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International Centre for Theoretical Physics



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**Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the
Quantum Regime**

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**Longitudinal Vibrations of Suspended Carbon Nanotubes - Franck-Condon effect,
cotunneling and nonequilibrium**

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Longitudinal vibrations of suspended carbon nanotubes - Franck-Condon effect, cotunneling, and nonequilibrium

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Carbon, as we know it

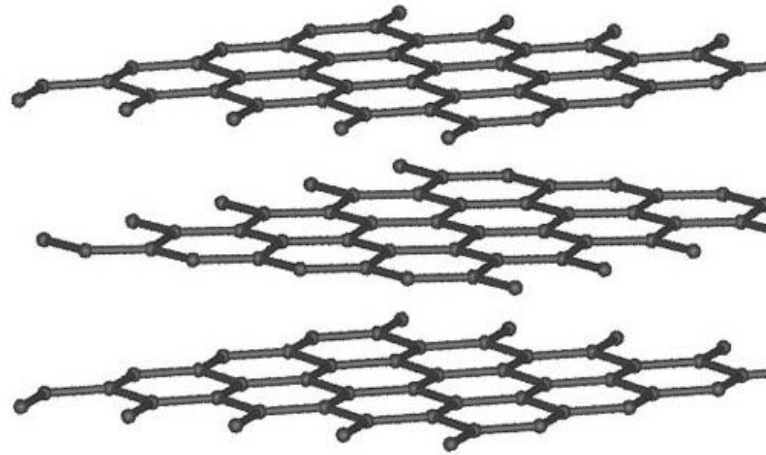
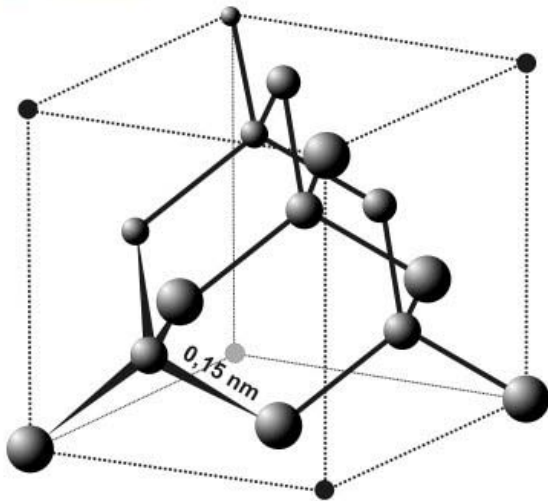
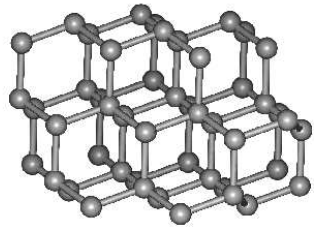
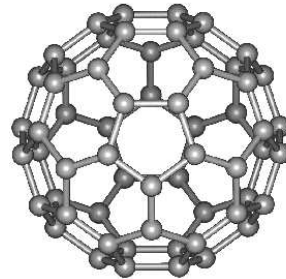


image source: Wikipedia

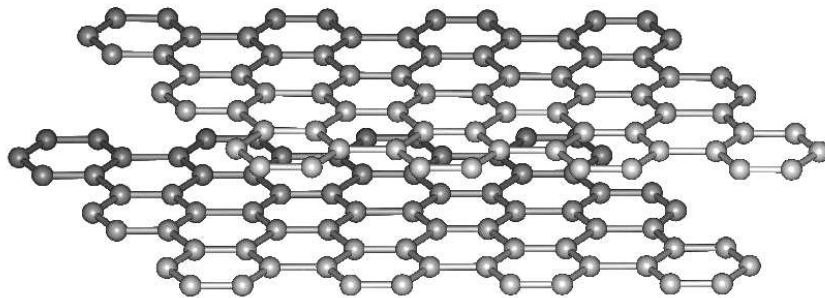
Carbon nanotubes: a more exciting form of carbon



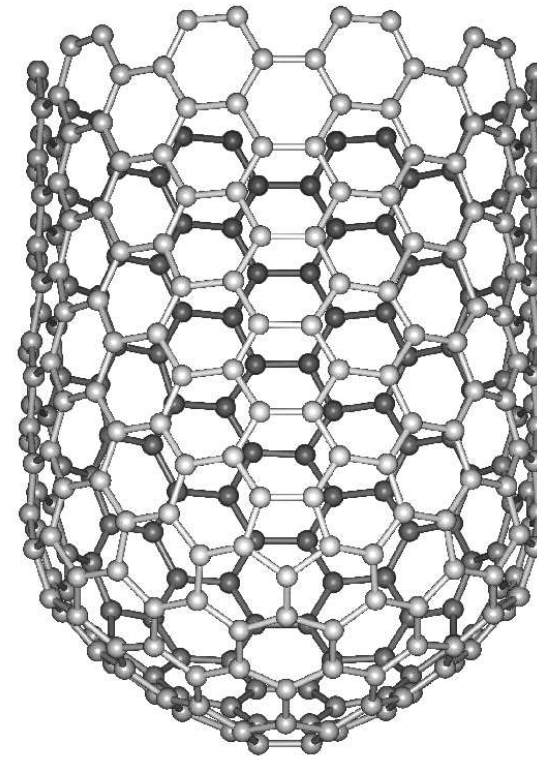
diamond



fullerene (C₆₀)



graphite / graphene



nanotube

Carbon nanotubes

- different production methods; often:
 - use small catalyst particles
 - hot gas, with carbon feed (e.g. CH_4)
 - nucleation of tube structure
- many different structures
 - single-wall, double-wall, multi-wall
 - zigzag, armchair, chiral (how the sheet is “wrapped together”)



