2164-4

Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

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Fluctuation-induced Switching and Power Spectra of Modulated Quantum Oscillators

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Fluctuation-induced switching and power spectra of modulated quantum oscillators

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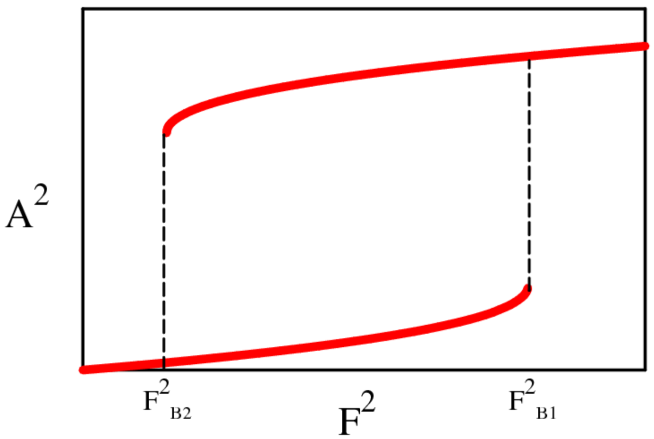
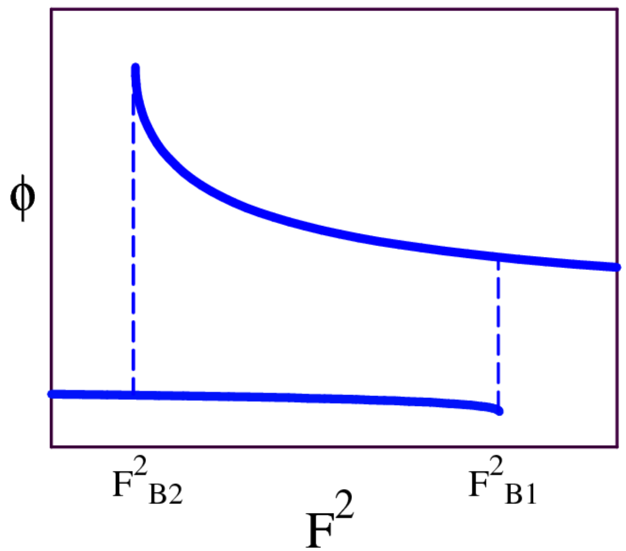
Steven Shaw, *Michigan State University*
Vadim Smelyanskiy, *NASA Ames*
Frank Wilhelm, *University of Waterloo*



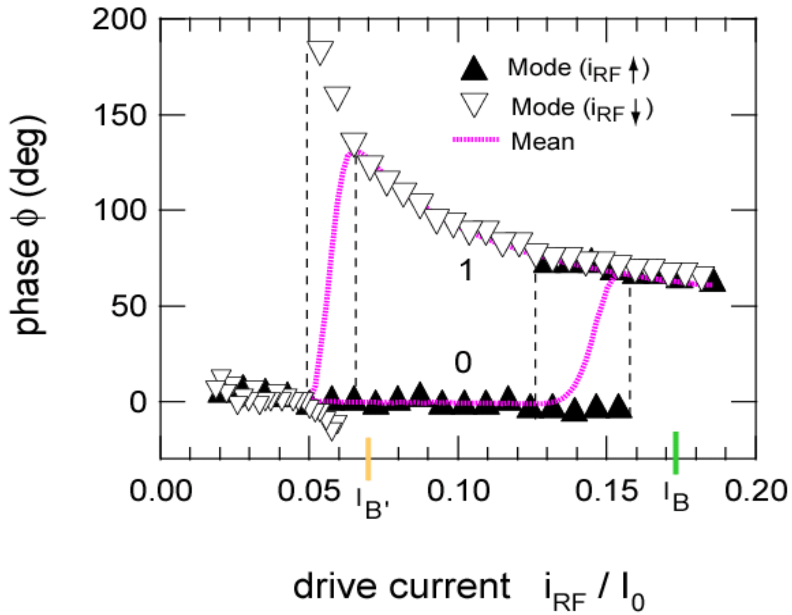
$$\ddot{q} + 2\Gamma\dot{q} + \omega_0^2 q + \gamma q^3 = F\cos\omega_F t$$

Periodic state: $q = A\cos(\omega_F t + \phi)$

Both A and ϕ **display hysteresis**



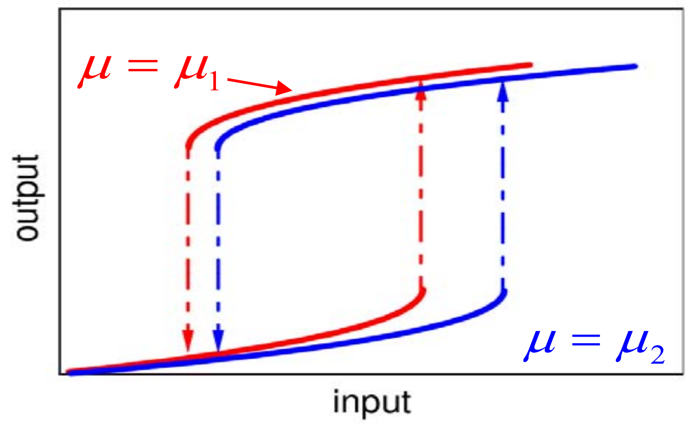
A Josephson junction based nonlinear oscillator
(Siddiqi *et al*. PRL 2004, 2005)



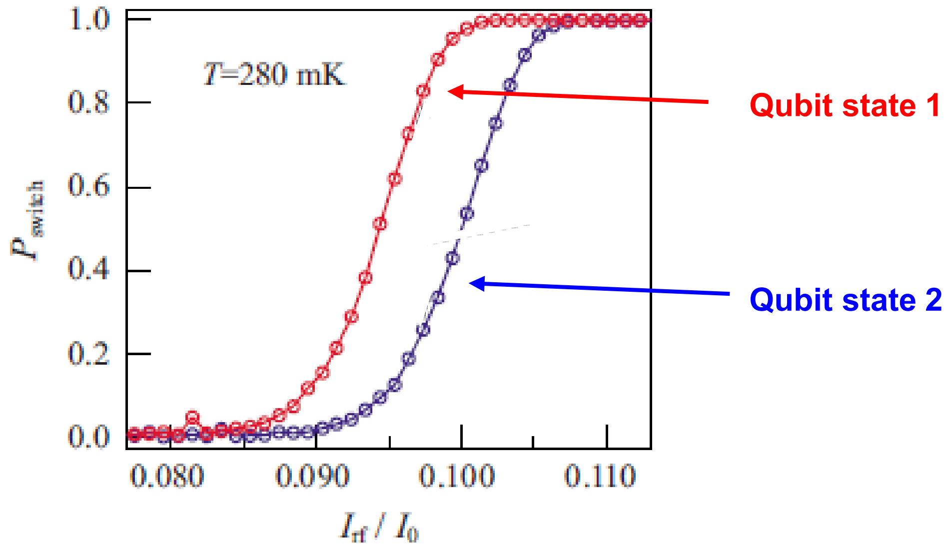
Bifurcation point:

$$\frac{d(\text{output})}{d(\text{input})} \rightarrow \infty$$

High sensitivity to control parameter



Real switching:
fluctuation-induced smearing



Siddiqi et al., 2005

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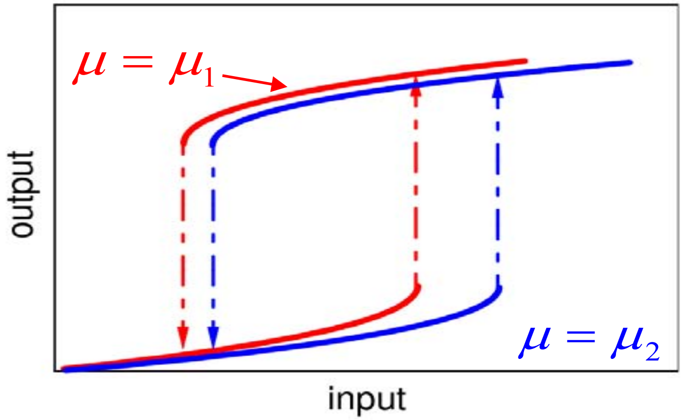
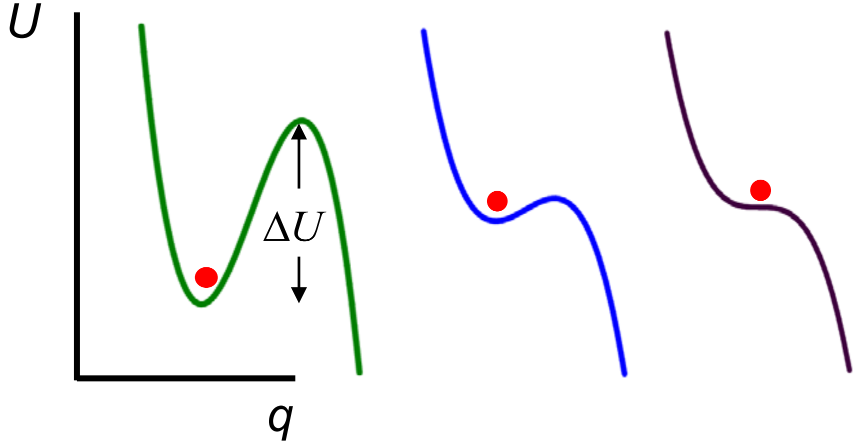
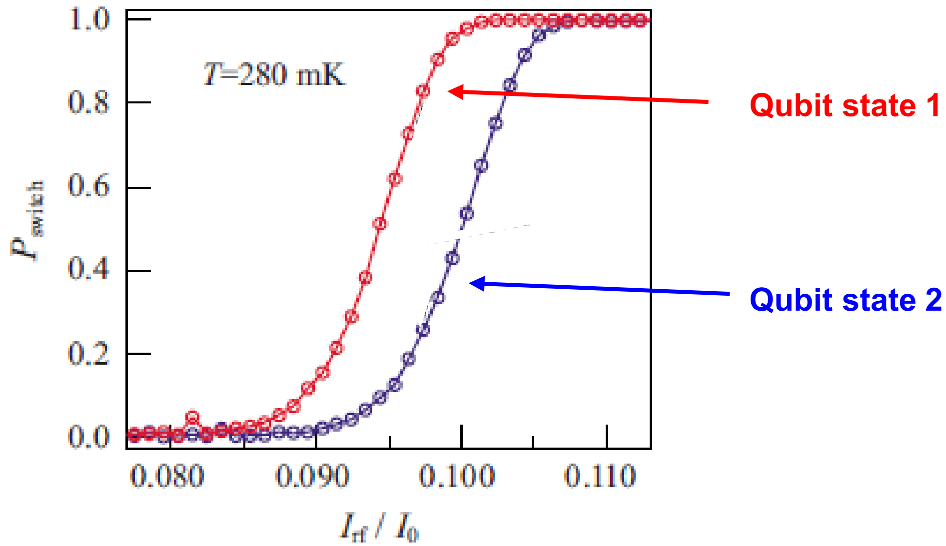




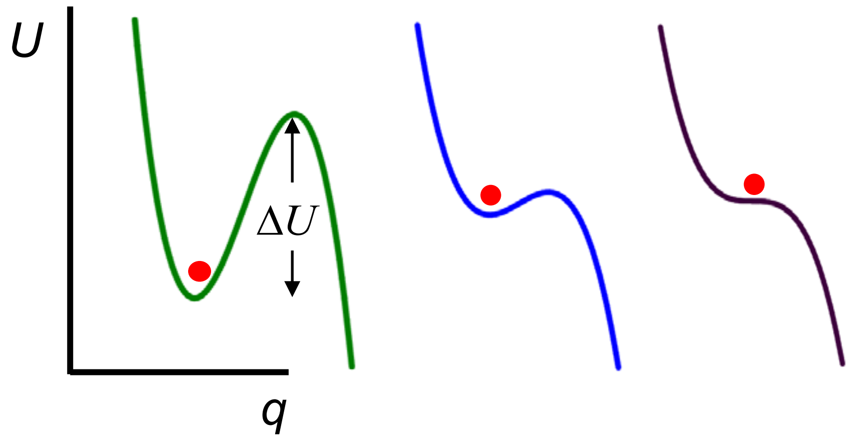


Siddiqi et al., 2005

Switching rate near bifurcation points

Near bifurcation points one of the motions is **slow, a soft mode** ➡ **universal behavior of the escape rate**

A **multivariable** system can be mapped onto a 1D overdamped Brownian particle



$$\dot q = -\partial_q U + f(\tau), \qquad U(q) = -\tfrac{1}{3}q^3 + \eta q, \qquad \langle f(\tau) f(\tau')\rangle = 2D\,\delta(\tau-\tau')$$

noise

thermal noise: $D = k_B T$

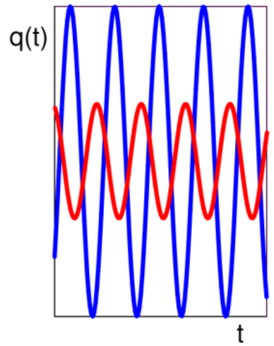
No noise: **relaxation time** $t_r = -1/\partial_q^2 U(q_a) = \eta^{-1/2}/2$ **is *large* for small** $|\eta|$

Noise: **delta-correlated in slow time.** **If noise is Gaussian, the switching rate is**

