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Insights from Plasma Focus Numerical Experiments: Scaling Properties to Scaling Laws

S. Lee and S.H. Saw
Institute for Plasma Focus Studies
Melbourne
Australia

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Plasma Focus Numerical Experiments-Scaling Properties to Scaling Laws (Part I & II)

S Lee and S H Saw

INTI International University, 71800 Nilai, Malaysia

Institute for Plasma Focus Studies, Chadstone, VIC 3148, Australia

e-mail: leesing@optusnet.com.au saw_sorheoh@intimal.edu.my

Plasma Focus Numerical Experiments- Scaling Properties and Scaling Laws

Part I: Scaling Properties & Scaling Laws

Outline of Part I

Recent numerical experiments uncovered new **insights** into plasma focus devices including :

(1) **Plasma current limitation effect**, as device static inductance Lo tends towards 0

(2) **Scaling laws** of neutron yield and soft x-ray yield as functions of Eo & I

These effects & scaling laws are a consequence of the scaling properties

(3) A by-product of the numerical experiments are diagnostic reference points.

Part II: Concepts into the Future

Outline of Part II

- Global Neutron scaling law
- Yield deterioration & saturation
- Dynamic Resistance-Cause of "Neutron Saturation"
- Beyond present saturation? Outline of part I

- New classification of plasma focus devices into T1 (Low L_0) & T2 (High L_0)
- T2 requires instability phase modeling
- Simulate by means of anomalous resistance(s)
- Result in new quantitative data of anomalous resistance

Experiments & Numerical Experiments 1\2

Experiments

- Consider the way experiments are carried out in the development of a pulsed dense plasma
- Initial experiments are to give the developers an idea of what the dense plasma looks like in terms of formation, dynamics, structures etc by various diagnostics
- Then some photographs and images- time-resolved would be very useful
- More advanced experiments would be to obtain scaling properties- which properties are important for the scaling of the machine
- These will lead to scaling laws e.g how important yields scale with selected important properties

Experiments & Numerical Experiments 2/2

Numerical Experiments

- Likewise for numerical experiments, modelling and simulation
- Initial experiments would be to see how the model would predict what the dense plasma would look like
- More advanced experiments: we expect the numerical experiments to obtain scaling properties
- Which lead to obtaining scaling laws from the numerical experiments

The Lee Model code-
Comprehensive Numerical Experiments

This is the approach of the Lee Model code

- To model the plasma dynamics & plasma conditions
- Then obtain insights into scaling properties
- Then scaling laws

Critical to the approach:

Model is linked to physical reality by the current waveform

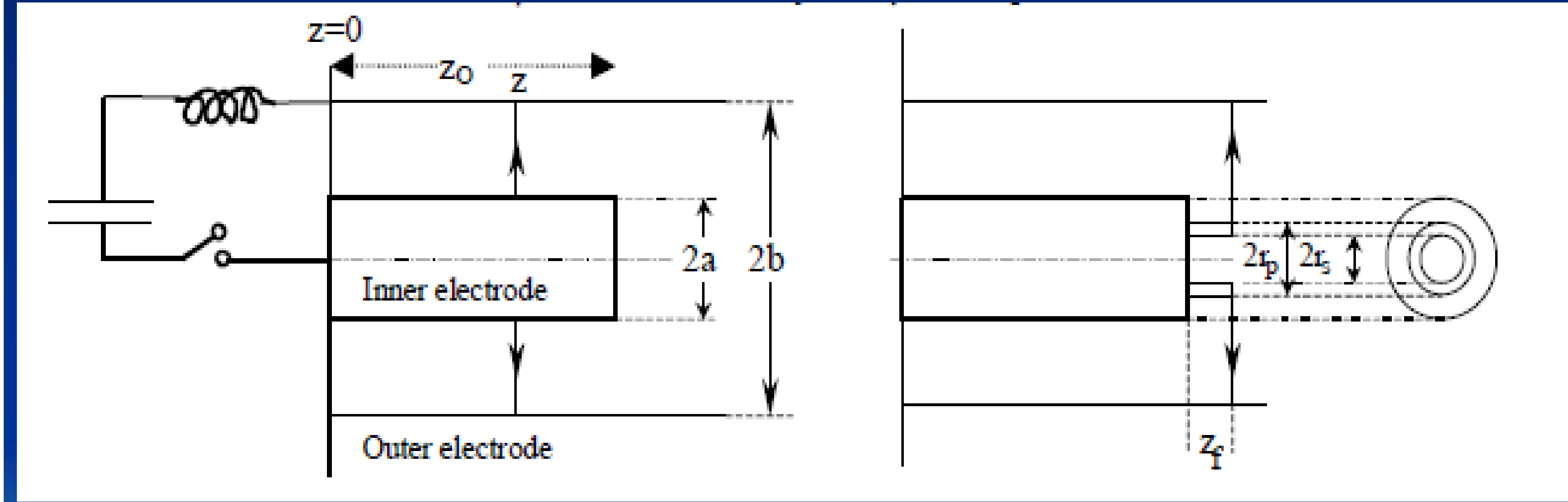
The Plasma Focus 1/2

- Plasma focus: small fusion device, complements international efforts to build fusion reactor
- Multi-radiation device - x-rays, particle beams and fusion neutrons
- Neutrons for fusion studies
- Soft XR applications include microelectronics lithography and micro-machining
- Large range of device-from J to thousands of kJ
- Experiments-dynamics, radiation, instabilities and non-linear phenomena