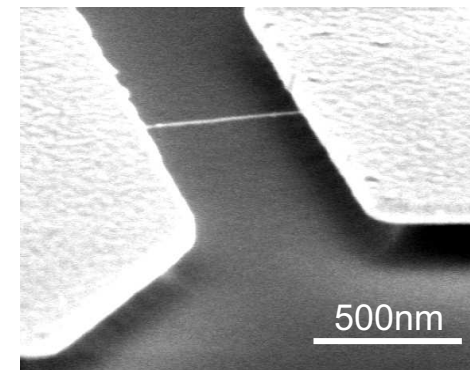
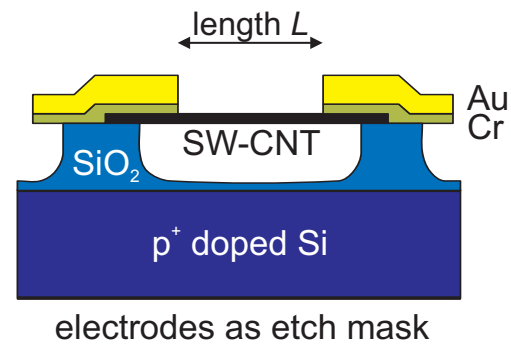
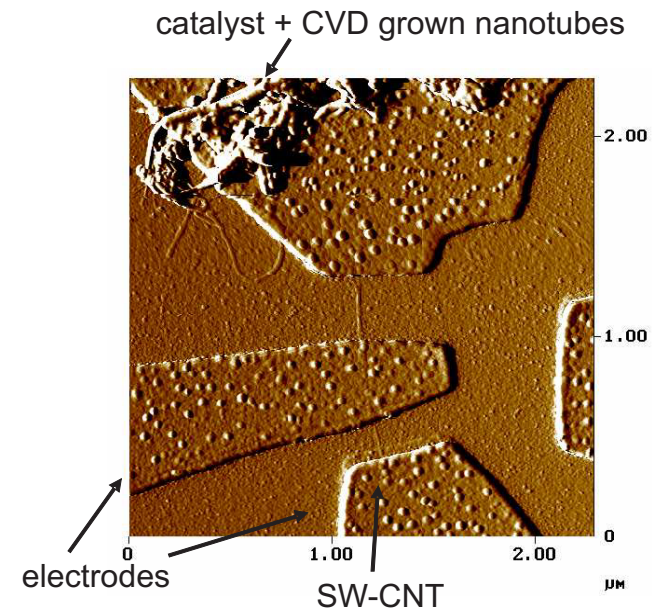
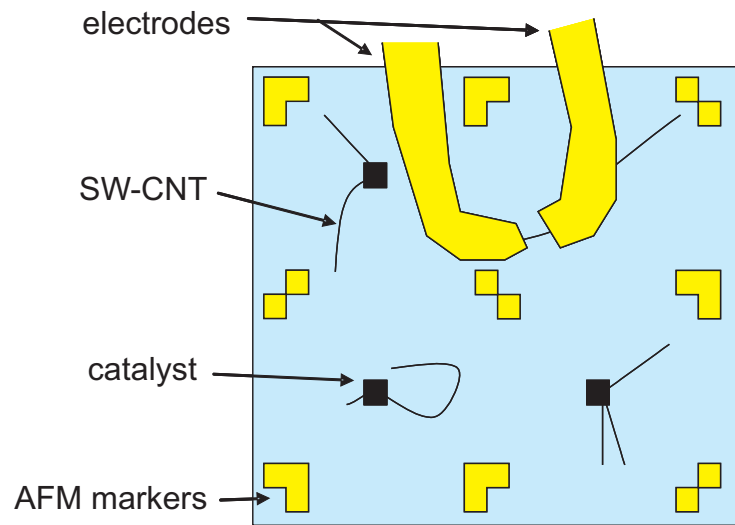
Mechanical properties of carbon nanotubes

- stiffer than steel
- resistant to damage from physical forces
- very light
- Young's modulus $E = \frac{F/A}{\Delta L/L}$:
 $E_{\text{CNT}} \simeq 1.2 \text{ TPa}$, $E_{\text{steel}} \simeq 0.2 \text{ TPa}$
- Density:
 $\rho_{\text{CNT}} \simeq 1.3 \frac{\text{g}}{\text{cm}^3}$, $\rho_{\text{Al}} \simeq 2.7 \frac{\text{g}}{\text{cm}^3}$
- (still) “material of dreams”



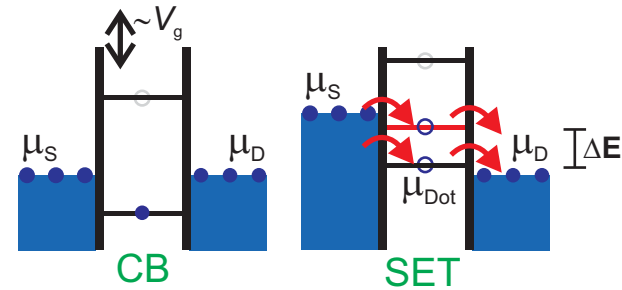
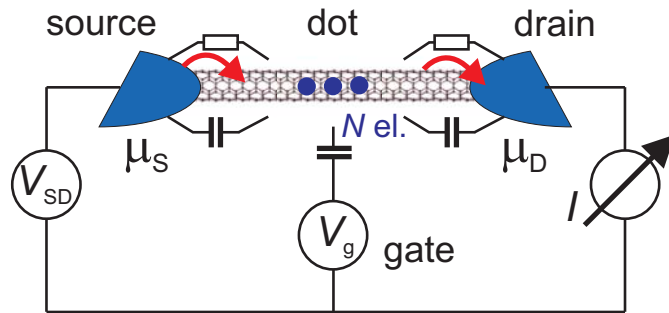
Suspended carbon nanotube sample fabrication

“the old way of doing things”

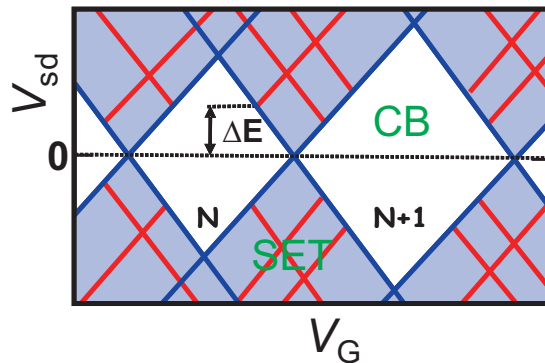


Low-temperature transport measurements

- Tunnel barriers between leads and nanotube
- Low temperature $k_B T \ll e^2 / C$: formation of a **quantum dot**