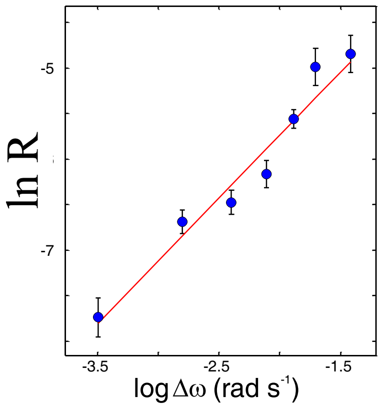
$$W = \Omega_e \exp(-R/D), \quad R = \Delta U = \tfrac{4}{3}\eta^{\xi}, \quad \Omega_e \propto \eta^{\zeta}. \qquad \xi = 3/2, \quad \zeta = 1/2$$

Thermal systems: Kurkijarvi (1972), Victora (1989). General case, far from equilibrium: MD & Krivoglaz (1979, 1980)

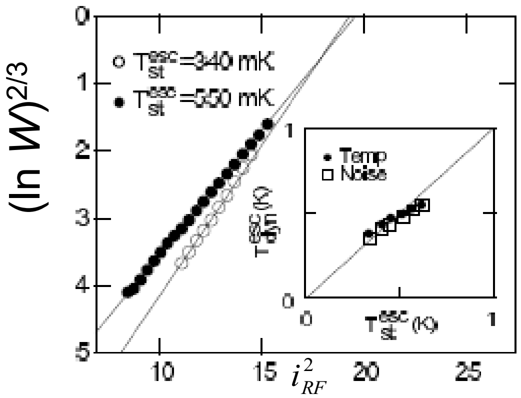
Scaling for a modulated classical oscillator



For resonant modulation, $R \propto \eta^{3/2}$ close to bifurcation points.



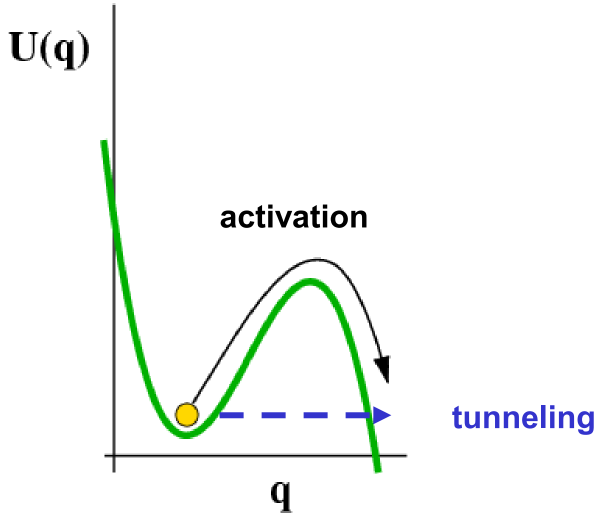
MEMS (Chan & Stambaugh, 2005/2006)



Josephson junctions (Siddiqi et al., 2005)

Different scaling, $R \propto \eta^2$, occurs near a cusp on the bifurcation curve (observed by Aldridge & Cleland, 2005) and for a parametrically modulated oscillator (observed by Stambaugh & Chan, 2006)

Low temperatures: conventionally, escape occurs via tunneling



$$W \propto \exp(-\Delta U / k_B T) \Rightarrow \exp(-2S_{\mathrm{tun}} / \hbar)$$

$W << \Gamma$, **the relaxation rate**

Near a saddle-node bifurcation point $U(q) = -\frac{1}{3} q^3 + \eta q$

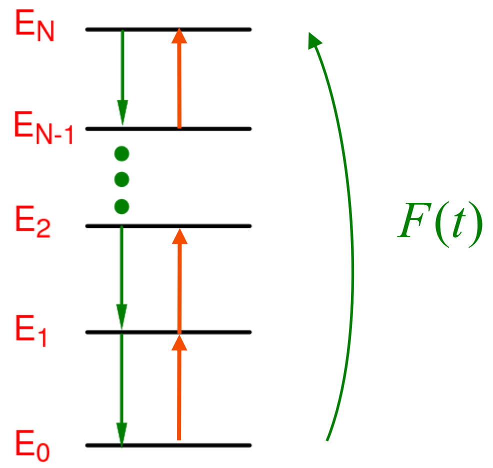
Weak damping: $S_{\mathrm{tun}} \propto \eta^{5/4}$

Strong damping, $\gamma_{\mathrm{r}} >> \eta^{1/4}$: $S_{\mathrm{tun}} \propto \gamma_{\mathrm{r}} \eta$ **(Caldeira & Leggett, 1983)**

Driven oscillator: quantum noise

Oscillator Hamiltonian: $H_0 = \frac{1}{2}p^2 + \frac{1}{2}\omega_0^2 q^2 + \frac{1}{4}\gamma q^4 - qF\cos\omega_F t$

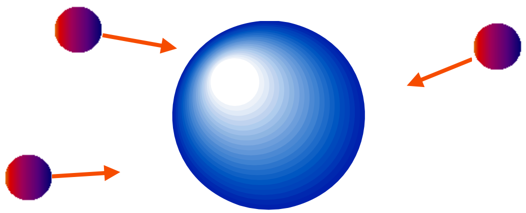
+ coupling to a thermal bath



$E_N \approx N\hbar\omega_0$

Relaxation: emission of excitations in the bath

An elementary collision is short, $\sim \omega_0^{-1}$. Classically, it gives a "kick" to the oscillator coordinate and momentum



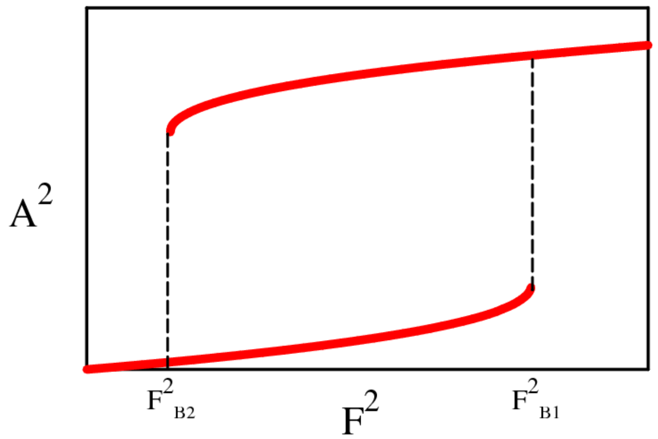
a Brownian particle colliding with molecules

"Noise intensity" for a quantum oscillator is the total rate of spontaneous emission / absorption

