2155-28

International Workshop on Cutting-Edge Plasma Physics

5 - 16 July 2010

Alfvén Modes and Alpha Particles in Burning Plasmas

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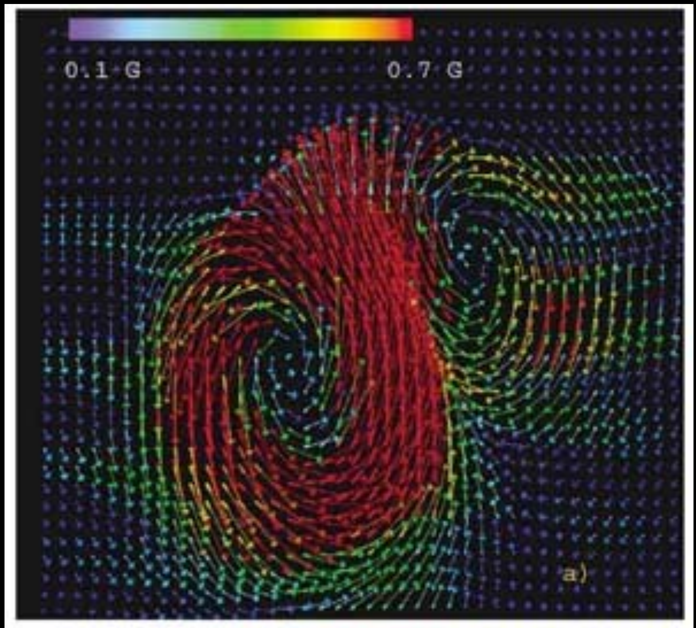

Shear Alfvén waves in LAPD
(Van Zeeland et al., PRL 2001)

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ICTP Trieste, 16 July 2010

Outline

- INTRODUCTION: Hannes Alfvén
- ALFVÉN WAVES
  - In general
  - In tokamaks
- BURNING PLASMAS (ITER)
  - Characteristics of burning plasmas
  - Dynamics of alpha particles
  - Energetic particles and stability
  - Nonlinear behavior
  - Cascade modes
  - Alfvén mode properties as diagnostics

INTRODUCTION:

HANNES ALFVÉN

"Father of Plasma Physics"

- Hannes Olof Gösta Alfvén
  - Born May 30, 1908 (Norrköping, Sweden); died April 2, 1995
- Career at a glance:
  - Professor of electromagnetic theory at Royal Institute of Technology, Stockholm (1940)
  - Professor of electrical engineering at UCSD (1967-1973/1988)
  - Nobel Prize (1970) for MHD work and contributions in founding plasma physics

Hannes Alfvén received the Nobel Prize in Physics in 1970 from the Swedish King Gustavus Adolphus VI

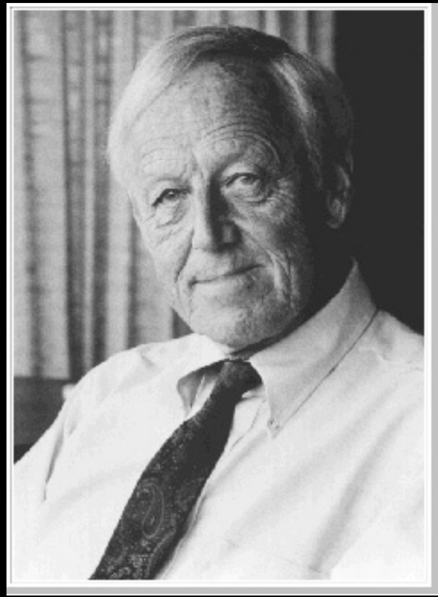
Huge Influence

- Contributions to plasma physics
  - Existence of electromagnetic-hydromagnetic (“Alfvén”) waves (1942)
  - Concepts of guiding center approximation, first adiabatic invariant, frozen-in flux
  - Acceleration of cosmic rays (--> Fermi acceleration)
  - Field-aligned electric currents in the aurora (double layer)
  - Stability of Earth-circulating energetic particles (--> Van Allen belts)
  - Effect of magnetic storms on Earth’s magnetic field
  - Alfvén critical-velocity ionization mechanism
  - Formation of comet tails
  - Plasma cosmology (Alfvén-Klein model)
  - Books: *Cosmical Electrodynamics* (1950), *On the Origin of the Solar System* (1954), *Worlds-Antiworlds* (1966), *Cosmic Plasma* (1981)

- Wide-spread name:
  - Alfvén wave, Alfvén layer, Alfvén critical point, Alfvén radii, Alfvén distances, Alfvén resonance, …

Factoids

- His youthful involvement in a radio club at school later led (he claimed) to his PhD thesis on "Ultra-Short Electromagnetic Waves"
- He had difficulty publishing in standard astrophysical journals (due to disputes with Sydney Chapman):
  - Enrico Fermi "Of course!" (1948)
- He considered himself an electrical engineer more than a physicist
- He distrusted computers
- The asteroid "1778 Alfvén" was named in his honor
- He was active in international disarmament movements
- The music composer Hugo Alfvén was his uncle

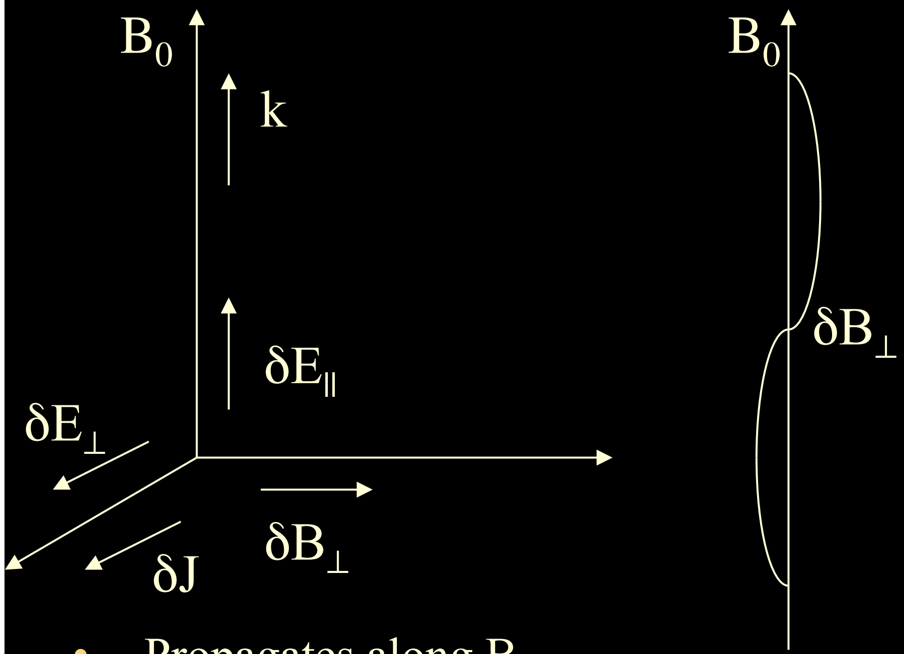


ALFVÉN WAVES:

- *IN GENERAL*
- *IN TOKAMAKS*

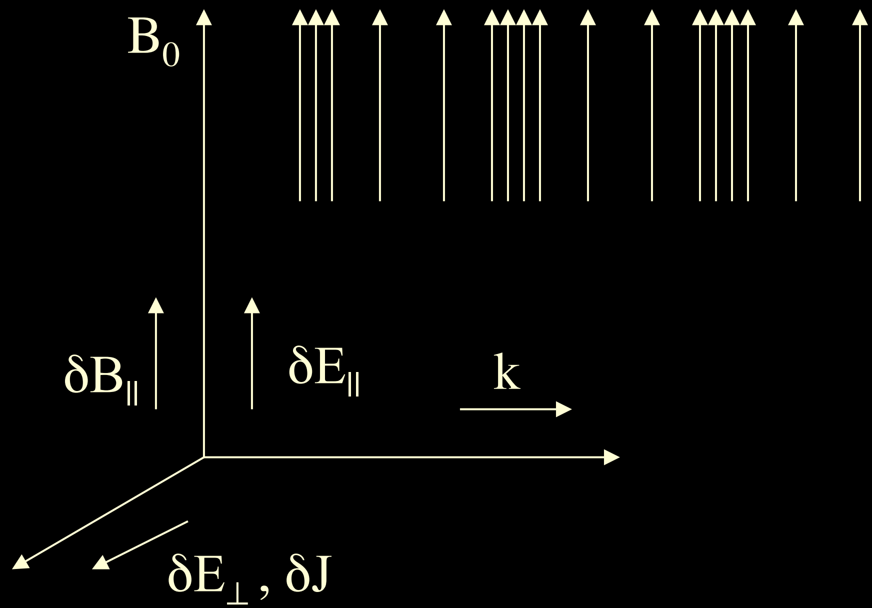
Alfvén Waves

Shear Alfvén $(k \| B_0, \delta B_{\|} \cong 0)$



- Propagates along B_0
- Oscillation resembles a plucked violin string (i.e., driven by B_0-line tension)

Compressional Alfvén $(k \perp B_0, \delta B_{\perp} \cong 0)$



- Propagates across B_0
- Compression-rarefaction wave (i.e., driven by magnetic/plasma pressure)
- Higher frequency, since $k_{\|} << k_{\perp}$

Shear Alfvén Continuum

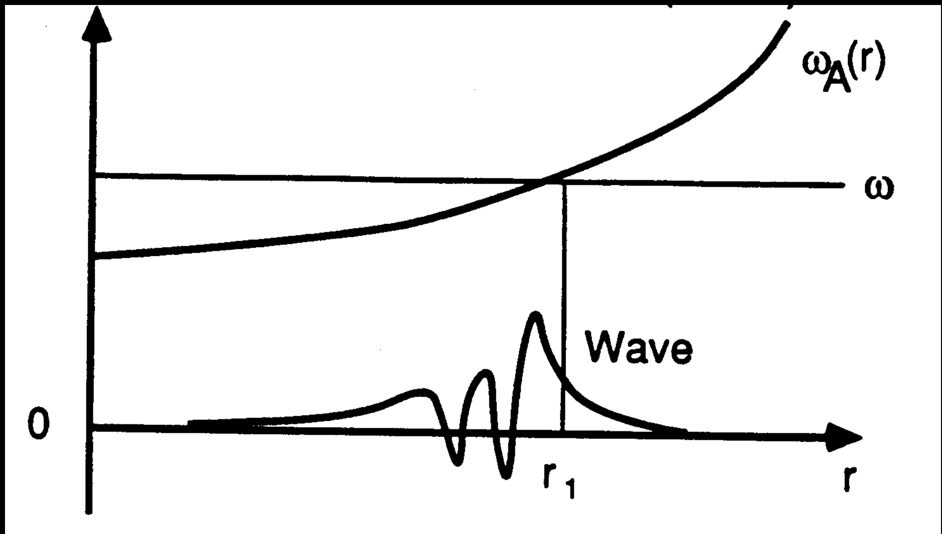
- Ideal-MHD eigenmode equation (large-aspect-ratio geometry):

$$(\ldots)\frac{d\xi_r^{\,4}}{dr^4}+\frac{d}{dr}\left\{\left[\frac{\omega^2-k_{\|}^2 v_A^{\,2}}{\omega^2-\left(k_\perp^{\,2}+k_{\|}^2\right)v_A^{\,2}}\right]\frac{B^2}{r}\frac{d}{dr}\left(r\xi_r\right)\right\}+\rho\left[\left(\omega^2-k_{\|}^2 v_A^{\,2}\right)+\ldots\right]\xi_r=0$$

- Coefficient of $d^2\xi_r/dr^2$ vanishes when $\omega^2 = k_{\|}^2 v_A^2$ (shear Alfvén continuum)
  - The mode structure is singular when the frequency satisfies the inequality $\mathrm{Min}\,(k_{\|}^2 v_A^2) \le \omega^2 \le \mathrm{Max}\,(k_{\|}^2 v_A^2)$
  - Alfvén velocity is a function of radius in an inhomogeneous plasma:

$$v_A(r)=\frac{B_0(r)}{\sqrt{4\pi n_0(r)M_i}}$$

Kinetic Alfvén Wave

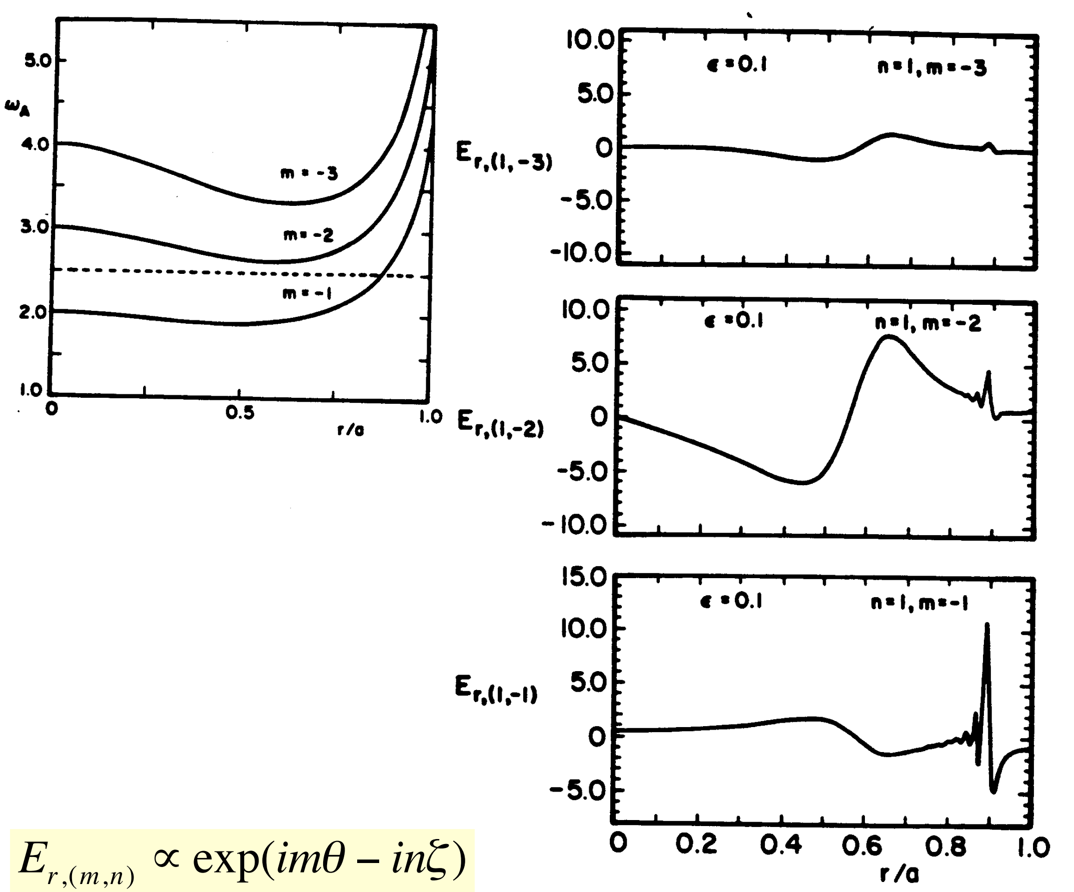


$$\omega_A(r) = k_{||} v_A(r) \propto \frac{1}{\sqrt{n_0(r)}}$$

- Solution is singular at position (r_1) of local Alfvén resonance where $\omega = \omega_A(r)$
  - Resonant absorption of wave energy (“continuum damping”)

- If electron parallel dynamics and ion FLR effects are included, a non-singular solution can be obtained: “Kinetic Alfvén Wave”
  - However, KAW experiences strong bulk plasma Landau damping, due to its short wavelength
  - Hence, the global-type Alfvén waves (GAE and TAE) are of more interest, since they have $\omega \neq \omega_A(r)$

Global Alfvén Eigenmode



$$E_{r,(m,n)} \propto \exp(im\theta - in\zeta)$$

- GAE is a radially extended, regular, spatially non-resonant discrete Alfvén eigenmode
  - Requires that the current profile be such that the Alfvén continuum have an off-axis minimum ($k_{\|}\neq 0$, thus $nm<0$):

$$\frac{d}{dr}\omega_A(r_1) = 0; \quad i.e., \frac{1}{k_{\|}}\frac{dk_{\|}}{dr} = -\frac{1}{v_A}\frac{dv_A}{dr}$$