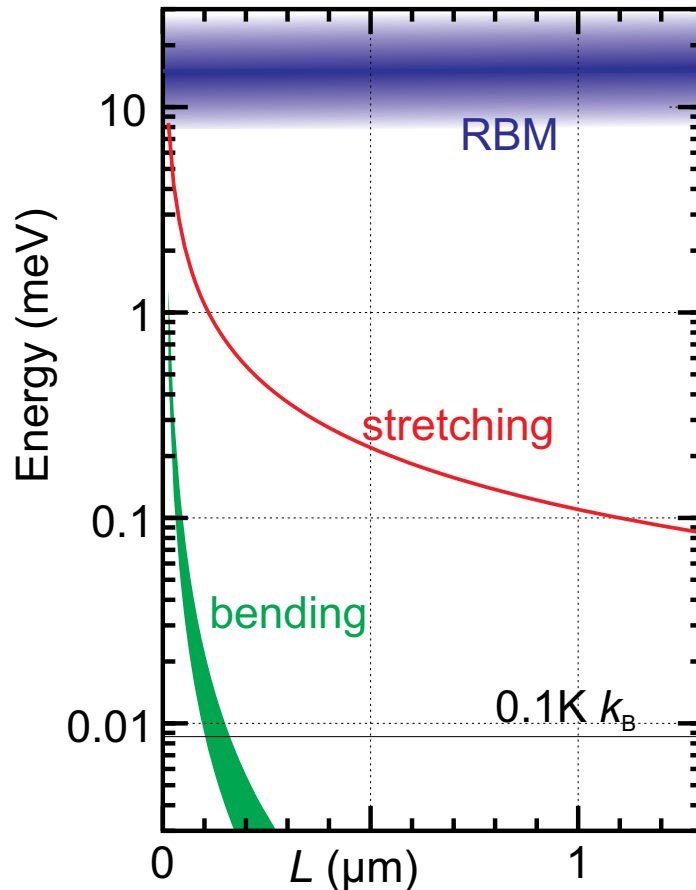
stability diagram: $\frac{dI}{dV_{SD}}(V_g, V_{SD})$



CB - Coulomb blockade “diamonds”
 SET - single electron tunneling

Excited states visible at finite bias!
 Spectroscopy of the electronic system

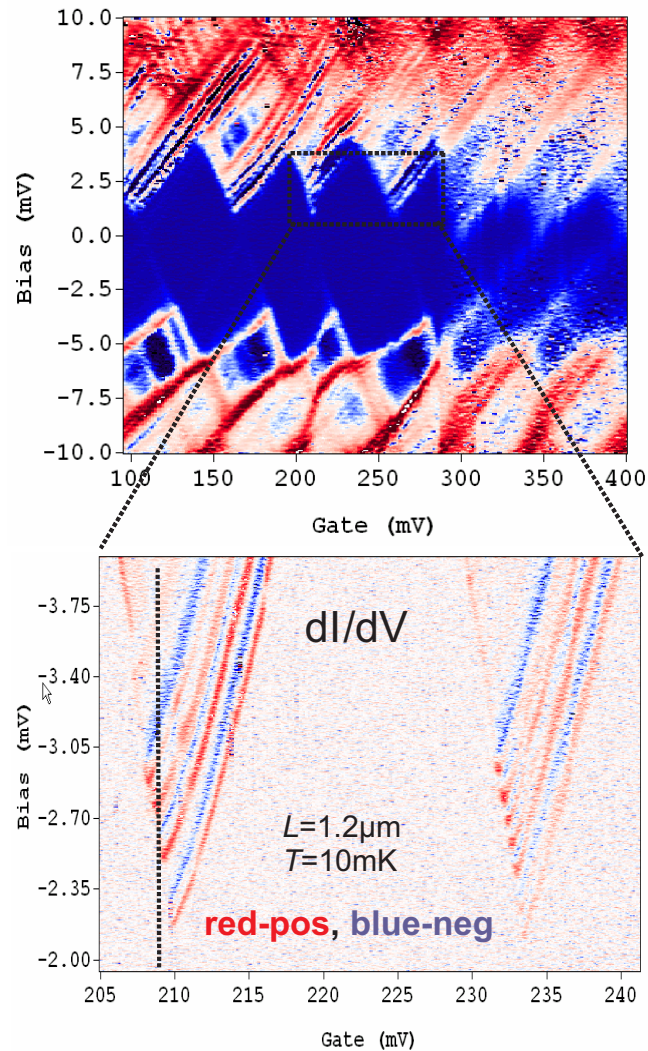
Vibration modes of carbon nanotubes



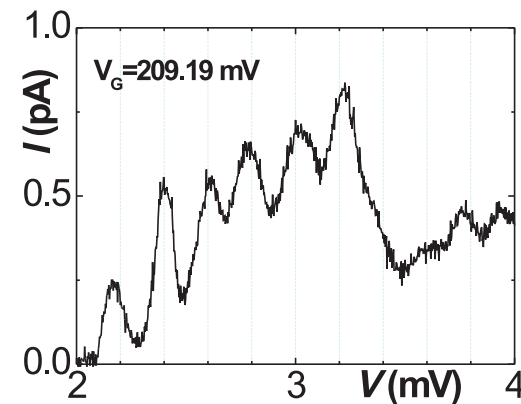
- radial breathing mode(s)
- **stretching** (longitudinal) mode:
 $h\nu \propto L^{-1}$
 $h\nu = 1100 \dots 110 \mu\text{eV}$,
 $\nu = 270 \dots 27 \text{ GHz}$
(for 100 nm ... 1 μm)
- **bending** (transversal) mode:
 $h\nu \propto L^{-2}$
 $h\nu = 10 \dots 0.1 \mu\text{eV}$,
 $\nu = 2.4 \text{ GHz} \dots 24 \text{ MHz}$
(for 100 nm ... 1 μm)
 $h\nu \propto d$, also tension-dependent

purely electronic excitations have different energy scale

The stretching mode – visible in electronic transport

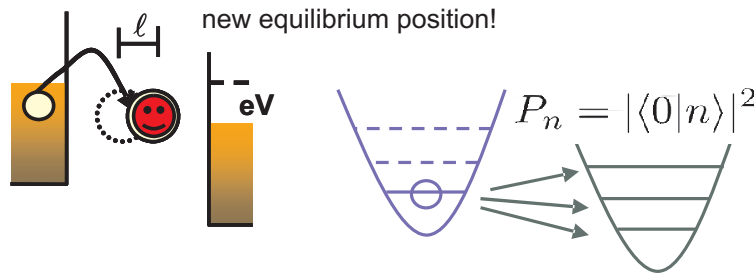


- Low-energy excitations
- Equally spaced, $\hbar\omega = 140 \mu\text{eV}$
- Identical for different charge states
- **Stretching mode** as harmonic oscillator