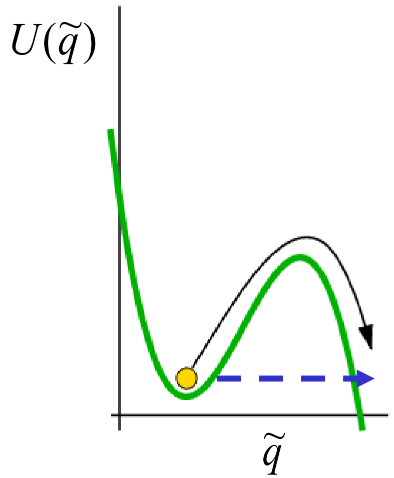
$$k_B T \Rightarrow k_B T_{\text{eff}} = \hbar\omega_0(2\bar{n}+1)/2, \quad \bar{n} = \left[\exp\left(\hbar\omega_0/k_B T\right) - 1\right]^{-1}$$

The picture applies only near bifurcation points; one dynamical variable + short collisions;
ultimate squeezing $\Longrightarrow$ **classical dynamics with quantum noise**

MD, 2007



semiclassical behavior near bifurcation points

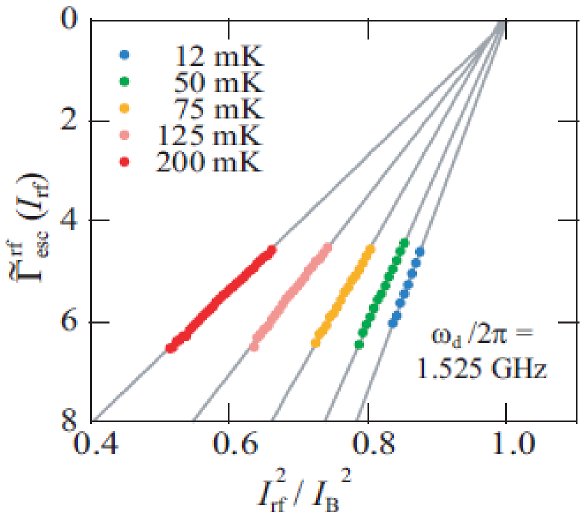


Quantum noise leads to switching via an "overbarrier transition" even for $T \to 0$ ⇒ ***"quantum activation"***

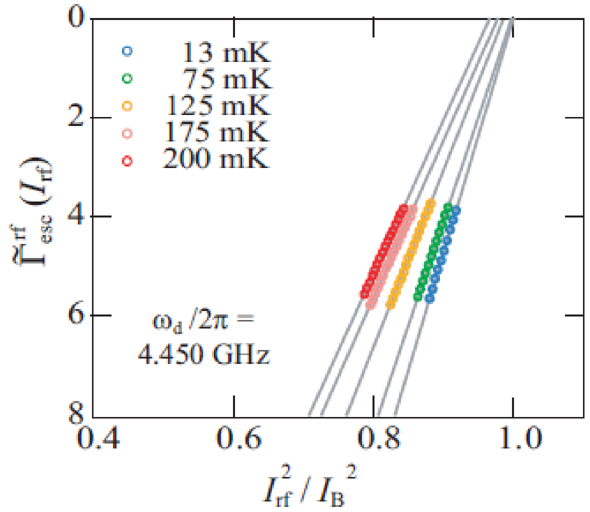
$$\boxed{W_A \propto \exp(-\Delta U_A / k_B T_{\text{eff}}), \qquad \Delta U_A \propto |F - F_B|^{3/2}}$$

$$T_{\text{eff}} \propto \hbar \quad \text{for} \quad T \to 0$$

Quantum tunneling rate near F_{B1} is $|\ln W| \propto S_{\text{tun}}/\hbar \propto (F\text{-}F_{B1})^{5/4}$ (5/4 < 3/2)



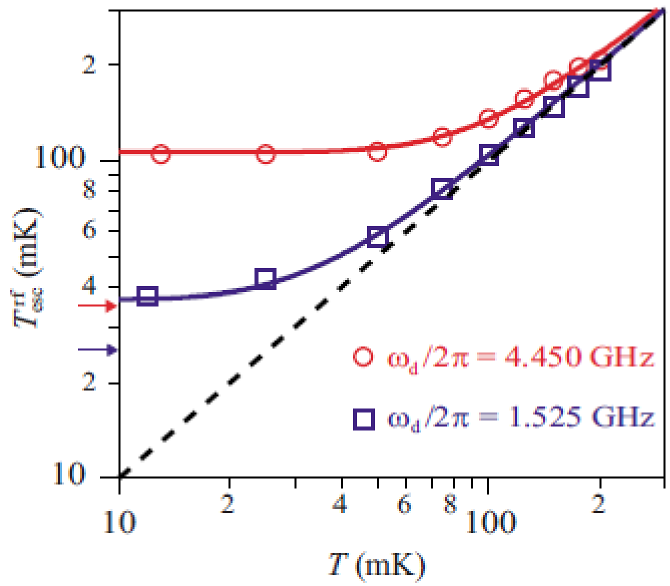
(b) I_0 = 3.71 μA I_B = 0.4 μA



$$\tilde{\Gamma}^{rf}_{esc} = [\log(\omega_{attempt} / W_A)]^{2/3}$$

$$\boxed{W_A \propto \exp(-\Delta U_A / k_B T_{eff}), \qquad \Delta U_A \propto |F - F_B|^{3/2}}$$

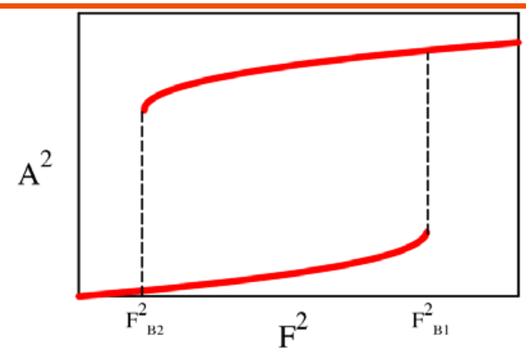
$$k_B T_{eff} = \hbar\omega_0(\bar{n} + \tfrac{1}{2}), \ \bar{n} = \left[\exp(\hbar\omega_0 / k_B T) - 1\right]^{-1}$$



Vijay et al. (2009)

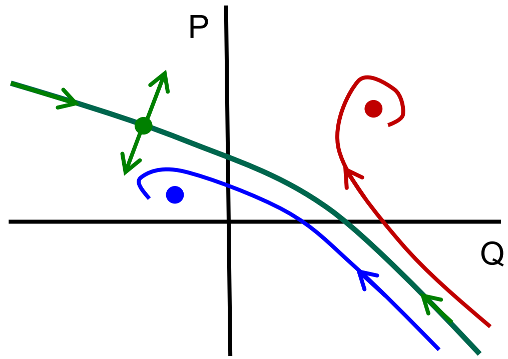
Classical picture

$$\ddot{q} + 2\Gamma\dot{q} + \omega_0^2 q + \gamma q^3 = F\cos\omega_F \mathrm{t}$$