The Plasma Focus

Axial Phase

Radial Phases



The 5-phases of Lee Model code

Includes electrodynamical- and radiation- coupled equations to portray the REGULAR mechanisms of the:

- axial (phase 1)
- radial inward shock (phase 2)
- radial RS (phase 3)
- slow compression radiation phase (phase 4)
- the expanded axial post-pinch phase (phase 5)

Crucial technique of the code: **Current Fitting**

Philosophy of Current fitting 1/3

- **The current trace of the focus is the best indicator of gross performance.**
- The exact time profile of the current trace is governed by the bank parameters, the focus tube geometry and the operational parameters.
- It depends on the mass swept-up and drive current fractions and their variations.
- These parameters determine the dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the current.
- There are many underlying mechanisms (see following 2 slides) which are not simple to model. The detailed current profile is influenced by these effects and during the pinch phase also reflects the Joule heating and radiative yields.
- At the end of the pinch phase the profile reflects the sudden transition from a constricted pinch to a large column flow.
- Thus the current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases.
- Conversely all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases affect the current.
- **The current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases.**
- This explains the importance attached to matching the computed total current trace to the measured total current trace in the procedure adopted by the Lee model code.
- **Once matched, the fitted model parameters assure that computation proceeds with all physical mechanisms accounted for, in the gross energy & mass balance sense.**

Philosophy of Current fitting 2/3

All inaccurate model effects are accounted for by the fitting:

Known effects that might deviate from our modelling include:

1. Geometrical, including our assumed geometry
2. Our assumed structures and distributions
3. Mass shedding & current sheet CS porosity
4. Current shedding, fragmenting, leakage & inclination
5. Non uniformity & inhomogeneity of CS and plasma; boundary layer effects
6. Radiation & thermodynamics
7. Ejection of mass caused by necking curvatures

Once current-fitted, also unspecified and unknown effect are also accounted for in terms of energy and mass.

Philosophy of Current fitting 3/3

- So we relate to reality through a measured current trace
- computed current waveform is adjusted to fit measured current waveform
- Adjustment by model parameters f_m, f_c, f_{mr}, f_{cr}; account for **all factors** affecting mass flow and force field flows not specifically modelled including all KNOWN and UNKNOWN effects.
- When adjustments are completed so that the computed waveform fit the measured waveform, the **computed system is energetically** and **mass-wise equivalent to the real system.**

Insights 1/2

- The Lee model code has produced ground-breaking insights no other plasma focus codes has been able to produce

Insights 2/2
Ground-breaking Insights published

- Limitation to Pinch Current and Yields- **Appl Phys Letts. 92 (2008) S Lee & S H Saw:** **an unexpected, important result**
- Neutron Yield Scaling-sub kJ to 1 MJ-**J Fusion Energy 27 (2008) S Lee & S H Saw-** multi-MJ- **PPCF 50 (2008) S Lee**
- Neon Soft x-ray Scaling- **PPCF 51 (2009) S Lee, S H Saw, P Lee, R S Rawat**
- Neutron Yield Saturation- **Appl Phys Letts. 95 (2009) S Lee**

  Simple explanation of major obstruction to progress

From Measured Current Waveform to Modelling for Diagnostics 1/2

Procedure to operate the code:

Step 1: Configure the specific plasma focus,

Input:

- **Bank parameters, L_0, C_0 and stray circuit resistance r_0;**
- **Tube parameters b, a and z_0 and**
- **Operational parameters V_0 and P_0 and the fill gas**

Step 2: Fitting the computed current waveform to the measured waveform-(connecting with reality) 2/2

- A measured discharge current I_{total} waveform for the specific plasma focus is required
- The code is run successively. At each run the computed I_{total} waveform is fitted to the measured I_{total} waveform by varying model parameters f_m, f_c, f_{mr} and f_{cr} one by one, one step for each run, until computed waveform agrees with measured waveform.

The 5-Point Fit:

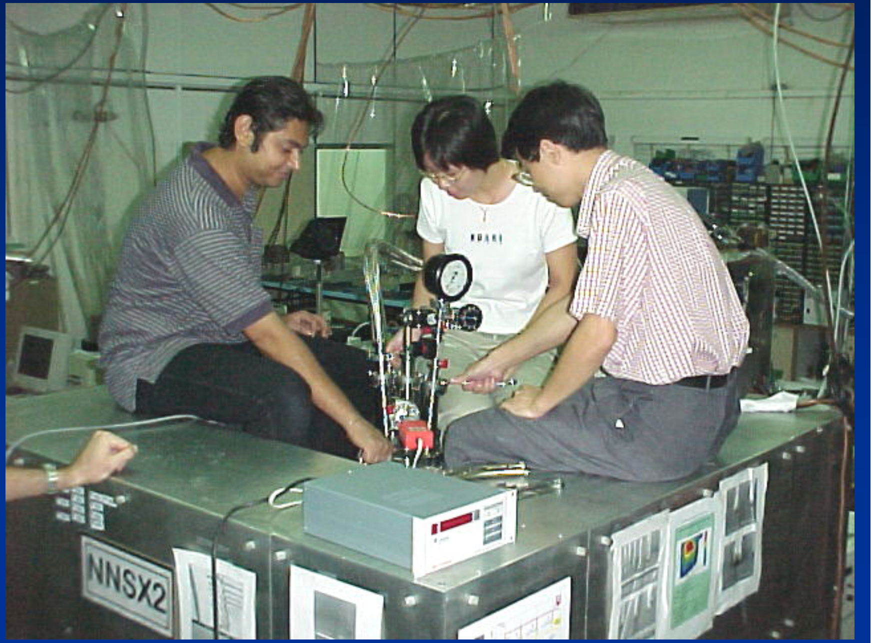
- First, the axial model factors f_m, f_c are adjusted (fitted) until
  - (1) computed rising slope of the I_{total} trace and
  - (2) the rounding off of the peak current as well as
  - (3) the peak current itself

  are in reasonable (typically very good) fit with the measured I_{total} trace.
- Next, adjust (fit) the radial phase model factors f_{mr} and f_{cr} until
  - (4) the computed slope and
  - (5) the depth of the dip

  agree with the measured I_{total} waveform.

Example : NX2-Plasma SXR Source 1/4

- **NX2**
- 11.5kV, 2 kJ
- **16 shots /sec**; 400 kA
- 20J SXR/shot (neon)
- 10^9 neutrons/shot (D)



Example of current fitting: Given any plasma focus : e.g. NX2 16 shots/sec Hi Rep 2/4

- Bank parameters: L_0=15nH; C_0=28uF; r_0=2 mΩ
- Tube parameters: b=4.1 cm, a=1.9 cm, z0=5cm
- Operation parameters: V_0=11kV, P_0=2.6 Torr in Neon

The UPFLF (Lee code) is configured (by keying figures into the **configuration panel** on the EXCEL sheet) as the NX2

INPUT:

Lo	Co	b	a	zo	ro mOhm
15	28	4.1	1.9	5	2
massf	currf	massfr	currfr	Model Parameters	
0.098	0.7	0.14	0.69		
Vo	Po	MW	A	At-1 mol-2	Operational
11	2.6	20	10	1	Parameters