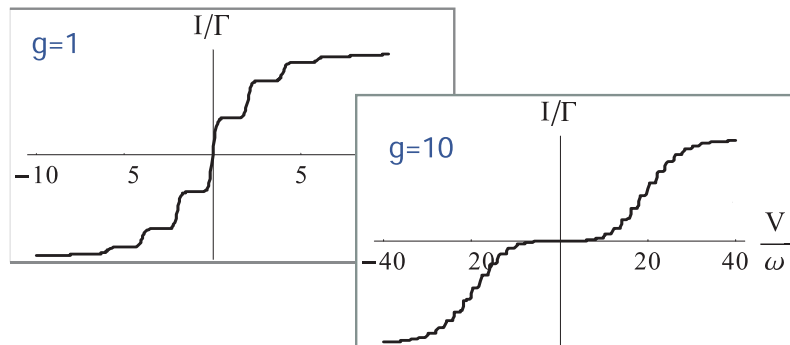


Electron-vibron coupling, Franck-Condon physics

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2\hat{x}^2 + \lambda\hat{x} \quad g = \frac{\lambda^2}{2} = \frac{1}{2} \left(\frac{\ell}{l_0} \right)^2 \quad l_0 = \sqrt{\frac{\hbar}{m\omega_0}}$$



$$\Gamma \rightarrow \Gamma_{el} \underbrace{|\langle \Psi_{\text{after}} | \Psi_{\text{before}} \rangle|^2}_{P_{nm}}$$



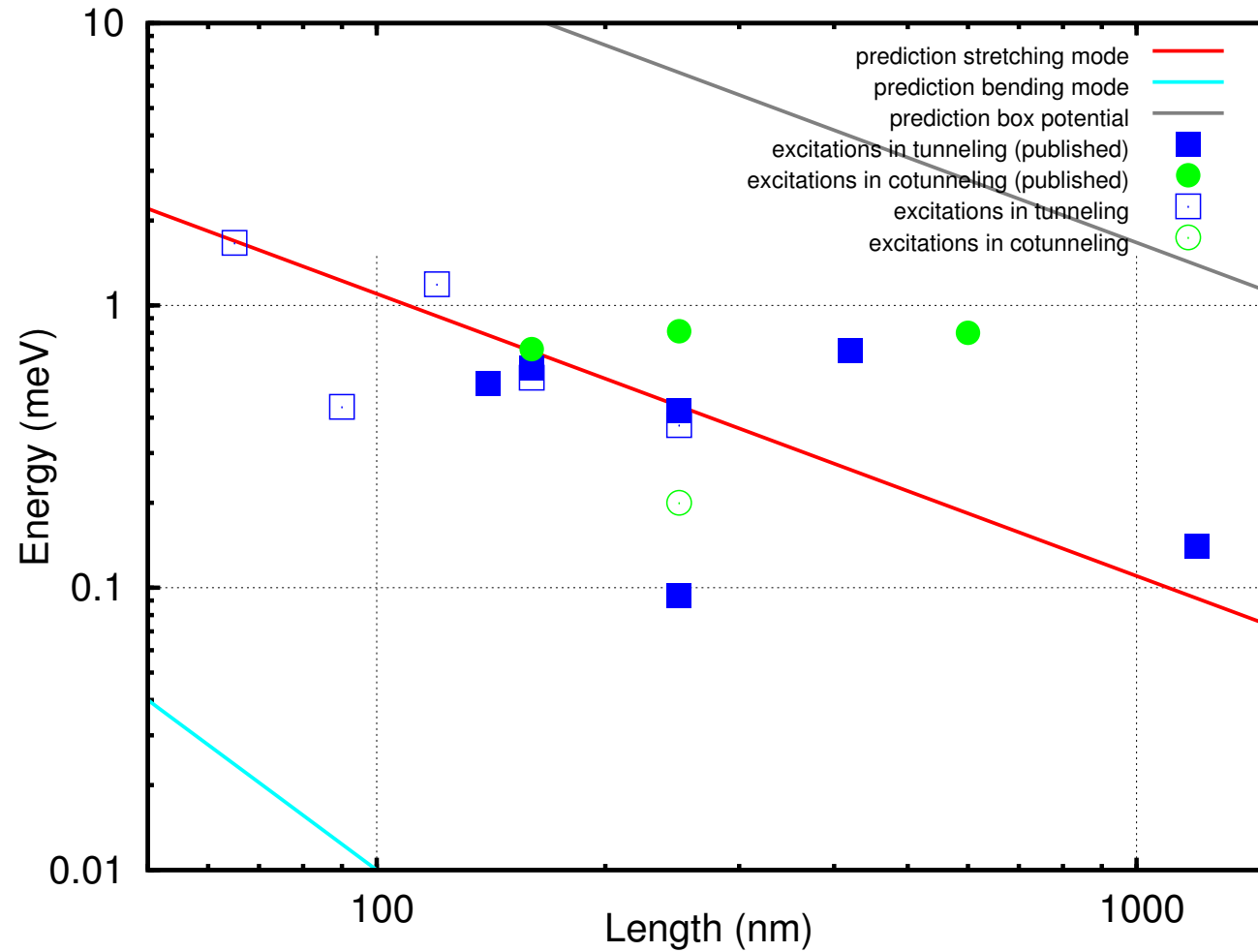
$$P_{n0} = |\langle \Psi(x - \ell) | \Psi(x) \rangle|^2 = \frac{e^{-g} g^n}{n!}$$