  - Frequency lies just below the lower edge of the continuum
  - Sidebands suffer continuum damping
  - Experiments tried to use GAE for "global" tokamak plasma heating

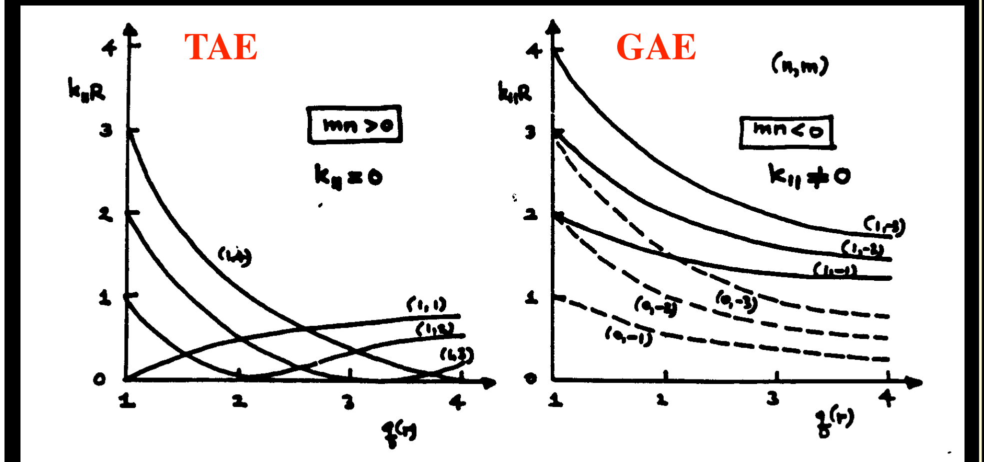
Alfvén Waves in Toroidal Geometry

- In a torus, wave solutions are quantized poloidally & toroidally:

$$\Phi(r,\theta,\zeta,t) \propto \exp(-i\omega t)\sum_m \Phi_m(r)\exp(im\theta - in\zeta)$$

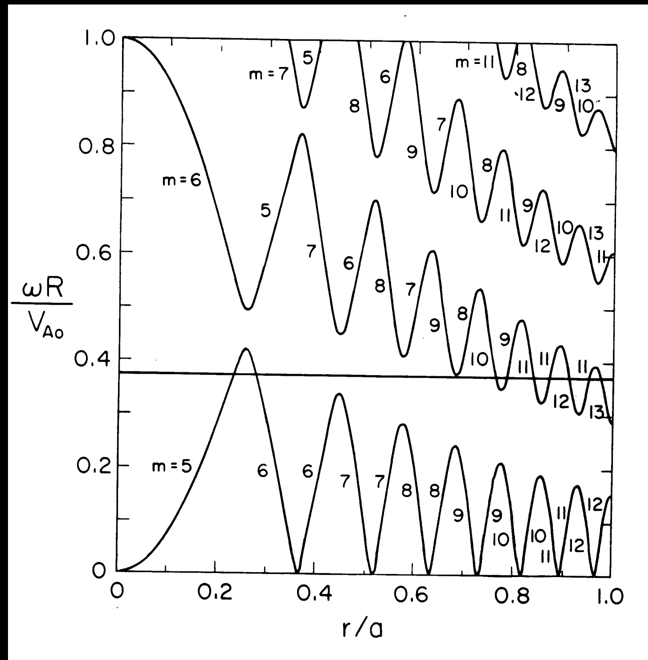
- Parallel wave number $k_{\|}$ determined by B-line twist $q(r) = rB_T/RB_p$ ("safety factor"):

$$k_{\|}(r) = \frac{1}{R}\left(\frac{m}{q(r)} - n\right)$$

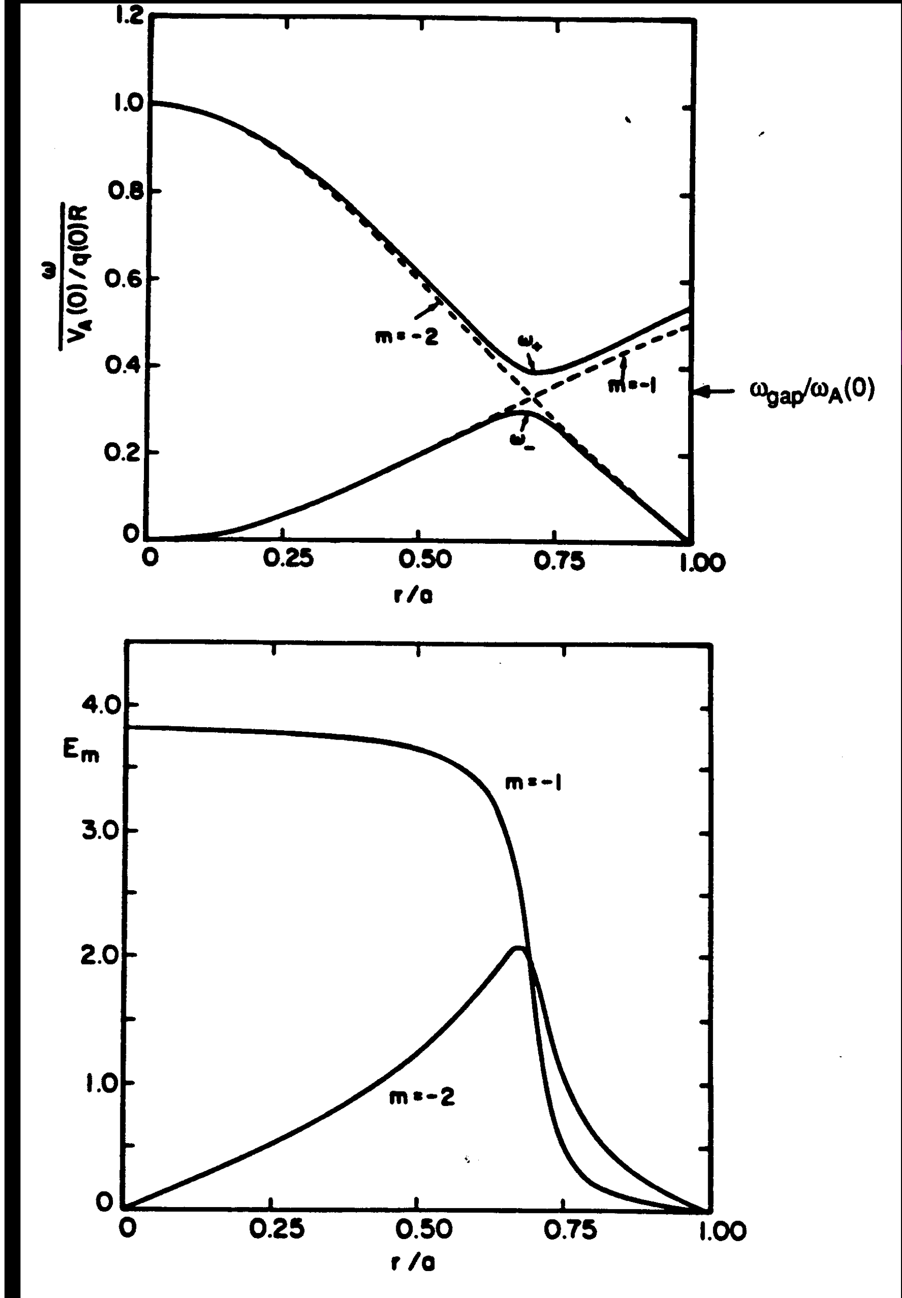


- "Gaps" occur in Alfvén continuum in toroidal geometry when

$$\omega = k_{\|m}(r)v_A(r) = -k_{\|m+1}(r)v_A(r)$$



- Discrete eigenmodes exist within gaps due to equilibrium poloidal dependence: e.g., $B_0 \propto 1 - (r/R)\cos\theta$



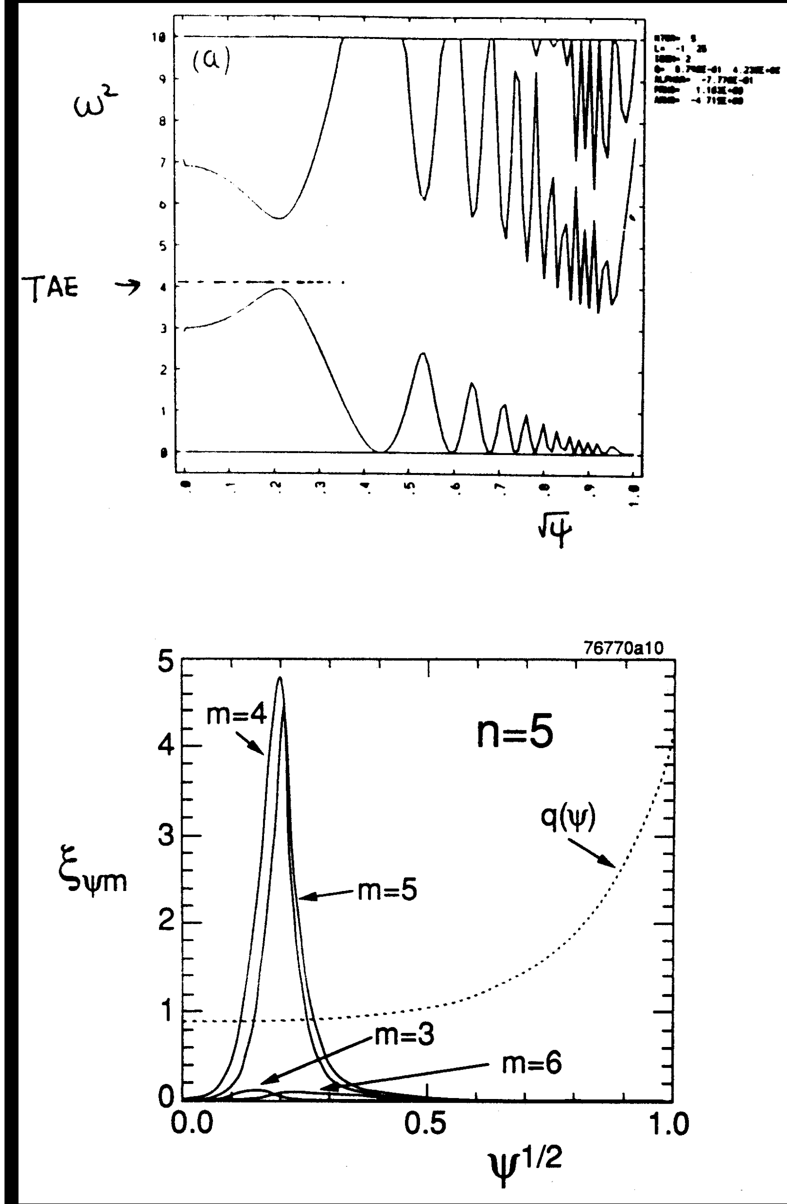
Toroidal Alfvén Eigenmode

- TAE is a discrete, radially extended, regular (non-resonant) eigenmode
  - Frequency lies in "gap" of width ~r/R <<1 at q=(m+1/2)/n, formed by toroidicity-induced coupling (m±1)

$$\omega_{TAE}^{m,m+1} = \left(\frac{n}{2m+1}\right)\frac{v_A}{R} \neq \omega_A(r)$$

  - Analogy to band gap theory in solid-state crystals (Mathieu equation, Bloch functions): "fiber glass wave guide"

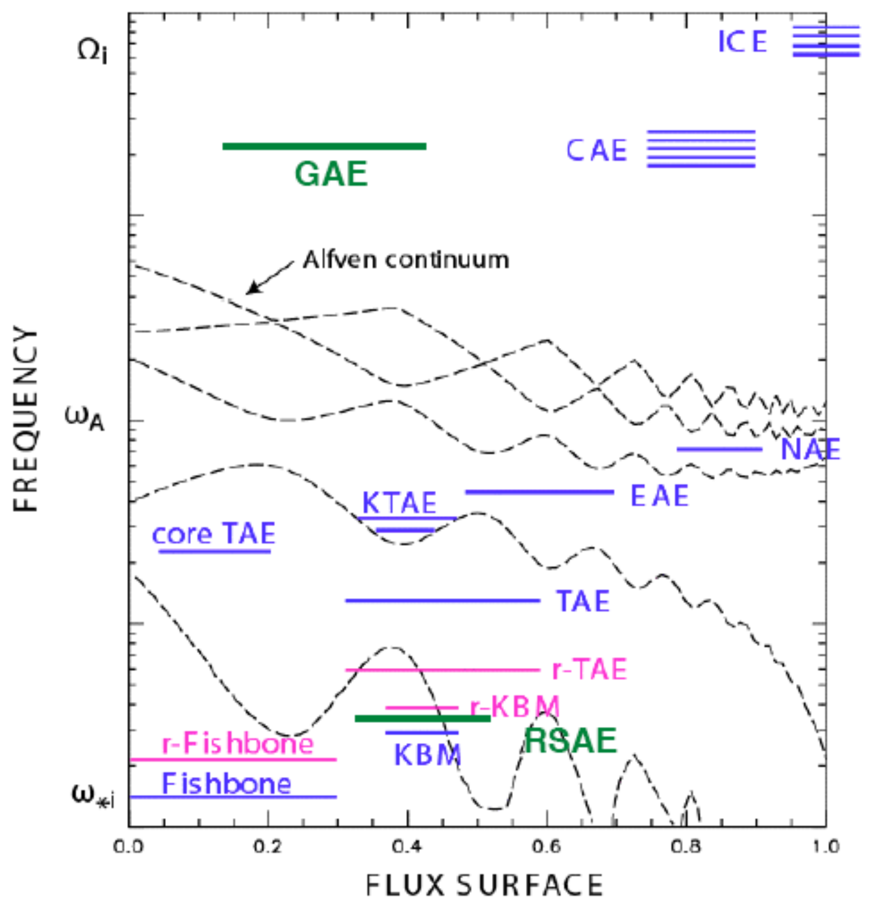
- Similarly there exist:
  - Ellipticity-induced Alfvén eigenmode (EAE): m±2
  - Triangularity-induced Alfvén eigenmode (NAE): m±3



Core-Localized TAE

- At low shear (near magnetic axis), the TAE moves near the bottom of the gap
  - Theoretical explanation requires retaining higher-order finite aspect ratio effects
  - In addition, a second core-localized TAE appears at the top of the gap