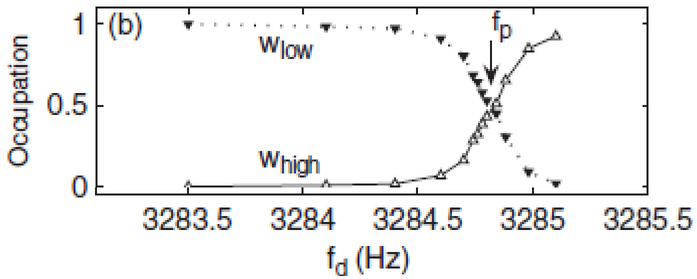
Change to the rotating frame, dimensionless variables:

$$q(t) = C[Q\cos(\omega_F t) + P\sin(\omega_F t)], \quad p(t) = -C\omega_F[Q\sin(\omega_F t) - P\cos(\omega_F t)] \qquad A^2 \propto Q^2 + P^2$$



Classical and quantum fluctuations:

- **Small-amplitude fluctuations about the attractors**
- **Inter-attractor switching; generally, the transition rates W_{12} and W_{21} are exponentially different**



Stambaugh & Chan, 2006

Classical picture

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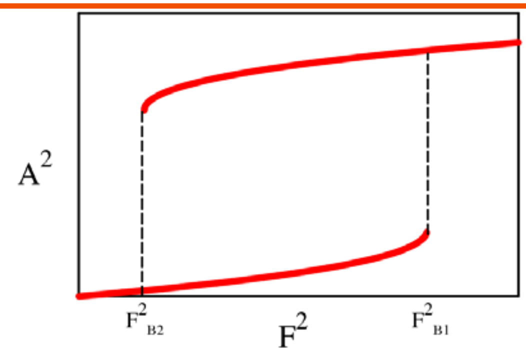

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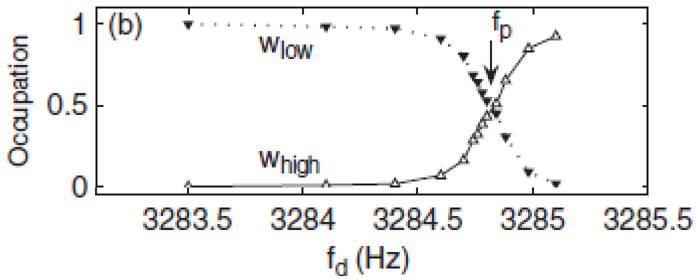
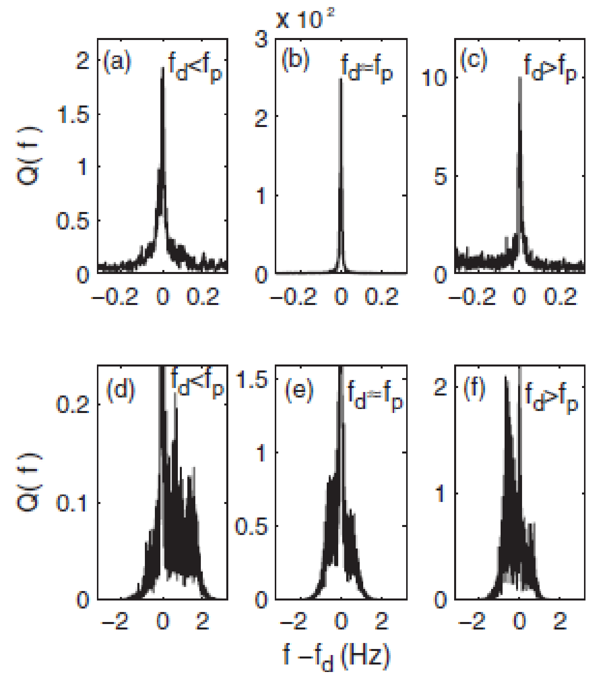
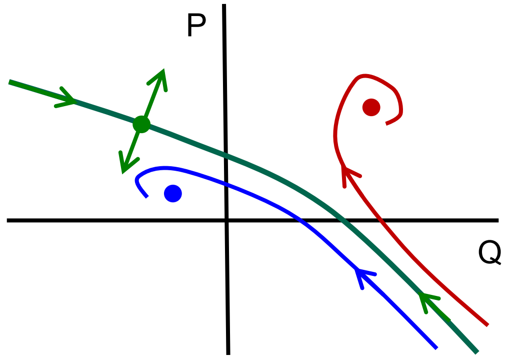


Stambaugh & Chan, 2006

Power spectra



Other spectra



✓Absorption of extra field (MD & Krivoglaz, 1979)

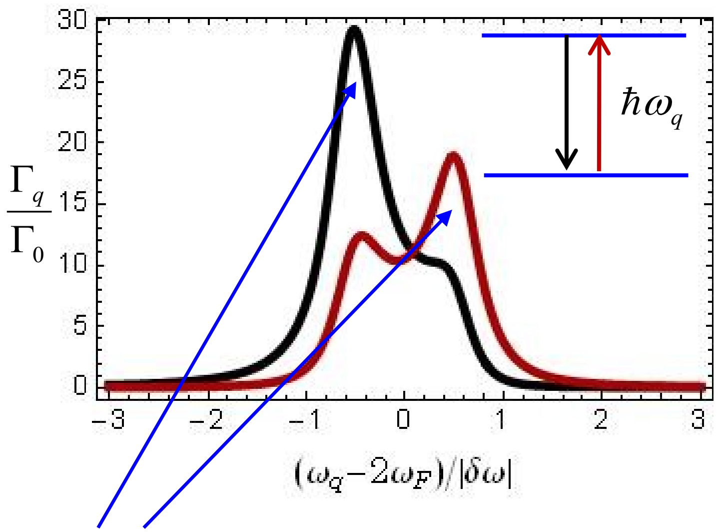
✓ Light transmission spectra (Drummond & Walls, 1980)
[optical cavity with two-level atoms, Bonifacio & Lugiato, 1978]

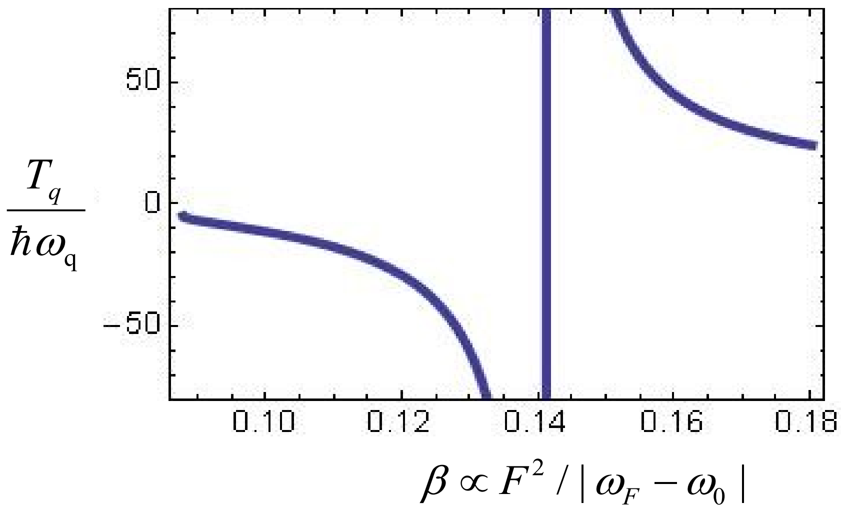
✓ Emission by a quantum oscillator

✓ Spectrum “seen” by a resonantly coupled qubit

Oscillator-to-qubit coupling: $H_i = g\sigma_x q$ (Jaynes-Cummings) or $H_i = g_2\sigma_x q^2$