no effect for $g < 0.1$

additional steps in $I(V_{sd})$ for $g > 0.1$

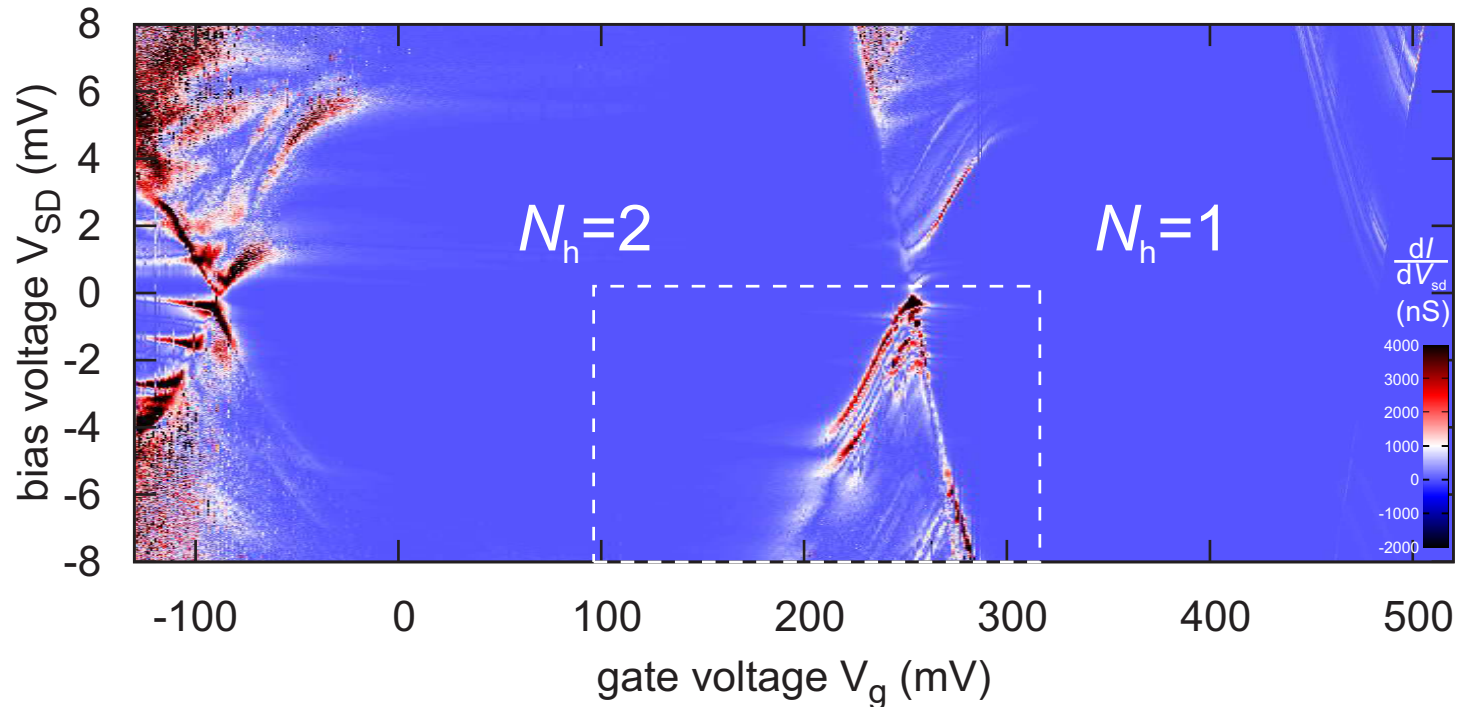
phonon blockade of transport for $g > 1, V_{sd} < g\hbar\omega_0$

Vibrational excitations observed so far



S. Sapmaz *et al.*, PRL **96**, 026801 (2006); A. K. Hüttel *et al.*, New J. Phys. **10**, 095003 (2008);
A. K. Hüttel *et al.*, PRL **102**, 225501 (2009); R. Leturcq *et al.*, Nat. Phys. **5**, 327 (2009)

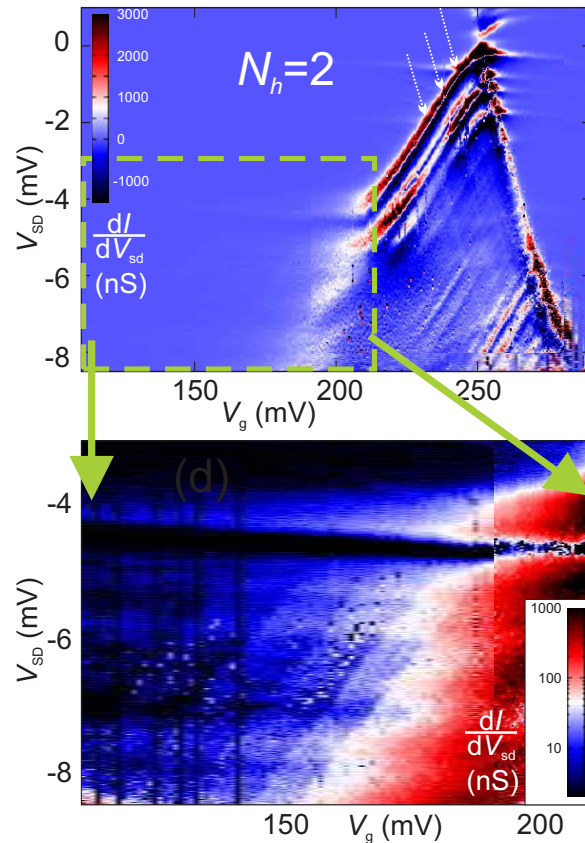
$L = 250$ nm SC nanotube, few-hole regime



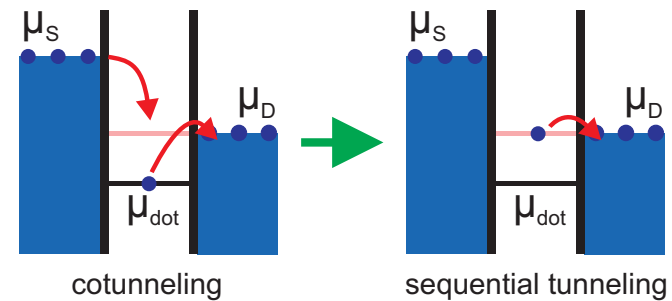
$$E_{\text{gap}} = 0.2 \text{ eV} \rightarrow d = 3.7 \text{ nm}, \quad \hbar\omega_{\text{RBM}} \simeq 7.8 \text{ meV}, \quad \text{maybe } (0,46), \quad \varepsilon \simeq 6.2 \text{ meV}$$
$$\text{length } L = 250 \text{ nm} \rightarrow \text{expected } \hbar\omega_{\text{bend}} \simeq 0.002 \text{ meV}, \quad \hbar\omega_{\text{stretch}} \simeq 0.44 \text{ meV}$$

bending lines \rightarrow shifting potential minima, DQD-like properties

Stretching mode in SET and cotunneling ($1 \leq N_{h^+} \leq 2$)

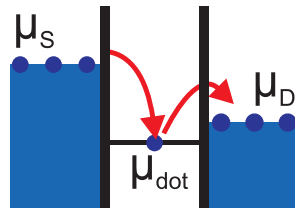


- excitations in SET, positive slope: harmonic, $\Delta\varepsilon = 0.42 \text{ meV} \simeq \hbar\omega_{\text{stretch}}$
- harmonic excitations in **cotunneling!**
- Cotunnel-assisted sequential tunneling, “CO-SET”