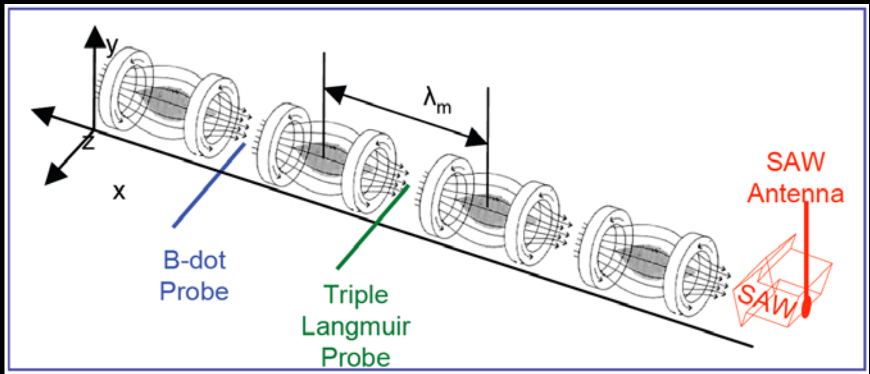
Zoology of *AE Modes

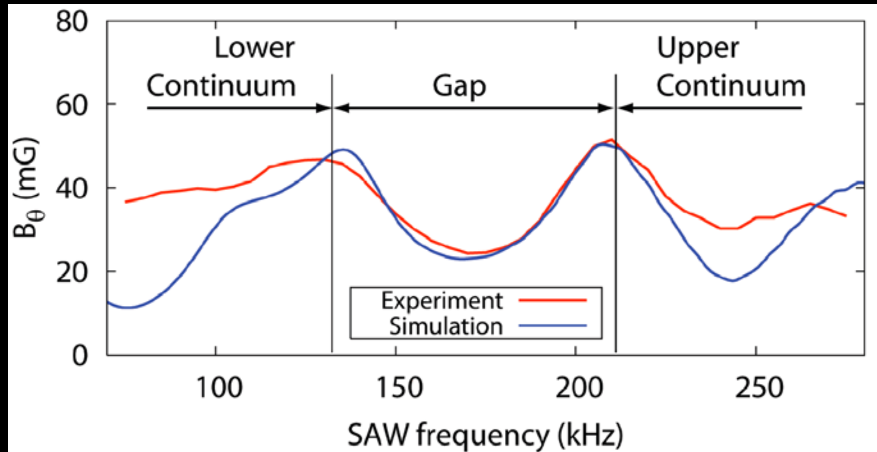


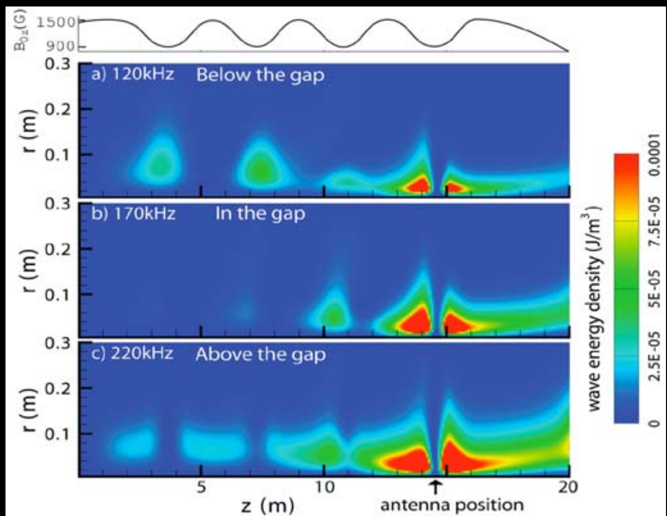
Heidbrink, Phys. Pl. 9 (2002) 2113

- Fast particles can destabilize a large variety of Alfvén modes (*AE)
  - D-T alphas have v (3.5 MeV) $\geq v_A$
- Mode identification is robust:
  - Frequency, mode structure, polarization
- Threshold is determined by balance of:
  - Growth rate (reliably calculate)
  - Damping rate (calculation is very sensitive to parameters, profiles, length scales—but can measure with active/ passive antennas)
- Also, Energetic Particle Modes
  - Exist only in presence of EP's

Alfvén Gap Modes in Linear Device







- Multiple mirror configuration in LAPD produces spectral gap due to Bragg reflection
- Standing shear Alfvén wave observed at gap frequencies (reproduced theoretically)

*IFS-UCLA-UC Irvine collaboration (B. Breizman), PoP **15**, 012103 (2008)*

DYNAMICS OF ALPHA PARTICLES

Energetic Particles

- In addition to thermal ions and electrons, plasmas often contain a supra-thermal species = "energetic particles" (or "fast particles," "hot particles")
  - Highly energetic ($T_f >> T_i$)
  - Low density ($n_f << n_i$), but comparable pressure ($n_f T_f \cong n_i T_i$)

- Energetic particles can be created from various sources:
  - Externally: ion/electron cyclotron heating or neutral beam injection —> high-energy "tails" of ions and electrons
  - Internally: fusion reaction alpha particles; runaway electrons

- The plasma physics of energetic particles is of interest to:
  - Laboratory fusion plasmas (alphas provide self-heating to sustain ignition)
  - Space and astrophysical plasmas (e.g., proton ring in Earth's magnetosphere)
  - High-energy-physics accelerators (collective effects)

Alpha Particle Characteristics

- **Plasma ions and electrons:**
  - $T_{i,e} \sim$ 10-20 keV
  - "Frozen-in" behavior to lowest order (MHD description)
  - Thermodynamic equilibrium (Maxwellian distribution)

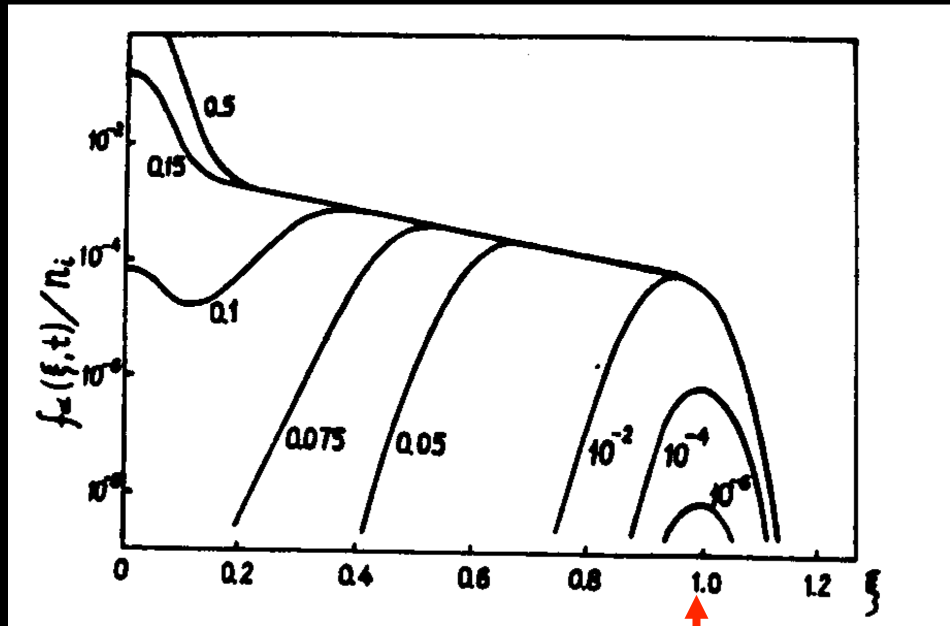
- **Alpha particles:**
  - High energy: $T_{\alpha,birth}^{DT} = 3.5$ MeV
  - Not "frozen" to B-field lines (require kinetic description)
  - Low density ($n_\alpha < n_{i,e}$), but comparable pressure ($p_\alpha \sim p_{i,e}$)
  - Non-Maxwellian "slowing down" distribution
  - Centrally peaked profile

$$\left|\nabla p_\alpha / p_\alpha\right|^{-1} \leq a/2$$

- **Other energetic particles:**
  - Supra-thermal ions from NBI and ICRH
    - Can simulate α particle effects without reactivity (although NBI/ICRH ions are anisotropic in pitch angle, whereas alphas are isotropic)
    - Also present in burning plasmas with auxiliary heating
  - Run-away electrons associated with disruptions

Birth, Life, & Death of α Particles

- **DT alphas are born in peaked distribution at 3.5 MeV at rate $\partial n_a/\partial t = n_D n_T \langle \sigma v \rangle$**
  – During time τ_s, they are slowed down by collisions with electrons to smoother distribution at ~ 1 MeV
  – After time τ_M, they thermalize against both electrons and ions to the plasma temperature ($T_e \sim T_i \sim 10$ keV)
  – Alphas are confined for time τ_α. In steady-state there are two alpha populations: slowing-down α's (n_s) and cool Maxwellian α's (n_M)



Birth velocity:

$$v_{\alpha 0}^{D-T} = 1.3 \times 10^9 \, cm/s$$

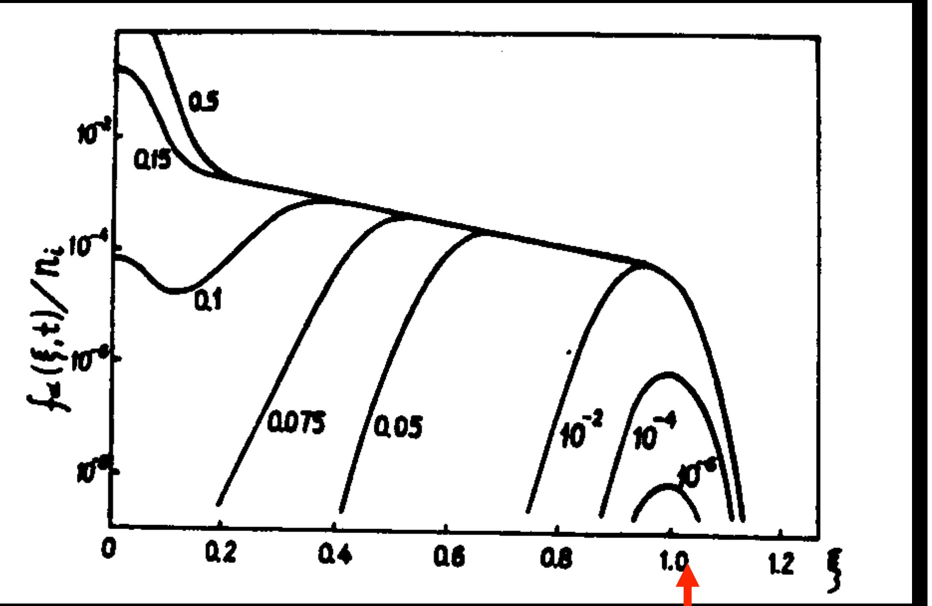
$$P_{fus} \propto n^2 \langle \sigma v \rangle \sim n^2 T^2 \propto p^2$$

- **Typically $\tau_\alpha \sim 10\, \tau_M \sim 10^3\, \tau_s$: hence α's have time to thermalize**
  – Since $n_s / n_\alpha \sim \tau_s / \tau_\alpha \sim 10^{-3}$, then $n_M \sim n_\alpha \sim n_e$ (for reactors); hence "ash" (slow α's) is a problem in reactors, because it will "poison" the plasma

Slowing-down Distribution

- The classical steady-state "slowing down" distribution (isotropic) is

$$F_s(r,v) = \begin{cases} S(r)\tau_s / 4\pi(v^3 + v_c^{\ 3}), & v < v_{\alpha 0} \\ 0, & v > v_{\alpha 0} \end{cases}$$



Slowing-down time:

$$\tau_s = \frac{3 m_e m_\alpha v_e^{\ 3}}{16\sqrt{\pi} Z_\alpha^{\ 2} e^4 n_e \ln\Lambda_e} \cong 0.37\,\mathrm{sec}$$

Birth velocity:

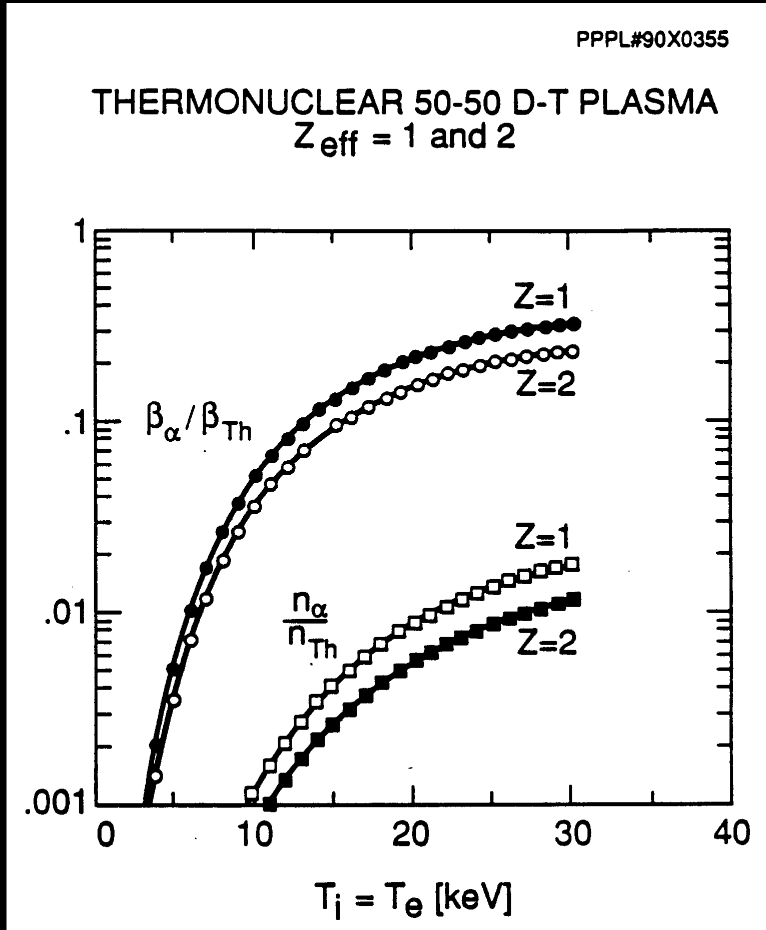
$$v_{\alpha 0}^{\ D-T} = 1.3\times 10^9\, cm/s$$

Critical velocity (balance ion/electron friction):

$$v_c = v_e \left[\frac{3\sqrt{\pi} m_e}{4 m_p \ln\Lambda_e} \sum_i \frac{n_i Z_i^{\ 2} \ln\Lambda_i}{A_i n_e} \right]^{1/3} \cong 4.6\times 10^8\, cm/s$$

Temperature Dependence

- Alpha parameters are determined by the plasma temperature
  - For ~10 keV plasma, α's deposit their energy into thermal electrons, with slowing-down time $\tau_s \propto T_e^{3/2} / n_e$
  - Since the alpha source ~ $n_e^2 <\sigma v>$, we have $n_\alpha/n_e \sim T_e^{3/2} <\sigma v> \propto T_e^{3/2}\ T_i^2$
  - For an equal-temperature Maxwellian plasma, the ratios n_α/n_e and $\beta_\alpha/\beta_{plasma}$ are unique functions of temperature T_e.



Significance for ITER

- Approximately 200-600 MW of alpha heating needed to sustain ignition
  - Huge amount of power to handle with no direct external control

- Experimental relevance of alpha loss:
  - Reduced self-heating
  - Damage to first wall and divertor plate structure (wall loading)
  - Impurity influx
  - Reduced efficiency of current drive or heating
  - Operational control problems
  - Quenching of ignition (α ash)

- Understanding of alpha physics is needed to:
  - Assure good alpha confinement
  - Optimize alpha heating efficiency
  - Avoid alpha-driven collective instabilities (*AE modes)

- Alpha dynamics integrated with overall plasma behavior
  - Macro-stability, transport, heating, edge, …

Parameter Comparison

Fast ion parameters in contemporary experiments compared with projected ITER values.

