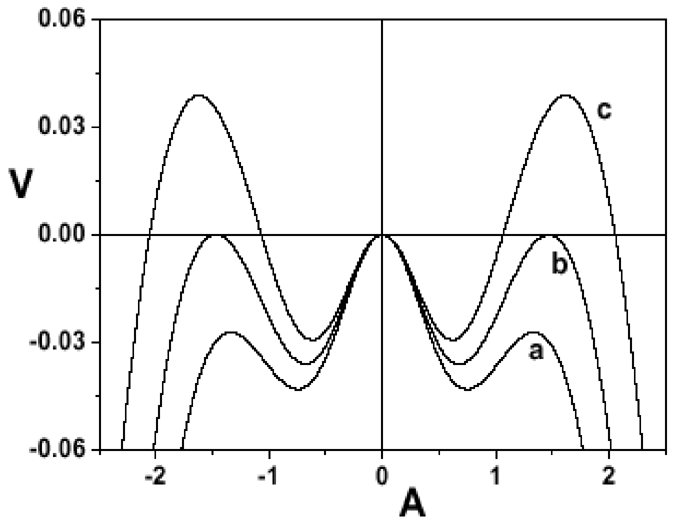


Fig.1 **Effective potential versus amplitude for different λ.**

"a" - $\lambda > \lambda_{cr}^{(1D)} \simeq 0.2162$

- ***no solitary solution***

"b" - $\lambda = \lambda_{cr}^{(1D)}$ critical value

"c" - $0 < \lambda < \lambda_{cr}^{(1D)}$

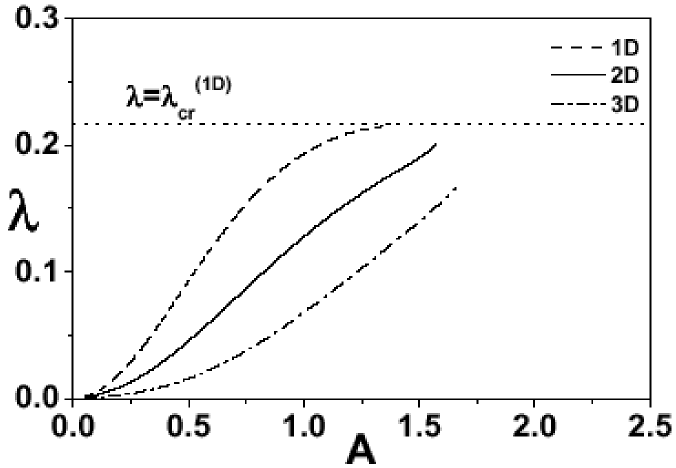


Fig.2 **Nonlinear Dispersion relations: the effective eigenvalue λ as a function of A_m.**

Boundary line (dotted) - $\lambda = \lambda_{cr}^{(1D)}$

In range $0 < \lambda < \lambda_{cr}^{(1D)}$

solitany solutions exist in 1, 2, 3 dimensions

A_m is found numerically for 2D, 3D. ID solution is restricted from above by

$$A_m \leq A_{mcr} \cong 1.4506$$

1D Stationary solitons

For small amplitudes Eq. (35) reduces to the standard NSE with a cubic nonlinearity, 1D soliton solution can be found analtically!

For large amplitude 1D solitons as

$$A \to A_{mcr} \quad (\lambda \to \lambda_{cr}^{(1D)})$$

we get the flat-top soliton - focusing-defocusing nonlinearity: ***the top part of the pulse with A >1 lies in the defocusing region with a tendency for diffraction while the wings of the soliton are in the focusing region preventing the total spread of the pulse.***

For $\lambda > \lambda_{cr}^{(1D)}$ system supports existence of dark soliton - antisymmetric function of coordinate with zero intensity at its center.

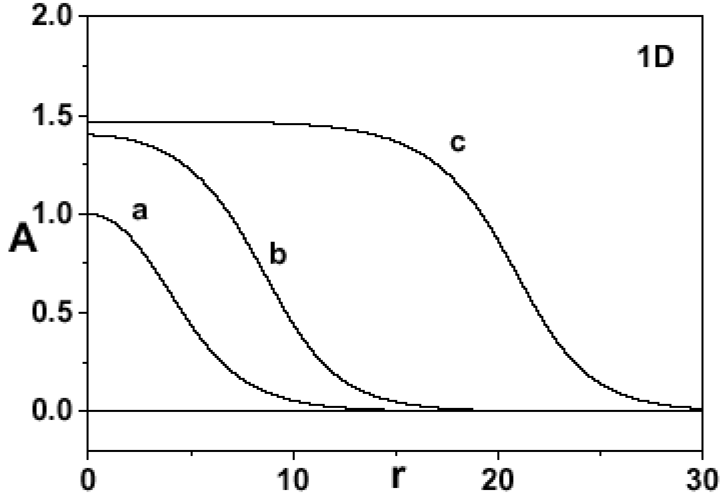


Fig.3 **Stationary solitons for 1D – different λ_{cr}**

"a" – $\lambda_{cr} = 0.19315$ with $A_m = 1$

"b" – $\lambda_{cr} = 0.21583$ with $A_m = 1.4$

"c" – $\lambda_{cr} = 0.21622$ with $A_m = 1.47$

"c" **-** ***flat-top soliton***

2D and 3D stationary solitons

2D , 3D - nonzero "friction force" forces corresponding critical values of $\boldsymbol{\lambda}$ ***to be*** $< \lambda_{cr}^{(1D)}$

