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Scaling in Soil Physics

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SCALING IN SOIL PHYSICS

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Introduction

Before introducing the subject of scaling, which in simple words can be understood as a set of operations to find scale factors to relate characteristics of one system to corresponding characteristics of another system, we will introduce the topic of dimensional analysis which allows us to determine these scale factors. It refers to the study of the dimensions that characterize physical quantities, like mass, force and energy. Classical Mechanics is based on three fundamental quantities, with dimensions MLT, the mass M, the length L, and the time T. The combination of these entities gives rise to derived quantities, like volume, speed and force, of dimensions L^3, LT^{-1}, MLT^{-2}, respectively. In other areas of Physics, other four fundamental quantities are defined, among them the temperature θ and the electrical current I.

To introduce the topics of Scaling and of Dimensional Analysis, let us look at a classical example of the romantic literature, in which *Dean Swift, in "The Adventures of Gulliver" describes the imaginary voyages of Lemuel Gulliver to the kingdoms of Liliput and Brobdingnag. In these two places life was identical to that of normal persons, their geometric dimensions were, however, different. In Liliput, man, houses, dogs, trees were twelve times smaller than in the country of Gulliver, and in Brobdingnag, everything was twelve times taller. The man of Liliput was a geometric model of Gulliver in a scale 12:1, and that of Brobdingnag a model in a scale of 1:12 (Figure 1).*

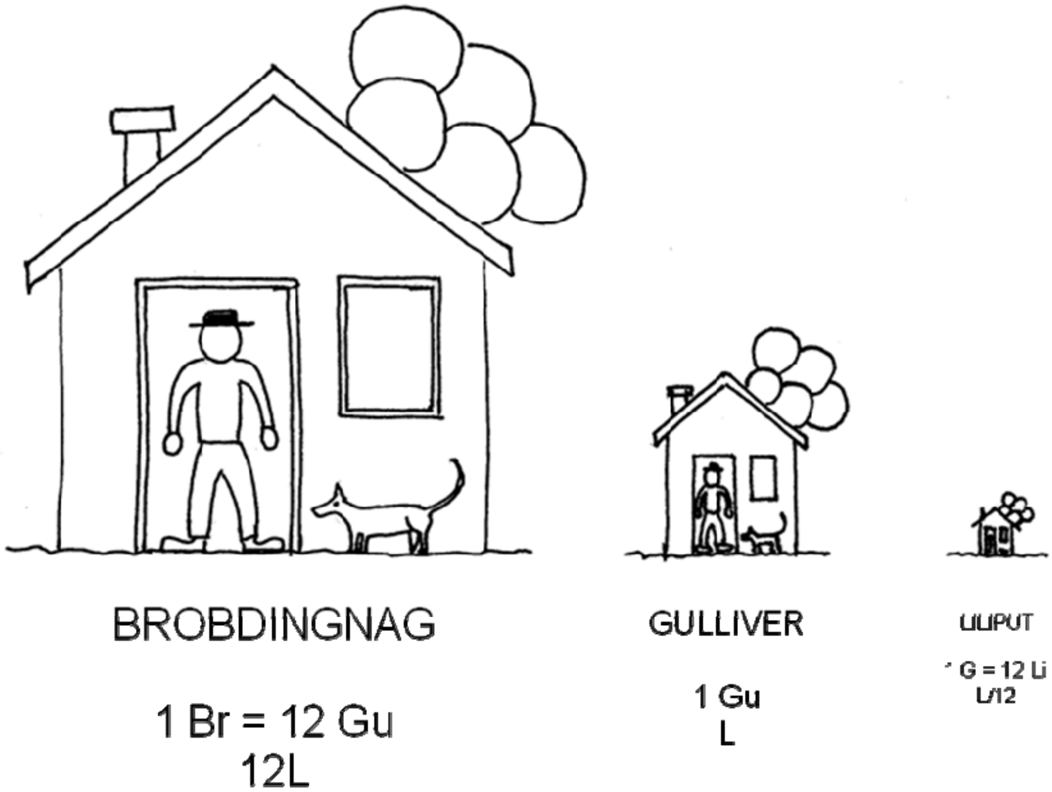


Figure 1. Schematic view of the kingdoms of Brobdingnag and Lilliput in comparison to Gulliver.

Analysing this dream of Dean Swift, one can come to interesting observations of these two kingdoms through dimensional analysis. Much time before Dean Swift, Galileus already found out that amplified or reduced models of man could not be like we are. The human body is built of columns, stretchers, bones and muscles. The weight of the body that the structure has to support is proportional to its volume, that is, L^3*, and the resistance of a bone to compression or of a muscle for traction, is proportional to* L^2*.*

Let´s compare Gulliver with the giant of Brobdingnag, which has all of his linear dimensions twelve times larger. The resistance of his legs would be 144 times larger than *that of Gulliver, and his (volume) weight 1728 times larger. The ratio resistance/weight of the giant would be 12 times less than Gullivers. In order to sustain its own weight, he would have to make an equivalent effort to that we would have to make to carry other eleven men.*

Galileus treated this subject very clearly, using arguments that deny the possibility of the existence of giants of normal aspect. If we wanted to have a giant with the same leg/arm proportions of a normal human, we would have to use a stronger and harder material to make the bones, or we would have to admit a lower resistance in comparison to a man of normal stature. On the other hand, if the size of the body would be diminished, the resistance would not diminish in the same proportion. The smaller the body, the greater its relative resistance. In this way, a very small dog could, probably, carry other two or three small dogs of his size on his back; on the other hand, an elephant could not carry even another elephant of his own size !