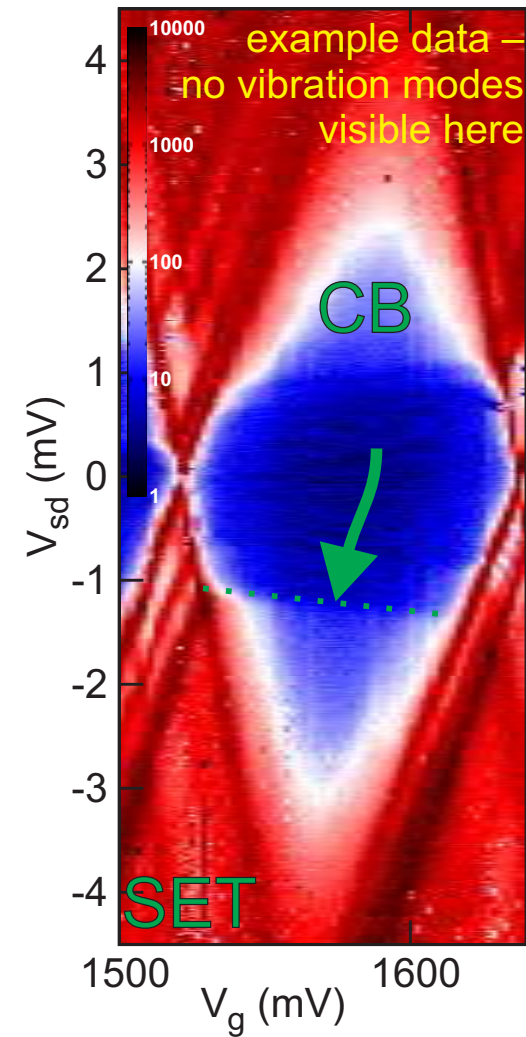
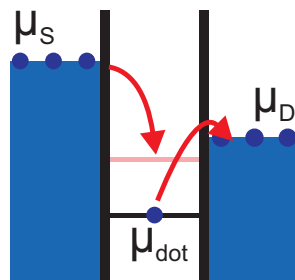


Reminder: cotunneling – second-order process

- current in Coulomb blockade:
“several electrons tunneling at once”
- two-electron processes:
 - elastic:

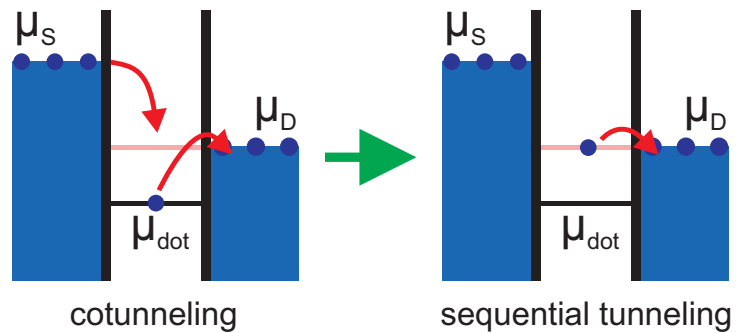


- inelastic (green arrow):

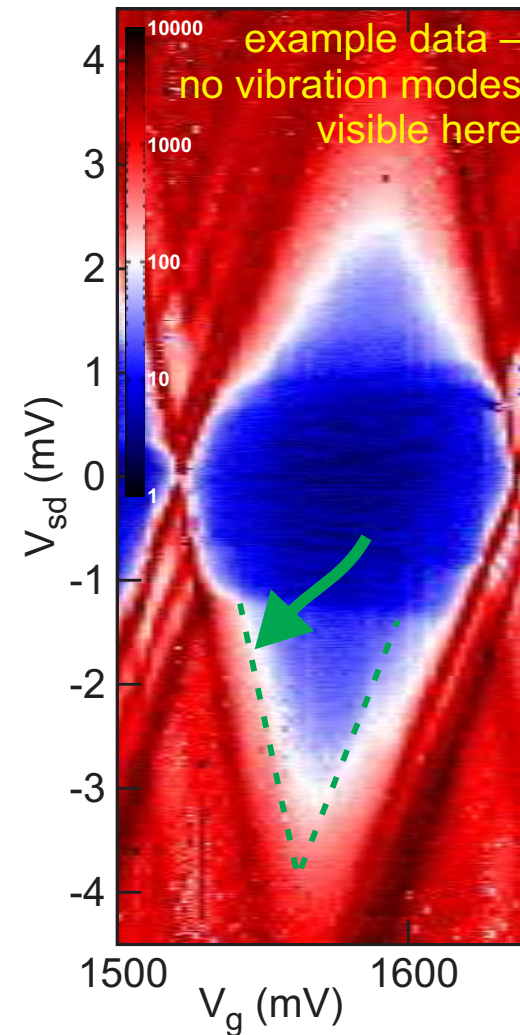


Cotunnel-assisted sequential tunneling (CO-SET)

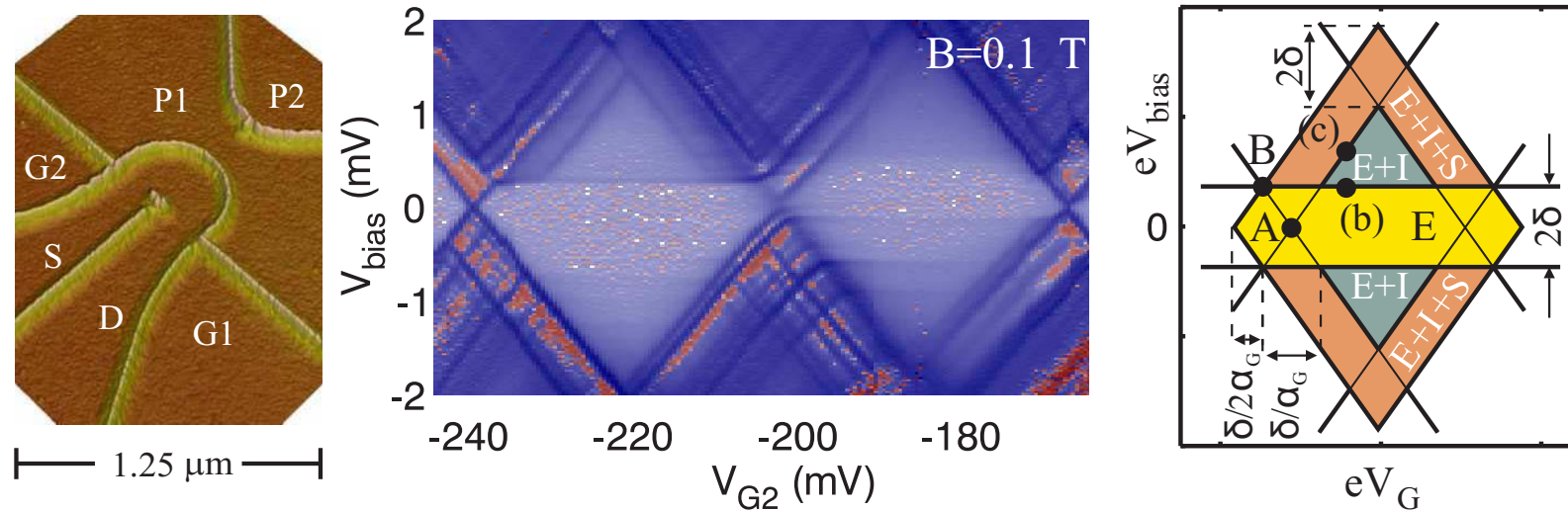
- inelastic cotunneling, followed by a tunnel-out process