Fundamental solitary solutions (without zero nodes)

Fundamental feature *of the soliton persists -* ***near the critical eigenvalues, the profile is endowed with the flat-top shape.***

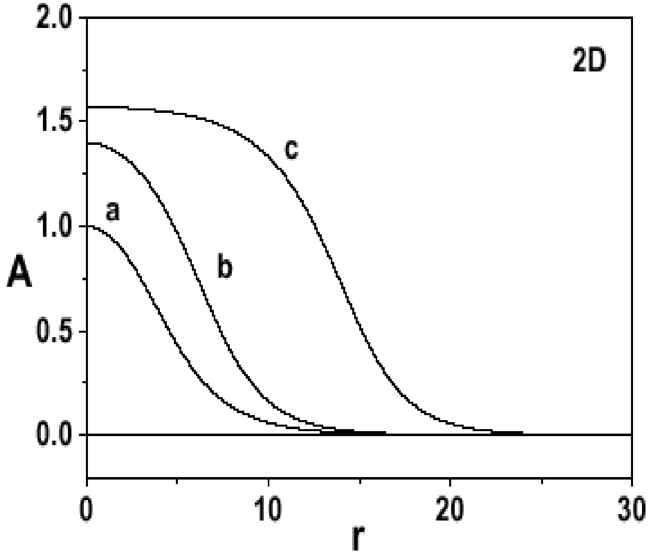


Fig.4

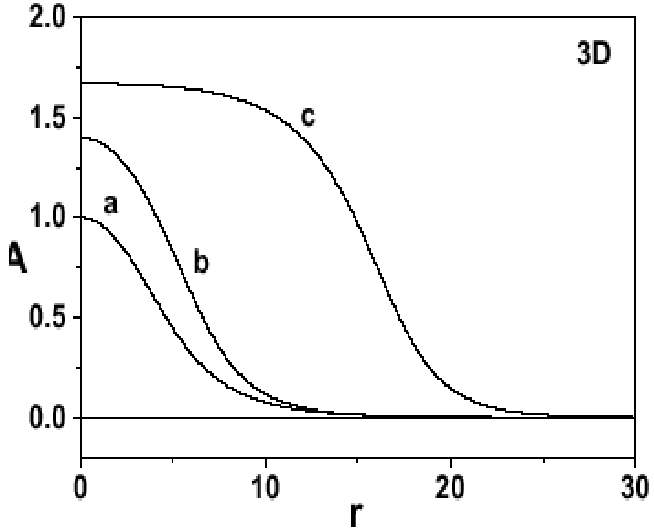


Fig.5

Stability of solutions

Vakhitov & Kolokolov criteria =>

Soliton is stable if $\partial N/\partial\lambda > 0$

$N = \int d\mathbf{r}_\perp d\xi \, |A|^2$ **- soliton energy ("photon number")**

1D - "photon number" is always a growing function of λ = such **solitons are stable against small perturbations.**

Soliton to exist in higher dimensions, its $N > N_{cr}$

Since $\partial N/\partial A_m > 0$ **then** $\partial N/\partial\lambda > 0$

Because A_m **is a growing function of** λ **(Fig.2)**

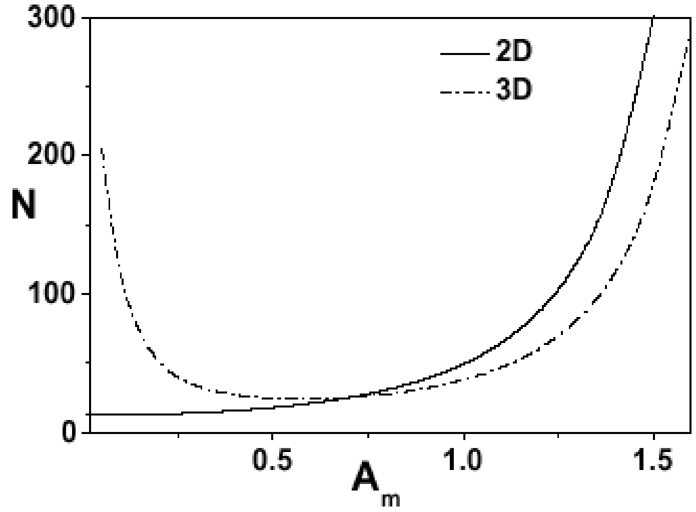


Fig.6 The dependence of *N* on *A* $_m$ for *2D & 3D.*

Normalized "photon numbers" correspond to $N^{(2D)}$ (solid line) ***and*** $10^{-1} N^{(3D)}$ (dashed line).