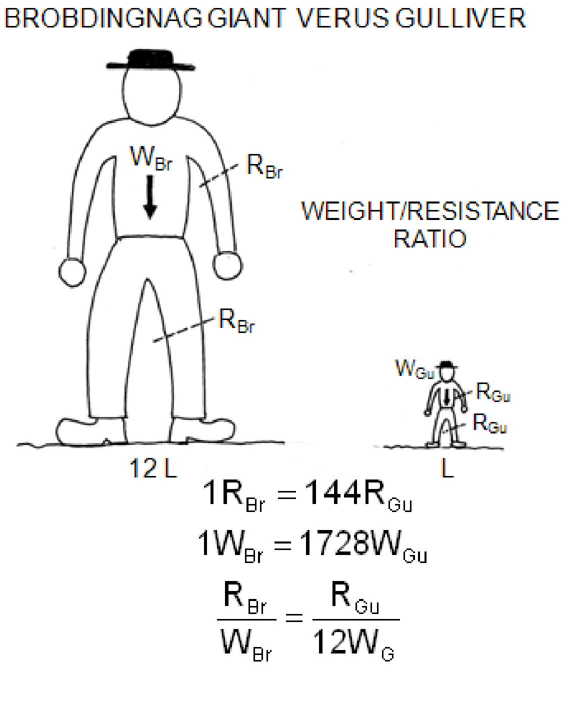


Figure 2. Comparison between the giant of Brobdingnag and Gulliver.

Let´s analyze the problem of the liliputans. The heat that a body loses to the environment goes through the skin, being proportional to the area covered by the skin, that is, L^2, considering constant the body temperature and skin characteristics. This energy comes from the ingestion of food. Therefore, the minimum volume of food to be ingested would be proportional to L^2. If Gulliver would be happy with a broiler, a bread and a fruit per day, a liliputan would get a $(1/12)^3$ smaller food volume. But a broiler, a bread, a fruit when reduced to the scale of his world, would be proporcional to a surface area $(1/12)^2$ smaller. He would, therefore, need twelve broilers, twelve breads and twelve fruits to be as happy as Gulliver.

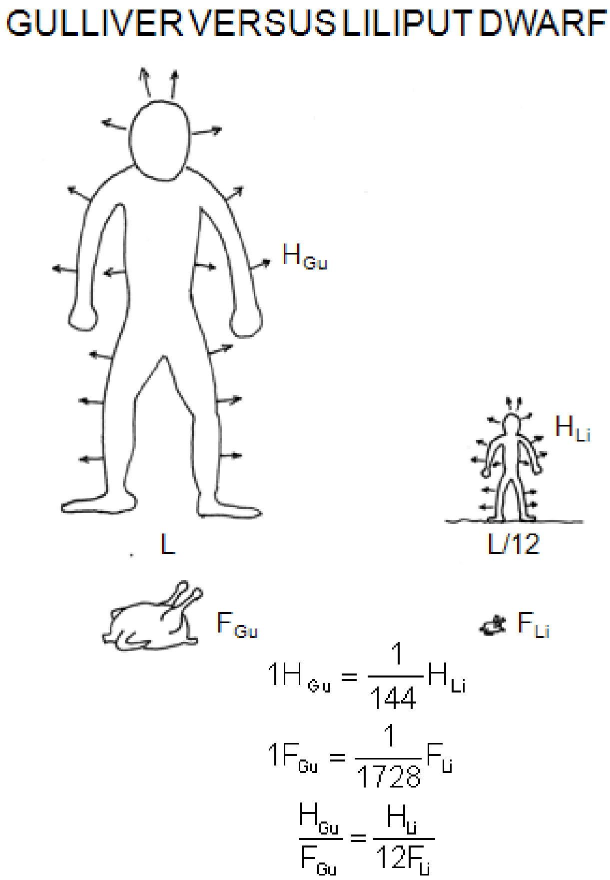


Figure 3. Comparison between the dwarf of Lilliput and Gulliver.

The liliputans should be famine and restless people. These qualities are found in small mammals, like mice. It is interesting to note that there are not many hot blood animals smaller than mice, probably in light of the scale laws discussed above, these animals would have to eat such a large quantity of food that would be difficult to obtain or, that could not be digested over a feasible time.

From all we saw, it is important to recognize that, although being geometric models of our world, Brobdingnag and Liliput could never be our physical models, since they would not have the necessary physical similarity which is found in natural phenomena. In the case of Brobdingnag, for example, the giant would be able to support his own weight having the stature of humans, if he would be living in a planet of gravity (1/12)g.