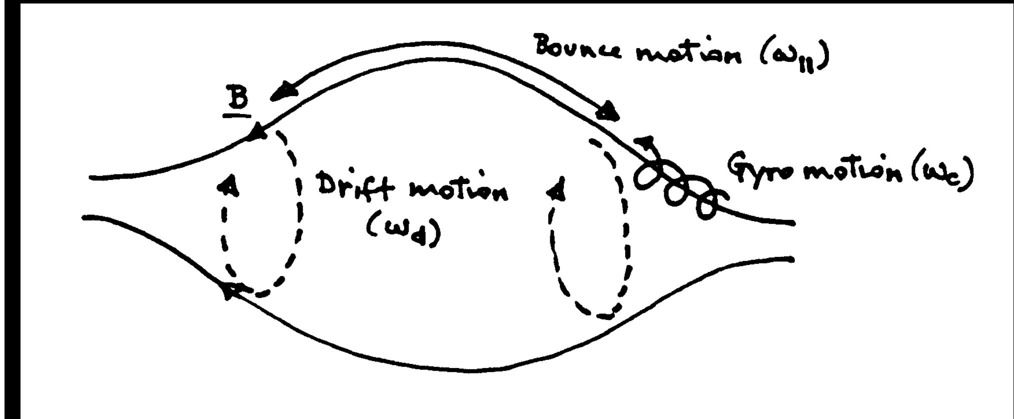
Tokamak	TFTR	JET	JT-60U	JET	ITER
Fast ion Source	Alpha Fusion	Alpha Fusion	Deuterium Co NBI	Alpha ICRF tail	Alpha Fusion
Reference	[3]	[3]	[34]	[20,52]	[52]
τ_S (s)	0.5	1.0	0.085	0.4	0.8
δ/a[a]	0.3	0.36	0.34	0.35	0.05
$P_f(0)$ ($\mathrm{MW\,m^{-3}}$)	0.28	0.12	0.12	0.5	0.55
$n_f(0)/n_e(0)$ (%)	0.3	0.44	2	1.5	0.85
$\beta_f(0)$ (%)	0.26	0.7	0.6	3	1.2
$\langle\beta_f\rangle$ (%)	0.03	0.12	0.15	0.3	0.3
max $\lvert R\nabla\beta_f\rvert$ (%)	2.0	3.5	6	5	3.8
$v_f(0)/v_A(0)$	1.6	1.6	1.9	1.3	1.9

- **Differences for fast ("f") ion physics in ITER:**
  - Orbit size δ/a in ITER is much smaller
  - Most of the other parameters (especially dimensionless) are comparable
  - No external control of alphas, in contrast to NBI and ICRH fast ions

Progress in ITER Physics Basis (2007)

ALPHA PARTICLES & STABILITY

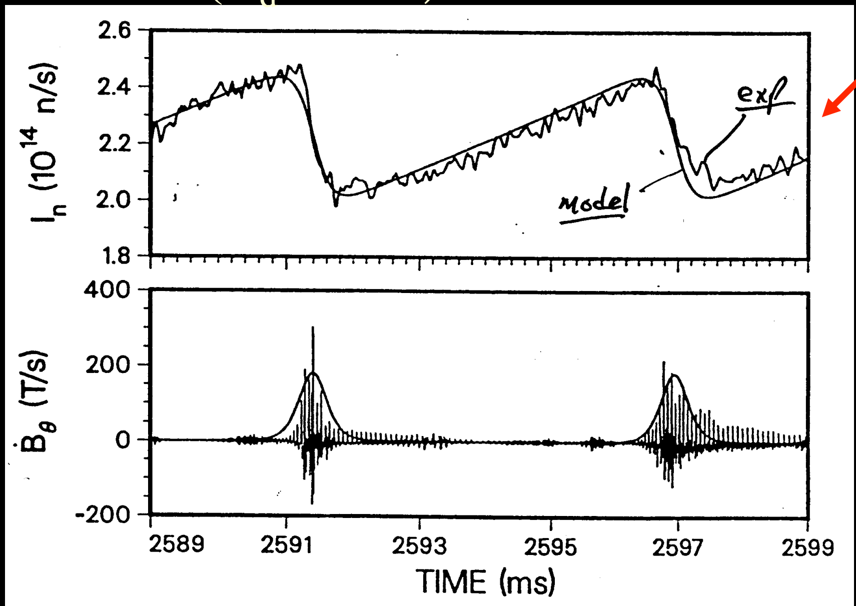
Kinetic Resonance & Stability

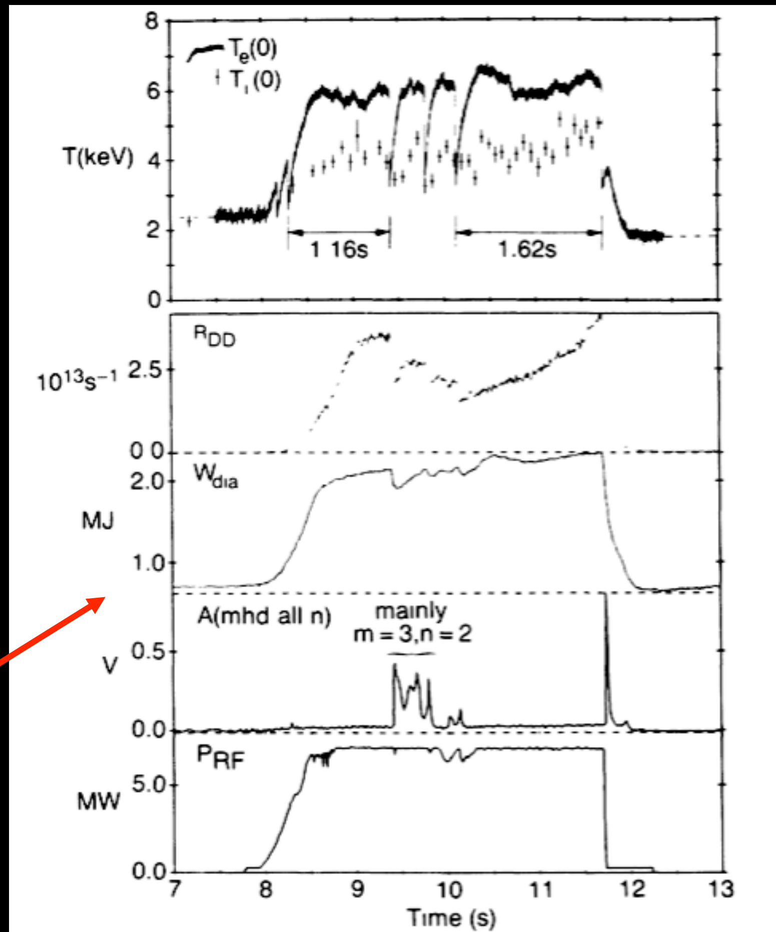


- Typically: $\omega_d << \omega_{||} << \omega_c$
- Most MHD modes are low frequency ($\omega << \omega_c$)
  - Hence magnetic moment $\mu = mv_{\perp}^2/2B$ is an adiabatic invariant
- Thermal ions/electrons do not drift across magnetic field ($\omega_d << \omega \sim \omega_{||}$)
  - "Frozen in" with field lines
  - If $\omega_d << \omega << \omega_{||}$, Kruskal-Oberman energy principle is valid (2nd adiabatic inv.)
- Energetic particles drift rapidly across magnetic field ($\omega_d \sim \omega << \omega_{||}$)
  - No longer "frozen in" with the field, since $\omega_d \propto \langle E_{fast} \rangle$
  - If ω **<<** $\omega_d << \omega_{||}$, Low Frequency energy principle is valid (3rd adiabatic invariant)

Fishbone Instability

Resonant destabilization of n=m=1 internal kink ($\omega_d^{fast} = \omega$): "fishbone instability"





Off-resonant stabilization of fishbone ($\omega_d^{fast} >> \omega$): "monster sawtooth"

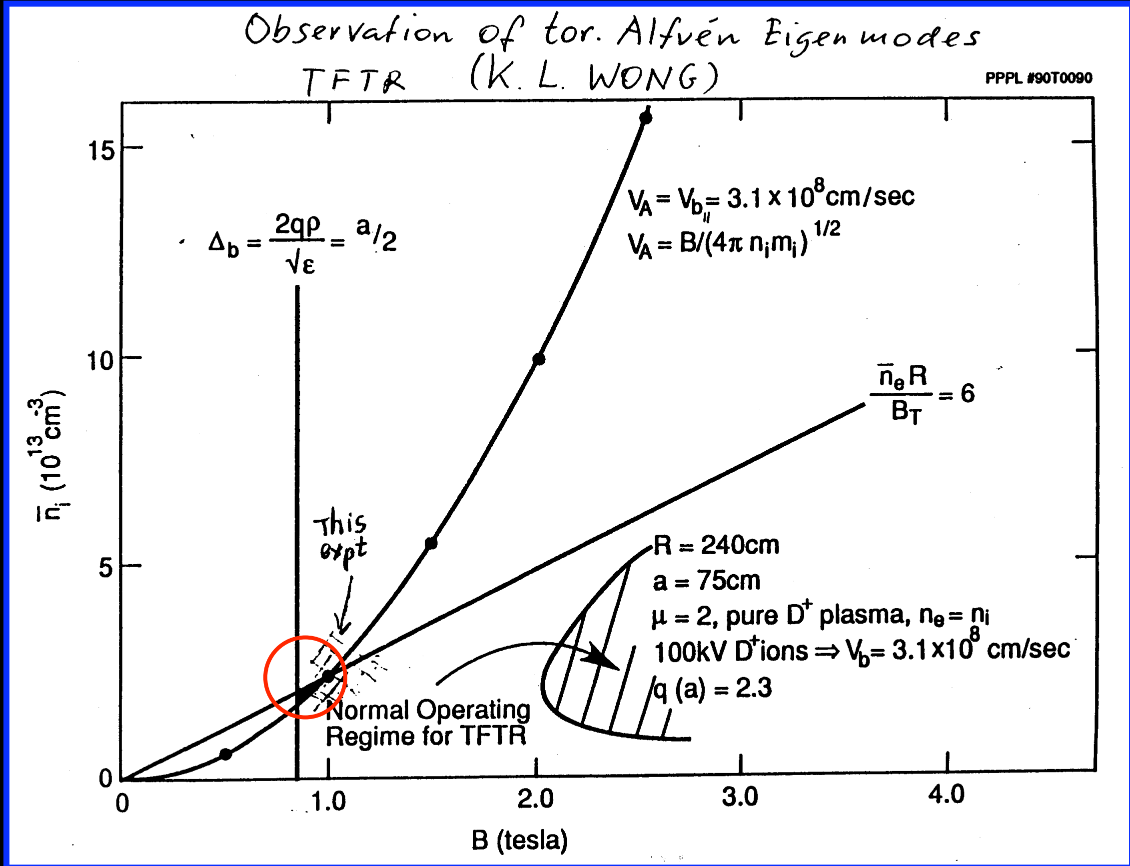
Nonlinear kinetic/fluid behavior of sawteeth is a challenging problem

TAE Instability

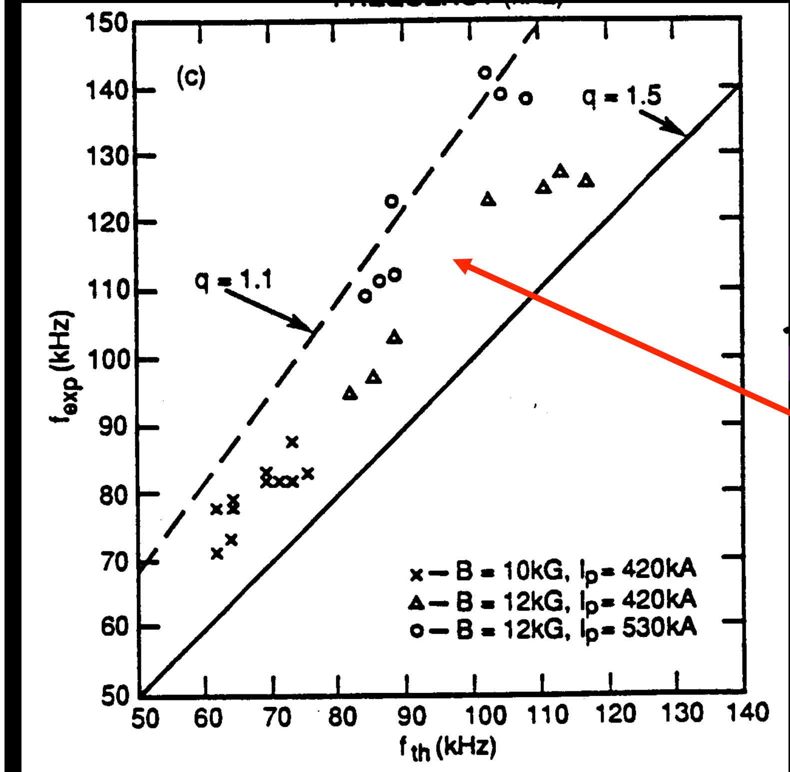
- Theoretical growth rate:

$$\frac{\gamma}{\omega_0} \cong \frac{9}{4}\left[\beta_\alpha\left(\frac{\omega_{*\alpha}}{\omega_0}-\frac{1}{2}\right)F-\beta_e\left(\frac{v_A}{v_e}\right)\right]$$

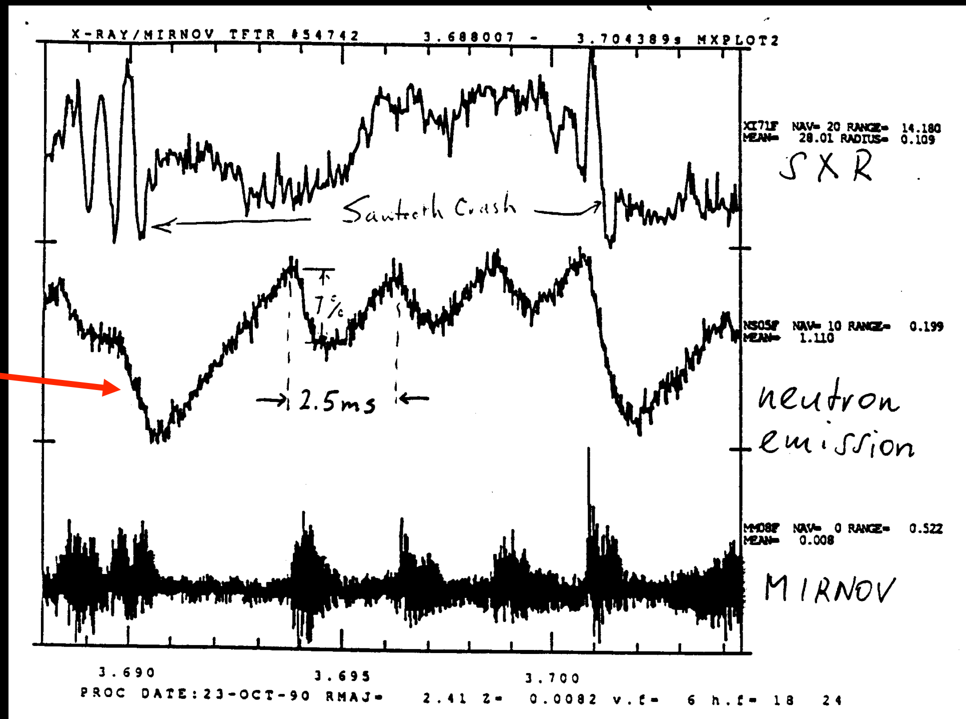
- Instability requires:
  - Wave-particle kinetic resonance ($v_\alpha \geq v_A$)
  - Inverse damping ($\omega_{*\alpha} > \omega_0$)
  - Growth overcomes damping (β_α/β_e > "small number" -- for electron Landau damping; other damping mechanisms also important)



TAE Observation



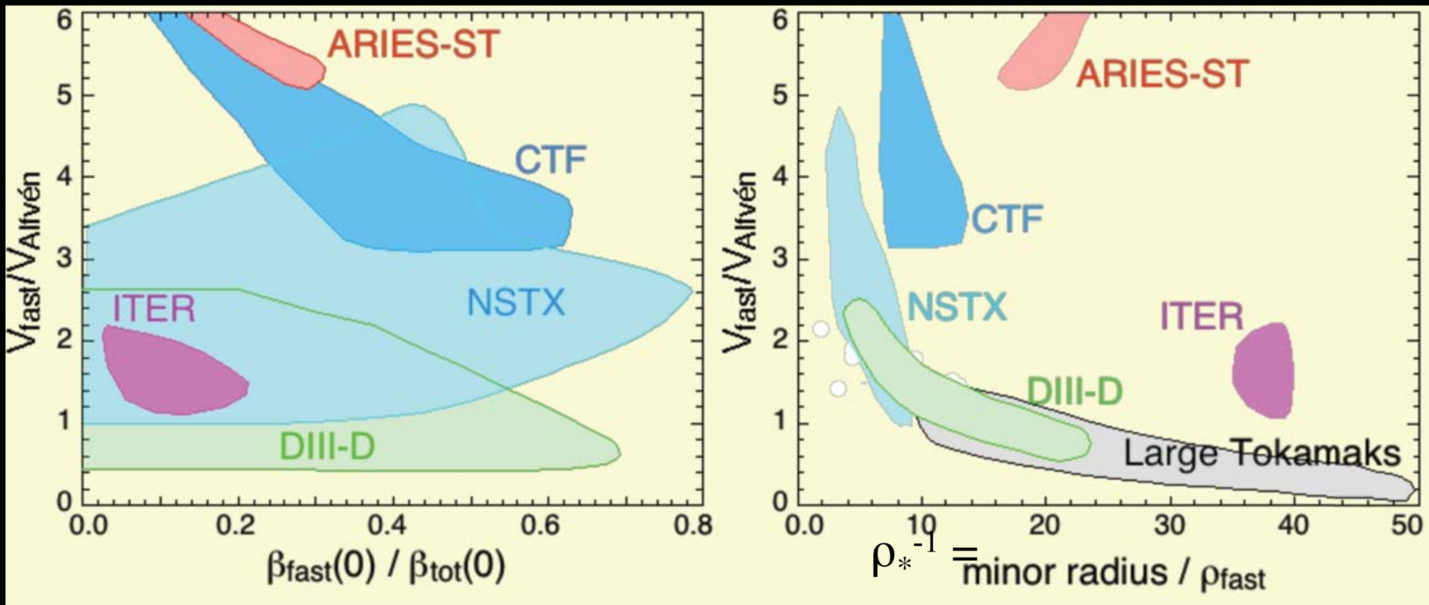
- Frequency scaled linearly with B
- Fluctuation amplitude increased with beam power