For 2D (3D) the threshold energy for the existence of soliton is $N_{cr} = 11.6$ (236.8).

Mass asymmetry between species

- **Another obvious source of asymmetry between species – a slight difference in the masses of positive & negative-charged particles.**

- **Different** from the one discussed above - **the mass asymmetry is initially given & is fixed as distinct from the dynamical asymmetry created due to temperature differences.**

- **Such a plasma can be created by the injection of appropriate ion-beams into a trap.**

- **Electron-hole plasma in certain semiconductors or e-p collider plasma created by slightly different Lorentz factor beams are also possible examples of such system.**

- **Appropriate conditions for such plasma production could readily appear in Dusty plasmas as well as in astrophysical Jets, and Pulsar magnetospheres.**

- ***Unmagnetized plasma*, equations (8) & (9) are used with changed Lorentz factors as:**

$$\gamma^- = \left[1 + (\mathbf{p}^-)^2\right]^{1/2} \quad \gamma^+ = \delta^{-1}\left[\delta^2 + (\mathbf{p}^+)^2\right]^{1/2} \quad \text{with} \quad \delta = m^+/m^-$$

Circularly polarized waves with wave frequency $\omega_0^2 = k_0^2 + (1+\delta^{-1})$

$\mathbf{P}_\perp^\pm = \mp\mathbf{A}$ & for **transparent plasma** $(\omega_0 \gg (1+\delta^{-1})^{1/2},\ v_g = k_0/\omega_0 \simeq 1)$

$$\frac{n^-}{\gamma^-} = \frac{1}{1+\phi}\,, \qquad \frac{n^+}{\gamma^+} = \frac{\delta}{\delta-\phi} \tag{38}$$

And

$$n^- = \frac{1}{2}\left[1 + \frac{1+|A|^2}{(1+\phi)^2}\right],\ n^+ = \frac{1}{2}\left[1 + \frac{\delta^2+|A|^2}{(\delta-\phi)^2}\right] \tag{39}$$

Leading to

$$2i\omega\frac{\partial A}{\partial\tau} + \frac{(1+\delta^{-1})}{\omega^2}\frac{\partial^2 A}{\partial\xi^2} + \nabla_\perp^2 A + A\phi\left[\frac{1}{1+\phi} - \frac{1}{\delta\,(\delta-\phi)}\right] = 0. \tag{40}$$

For small mass difference $\delta = 1+\eta\ (\eta \ll 1)$ **and characteristic length** $L \gg 1$

$$\phi = \frac{\eta|A|^2}{2(1+|A|^2)} \qquad \phi \ll 1 \text{ for } \eta \ll 1 \tag{41}$$

Compare with equations (27), (32).

Eq.-s (40), (41) *with appropriate normalization of variables & inclusion of transverse field variations,* **constitute an NLSE (35) with following saturating nonlinearity** (*η - absorbed*):

$$F(|A|^2) = 1 - \frac{1}{(1+|A|^2)^2} \, . \tag{42}$$

Notice: **this form of saturation nonlinearity function coincides with the one obtained in for e-p plasma with a small fraction of heavy ions.**

Equation (35) (with (42)) admits a "spherically" symmetric solitary wave solution. i.e. "light bullet", a concentration of mass and energy.

If the "bullet" $A_m >$ 0 7 , this "bullet"is stable.

These "light bullets" are found to be exceptionally robust

The total **plasma density variation associated with the soliton** $\delta n \sim A^2$ **is large for** $A^2 >> 1$

Some discussions

The saturating nonlinearity (36) caused by an initial temperature asymmetry - of a new type (vanishing for intense pulses).

It differs quite fundamentally from the one found for density asymmetry (identical to the one originating in a initial mass asymmetry).

Translating the **temperature asymmetry** into a difference in the "effective masses" $G^{\pm}m^{\pm}$

one finds that it **is dynamical and dependent on** ϕ : $m_{\text{eff}}^{+}/m_{\text{eff}}^{-} \sim (1-\epsilon+0.5\,\nu\,\phi)$

One originating in an initial mass asymmetry $(\delta = 1+\eta)$ **is constant and never leads to heating/cooling!**

The electrostatic potential ϕ **is important for:**

maintaining nonlinearity + to create dynamical temperature asymmetry for hot plasma conditions => new type of nonlinearity

(42) & corresponding results – valid for cold plasmas only!

Main property: density bunching & energy localization is always there in pair plasmas with different type initial asymmetries; it is just the character of localization that changes

Stable optical vortex solitons in PPs – formation & stability

A new focusing-defocusing nonlinearity (36) belongs to the general class of saturating nonlinearities (whose magnitude tends to a constant as the wave amplitude becomes large).