- requires energy storage
- this is the process we've seen
- requires energy storage: tunnel-out must be faster than relaxation

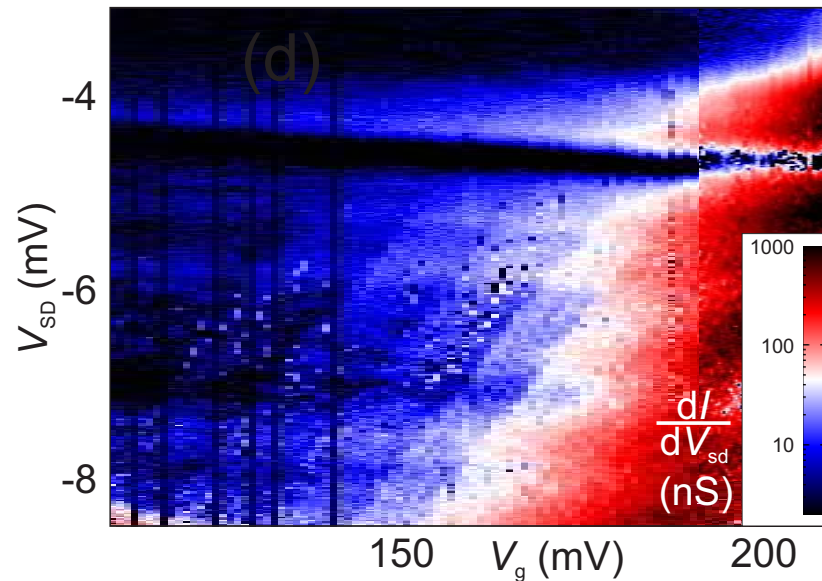


Cotunnel-assisted sequential tunneling (CO-SET)



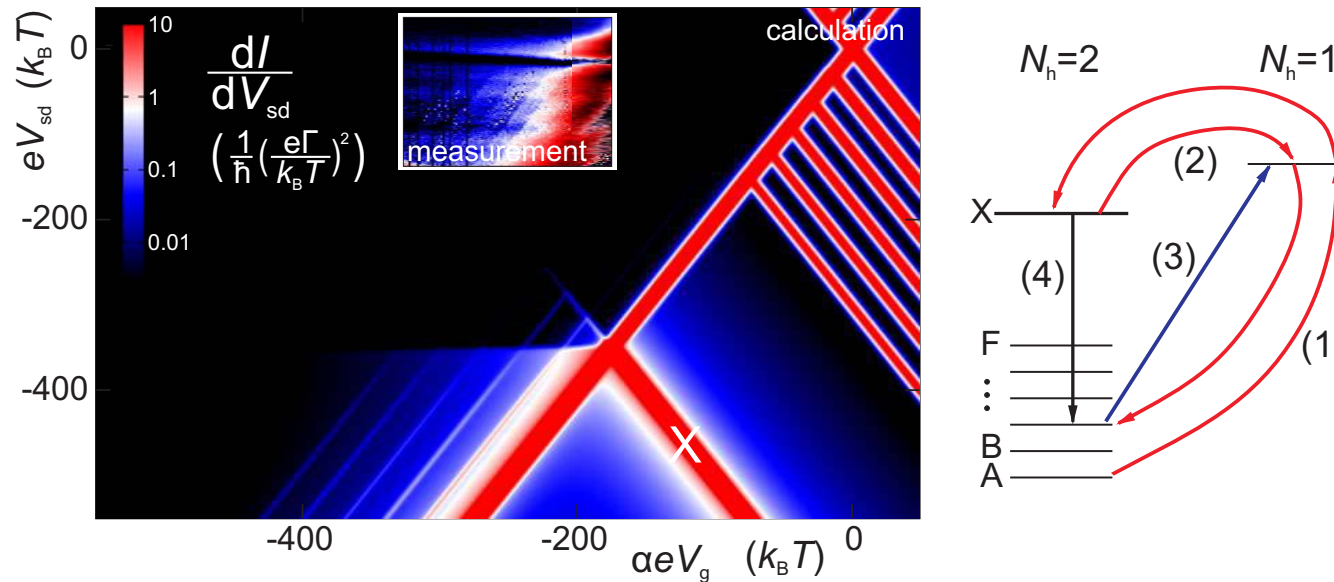
- first observed and explained by Schleser *et al.* \sim 2005
electronic excitations in GaAs/AlGaAs quantum dots

Measurement detail



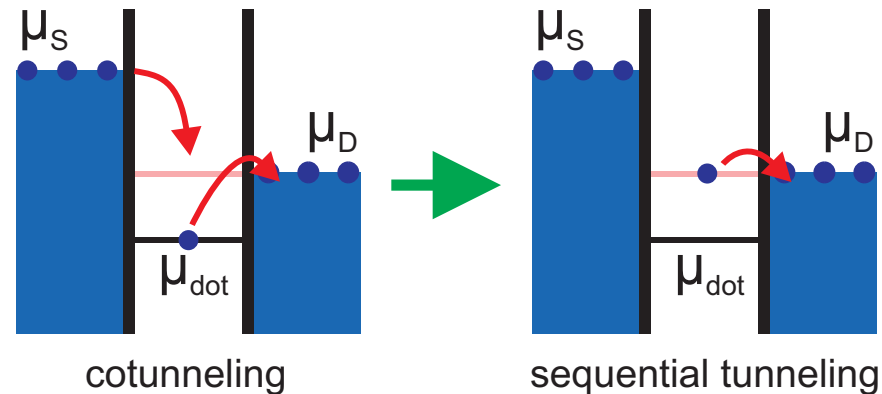
- CO-SET current sets in at additional (electronic) excited state X
- Tunnel rates coupling an 2h state to 1h ground state:
 small for 2h ground state, **large** for 2h excited state X
- Real-time transport theory calculations, M. Leijnse & M. Wegewijs
- Vibration mode is **pumped** by multiple inelastic cotunnel processes involving X (e.g. sequence (1) \rightarrow (2) \rightarrow (3))