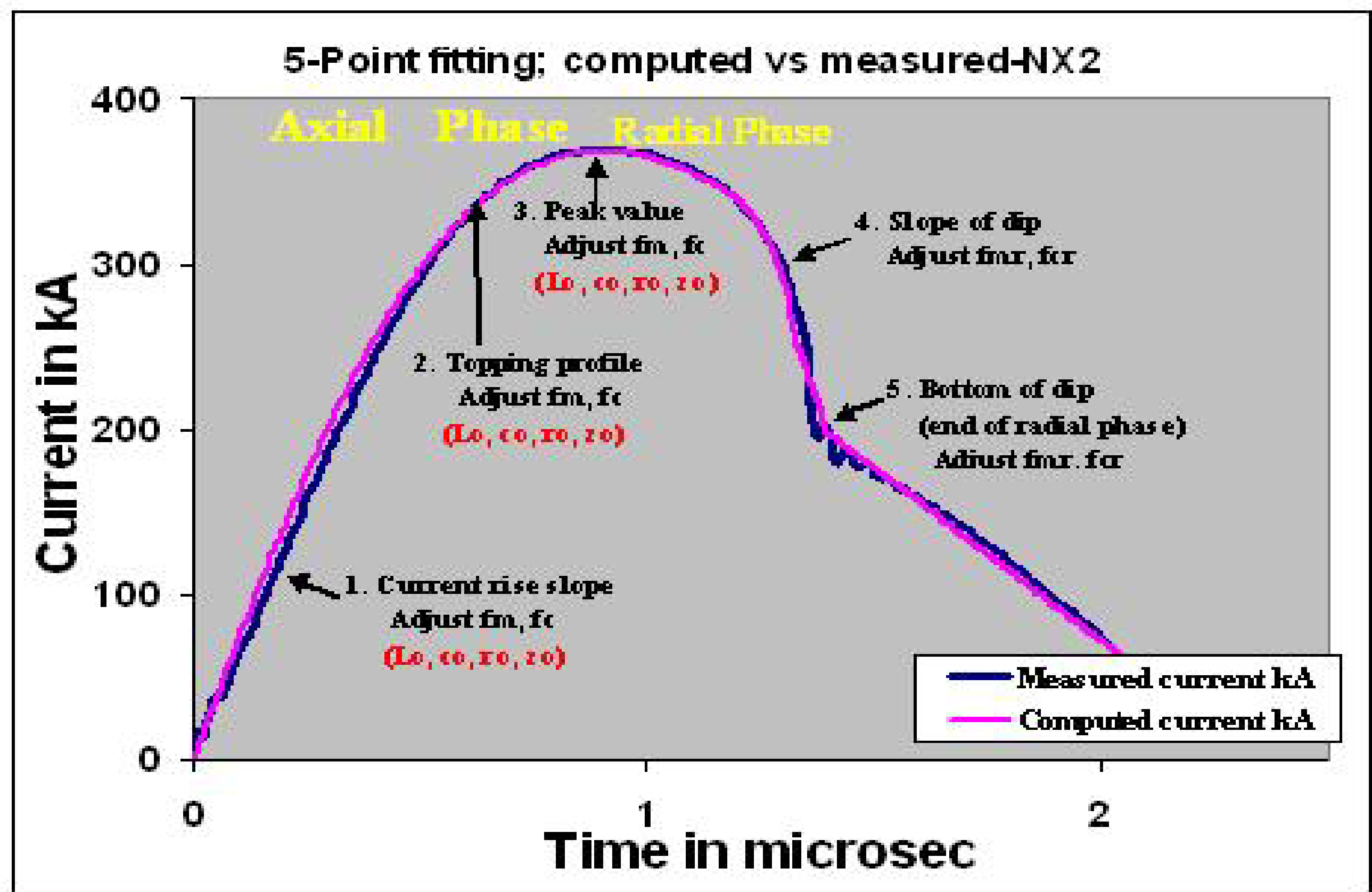
OUTPUT: NX2 **current waveform**

NX2 **dynamics & electrodynamics**

NX2 **plasma pinch dimensions & characteristics**

NX2 **Neon SXR yield**

Fitting computed I_{total} waveform to measured I_{total} waveform: the 5-point fit 3/4



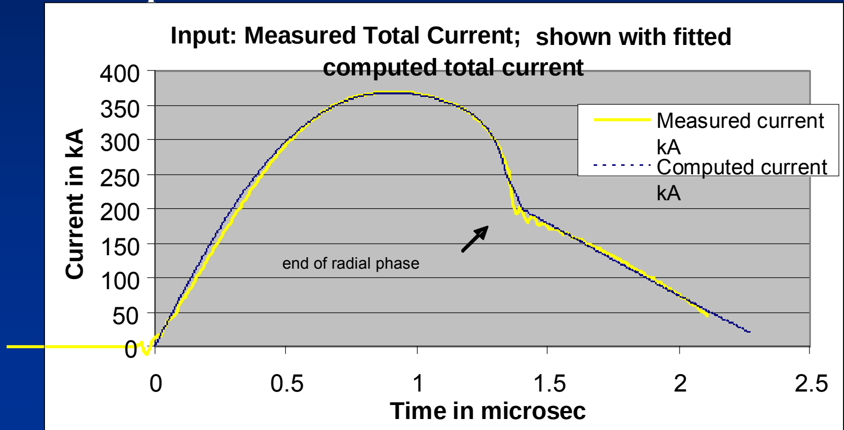
Once fitted: model is energy-wise & mass-wise equivalent to the physical situation 4/4

- All dynamics, electrodynamics, radiation, plasma properties and neutron yields are realistically simulated; so that the code output of these quantities may be used as reference points for diagnostics

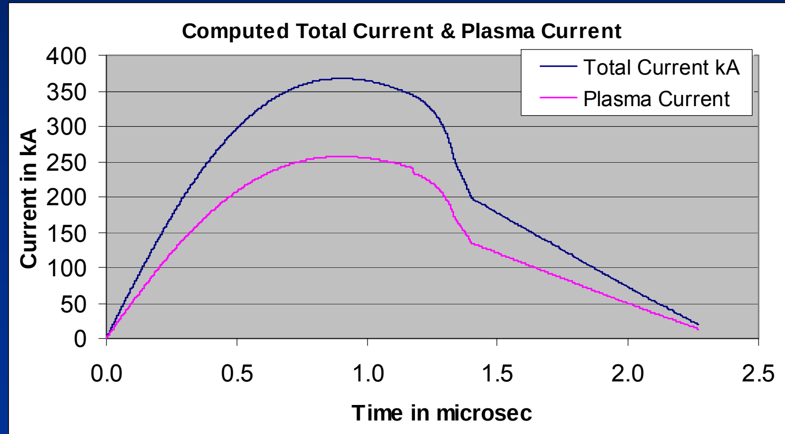
Numerical Diagnostics- Example of NX2 Time histories of dynamics, energies and plasma properties computed by the code 1/3

Last adjustment, when the computed I_{total} trace is judged to be reasonably well fitted in all 5 features, computed times histories are presented (NX2 operated at 11 kV, 2.6 Torr neon)