Saturating nonlinearities appear in theories of large amplitude wave propagation in pair plasmas in which the pair symmetry is broken by some physical mechanism:

- **a small amount of Baryonic matter (protons) may break the symmetry of an (e-p) plasma in the MEV era of the early universe.**
- in recently created **pair ion (PI) plasmas in the laboratory**, **a variety of symmetry breaking mechanisms** like the small contamination by a much heavier immobile ion, or a small mass difference between the two constituent species, **could exist.**
- **in the laboratory a temperature difference could be readily engineered (in a controlled way)**
- there are reasons to believe that **species temperature difference could exist in cosmic and astrophysical setting where one encounters e-p plasmas**.

Formation of optical vortices in Pair Plasmas

In a self-defocusing medium an optical vortex soliton (OVS) is a (2 + 1)–dimensional *(two transverse dimensions and time)* stationary beam structure with phase singularity.

An OVS is a dark spot (a zero intensity center surrounded by a bright infinite background).

Self-focusing media also support localized optical vortex soliton solutions (LOVS) with phase dislocation surrounded by a bright ring.

In self-focusing medium, LOVS are unstable against symmetry breaking perturbations that lead to the breakup of rings into filaments.

Exploring the possibility for the formation of 2D stable soliton-structures carrying a screw type of dislocation, i.e., optical vortices in Pair Plasmas.

The new nonlinearity (36) has focusing-defocusing features - both OVS & LOVS may exist.

In contrast to cubic-quintic model sign-changing nonlinearity ($f(|A|^2) = |A|^2 - |A|^4$), focusing-defocusing satur. nonl. $f(|A|^2) = |A|^2(1+|A|^2)^{-2}$ has same sign for all values of $|A|$.

Assumption: **pulse is sufficiently long & effects related to the group velocity dispersion** $(\sim \omega_0^{-2}\, \partial^2 A/\partial\xi^2)$ **can be ignored**

We look for solutions in polar coordinates as: $A = A(r)\exp(i\lambda t + im\theta),$ (43)

where integer ***m*** defines the topological charge of vortex.

Ansatz (43) converts (35) to ODE ($r = \left(\mathbf{r}_\perp^2 + Z^2\right)^{1/2}$):

$$\frac{d^2A}{dr^2} + \frac{1}{r}\frac{dA}{dr} - \frac{m^2}{r^2}A - \lambda A + \frac{A^3}{(1+A^2)^2} = 0. \qquad (44)$$

Using numerical methods to solve (44), mapping it in (A, A_r) **plane (phase plane) one finds:**

- **(44) has both OVS & LOVS solutions;**

- **LOVS can exist in the form of an infinite number of discrete bound states** where the radial quantum number ***n*** denotes the finite ***r*** zeros of the eigenfunction.

  $A_n(r)\ (n = 1, 2, ...)$

We consider only the lowest radial eigensolution (n = 1) of Eq.(44).

m* ≠ *0 - ground state LOVS is positive amplitude, has a node at the origin ***r* = *0***, reaches a maximum, & then monotonically decreases as $r \to \infty$

LOVS exists for λ > 0 & $A_{r\to 0} \to r^{|m|} A_0$, $A_{r\to\infty} \to \exp(-r\sqrt{\lambda})/\sqrt{r}$

A_0 is a constant measuring slope of A at origin .

OVS has same behavior at $r \to 0$, while at $r \to \infty$ it has a nonzero value

$$A(r) = A_\infty + m^2/(r^2 f'(A_\infty)) \qquad \lambda = f(A_\infty)$$

OVS exists only if $f'(A_\infty) < 0$ - background intensity of the soliton (far beyond vortex core) is still relativistic $A_\infty > 1$

i.e. $d\delta n_{nl}/dI < 0$ **medium is defocusing in asymptotic region of the solution!**

Constant background field with $A_\infty > 1$ **is modulationally stable.**

Simulation results

Shooting code was used to solve Eq. (44). **Analogy to non-conservative motion of particle:**

$$\frac{d}{dr}\left[\left(\frac{dA}{dr}\right)^2 + V(A)\right] = \frac{m^2}{r^2}\frac{dA^2}{dr} - \frac{2}{r}\left(\frac{dA}{dr}\right)^2 \tag{45}$$

With **effective potential**:

$$V(A) = -\lambda A^2 + \ln(1 + A^2) - A^2/(1 + A^2)$$

Has two maxima at $A = 0$ &

$$A_{\max} = \sqrt{\left[1 - 2\lambda + \sqrt{1 - 4\lambda}\right]/2\lambda}$$

Bounded solution exists only with $A_{\max} > 1$

for $0 < \lambda < 0.25$

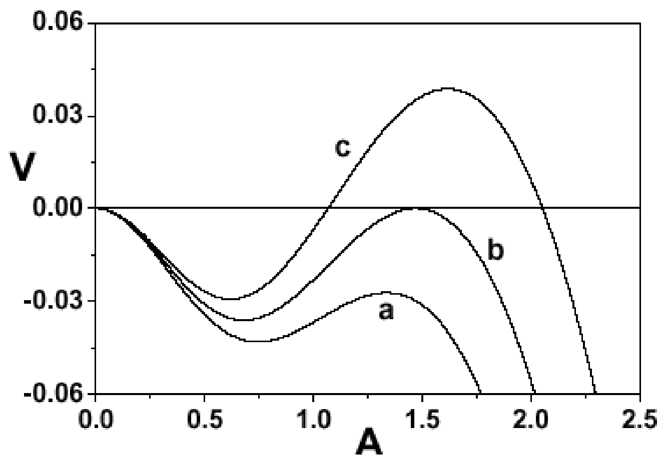


Fig. 7 "a" $\lambda > \lambda_{cr} \simeq 0.2162$ "b" - $\lambda = \lambda_{cr}$ "c" $0 < \lambda < \lambda_{cr}$