- Energetic beam ions were ejected: X-rays dropped 7% in periodic bursts
- A later experiment found intense fast ion fluxes that damaged vacuum vessel (ripple trapping caused by TAE resonance)

ITER Stability–1

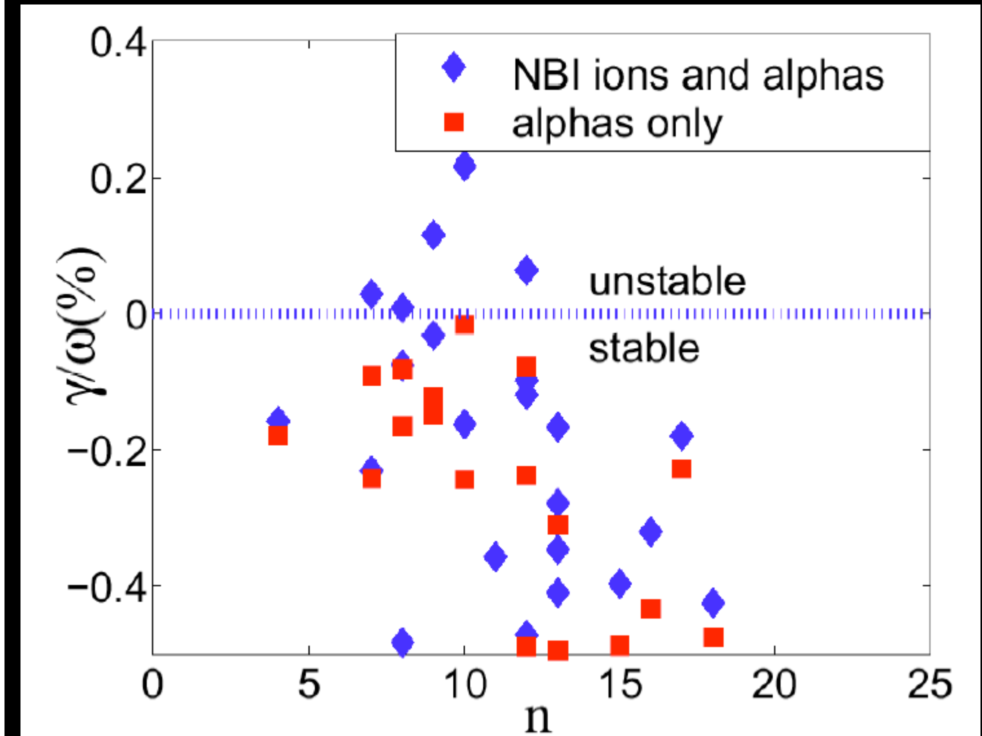
- **Alfvén Mach number** (v_α/v_A) and **pressure** (β_α) for ITER α-particles have similar values as in existing experiments
- However, ITER's **large size** [i.e., small-wavelength (a/ρ_{*fast} >> 1) regime] implies "sea" of many high-n potentially unstable modes (n^2 problem)
  - Could cause outward redistribution/loss of α's (domino-effect "avalanche")



ρ_*^{-1} =

E. Fredrickson

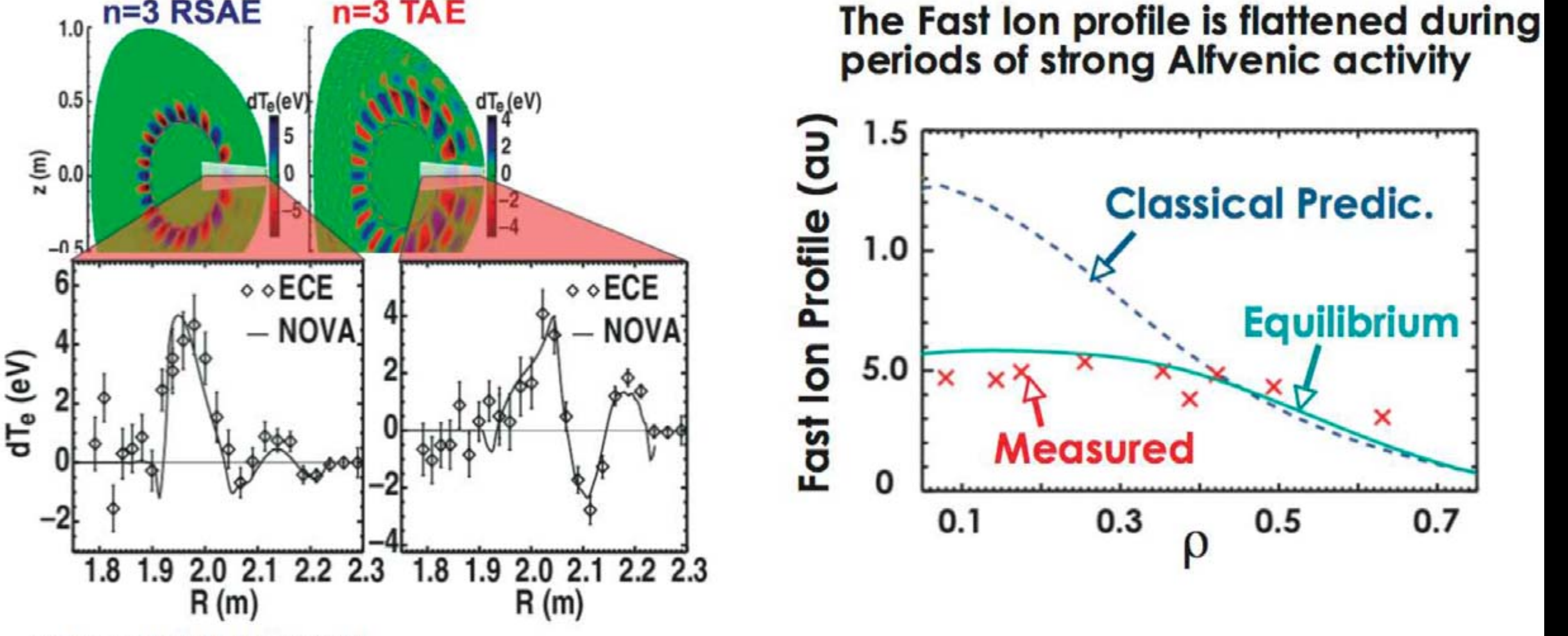
ITER Stability–2



- ITER will have 1 MeV negative ion neutral beams for heating and current drive
  - Theory predicts that these NB energetic ions can drive TAE instability, comparable to alpha particles
  - Including the NB ion drive changes the stability prediction for ITER (at 20 keV) from marginality to definite instability
  - A model quasi-linear calculation predicts negligible to modest losses

Beam ion losses

- Fast ions from neutral beams serve as a proxy in present-day experiments for investigating the behavior of fusion alpha particles
- Under certain conditions, fast ions excite Alfvén instabilities, which in turn cause loss of fast ions



Van Zeeland PRL 97, 135001 (2006)

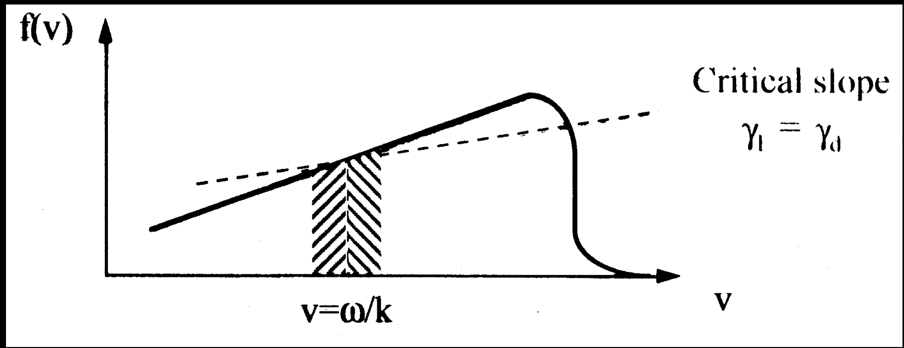
NONLINEAR ALFVÉN MODES

TAE Saturation

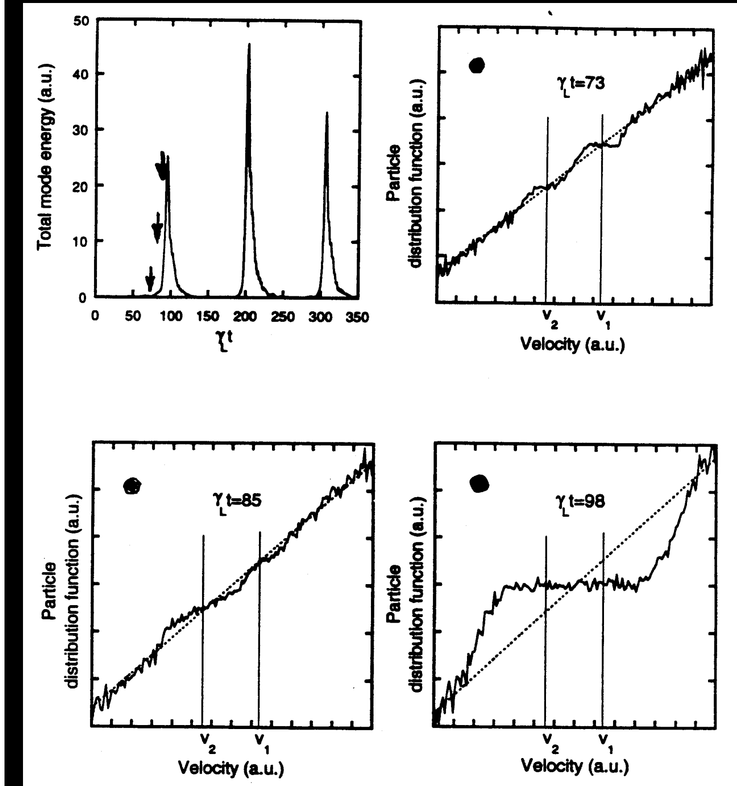
- Characteristic time scales:
  - Bounce frequency of resonant particles $\omega_b \propto E^{1/2}$
  - Linear growth rate γ_L
  - Background damping rate γ_d
  - Reconstitution rate ν for resonant particles (inverted distribution)

- Typically: $\gamma_L \geq \gamma_d \geq \nu$

- Pulsation scenario:
  - Waves are unstable until the resonant particle distribution function flattens; instability saturates when $\omega_b \sim \gamma_L$
  - Excited wave damps at rate $\gamma_d << \gamma_L$, while distribution remains flat
  - The source restores the distribution function, bringing new free energy into the resonant region
  - The cycle repeats

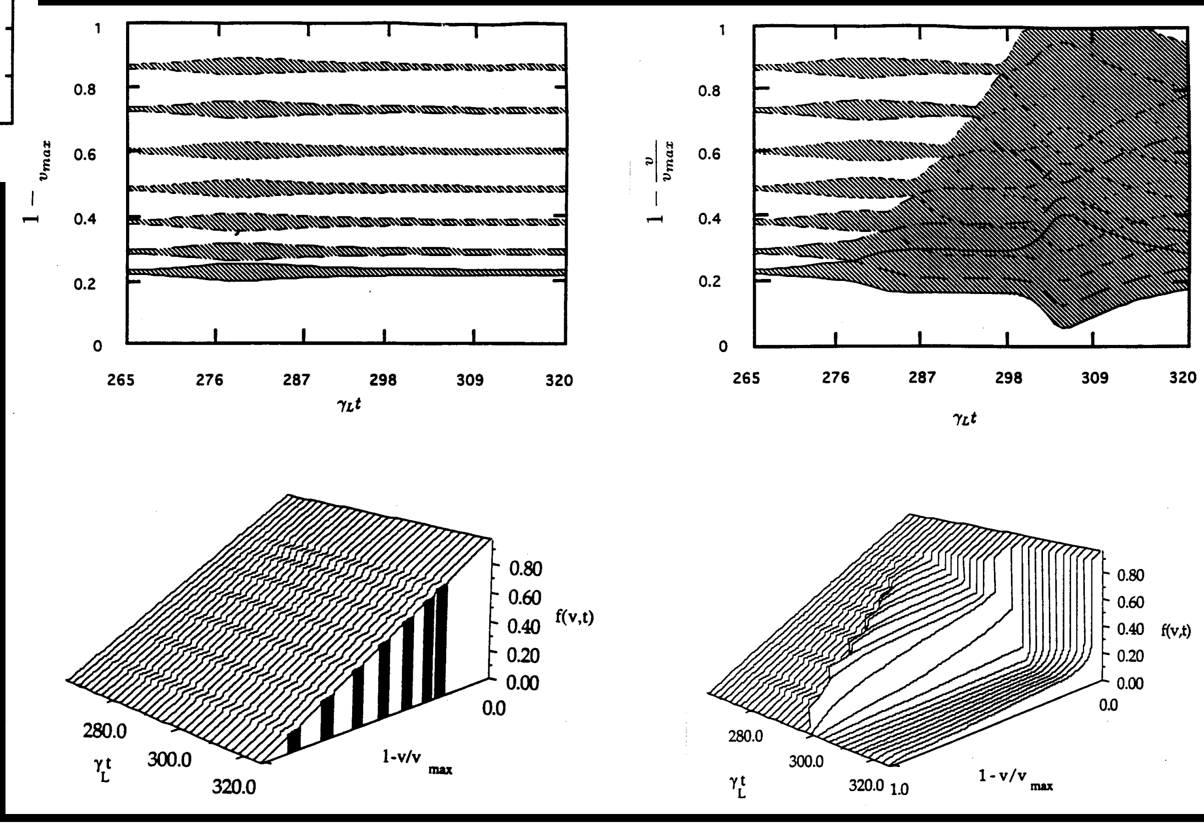


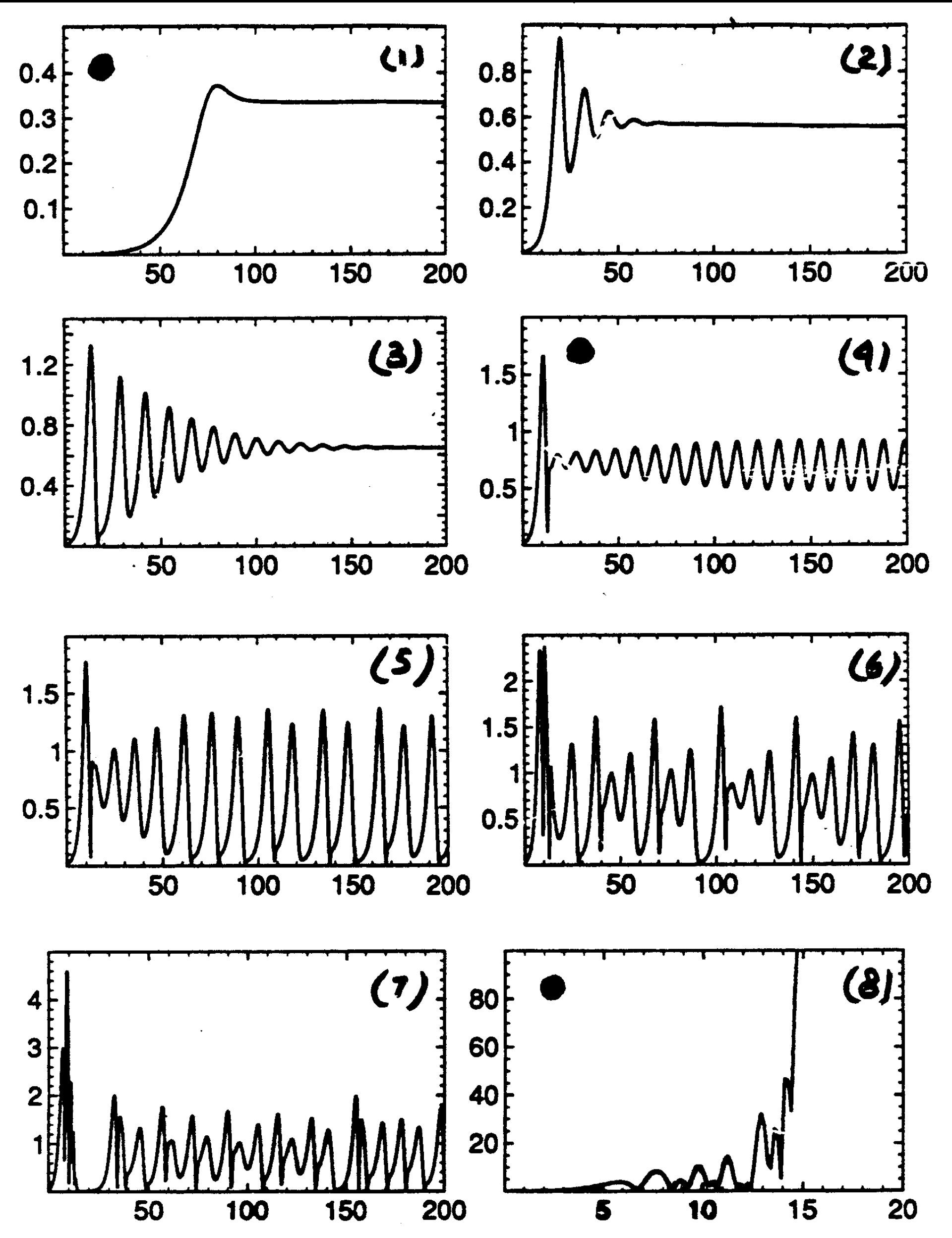
Avalanches



- Mode overlap can lead to a rapid transition from low-level benign oscillation, to larger amplitude oscillations accompanied by rapid diffusion of energetic particles over large area of the plasma