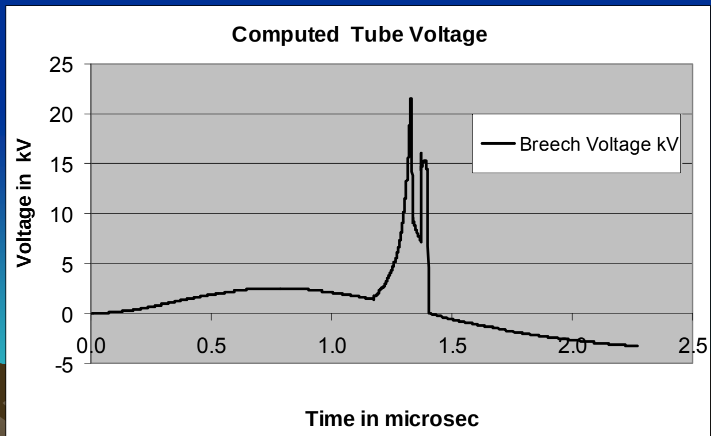
Computed Itotal waveform fitted to measured



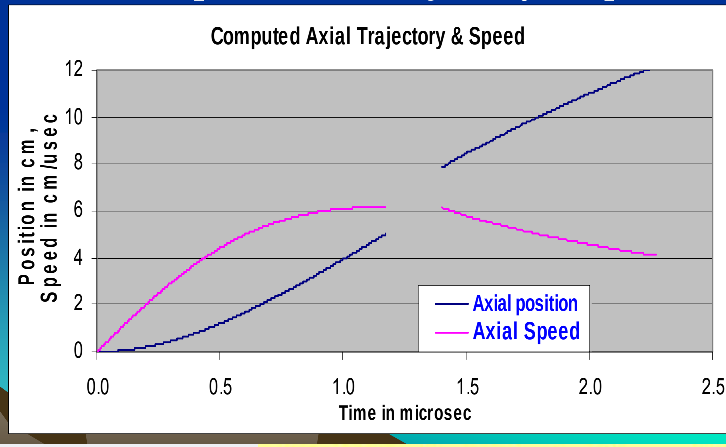
Computed Itotal & Iplasma



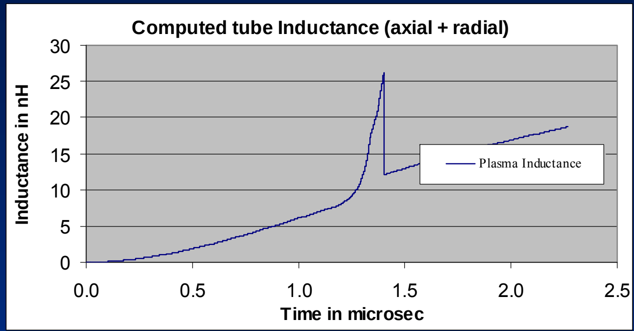
Computed Tube voltage

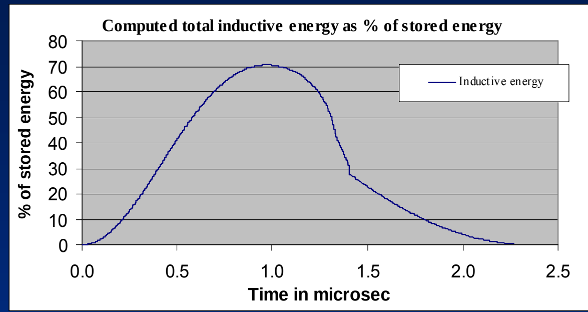


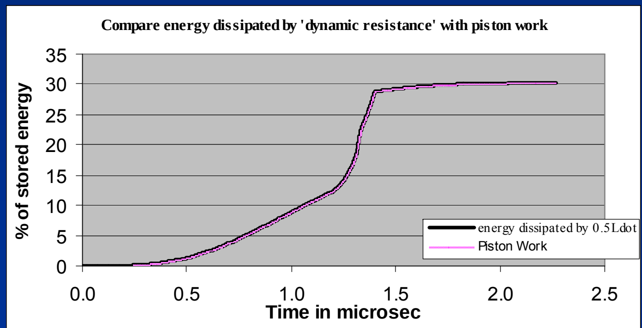
Computed axial trajectory & speed

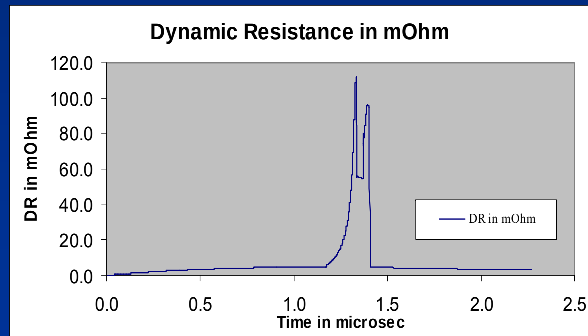


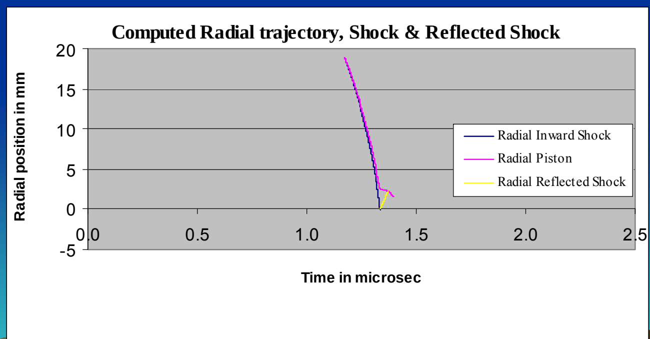
Numerical Diagnostics- Example of NX2

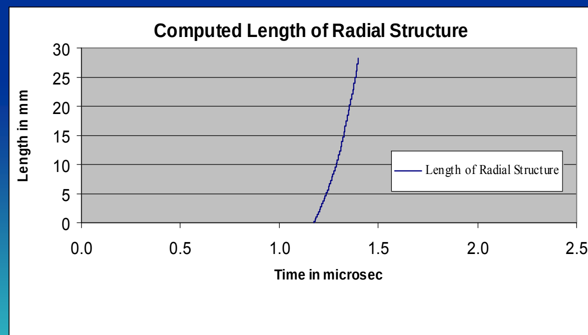