Physical Quantities and Dimensional Analysis

The parameters that characterize physical phenomena are related among themselves by laws, in general of quantitative nature, in which they appear as measures of the considered physical quantities. The measure of an quantity is the result of its comparison with another one, of the same type, called unit. In this way, a quantity (G) is given by two factors, one being the measure (M) and the other the unit (U). When we write $V = 50\ m^3$, V is the quantity G, 50 is the ratio between the measures (M), and the unit U is m^3. Therefore:

$$\mathbf{G = M\ (G)\ .\ U\ (G)}$$

M(G) being the measure of G and U(G) the unit of G. In addition, the quantity G has a dimensional symbol, which is the combination of the fundamental units that built up the entity. Some examples are given below:

Entity (G)	**M (G)**	**U (G)**	**Dimensional symbol**
Area	200	m^2	L^2
Speed	40	$m\ s^{-1}$	LT^{-1}
Force	50	$N = kg\ m\ s^{-2}$	MLT^{-2}
Pressure	1,000	$Pa = kg\ m^{-1}\ s^{-2}$	$ML^{-1}\ T^{-2}$
Flow	5	$m^3\ s^{-1}$	$L^3\ T^{-1}$

The International Units System has seven fundamental quantities:

a) Mass (M): quilogram (kg);

b) Lenght (L): meter (m);

c) Time (T): second (s);

d) Electrical current (I): Ampere (A);

e) Thermodynamic temperature (θ): Kelvin (K);

f) Light intensity (Iv): candela (cd);

g) Quantity of matter (N): mol (mol).

Derived Physical quantities are, in general, expressed by a relation involving the fundamental or derived quantities X, Y, Z, ... which take part in their definition:

$$G = k\ X^a \, . \, Y^b \, . \, Z^c \ldots\ldots\ldots$$

where k is a non dimensional constant, and a, b, c, ... are integers or real exponents.

If, for example, we would have doubts on the formula F = m.a, we could make a check and admit, at least, that F is a function of m and a:

$$G = k\,X^a.Y^b \quad \textbf{or} \quad F = k\,m^a.a^b$$

since F has dimensions MLT^{-2}, the right hand side member has also to have dimensions MLT^{-2}, that is:

$$MLT^{-2} = k\,M^a.\left(LT^{-2}\right)^b$$

remembering that the dimension of acceleration is LT^{-2}. So, $MLT^{-2} = k\,M^a.L^b.T^{-2b}$, and we can see that the only possibility is k=1, a=1 and b=1, thus confirming F=m.a.

Products P are any products of the variables that involve a phenomenon. The fall of bodies from an origin 0 with no initial velocity in the vacuum involves the variables space S, acceleration of gravity g and time t, according to:

$$S = \frac{1}{2}g\,.\,t^2$$

For this phenomenon we can write an infinite number of products P, as for example:

$P_1 = S^2.\,t^{-2}.g$, with dimensions $L^2.T^{-2}.\,L\,.T^{-2} = L^3.T^{-5}$

$P_2 = S^0.\,t^{2}.g$, with dimensions $1.T^2.\,L\,.T^{-2} = L$

$P_3 = S^{-3}.\,t^{4}.g$, with dimensions $L^{-3}.T^{4}.\,L\,.T^{-2} = L^{-2}.T^2$

$P_4 = S^{-2}.\,t^{4}.g^2$, with dimensions $L^{-2}.T^4.\,(L\,.T^{-2})^2 = L^0.T^0 = 1$

When a chosen product is non-dimensional, as P_4, it is called a non-dimensional product and is symbolized by π, in this case $P_4 = \pi_4$. The Buckingham Pi Theorem states that: "given n dimensional quantities $G_1, G_2, ..., G_n$ generated through products of k fundamental entities, if a phenomenon can be expressed by the function $F(G_1, G_2, ..., G_n) = 0$, it can also be described by $\phi(\pi_1, \pi_2, ..., \pi_{n-k}) = 0$", a function with less variables.

The problem mentioned in the introduction about the Kingdoms of Liliput and Brobdingnag, is of physical similarity. Every time we work with models of objects in different scales, it is necessary that there is a physical similarity between the model (a prototype, in general smaller) and the real object of study. Depending on the case, we talk about kinematic similarity, which involves relations of velocity and acceleration between model and object; or about dynamic similarity, which involves relations between the forces that act on the model and on the object . In the similarity analysis we use the π products, like the known "numbers" of Euler, Reynolds, Froude and Mach. In this analysis we have:

OBJECT:

$$F(G_1, G_2, ..., G_n) = 0 \longrightarrow \phi(\pi_1, \pi_2, ..., \pi_{n-k}) = 0$$

PROTOTYPE:

$$F(G'_1, G'_2, ..., G'_n) = 0 \longrightarrow \phi(\pi'_1, \pi'_2, ..., \pi'_{n-k}) = 0$$

and the G_i s can be different of G'_i s. There will be physical similarity between object and prototype, only if $\pi_1 = \pi'_1$; $\pi_2 = \pi'_2$; ...; $\pi_{n-k} = \pi'_{n-k}$.

This analysis is frequently used in hydrodynamics, studies of machines, engineering, etc., and it has not many applications in Soil-Plant-Atmosphere systems. The study of Shukla et al. (2002) which utilizes the non dimensional products π to describe miscible displacement, is an exception. Texts of Maia (1960), Fox & McDonald (1995) e Carneiro (1996) are good references on this subject.

Non dimensional quantities, like the π products, have a numerical value k of dimension 1:

$$M^o L^o T^o K^o = 1$$

It is also common to produce non-dimensional variables through the ratio of two entities G_1 and G_2 of the same dimension: $G_1/ G_2 = \pi$. This is the case of the number π = 3,1416..... which is the result of the ratio of the length of any circle (πD, of dimension L) and the respective diameter (D, also dimension L).