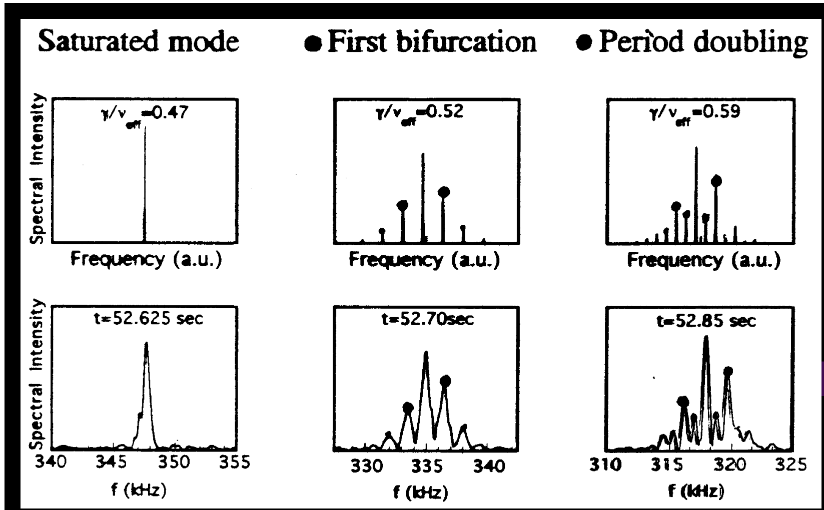
- Overlapping resonances
- Sandpile scenario





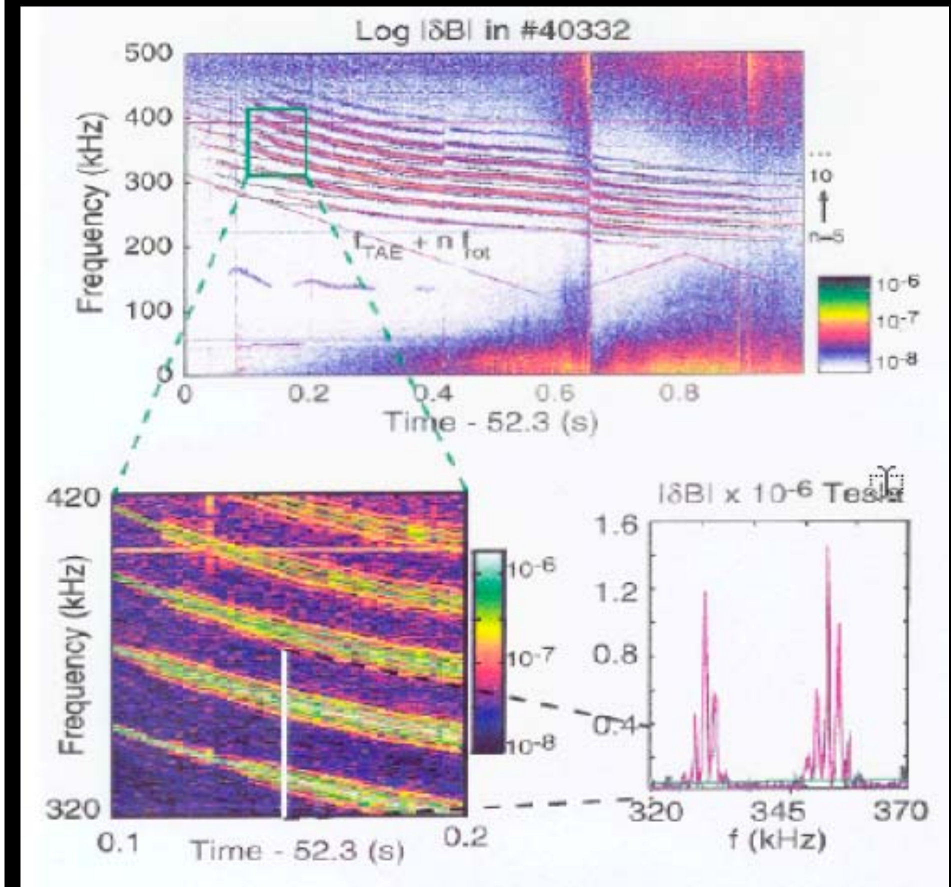
NL Behavior

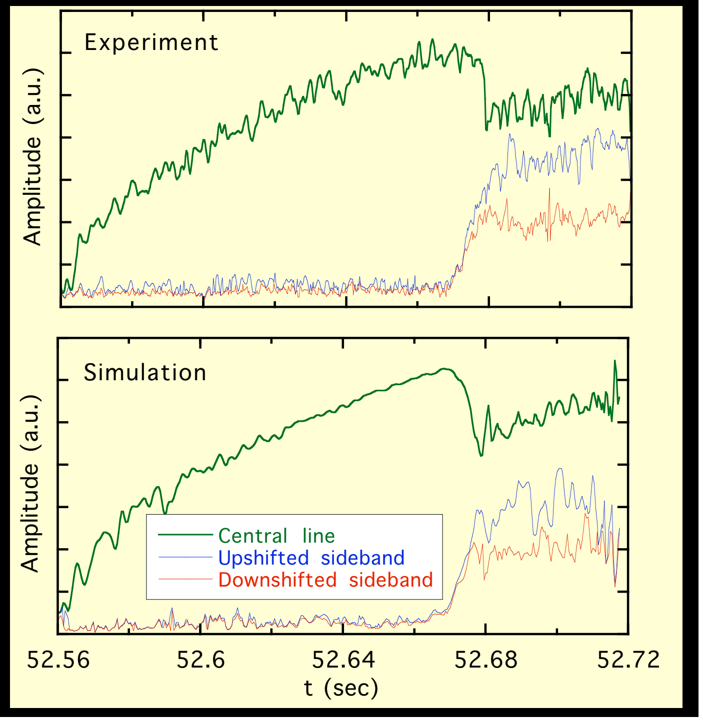
- Near marginal stability, different types of nonlinear behavior occur, depending on relative size of parameters γ_L, γ_d, ω_b, and ν,
  - Steady-state saturation (1), when $\nu > \gamma_L - \gamma_d$
  - Oscillatory behavior (4, 5)
  - Explosive nonlinear regime (8), with frequency chirping



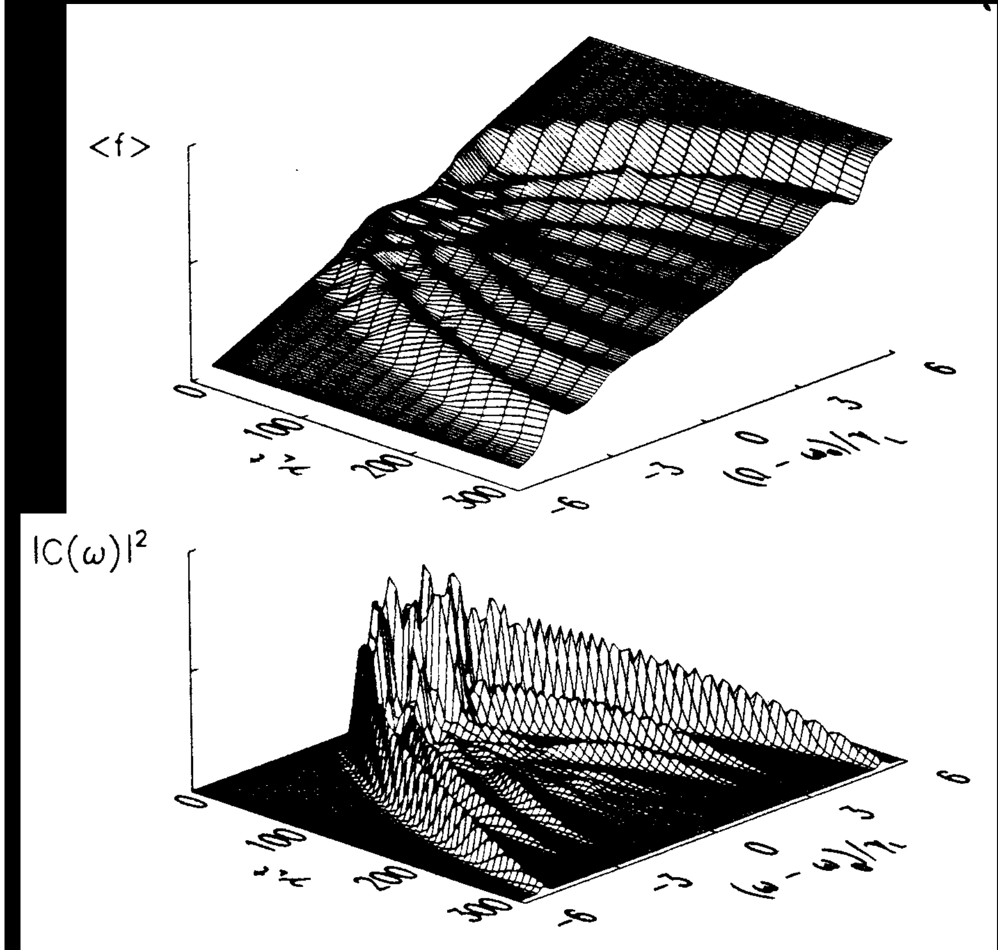
Pitchfork Modes

- Saturated steady-state solution is a fixed point of the system dynamics; the path to chaos occurs through nonlinear bifurcations of this initially stable fixed point.
- Frequency splitting corresponds to sideband formation

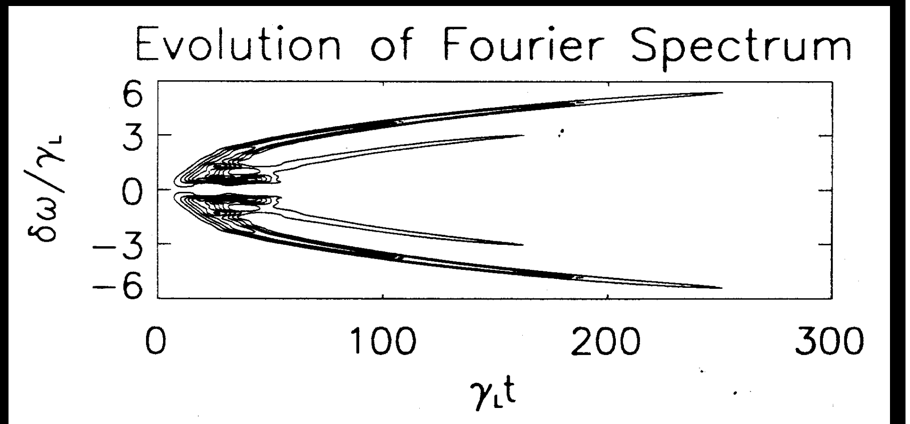




Holes & Clumps



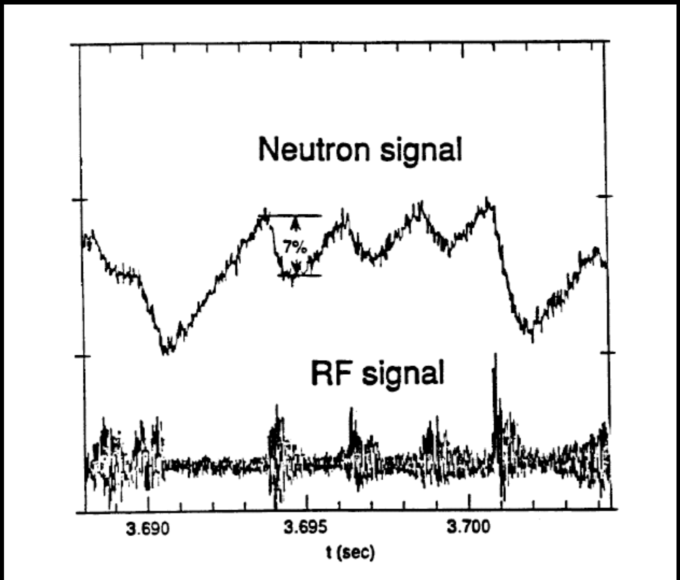
- Frequency sweeping arises from BGK-like phase space structures that grow from unstable kinetic resonances