OVS & LOVS solutions

OVS solution – starts at A = 0 with initial A_0 (velocity if m=1 , acceleration if m=2 & etc)

It dissipates its initial energy as asymptotically approaches potential maximum A_{max}

Background of OVS $A_\infty = A_{max} > 1$ always; can become arbitrarily large for $\lambda \to 0.$

OVS also exists for $0.25 > \lambda > \lambda_{cr} \simeq 0.2162$

i.e when $V(A_{\max}) < 0$

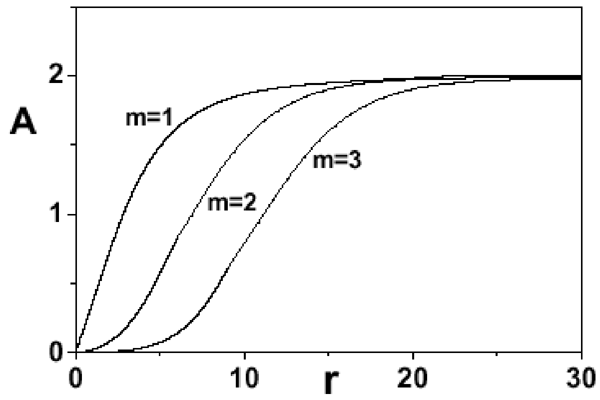


Fig. 8 OVS-s for m = 1; m = 2; m = 3; λ= 0:16.

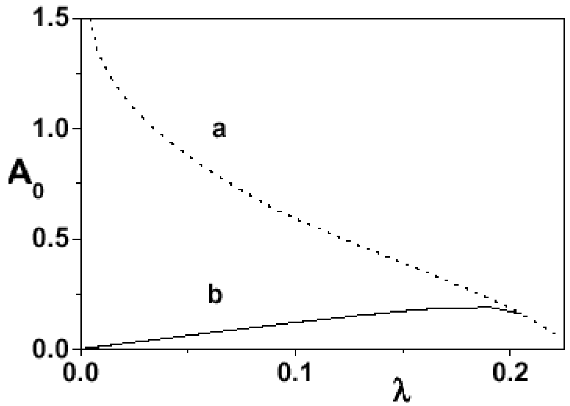


Fig.9 m = 1, “a” – OVS, “b” - LOVS

LOVS solutions - particle returning back asymptotically to initial position at *A = 0*.

Due to "friction" the "particle" can not make its way back if $\lambda > \lambda_{cr}$

LOVS may exist if $0 < \lambda < \lambda_{cr}$ **, its amplitude is a growing function of λ**

Amplitude of LOVS is bounded from above by $A_{cr}(\simeq 1.5)$

- is moderately relativistic!

For $0.16 \leq \lambda \leq \lambda_{cr}$ amplitude varies in the range

$$1 \leq A_m \leq A_{cr}$$

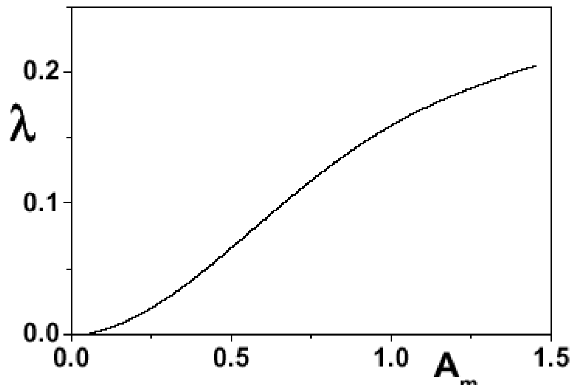


Fig.10 λ versus *Am*.

It is possible to create flat-top LOVS with large transverse width.

Convergence of LOVS to OVS is possible at $\lambda \approx \lambda_{cr}$

Similar behavior for large charge (m = 1, 2, …)

Fig.11 "a", "b", "c" – LOVS; "d" - OVS

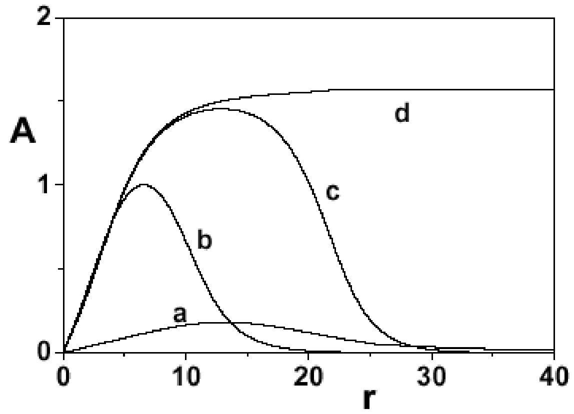


Fig.11.