Assume fast oscillator relaxation compared to the qubit. Coupling-induced qubit decay rate and T_q:





frequency of decayed vibrations about the attractor

Serban, MD & Wilhelm, 2010

Quasienergy spectrum and quantum temperature

Underdamped oscillator in the rotating frame: decay rate $\Gamma \ll |\omega_F - \omega_0|$

$$H(t) = \tfrac{1}{2}(p^2 + \omega_0{}^2 q^2) + \tfrac{1}{4}\gamma q^4 - qF\cos(\omega_F t)$$

$$i\hbar\dot{\psi} = H(t)\psi, \quad \psi_\varepsilon(t+\tau_F) = \exp(-i\varepsilon\tau_F/\hbar)\psi_\varepsilon(t)$$

quasi-energy

Quasienergy spectrum and quantum temperature

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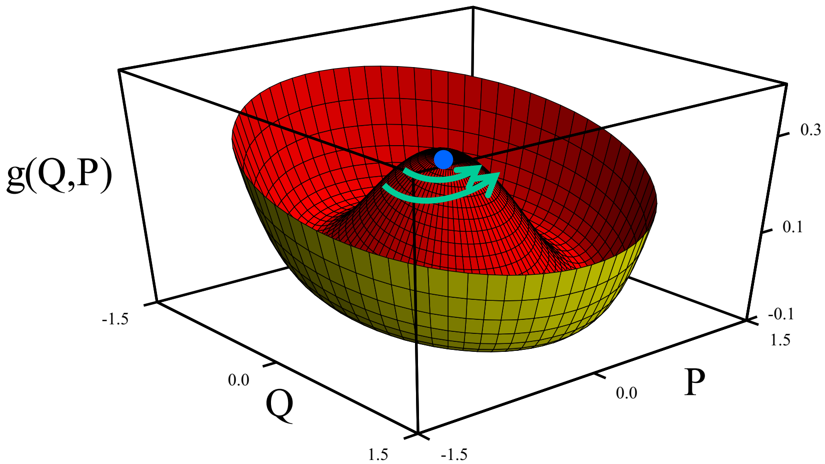
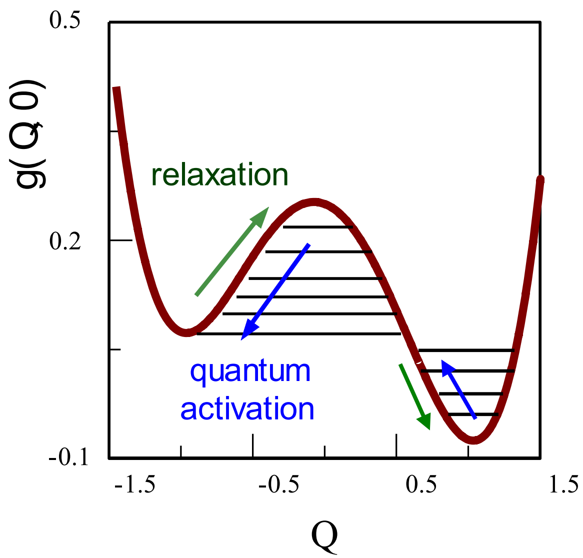
$$H(t) = \tfrac{1}{2}(p^2 + \omega_0^2 q^2) + \tfrac{1}{4}\gamma q^4 - qF\cos(\omega_F t) \xrightarrow{\text{RWA}} C_g g(Q,P), \qquad [P,Q] = -i\lambda, \quad \lambda = \frac{3\hbar\gamma}{8\omega_F^2(\omega_F - \omega_0)}$$

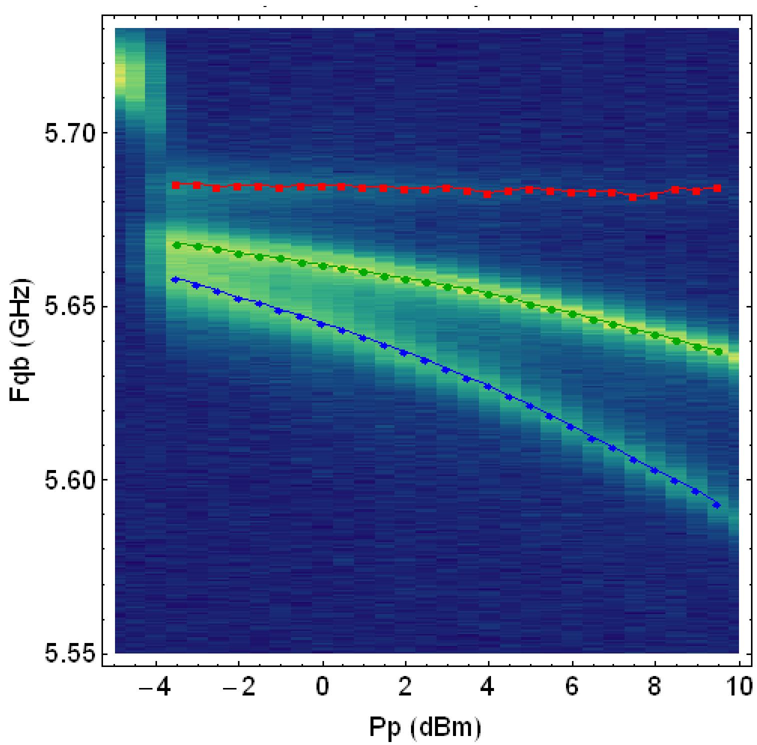
time-independent $\quad g(Q,P) = \frac{1}{4}(Q^2 + P^2 - 1)^2 - Q\beta^{1/2} \quad (\beta \propto F^2)$