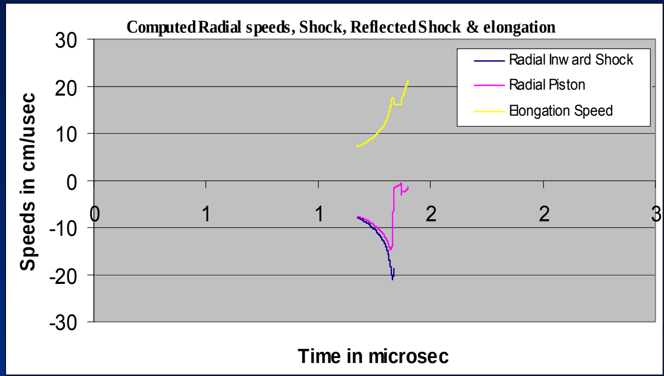


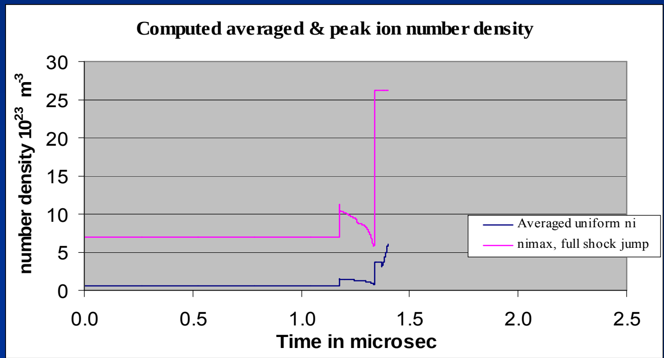


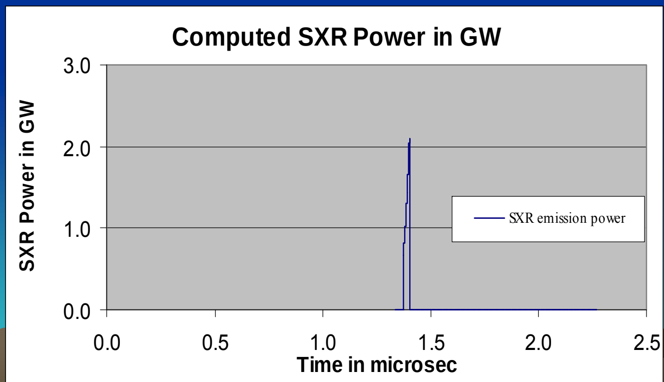


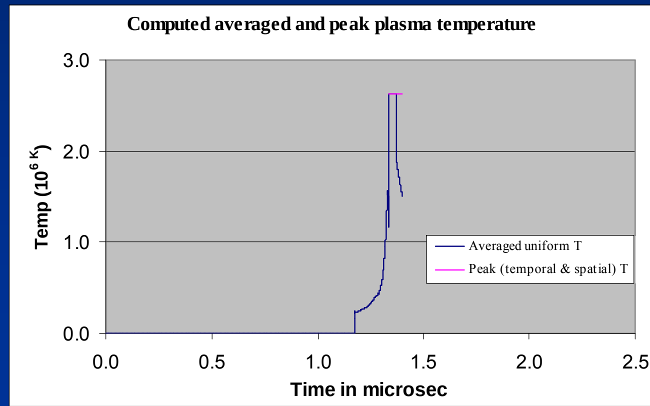


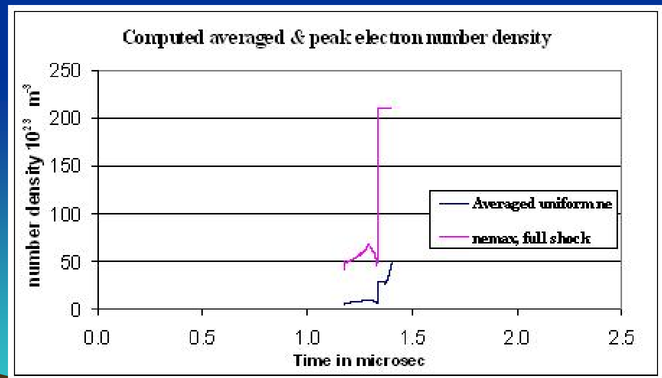
Numerical Diagnostics- Example of NX2 3/3











Scaling Properties

3 kJ machine

Small Plasma Focus

1000 kJ machine

Big Plasma Focus

Comparing small (sub kJ) and large (thousand kJ) Plasma Focus

Scaling Properties: size (energy) , current, speed and yield

Scaling properties-mainly axial phase 1/3

Table 1.