In the Soil-Plant-Atmosphere system, several variables are non dimensional by nature (or definition), and are represented in % or parts per million (ppm). Soil water content u (on mass basis), θ (on volume basis), porosities, etc., are examples of π products. Important is the procedure of turning dimensional variables into non dimensional ones. The simplest case is dividing the variable by itself, in two different conditions. For instance, in experiments using soil columns, each researcher uses a different column length L. How can we compare results? If the space coordinate x or z (along the column) is divided by its maximum value L, we have a new variable: X = x/L, with the advantage that, for any L, at x = 0, X = 0; at x = L, X =1, varying, therefore, within the interval 0 to 1.

This procedure can also be used for variables which already are dimensionless, like the soil water content θ. If we divide $(\theta - \theta_s)$ by its largest interval $(\theta_o - \theta_s)$, where θ_s e θ_o are, respectively, initial and saturation values, we obtain a new variable $\Theta = (\theta - \theta_s)/(\theta_o -$

θ_s), for which $\Theta = 0$ for $\theta = \theta_s$ (dry soil) and $\Theta = 1$ for $\theta = \theta_o$ (saturated soil). In this way, for any type of soil, Θ varies from 0 to 1 and comparisons can be made more adequately. The same is also made when studying coil solution concentrations C_i in miscible displacement experiments, where a non-dimensional concentration $C = (C_i - C_o)/(C_{max}-C_o)$ is used, in which C_o is the initial concentration and C_{max} the maximum.

Another interesting example of making variables dimensionless is presented by Hui *et al.* (1998), used in statistical analyses. If a X_t data set is transformed with respect to its mean (m) and standard deviation (s), according to $x_t = [X_t - (m - 2s)]/4s$ the transformed values x_t become dimensionless with mean $m = 0.5$ and standard deviation $s = 0.25$. This transformation allows comparisons among different populations and is successfully used in state- space analyses.

Tillotson and Nielsen (1984) in their review on DIMENSIONAL ANALYSIS and SCALING present several examples related to their applications in soil science. According to them, the foundation of dimentional techniques rests in the concept of similarity, and mention that three types of similarity are possible: geometric, kinematic and dynamic. The GEOMETRIC SIMILARITY refers to the size relationship between systems, as it was the case of the kingdoms of Brobdingnag and Lilliput. The KINEMATIC SIMILARITY refers to the relationships among motions in two systems. Tillotson and Nielsen (1984) present an example on the dispersion velocities of a solute in soil columns. The DYNAMIC SIMILARITY refers to force relationships in two systems, very much employed in engeneering anf hydrodynamics. They also describe the INSPECTIONAL ANALYSIS, which is another method to obtain non-dimensional quantities and scale factors. This analysis requires that physical laws governing the system are known. A very good illustrative example is given by these authors on vertical movement of a solute in a soil column.

Scales and Scaling

We already mentioned scales when presenting the “Adventures of Gulliver” and discussing physical similarity between object and prototype. Maps are also drawn in scale, for example, in a scale of 1:10,000, 1 cm^2 of paper can represent 10,000 m^2 in the field. Entities that differ in scale cannot be compared in a simple way. As we have seen, there is the problem of physical similarity.“Scaling”, frequently used in Soil Physics, is based on similarity concepts and on dimensional analysis. Miller & Miller (1956) were among the first giving the needed emphasis on these important tools through the concept of similar media applied to “capillary flow” of fluids in porous media. According to these authors, two media M_1 and M_2 are similar when the variables that describe the physical phenomena that occur within them, differ of a linear factor λ, which they called microscopic characteristic length, that relates their physical characteristics.

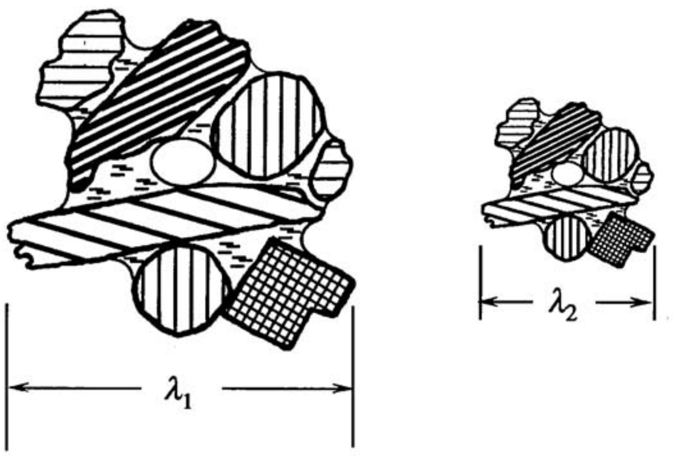
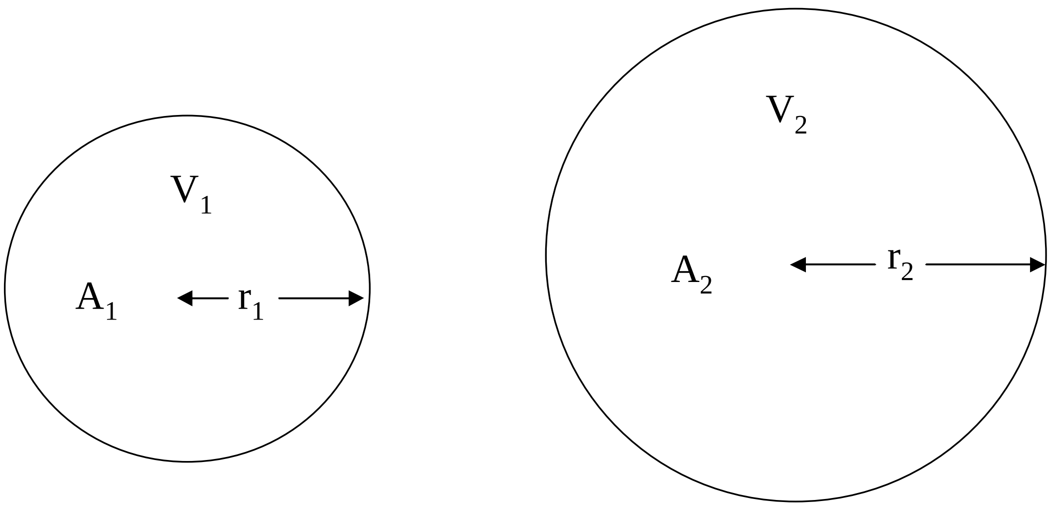