Stability

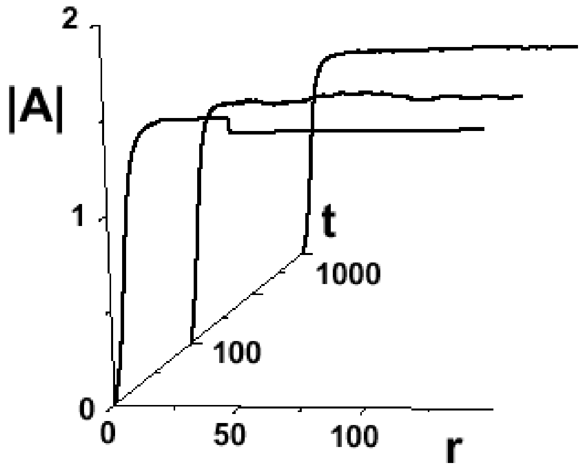
- **OVSs with m = 1 are stable**
- **OVSs with m >1 may decay into m = 1 vortices in self-defocusing media.**

Bulk of OVS – defocusing, stable; core of vortex – focusing = > Stability is not guaranteed!

Numerical solution of (44) following the **linear stability procedure:**

perturbations acting along a ring of mean radius r_*, $A(r_*) = A_m$

Fig.12 Initially perturbed OVS dynamics; t=0; 100; 1000



Assuming constant intensity & spatial uniformity ring, $\nabla_\perp^2 = r_*^{-2}\partial^2/\partial\theta^2$

introducing azimuthal perturbation with a phase factor

$\Psi = \Omega t + M\theta$ (M integer) ; we derive:

$$\mathrm{Im}(\Omega) = \frac{M}{r_*}\mathrm{Re}\left[\frac{2(1-A_m^2)}{(1+A_m^2)^3} - \frac{M^2}{r_*^2}\right]. \tag{46}$$

Large amplitude LOVS with $A_m > 1$ is always stable.

lower amplitude LOVS decay into M_{max} multiple filaments,

M_{max} ≈ the number for which maximal growth rate is maximal.

Instability splits pulse into filaments with their number being 2, 4, & 5 (or 6) for m = 1; 2; 3.

Vortices have a topological sense - branch points (Re & Im parts of field become strictly zero), **topological charge represents number of intersecting pairs of zero lines of Re & Im parts of field *A*.**

The circulation of the field's phase gradient is conserved along the closed path which encloses the branch point.

Vortex nested in the EM beam cannot disappear even when the EM beam undergoes a structural change.

Fusion of filaments is forbidden for topological reasons

These filaments must conserve total angular momentum, They can eventually spiral about each other or fly off tangentially to the initial ring .

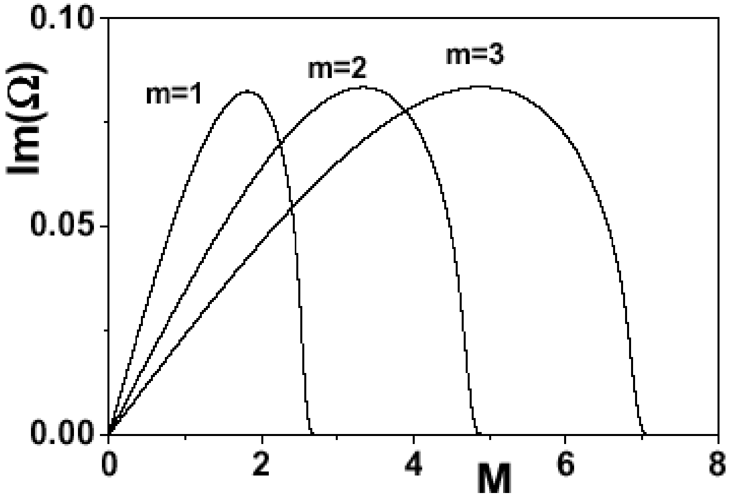


Fig.13 $Im(\Omega)$ versus M for λ= 0,1 & for m = 1; 2; 3 . A_m – 0,66; 0,65; 0,63 & r_* = 6,3; 11,6; 16, 9

Vortex dynamics

Numerical simulations for $A_m < 1$ - evidence of a quickly developing Instability in agreement with predictions of linear stability analysis.

the LOVS with m = 1 (m = 2) break up into 2 (4) filaments.

The filaments are running away tangentially without spiraling.

All filament like spatial solitons remain stable.