	E_0	a	z_0	V_0	P_0	I_{peak}	v_a	ID	SF	Y_n
	kJ	cm	cm	kV	Torr	kA	cm/μs	kA/cm	(kA/cm) torr$^{0.5}$	10^8
PF1000	486	11.6	60	27	4	1850	11	160	85	1100
UNU ICTP	2.7	1.0	15.5	14	3	164	9	173	100	0.20
PF-400J	0.4	0.6	1.7	28	7	126	9	210	82	0.01

Scaling of anode radius, current and Y_n with energy E_0

Scaling properties-mainly axial phase 2/3

- Peak current I_{peak} increases with E_0.
- Anode radius 'a' increases with E_0.
- Current per cm of anode radius (ID) I_{peak} /a : narrow range 160 to 210 kA/cm
- SF (speed factor) (I_{peak} /a)/$P^{0.5}$:

narrow range 82 to 100 (kA/cm) per Torr $^{0.5}$ D
Observed Peak axial speed v_a : 9 to 11 cm/us.

- Fusion neutron yield Y_n :

10^6 for PF400-J to 10^{11} for PF1000.

Variation of ID SF and Y_n

Scaling properties-mainly axial phase 3/3

- ID and SF are practically constant at around 180 kA/cm and 90 (kA/cm) per $torr^{0.5}$ deuterium gas throughout the range of small to big devices (1996 Lee & Serban IEEE Trans)

- Y_n changes over 5 orders of magnitude.

Comparing small (sub kJ) & large (thousand kJ) Plasma Focus

Scaling Properties: size ('a') , T, pinch dimensions & duration

Scaling properties-mainly radial phase 1/2

Table 2.

	c= b/a	a	T_{pinch}	v_p	r_{min}	z_{max}	Pinch duration	r_{min}/a	z_{max}/a	Pinch duration/a
		cm	10^6K	cm/μs	cm	cm	ns			ns/cm
PF1000	1.4	11.6	2	13	2.2	19	165	0.17	1.6	14
UNU ICTP PFF	3.4	1.0	8	26	0.13	1.4	7.3	0.14	1.4	8
PF400J	2.6	0.6	6	23	0.09	0.8	5.2	0.14	1.4	9

Focus Pinch T, dimensions & lifetime with anode radius 'a'

Scaling properties-mainly radial phase 2/2

- Dimensions and lifetime scales as the anode radius 'a'.
- r_{min}/a (almost constant at 0.14-0.17)
- z_{max}/a (almost constant at 1.5)
- Pinch duration narrow range 8-14 ns/cm of 'a'

- **T_{pinch} is measure of energy per unit mass.**

Quite remarkable that this energy density varies so little (factor of 5) over such a large range of device energy (factor of 1000).

- **Scaling Properties: Pinch Dimensions & Duration: Compare D & Ne**
- **(Lee, Kudowa 1998, Cairo 2003)**

Table 3.

		Deuterium	Neon (for SXR)
minimum radius	r_{min}	0.15a	0.05a
max length (hollow anode)	z	1.5a	1.6a
radial shock transit	t_{comp}	5×10^{-6}a	4×10^{-6}a
pinch lifetime	t_p	10^{-6}a	10^{-6}a
Speed factor	SF	90	

Rule-of-thumb scaling properties, (subject to minor variations caused primarily by the variation in c=b/a) over whole range of device

- **Axial phase energy density (per unit mass)** — **constant**
- **Radial phase energy density (per unit mass)** — **constant**
- **Pinch radius ratio** — **constant**
- **Pinch length ratio** — **constant**
- **Pinch duration per unit anode radius** — **constant**

Further equivalent Scaling Properties

- Constant axial phase energy density (Speed Factor $(I/a)/\rho^{0.5}$, speed) equivalent to constant dynamic resistance
- I/a approx constant since ρ has only a relatively small range for each gas
- Also strong relationship requirement between plasma transit time and capacitor time $t_0 = (L_0 C_0)^{0.5}$
- E.g. strong interaction between t_0 and ‘a’ and I for a given bank.