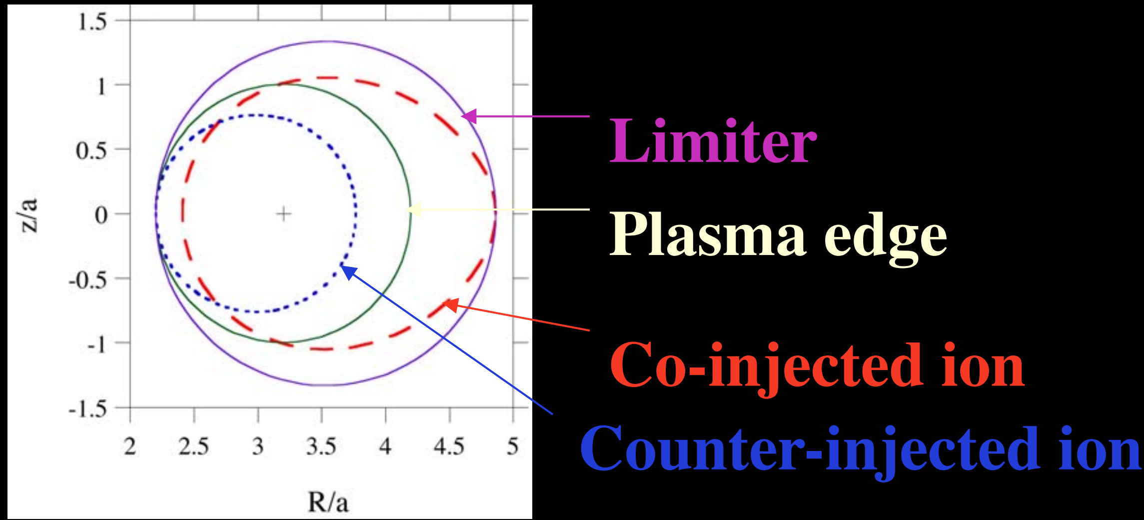


$\delta\omega \propto t^{1/2}$

TAE Bursts–1

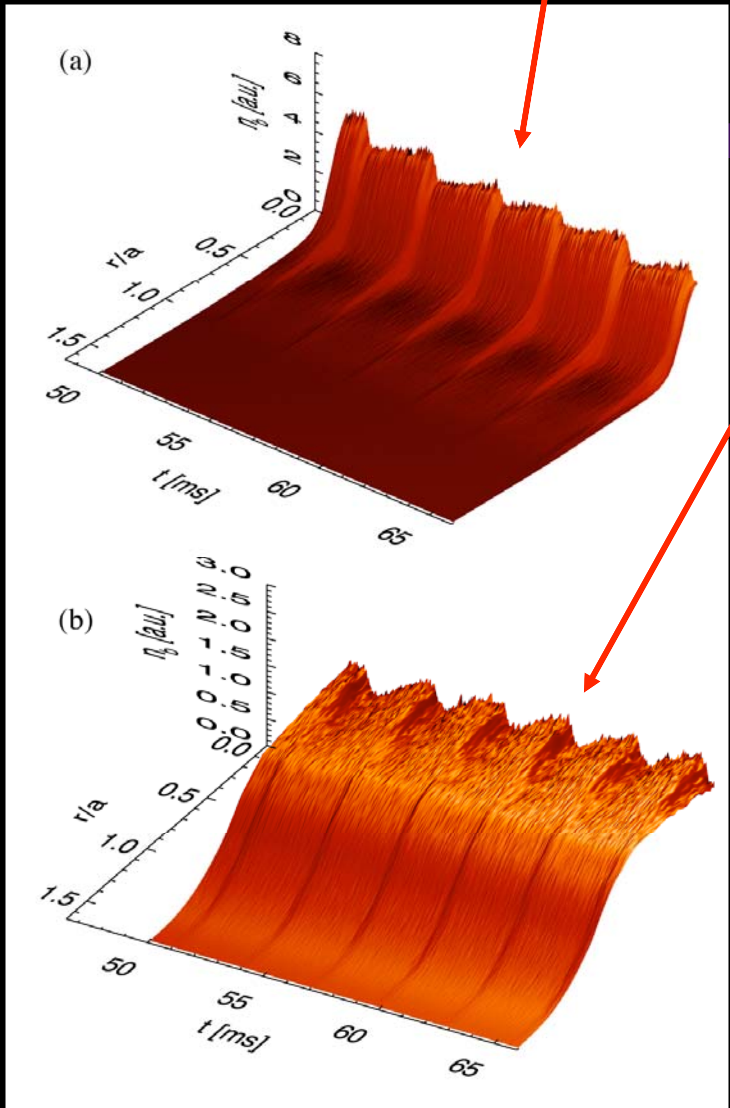
- NL simulations can explain TAE bursts and fast ion losses observed in TFTR





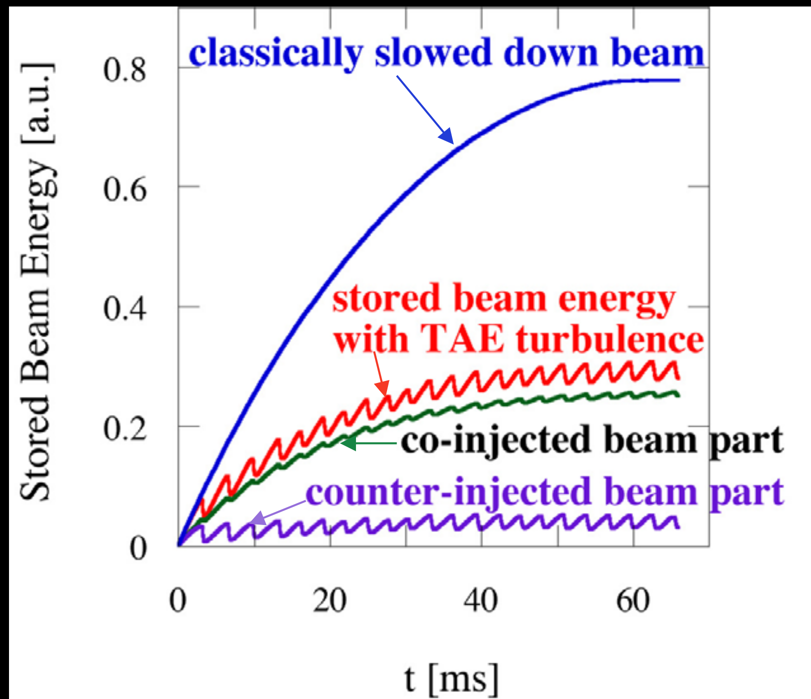
- Co-injected particles shift outward and can leave plasma
- Counter-injected particles immediately hit inner limiter

- Counter-injected beams are confined only near the magnetic axis



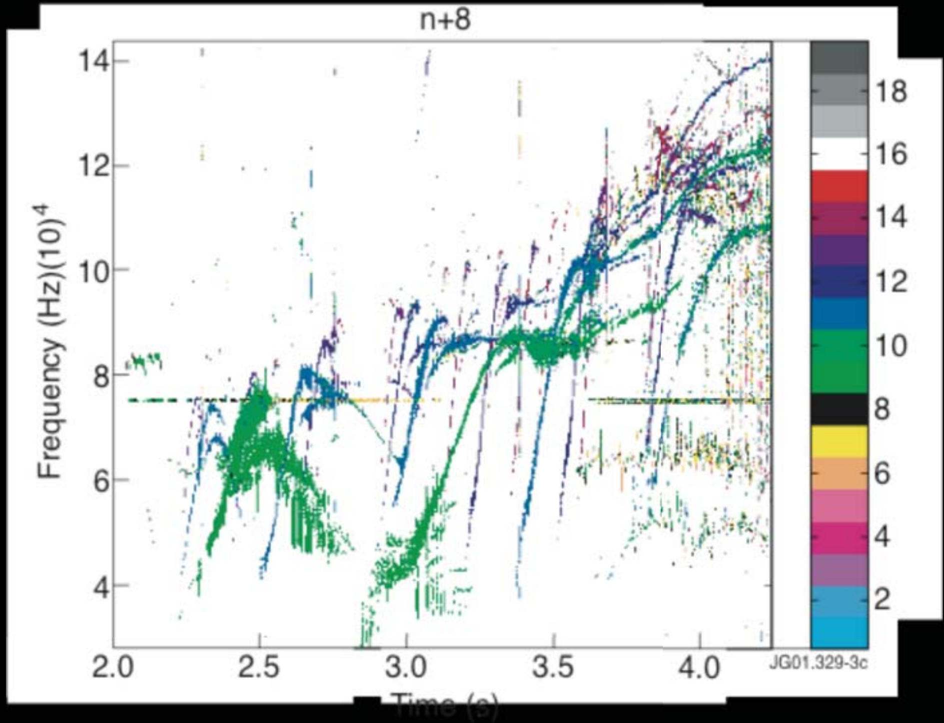
TAE Bursts–2

- Co-injected beams are confined efficiently
  - Central pressure gradient collapses periodically at criticality
  - Large pressure gradient maintained at edge



CASCADE MODES

Alfvén Cascades–1



Alfvén Cascade modes (aka RSAE)

- Occur in reversed shear plasma at zero-shear ($q'=0$) location $q(r)=q_0$
- Frequency initially below TAE gap and sweeps upward (usually never downward) as safety factor decreases in time
- Frequency signals are quasi-periodic for a large number of n-values (typically, n=1–6)
- Higher-n modes recur more frequently and have more rapid frequency sweeping ($\partial\omega/\partial t \propto n$)
- Sometimes the frequency merges into the TAE gap
- Modes are suppressed at low frequencies

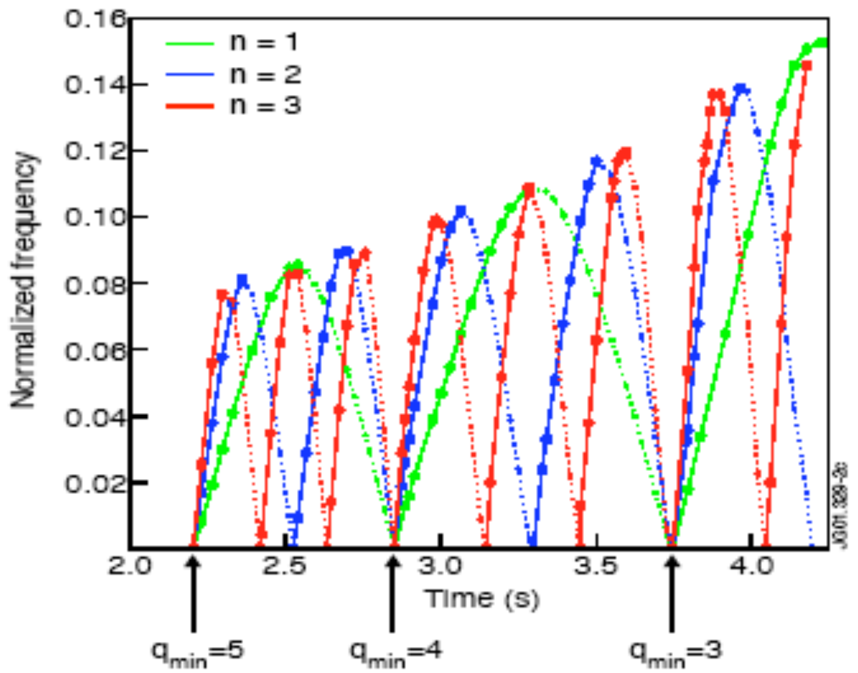
Alfvén Cascades–2

- Cascade mode is associated with the extremum of the Alfvén continuum at shear reversal q_{min}

- As $q_{min}(t)$ evolves in time due to changes in the current, the Alfvén Cascade frequency ω_{AC} varies:

$$\omega_{AC}(t) \cong \left| \frac{m}{q_{\min}(t)} - n \right| \frac{v_A(t)}{R} + \Delta\omega$$

$$\Rightarrow \quad \frac{d}{dt}\omega_{AC}(t) = m\frac{v_A}{R}\frac{d}{dt}q_{\min}^{-1}(t)$$



Time evolution n=1, n=2, and n=3 continuum tips during $q_{min}(t)$ evolution

Alfvén Cascades–3

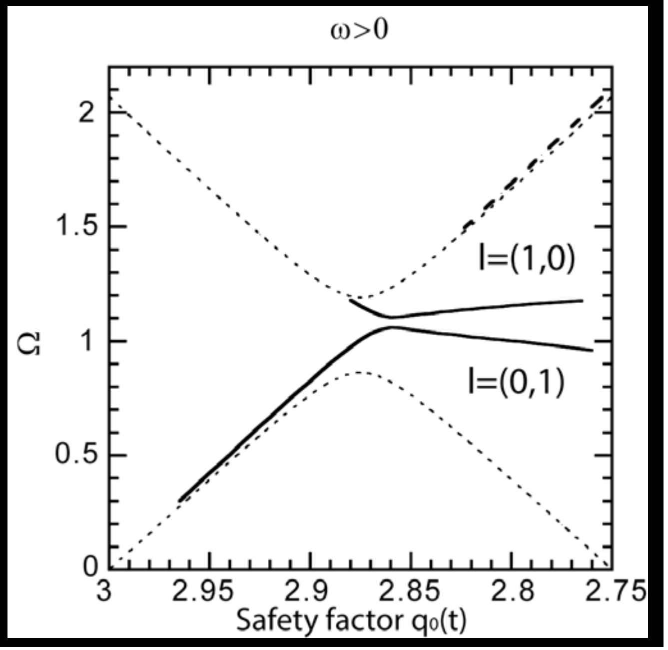
- Cascade explained by single-mode theory expanded near shear reversal surface:

$$\frac{m^2}{r^2}\left(\frac{\omega^2}{V_A^2}-k_\parallel^2\right)\Phi_m-\frac{\partial}{\partial r}\left(\frac{\omega^2}{V_A^2}-k_\parallel^2\right)\frac{\partial\Phi_m}{\partial r}=-\frac{4\pi e}{cB}\frac{m}{r}\Phi_m\frac{\partial}{\partial r}\left[\omega\left\langle n_{\text{fast ion}}\right\rangle-\left\langle\frac{k_\parallel}{e}j_{\parallel\,\text{fast ion}}\right\rangle\right]$$

$$k_\parallel\approx\frac{1}{R}\left(n-\frac{m}{q_0}\right)+\frac{1}{R}\frac{mq_0^{//}}{2q_0^2}\left(r-r_0\right)^2$$

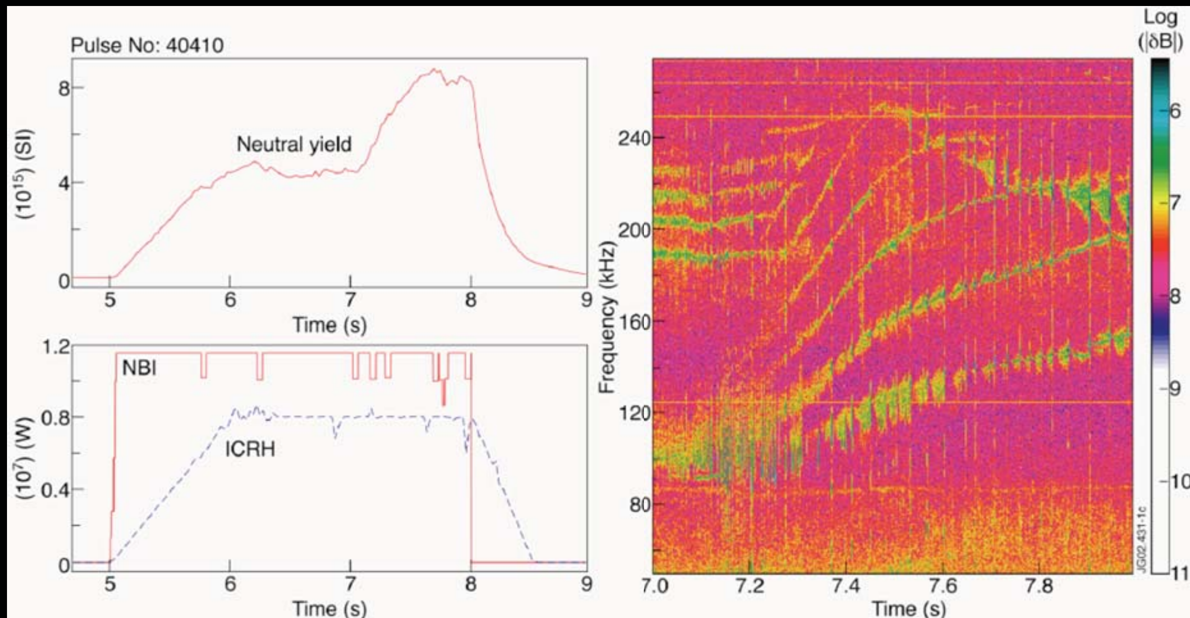
- Frequency is above the Alfvén continuum frequency if m-nq>0 and only shifts upward as $q_0(t)$ decreases

- Toroidal mode coupling increases near the TAE frequency, and causes the Cascade mode to merge into the TAE band

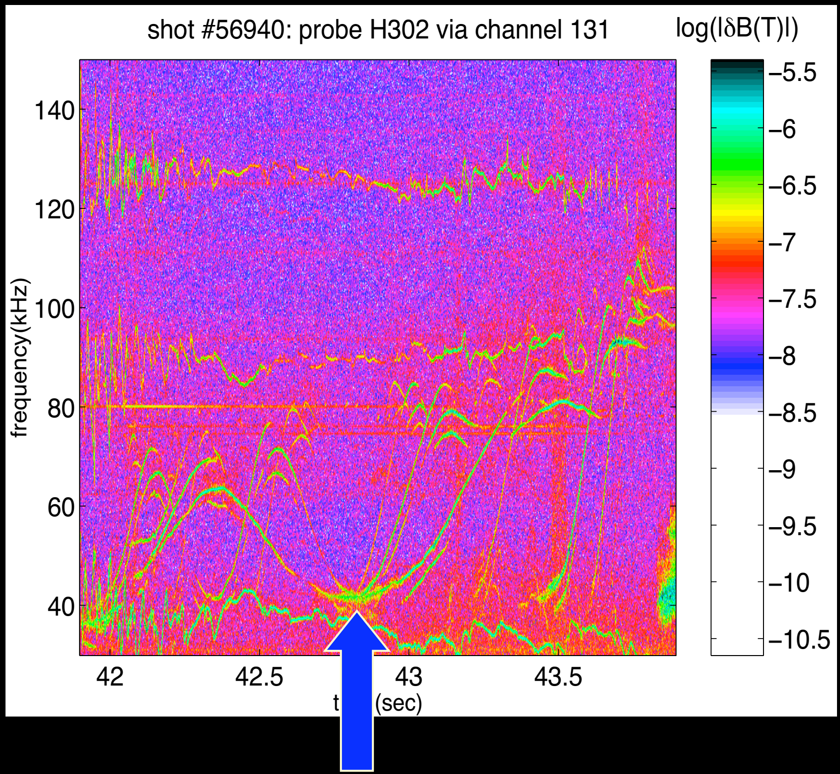


Grand Cascades

- Internal transport barrier (ITB) triggering event
  - "Grand Cascade" (many simultaneous n-modes) occurrence is coincident with ITB formation (when q_{min} passes through integer value)
  - Being used on JET as a diagnostic to monitor q_{min}
  - Can create ITB by application of main heating shortly before a Grand Cascade is known to occur



Frequency Roll-Over



- In rare events, the Cascade frequency downshifts as q_{min} decreases, with a roll-over that suggests a continuum lower boundary
  - Theory finds plasma compressibility induced by geodesic curvature leads to a minimum Cascade frequency at $\omega = 2^{1/2}\ c_s/R$. For $q_{min} \geq 2$, this frequency is above acoustic resonance ($\omega = c_s/q_{min}R$); hence fluid treatment is valid at lowest order.
  - This also explains why Cascade modes begin at finite frequency.
  - Finite beta continuum sets the minimum frequency for Cascades.