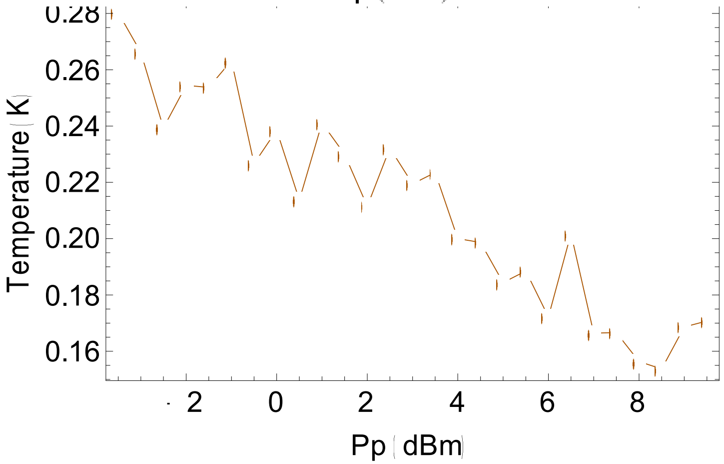




Quantum fluctuations: distribution over the quasienergy states = the eigenstates of g, with effective *quantum temperature* T_{eff} --- different spectral line intensities for transitions toward/away from the stable state ("*anti-Stokes*" and "*Stokes*" sidebands)

Effective temperature of the pumped resonator





M. Ong et al., experiment and theory, to be published
[thanks to P. Bertet for the slide and the discussion of the spectra of driven oscillators]

➡ Mode temperature strongly dependent on resonator pumping

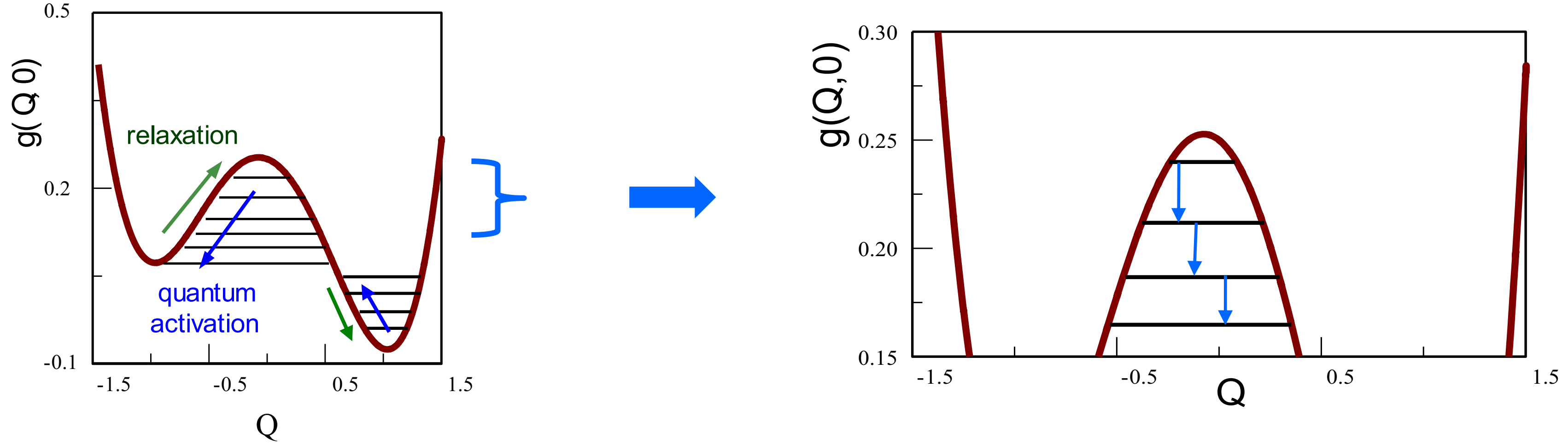
➡ Parametric spontaneous down-conversion !

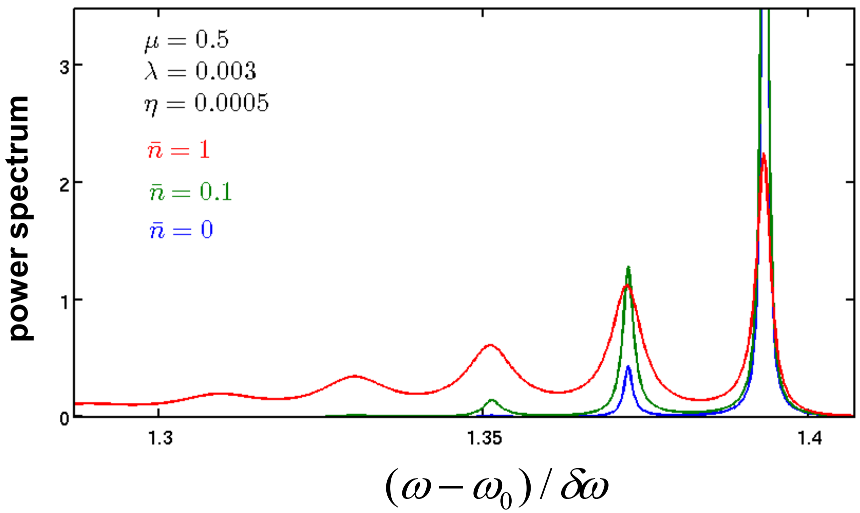
➡ Related to « quantum activation »

M.I. Dykman and V.N. Smelyanskiy, JETP 67, 1769 (1988)

M. Marthaler and M.I. Dykman, PRAL 73, 042108 (2006)

Quasienergy levels are nonequidistant





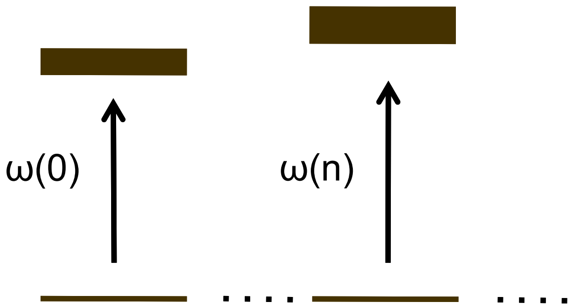
Line spacing is proportional to the dimensionless Planck constant

$$\lambda = \frac{3\hbar\gamma}{8\omega_F^2(\omega_F - \omega_0)}$$

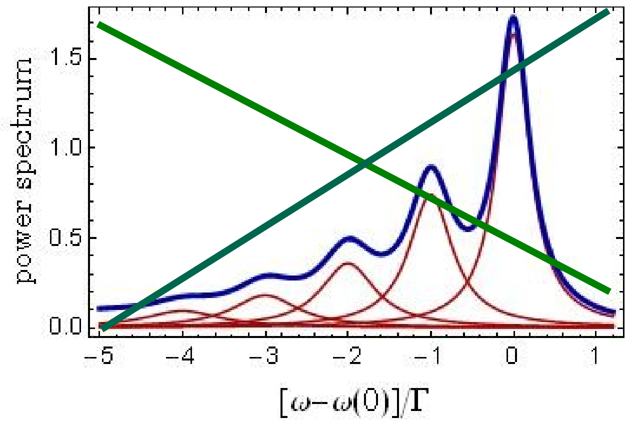
Linewidth linearly increases with the line number. What if it is not that small?