Numerical calculation



- CO-SET current sets in at additional (electronic) excited state X
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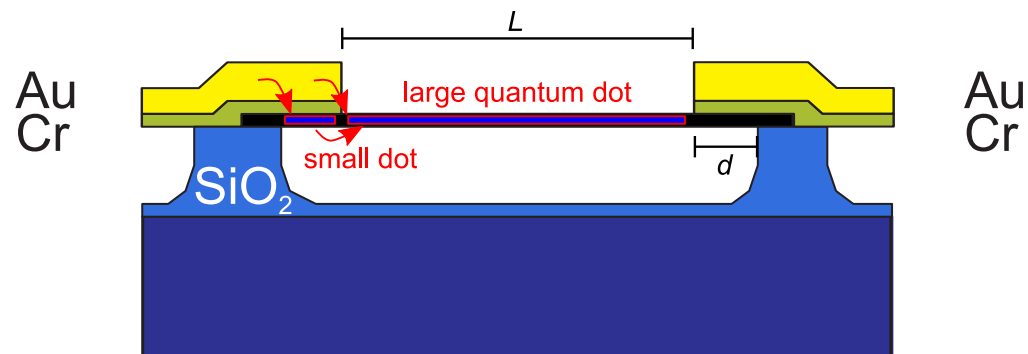
CO-SET process requires energy storage, nonequilibrium



- Vibration mode must remain excited until tunnel-out
- Vibrons are pumped as in a three-level laser!
- Comparison of timescales & tunnelling rates
 - first (weak) lower boundary for mechanical quality factor
 - $Q_{\text{stretch}} \gtrsim 30$
- Known values for transversal CNT mode:
 - $Q_{\text{bend,RT}} \lesssim 2000$, $Q_{\text{bend,20mK}} \lesssim 150000$

Open question #1: Nature of the excited state X

- simplest possibility: orbital excited state of the nanotube quantum dot
 - different orbital wavefunction
 - different tunnel couplings
 - our model idea should work fine
- alternative explanation: potential side minimum / double quantum dot
 - possible since the suspended nanotube is partially covered by the contacts
 - bending resonance lines in Coulomb diamonds: shifting potential minima
 - our model idea should still work fine!

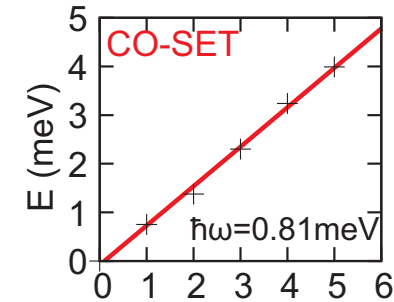
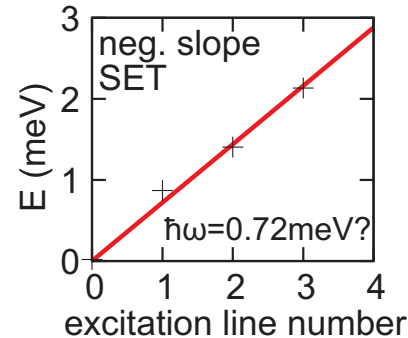
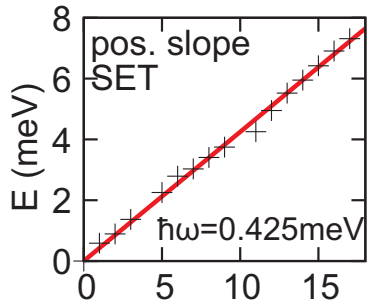


Some speculations about a “phaser”



- idea: use analogy with the 3-level laser
- current pumps vibration via the electronic excited state
- use a double quantum dot to generate this level structure
- feed a mechanical mode faster than it can decay, population inversion
- ?

Open question #2: Frequency doubling



- pos. slope SET excitations: $\hbar\omega = 0.42\text{ meV}$
- CO-SET and neg. slope SET excitations: $\hbar\omega \simeq 0.8\text{ meV}$
- few-carrier system
- adding a hole redistributes the entire charge on the nanotube
- up to now, all measurements of the Franck-Condon sidebands were in the metallic limit

Some speculations about 1D Wigner crystals

- few charge carrier limit
- one-dimensional chain of electrons or holes
- different charge number \longrightarrow completely different charge distribution
- influence of charge distribution on the electrostatic forces inducing the vibrations
- **dynamic interaction?**
- electronic system much faster than vibration
 \longrightarrow can regard mechanical oscillator fixed for each point in time
- ?

The team at Molecular Electronics & Devices, Delft and theory friends

Thanks!



Herre van der Zant



Benoit Witkamp



Hari Pathangi



Menno Poot



Martin Leijnse



Maarten Wegewijs

& Samir Etaki, Yaroslav Blanter, Fabio Pistolessi, Ivar Martin, Sami Sarmaz, Pablo Jarillo-Herrero, Raymond Schouten, Hidde Westra, ...

Meanwhile, things have moved on a bit...

... and me as well, back to Bavaria

- new research group at Universität Regensburg
- spin injection and spin transport in carbon nanotubes
- carbon nanotubes with superconducting contacts
- and now (since funding has finally come):
carbon nanotubes as nano-electromechanical hybrid systems
- **postdoc position available**





Universität Regensburg

FAKULTÄT FÜR PHYSIK

Institute for experimental and applied physics

Postdoc position in NEMS available!

You have already been working successfully with **millikelvin RF** equipment in your PhD research, and have a good understanding of **low temperature physics** as well as **gigahertz technology**? Ideally, you are coming from a research group specialized in **superconductor-related mesoscopic physics**, quantum information, or cavity QED? You are interested in contributing to a young and dynamic team, trying to push the limits of what is doable in nano-electromechanical systems?

Then you might be just about right here. Your job will be to build up a low-temperature high frequency measurement setup in a state-of-the-art dilution refrigerator, and conduct measurements on **coupled superconductor-carbon nanotube systems**. You will be supported by a PhD student and a MSc student. We expect your work to lead to reeeally great publications!

Your salary will be based on the German TV-L E13. Regensburg university has a strong focus on nanophysics, in particular on spin phenomena and carbon-based systems. The natives are friendly, and while our university buildings feature classic 1965 concrete, the medieval city of Regensburg is a jewel on its own, with a vibrant young atmosphere. Both mountains and Munich airport are not far away.

Interested? Have a look at <http://www.physik.uni-r.de/forschung/huettel/> and contact **Andreas K. Hüttel** (e-mail: andreas.huettel@physik.uni-r.de) for more information!