Figure 4. A classical example of similarity in porous media arrangement.

The best way to visualize this concept is to consider M_2 as an amplified (or reduced) photography of M_1 by a factor λ. For these media, the particle diameter of one is related to the other by: $D_2 = \lambda D_1$. The surface of this particle by: $S_2 = \lambda^2 S_1$, and its volume by $V_2 = \lambda^3 V_1$ (Figure 5). Under these conditions, if we know the flow of water through M_1, would it be possible to estimate the flow through M_2, based only on λ ? Using artificial porous media (glass beads **)**, Klute & Wilkinson (1958) and Wilkinson & Klute (1959) obtained results on water retention and hydraulic conductivity that validated the similar media concept.



$r_2 = 1.5\ r_1$

$r_1 = 3$ cm	$r_2 = 4{,}5$ cm
$A_1 = \pi r_1^2 = 28.27\ \text{cm}^2$	$A_2 = \pi r_2^2 = 63.62\ \text{cm}^2$
$V_1 = \dfrac{3\pi r_1^3}{4} = 63.62\ \text{cm}^3$	$V_2 = \dfrac{3\pi r_2^3}{4} = 214.71\ \text{cm}^3$

$$\frac{A_2}{A_1} = 2.25 \quad \rightarrow \quad \sqrt{2.25} = 1.5 \quad \text{ou} \quad A_2 = (1.5)^2 A_1$$

$$\frac{V_2}{V_1} = 3.37 \quad \rightarrow \quad \sqrt[3]{3.37} = 1.5 \quad \text{ou} \quad V_2 = (1.5)^3 V_1$$

Figure 5 – Spheres seen under the similar media concept.

After Miller and Miller's contribution several studies appeared in the soil science literature which, however, did not significantly push ahead this concept. In a general way it was concluded that the similar media concept could not be applied to such heterogeneous materials as soils, since even working with very homogeneous glass beads media they had no full success. More than 10 years later, Reichardt et al. (1972) reappear with the subject, having success even with natural porous media, i.e., soils of a wide range in texture. We will present here their study in a very complete manner because it is a very good example of the use of the scaling technique.

REICHARDT, NIELSEN AND BIGGAR SCALING

They assumed that soils can be considered similar media, each one characterized by its factor λ which, at the beginning, they did not know how to measure. They started using **inspectional analysis** on the concept on horizontal water infiltration studies, using homogeneous soil columns of initial soil water content θ_i, applying free water at the entrance ($x = 0$) so that at this point the saturation water content θ_o was maintained thereafter:

$$\theta = \theta_i \ , \ x > 0 \ , \ t = 0 \qquad (1)$$

$$\theta = \theta_o \ , \ x = 0 \ , \ t > 0 \qquad (2)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta)\frac{\partial \theta}{\partial x} \right] \qquad (3)$$

where $D(\theta) = K(\theta).dh/d\theta$; $K(\theta)$ is the soil hydraulic conductivity and h the soil water matric potential.

The results obtained in such experiments is summarized in Figure 6, showing the wetting front (x_{wf}) advance which is linearly related to the square root of time t.

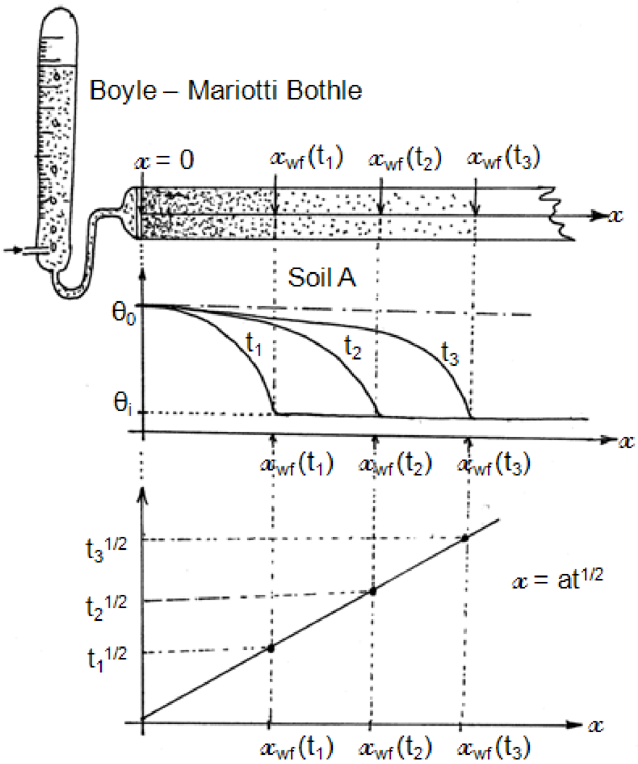


Figure 6 – The experimental arrangement to study the horizontal infiltration using transparent acrylic columns for soil A and the advancement of wetting front as a function of the square root of time.