Interference of transitions

Oscillator "hops" between overlapping partial spectra



$$\omega(n) \approx \omega(0) + n\Delta\omega$$



need time ~($\Delta\omega$)$^{-1}$ to resolve transitions ; $\Gamma \sim \Delta\omega$ ➡ ***interference of transitions***

To find a spectrum, add complex transition amplitudes and square the absolute value, not just add squared amplitudes

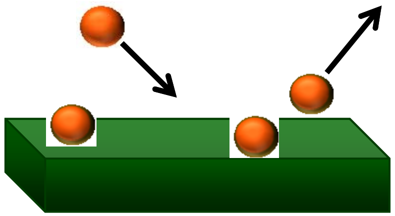
The spectrum is a superposition of coupled complex partial spectra

$$\chi(\omega) = \mathrm{Re}\sum_n \varphi(\omega, n)$$

$$[\Gamma_n - i(\omega - \omega_0 - n\Delta\omega)]\varphi(\omega, n) = i\rho_n + \Gamma\sum_i C_{ni}\varphi(\omega, n+i)$$

nth level occupation weighted with the transition matrix element

Explicit solution is obtained for the power spectrum of the driven oscillator



The method of coupled partial spectra applies both where the coupling is due to dissipation and/or to *dephasing*. Example: attachment/detachment of molecules to a nanoresonator

Attachment: molecule influx $N \overset{WN_0}{\Rightarrow} N+1$ **Detachment:** $N \overset{WN}{\Rightarrow} N-1$

Frequency change per molecule: $\Delta\omega \approx \omega_0 m_{\mathrm{mol}} / 2M_{\mathrm{res}}$

Eigenfrequency with N molecules attached: $\omega(N) \approx \omega_0 + N\Delta\omega$ ➡ **frequency fluctuations**

Classical equation of motion

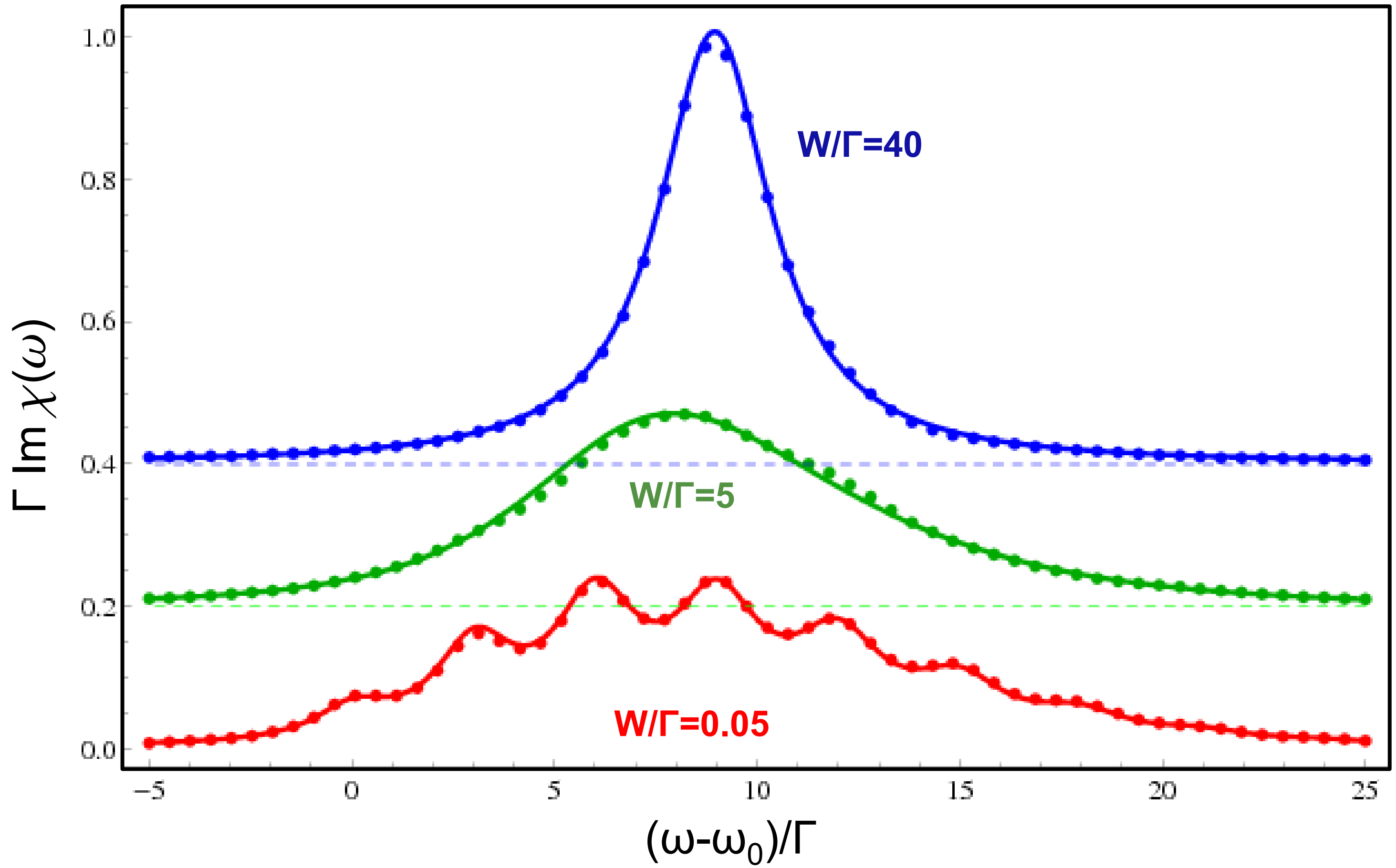
$$\frac{d}{dt}\left(M(t)\dot{q}\right) + 2\Gamma M_{\mathrm{res}}\dot{q} + M_{\mathrm{res}}\omega_0^2 q = (Fe^{-i\omega t} + \mathrm{c.c})$$

Poisson noise

The susceptibility

$$\boxed{\langle q(t)\rangle = 2\,\mathrm{Re}\left(\chi(\omega)Fe^{-i\omega t}\right)}$$

$\chi(\omega)$ **is formed as a result of *interference of partial spectra* with different *N*. It could be found in the explicit form** (MD, Khasin, Portman, & Shaw, 2010)



$\Delta\omega/\Gamma=3$

$N_0=3$

For $\boxed{W >> \Delta\omega}$ **- a single Lorentzian peak at** $\omega_0 + N_0\Delta\omega$ **with halfwidth** $\mathbf{\Gamma}$

Conclusions

- Escape of modulated quantum oscillators occurs via ***quantum activation***
- **The exponents and prefactors of switching rates *scale* as a power of the distance to the bifurcation point, both for classical Gaussian noise and for quantum activation**
- **The spectra of modulated oscillators can vary from a single peak to a peak with pronounced fine structure. A similar behavior is displayed by the spectra of nanomechanical resonators with attaching-detaching molecules.**
- **The method of coupled partial spectra can be used to describe both relaxation and dephasing of high-Q oscillators**