$\lambda = 0{,}1$

left panel – for $m = 1$; $A_{max} = 0{:}66$, vortex splits into 2 filaments;

right panel – for $m = 2$; $A_{max} = 0{:}6580$, vortex splits into 4 filaments;

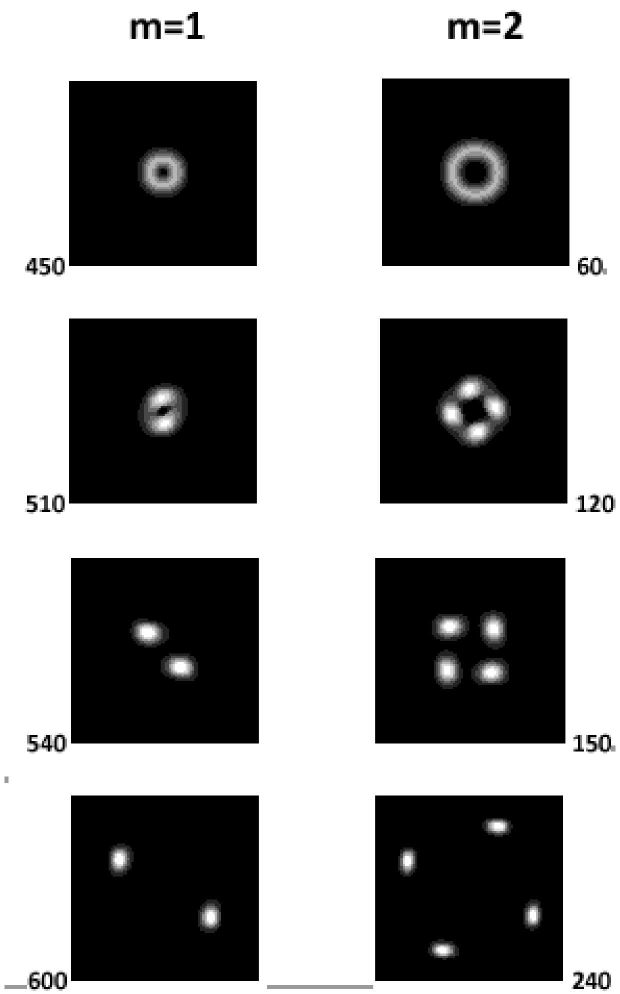


Fig.14

Evolution of higher intensity LOVS
for $\lambda = 0{,}2$, $m = 1$ & $m = 2$:

corresponding **amplitudes** for the soliton solutions are
$A_m = 1.39$ & $A_m = 1.37$, respectively.

The initial input LOVS solution was modulated by a Gaussian noise (level of noise - 5%).

LOVS maintain their fidelity - no breaking takes place.

Simulations were carried out for long times,
$t = 4000$, *i.e.,* for 130 soliton periods, $T_{sol} (= 2\pi / \lambda \approx 30)$.

Single & multi-charged large amplitude LOVS are stable!

Caution: from general topological reasons the multi-charged vortices are supposed to be unstable; they should break into single charge vortices.

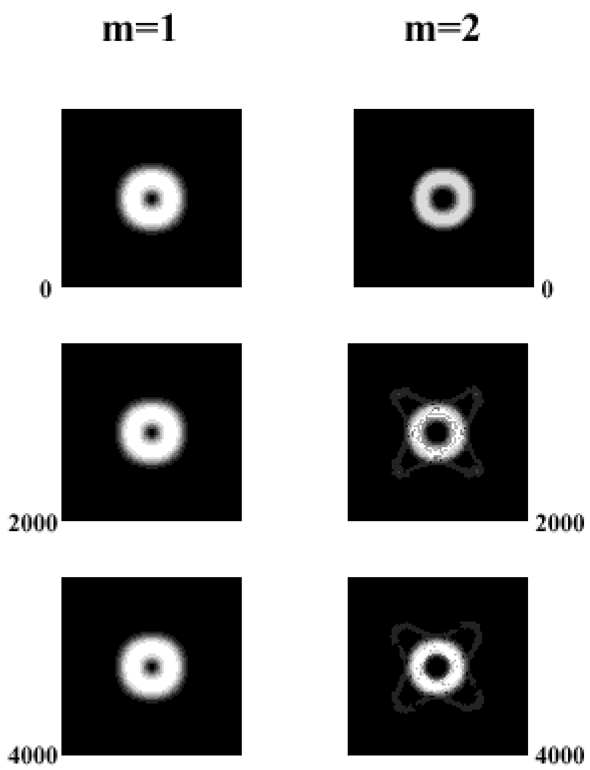


Fig.15 Vortex dynamics (*for different time-moments*) when $\lambda = 0{,}2$, **vortex is robust towards perturbations;**
left panel - m = 1; $A_{max} = 1.386$
right panel - m = 2; $A_m = 1.3729$

Light bullets in pair plasmas

Direct simulation of (44) for its fundamental solitary solution (n = 0) = >

Dispersion relation displayed in Fig.10 shows:

- **Localaized solution exists in range** $0 < \lambda < 0.193$
- **Soliton amplitude is bounded from above** $A_m < 1.68$
- **Vanishing saturating nonlinearity does not sustain solitonic solutions with ultrarelativistic amplitudes** $(A_m \geqslant 1)$
- **When pulse amplitude approaches the critical value the profile acquires the flat-top!**
- **If an initial profile of the pulse is close to the stable equilibrium**, pulse quickly attains the profile of ground state soliton & **propagates for a long distance without distortion of its shape.**
- Even if initial **pulse** is in a parameter domain far from equilibrium, it, **will find its way (by focusing / defocusing) to ground state equilibrium exhibiting damped oscillations around it.**

Spatiotemporal spinning solitons in pair plasmas

Generalization of the results by keeping the term $\sim \partial^2 A / \partial \xi^2$

(inclusion of effects related with group velocity dispersion)

- **Transparent plasma:** this term can affect the long time dynamics of self-guiding vortex solitons.
- **Due to weak modulation instability the self-trapped beam eventually breaks into a train of spatiotemporal solitons along the propagation direction, i.e. the ”light bullets”** (just as an ordinary self-trapped beam does).
- **Topological reasons, however, will let the vortex line survive structural changes.**
- ***Expectation:*** **longitudinal instability of 2D vortex soliton & reshaping of the profile along the propagation direction could result in the generation of fully localized bullets of vortex solitons (spinning-bullets).**

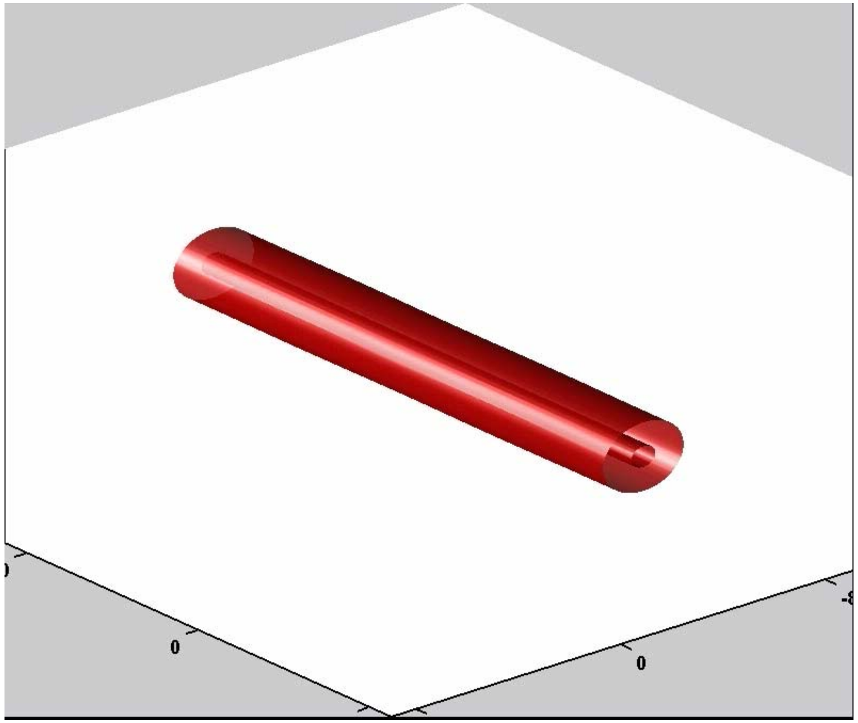
Conjecture was verified by numerical simulations.

Initial pulse was chosen to be a 2D vortex soliton with amplitude $A_m > 1$.

The breaking up of the vortex beam into a chain of stable spatiotemporal pulses is illustrated ====>

Note: **the zero vortex line survive structural changes.**

The breakup dynamics for the input vortex soliton with $m = 1$ & $A_m = 1{,}38$.



Intensity donat-shape contours at $t = 4300$ correspond to the spinning bullets.

Summary

- **The asymmetries originating in small temperature differences in the constituent species of an electromagnetically active medium may be always available for structure formation both in laboratory and cosmic/astrophysical settings.**

- **A fundamentally new type of saturating focusing-defocusing nonlinearity is derived for a physical system.**

- **This composite nonlinearity, originating in a small temperature asymmetry in the constituent fluids of a pair plasma, promises the existence of interesting structures that intense electromagnetic waves can acquire in such plasmas.**

- **Different parts of a high amplitude pulse are effected differently - the simultaneous expansion of the peak region & scrunching of the wings imparts flat top shape to the pulse.**

- **Pair plasmas, then, can support multi-dimensional stable large amplitude optical vortex (OVS) & localized vortex solitons (LOVS).**

- **Localized structures for certain parameters may have the flat-top shapes.**

- **The coexistence of LOVS & OVS solutions & their stability in such medium is due to the specific form of saturating nonlinearity switching from the self-focusing to the self-defocusing regime and vice versa.**

- **Pair plasmas, through this new nonlinearity, support multidimensional stable large amplitude light bullets as well as bullets carrying vortices, i.e.spinning bullets.**

- **The consequences of this nonlinearity can be observed in a variety of situations, in particular in the laboratory settings. An investigation of such a nonlinearity is likely to advance our understanding of many naturally occuring physical systems.**