Since for any soil the solution of this boundary value problem BVP is of the same type: $x = \phi(\theta).t^{1/2}$, in which $\phi(\theta)$ depends on the characteristics of each porous medium, would it not be possible to find a generalized solution for all media (considered similar) if λ of each soil would be known ? The procedure they used included the process of making all involved variables dimensionless, using the similar media theory (**geometric and kinematic similarities**) applied to each of the i soils, each with its λ_1, λ_2, ...λ_i. The soil water content θ and the space coordinate x were transformed as mentioned above in this text**:**

$$\Theta = \frac{(\theta - \theta_i)}{(\theta_o - \theta_i)} \tag{4}$$

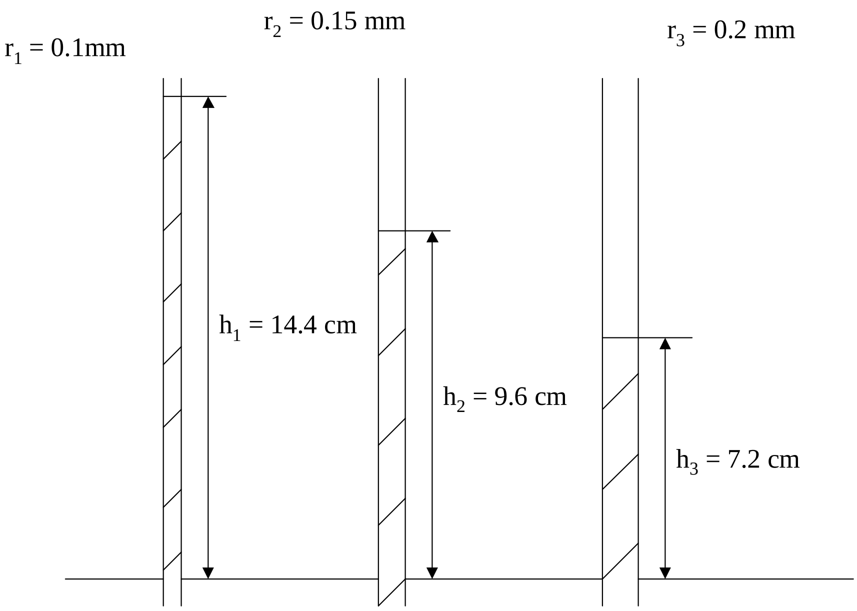
$$X = \frac{x}{x_{max}} \tag{5}$$

The matric soil water potential h was considered to be only the result of capillary forces: $h = 2\sigma/\rho gr$ or $hr = 2\sigma/\rho g$ = constant. If each soil i would have only capillaries of radius r_i, and if the characteristic length λ_i would be proportional to r_i, we would have:

$$h_1r_1 = h_2r_2 = ...= h_ir_i = \text{constant}$$

If, among the i soils, we choose one as a standard soil, for which we make, arbitrarily, $\lambda^* = r^* = 1$ (one μm, or any other value), the constant above becomes $h^*r^* = h^*$, which is the matric potential h^* of the standard soil (Figure 7). Through dimensional analysis we can also make h^* non-dimensional:

$$h^* = \frac{\lambda_1 \rho g h_1}{\sigma} = \frac{\lambda_2 \rho g h_2}{\sigma} = ... = \frac{\lambda_i \rho g h_i}{\sigma} \qquad (6)$$



$$h_1 r_1 = h_2 r_2 = h_3 r_3 = \text{constant}$$

$$14.4 \times 0.1 = 9.6 \times 0.15 = 7.2 \times 0.2 = 1.44$$

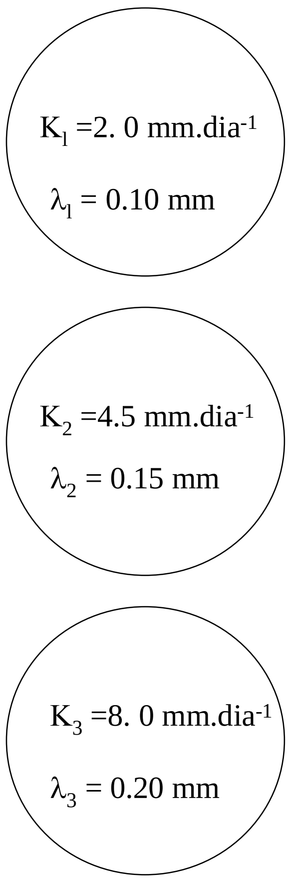
Figure 7 – Similar glass capillaries in water.