WAVE PROPERTIES AS DIAGNOSTICS

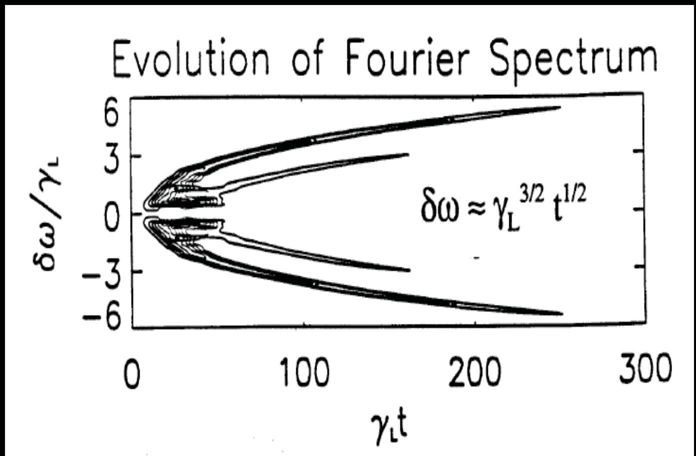
Using Waves as Diagnostics-1

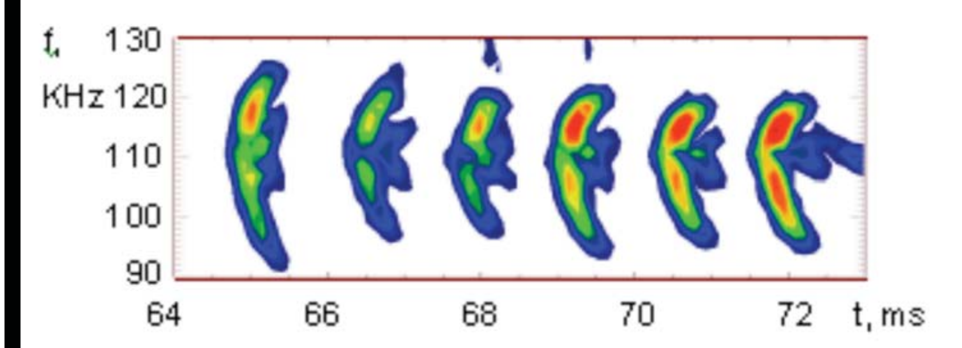
"…generating and observing these [hydromagnetic] waves at low power may be a useful diagnostic technique for determining plasma characteristics which are important parameters of these waves such as magnetic field strength, plasma density and distribution function, plasma temperature and composition, and the state of ionization of the plasma…."

(T. Stix, 1958)

"MHD spectroscopy"

Using Waves as Diagnostics-2

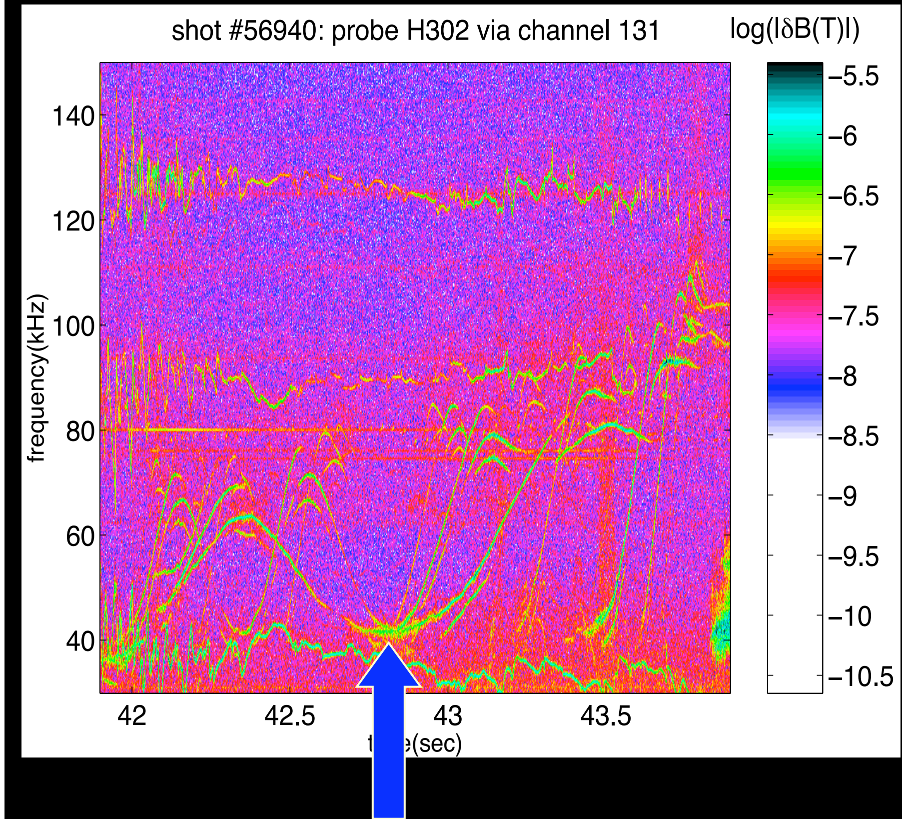




TAEs in MAST

- Determine internal fields from frequency sweeping
- Sweep rate $\delta\omega$ = const. $\omega_b^{3/2}\ t^{1/2}$, where $\omega_b \propto \delta B_r^{1/2}$
- Processing the observed frequency sweep gives $\delta B_{max} = 2 \times 10^{-4}$ T, compared to Mirnov coil measurement $\delta B_{max} = 5 \times 10^{-4}$ T

Using Waves as Diagnostics-3



- Temperature inferred from low-frequency suppression of Cascade modes
  - Geodesic deformation of Alfvén continuum determines the minimum frequency:

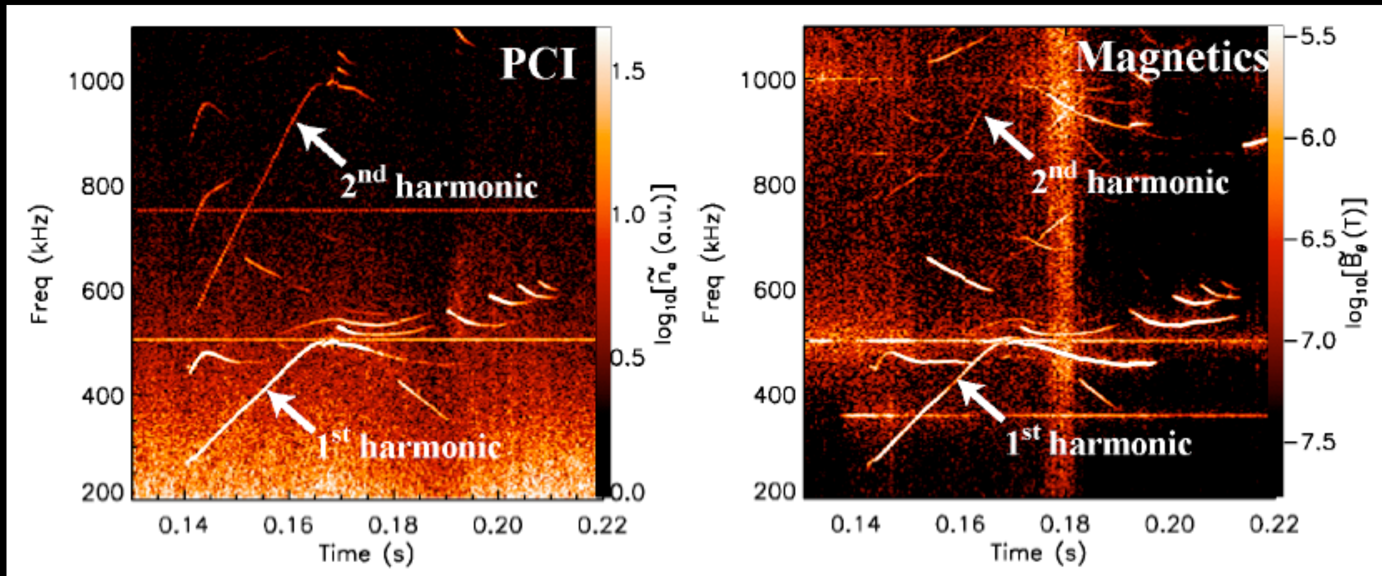
$$\omega_{\min}{}^2 \cong \frac{2c_s{}^2}{R^2}\left(1+\frac{7}{4}\frac{T_i}{T_e}\right)$$

  - This can be inverted for temperature:

$$T_e[eV]\left(1+\frac{7}{4}\frac{T_i}{T_e}\right) = 3.77 \times \left(f_{\min}\left[kHz\right]\right)^2$$

Using Waves as Diagnostics-4

Phase contrast imaging (internal) | Magnetic coils (external)



- Determine internal fields from 2nd harmonic Alfvén Cascade perturbed density
  - Ratio of the measured fundamental and second harmonic PCI signals provides independent information about internal amplitudes of perturbed B field

References

- *ITER Physics Basis Document*, Chap. 5 "Energetic Particles," Nucl Fusion **29**, 3471 (1999); *Progress in the ITER Physics Basis*, Nucl Fusion **47**, 1-414 (2007)

- "The Scientific Challenge of Burning Plasmas": http://burningplasma.org/reference.html

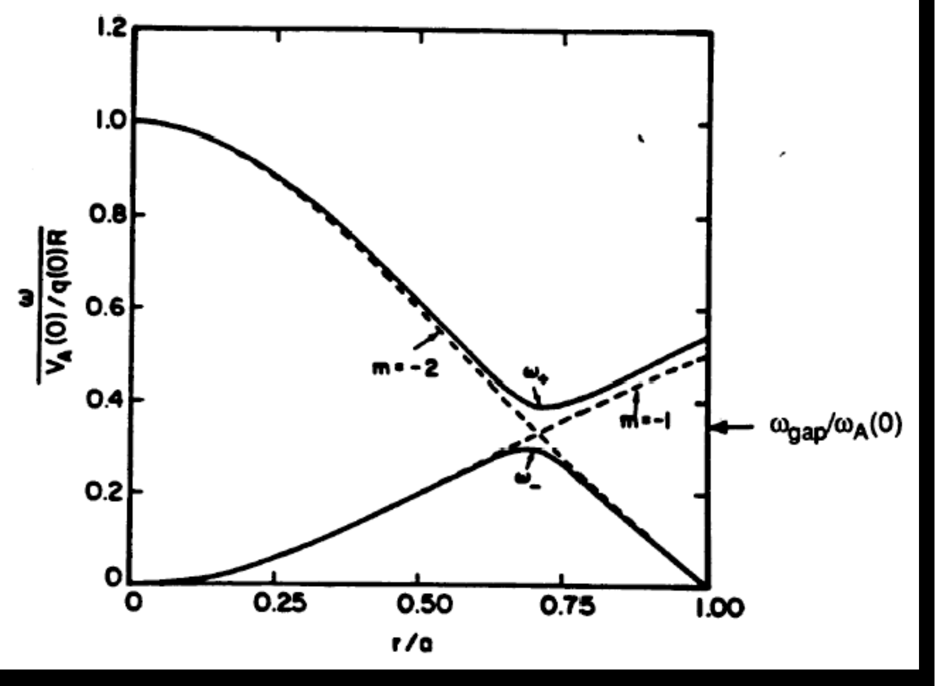
- U.S. Burning Plasma Workshop (Oak Ridge, TN, 2005): www.burningplasma.org/WS_05/html
  - Energetic particle physics plenary talk, break-out group presentations, and summary

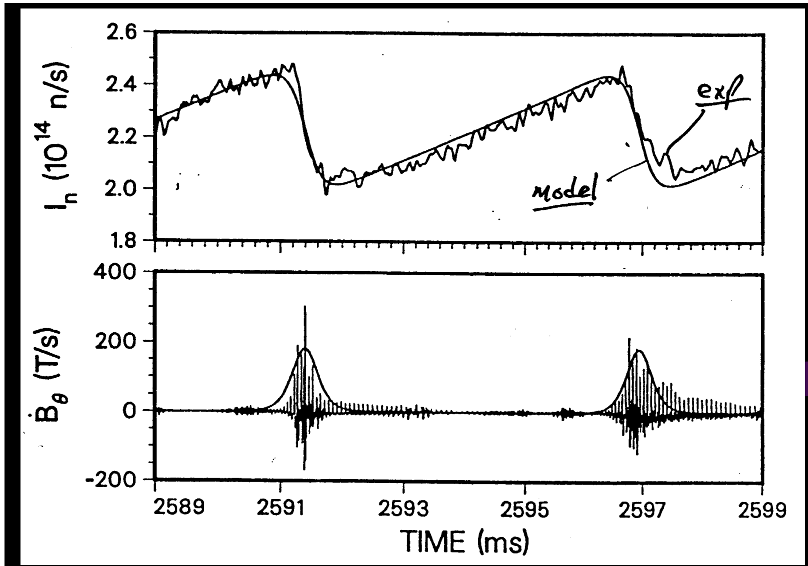
- 9th IAEA Technical Meeting on Energetic Particles in Magnetic Confinement Systems (Takayama, Japan, 2005): http://htpp.lhd.nifs.ac.jp/IAEATM-EP2005/index.html

- Joint Transport Task Force/US-Japan JIFT Workshop on Energetic Particles (Napa, CA, 2005): www.mfescience.org/TTF2005/

Exercise #1

- For the Toroidal Alfvén Eigenmode, calculate the q-value where the gap is located and estimate the typical TAE mode frequency.
  - Repeat this exercise for the Ellipticity- and Triangularity-induced Alfvén Eigenmodes





Exercise #2

- Consider a simple model for the fishbone cycle [R. White]. Assume the trapped fast ions are deposited at rate S until the threshold beta β_c is reached. Model the losses as a rigid displacement of the trapped particles toward the wall. Equations for the trapped particle beta β and the mode amplitude A are:

$$\frac{d\beta}{dt} = S - A\beta_c \ , \quad \frac{dA}{dt} = \gamma_0 \left(\frac{\beta}{\beta_c} - 1 \right) A$$

  - Show that the solution of these equations will be cyclic. [Hint: Approximately plot β and A as functions of time. Then, from the equations, construct a function $F(\beta,A)$ that satisfies $\partial F/\partial t = 0$. Approximately plot the contours F = constant in β-A phase space. Show that F is minimum when $\beta = \beta_c$ and $A=S/\beta_c$.]

- If the losses are diffusive, rather than rigid, then $d\beta/dt = S - A\beta$. In this case, show that $\partial F/\partial t \leq 0$, with $\partial F/\partial t=0$ only at $\beta=\beta_c$, and that the solution spirals toward this fixed point.