The hydraulic conductivity K is proportional to the area (λ^2) available for water flow (k = intrinsic permeability, L^2), and using the known relation $K = k\rho g/\eta$ or $K/k = \rho g/\eta$ = constant, we have for the i soils:

$$\frac{K_1}{k_1} = \frac{K_2}{k_2} = \ldots\ldots\ldots\ldots \frac{K_i}{k_i} = \text{constant}$$

$$K^* = \frac{\eta K_1}{\lambda_1^2 \rho g} = \frac{\eta K_2}{\lambda_2^2 \rho g} = \ldots\ldots\ldots \frac{\eta K_i}{\lambda_i^2 \rho g} \qquad (7)$$

where K* is the hydraulic conductivity of the standard soil, assuming $\lambda^* = r^* = k^* = 1$ (Figure 8).



$$\frac{K_1}{\lambda_1^2} = \frac{K_2}{\lambda_2^2} = \frac{K_3}{\lambda_3^2} = \text{constant}$$

$$\frac{2}{(0.10)^2} = \frac{4{,}5}{(0.15)^2} = \frac{8}{(0.20)^2} = 200$$

Figure 8 – Cross-sections of soil columns with their respective conductivities.

Through the definition of soil water diffusivity D = K.dh/dθ, it is possible to verify that the soil water difusivity D* is given by:

$$D^* = \frac{\eta\, D_1}{\lambda_1\, \sigma} = \frac{\eta\, D_2}{\lambda_2\, \sigma} = ... = \frac{\eta\, D_i}{\lambda_i\, \sigma} \qquad (8)$$

To make equation 3 dimensionless it is now needed to make the time t dimensionless. In accordance to all other variables, we can have a time t* for the standard soil, as follows:

$$t^* = \frac{\lambda_1\, \sigma\, t_1}{\eta\left(x_{1max}\right)^2} = \frac{\lambda_2\, \sigma\, t_2}{\eta\left(x_{2max}\right)^2} = ... = \frac{\lambda_i\, \sigma\, t_i}{\eta\left(x_{i\,max}\right)^2} \qquad (9)$$

It can now be seen that if we substitute θ by Θ, x by X, t by t_i and D by D_i in equation 3, we obtain the differential equation for the standard soil, which differs from the equations of all other soils by factors λ_i, not seen in equation 10, but built-in the definitions of t* and D*:

$$\frac{\partial \Theta}{\partial t^*} = \frac{\partial}{\partial X}\left[D^*(\Theta) \frac{\partial \Theta}{\partial X} \right] \qquad (10)$$

subject to conditions:

$$\Theta = 0 \ , \ X \geq 0 \ , \ t^* = 0 \qquad (11)$$

$$\Theta = 1 \ , \ X = 0 \ , \ t^* > 0 \qquad (12)$$

the solution of which is:

$$X = \phi^*(\Theta).\left(t^*\right)^{1/2} \qquad (13)$$

It is interesting to analyze the non dimensional infiltration time of equations (9) and (13), in light of the physical similarity of the kingdoms of Liliput and Brobdingnag, which shows that to compare different soils (considered similar media), their times have to be different and dependent of λ which is a length ! We could even suggest that this fact contributes to explain how time is considered the forth coordinate, together with x, y and z, in the theories of Modern Physics.

By analogy with what was made with h and K, we can write:

$$t_1\lambda_1 = t_2\lambda_2 = ... = t_i\lambda_i = \frac{t^*\eta\left(x_{max}\right)^2}{\sigma} = \text{constant}$$

Once the theory was established, Reichardt et al. (1972) looked for ways to measure λ for the different soils. The “Columbus Egg” was found when they realized that if the linear regressions of x_i versus $t_i^{1/2}$ for the position of the wetting front for each soil, should overlap to one single curve for the standard soil (X versus $t^{*1/2}$), and that the factors used to rotate the line of each soil to the position of the line of the standard soil, could be used as characteristic lengths λ_i. This procedure is called **FUNTIONAL SCALING**. We know that straight lines passing through the origin: $y = a_i x$ can be rotated over each other using the relation a_i/a_j of their slopes. Since in our case the lines involve a square root, the relation to be used is:

$$\frac{\lambda_i}{\lambda^*} = \left(\frac{a_i}{a^*}\right)^2 \tag{14}$$

Figure 9 shows na example of the functional scaling for three soils A, B and C. It is interesting to note that after scaling the water content profiles overlap so that only the shapes of these profiles become apparent. They show the intrinsic differences of the soil water retention characteristics of the different soils, since these profiles are different for sany, silty or clayey soils.

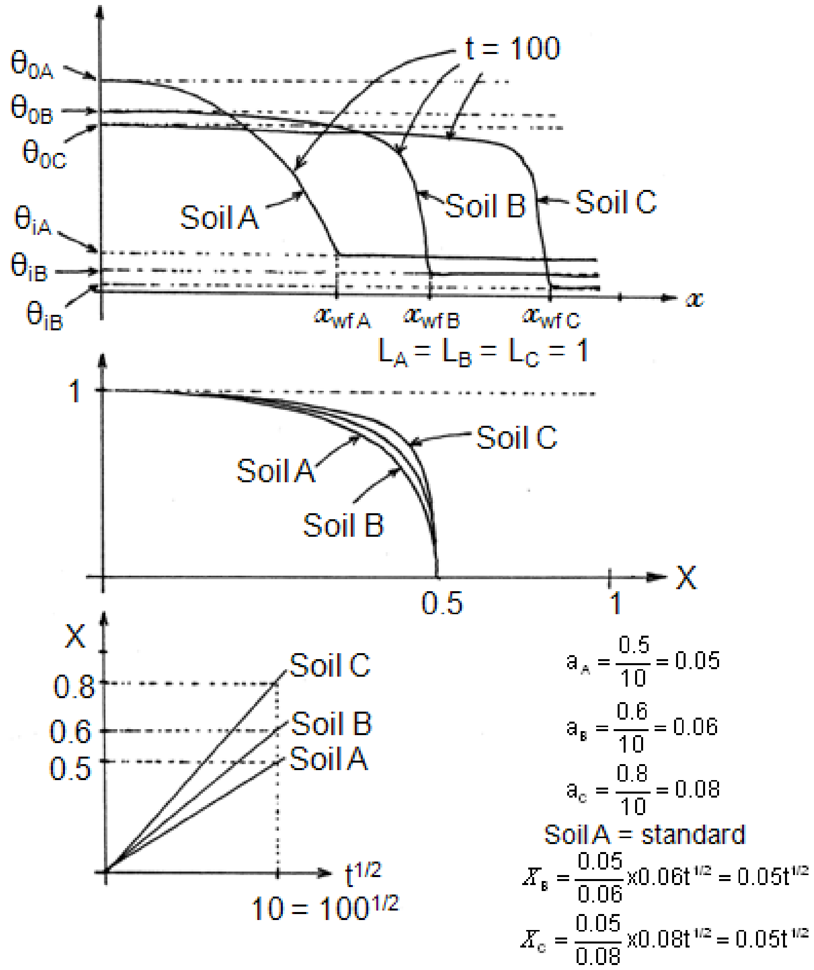


Figure 9 – Horizontal infiltration profiles for soils A, B and C, and their scaled profiles.

With this relation Reichardt et al. (1972) found the values λ_i for each soil, taking arbitrarily as a standard the soil of fastest infiltration, for which they postulated $\lambda^* = 1$. In this way, the slower the infiltration rate of soil i, the slower its λ_i. This way of determining λ as a scaling factor and not as a physical soil characteristic like the microscopic characteristic length of Miller & Miller (1956), facilitated the experimental part of the study and, more than that, opened the door for a much wider concept of scaling applied in other areas of Soil Physics. Reichardt et al. (1972) had only success in scaling $D(\theta)$ and a partial success in scaling h(θ) and K(θ), the reason for this being the fact that soils are not true similar media. The success of scaling D(θ) lead Reichardt & Libardi (1973) to establish a general equation to estimate D(θ) of a given soil, by measuring only the slope a_i of the wetting front advance x versus $t^{1/2}$:

$$D(\Theta) = 1{,}462 \text{ x } 10^{-5} a_i^2 \exp(8{,}087.\Theta) \qquad (15)$$

Reichardt et al. (1975) also presented a method to estimate K(Θ) through the coefficient a_i of equation (15); Bacchi & Reichardt (1988) used scaling techniques to evaluate K(θ) measurement methods.

More recent developments in scaling

Sposito (1998) edited a book on scale dependence and scale invariance with several applications of scaling techniques in land-surface hydrology, river networks, field soil water behavior, Richards equation, watershed modeling, heterogeneity in vadose-zone hydralogy, solute transport and several other topics. One important aspect that has to be mentioned is the application of scaling to problems related to spatial variability of soil properties. If a scaling factor is assigned to each site, like deviations from the mean, very disperse measurements coalesce to a single curve, as it can be seem on Figure 10 for saturated hydraulic conductivity.

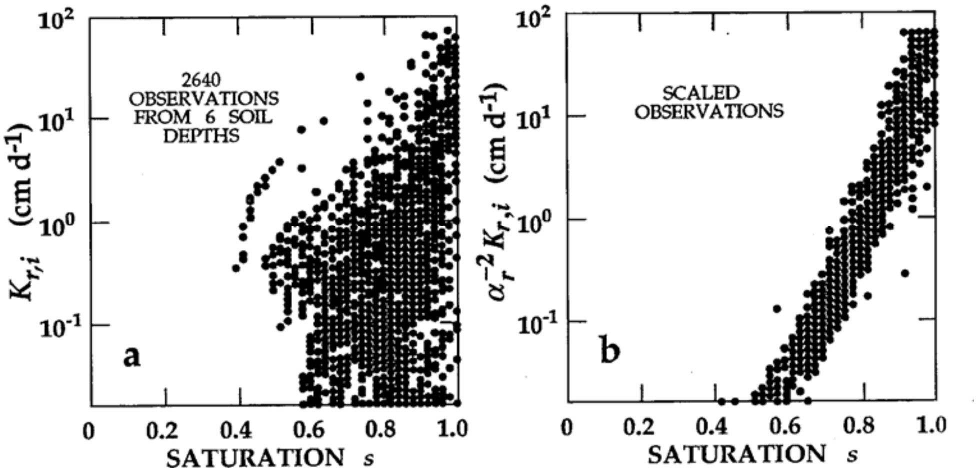


Figure 10. Left, unscaled hydraulic conductivity data; right, well coalesced scaled data. Taken from Sposito (1998).

Another example of the use of scaling in miscible displacement experimentation was given more recently by Shukla et al. (2002), using the π products.

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