Thinking in terms of scaling properties: Example: consider the following Gedanken situation

- L_0 tends to zero; Lo is the static inductance up to the start of the axial phase.
- In such a case $t_0 = (L_0C_0)^{0.5}$ tends to zero
- As soon as the capacitor is switched onto the Plasma Focus tube, the current starts to tend towards huge (infinite values)
- Immense axial acceleration will occur, axial speed reaches designed values very quickly and dynamic resistance will settle the current down to steady values.
- Such a situation will give rise to an early overshoot of current to values beyond the values when dynamic resistance takes over. (Lee 2008 PPCF)
- Because of the sharp rise of current it is advantageous to have a short anode; moreover to accommodate the large current, anode radius will need to be increased accordingly.
- Such a situation is consistent with numerical experiments in which when L_0 is reduced to small values a short anode with large radius is required for matching.
- From thinking about this situation it is also clear that the situation of zero L_0 is impossible to match; which leads to the conclusion that there is a minimum L_0 beyond which it is not advantageous to reduce Lo further to improve any yield performance (Lee & Saw 2008 Appl Phys Letts)

The Lee Model Code

- Realistic simulation of all gross focus properties
- Couples the electrical circuit with plasma focus dynamics, thermodynamics and radiation (Lee 1983, 1984)
- 5-phase model; axial & radial phases
- Includes plasma self-absorption for SXR yield (Lee 2000)
- Includes neutron yield, Y_n, using a beam–target mechanism (Lee & Saw 2008, J Fusion energy)

The Lee Model code- 5 Phases 2/3

- **Axial Phase**
- **Radial Inward Shock Phase**
- **Radial Reflected Shock (RS) Phase.**
- **Slow Compression (Quiescent) or Pinch Phase**
- **Expanded Column Phase**

The Lee Model code 3/3

- Institute for Plasma Focus Studies
  - http://www.plasmafocus.net/
- Internet Workshop on Plasma Focus Numerical Experiments (IPFS-IBC1) 14 April-19 May 2008
  - http://www.plasmafocus.net/IPFS/Papers/IWPCAkeynote2ResultsofInternet-basedWorkshop.doc
- Lee S Radiative Dense Plasma Focus Computation Package: RADPF
  - http://www.intimal.edu.my/school/fas/UFLF/File1RADPF.htm
  - http://www.plasmafocus.net/IPFS/modelpackage/File1RADPF.htm

Computation of Neutron yield (1/2)

- Adapted from Beam-target neutron generating mechanism
  (ref Gribkov et al)
- A beam of fast deuteron ions close to the anode
- Interacts with the hot dense plasma of the focus pinch column
- Produces the fusion neutrons
- Given by:

$$Y_{b-t} = C_n \, n_i I_{pinch}^2 z_p^2 (\ln(b/r_p)) \sigma / U^{0.5}$$

where
n_i = ion density
b = cathode radius,
r_p = radius of the plasma pinch column with length z_p,
σ = cross-section of the D-D fusion reaction, n- branch,
U= beam energy, and
C_n = calibration constant

Computation of Neutron yield (2/2)

Note:

- The D-D cross-section is sensitive to the beam energy in the range 15-150 kV; so it is necessary to use the appropriate range of beam energy to compute σ.
- The code computes induced voltages (due to current motion inductive effects) V_{max} of the order of only 15-50 kV. However it is known, from experiments that the ion energy responsible for the beam-target neutrons is in the range 50-150keV, and for smaller lower-voltage machines the relevant energy could be lower at 30-60keV.
- In line with experimental observations the D-D cross section σ is reasonably obtained by using $U = 3V_{max}$.
- The model uses a value of $C_n = 2.7 \times 10^7$ obtained by calibrating the yield at an experimental point of 0.5 MA.

Computation of Neon SXR yield (1/2)

Neon SXR energy generated Y_{SXR} = Neon line radiation Q_L

Q_L calculated from:

$$\frac{dQ_L}{dt} = -4.6x10^{-31} n_i^{\,2} Z Z_n^{\,4} \left(\pi r_p^{\,2}\right) z_f / T$$

where :

Z_n = atomic number,

n_i = number density ,

Z = effective charge number,

r_p = pinch radius,

z_f = pinch length and

T = temperature

Q_L is obtained by integrating over the pinch duration.

Computation of Neon SXR yield (2/2)

Note:

- The SXR yield is the reduced quantity of generated energy after plasma self-absorption which depends primarily on density and temperature
- The model computes the volumetric plasma self-absorption factor A derived from the photonic excitation number M which is a function of the Z_n, n_i, Z and T.
- In our range of operation the numerical experiments show that the self absorption is not significant.
- Liu Mahe (1999) first pointed out that a temperature around 300 eV is optimum for SXR production. Shan Bing's (2000) subsequent work and our experience through numerical experiments suggest that around 2×10^6 K (below 200 eV) or even a little lower could be better.
- Hence for SXR scaling there is an optimum small range of temperatures (*T* window) to operate.

Numerical Experiments (1/2)

As shown earlier, Procedure is as follows:

- The Lee code is configured to work as any plasma focus:
- **Configure**
  - o bank parameters: L_0, C_0 and stray circuit resistance r_0;
  - o tube parameters: b, a and z_0
  - o operational parameters: V_0 and P_0 and the fill gas.
- **FIT:** the computed total current waveform to an experimentally measured total current waveform using four model parameters :
  - mass swept-up factor f_m;
  - the plasma current factor f_c
  - for the axial phase; and
  - factors f_{mr} and f_{cr} for the radial phases.

Scaling laws for neutrons from numerical experiments over a range of energies from 10kJ to 25 MJ (1/4)

- To study the neutrons emitted by PF1000-like bank energies from 10kJ to 25 MJ.
- 1) Apply the Lee model code to fit a measured current trace of the PF1000:

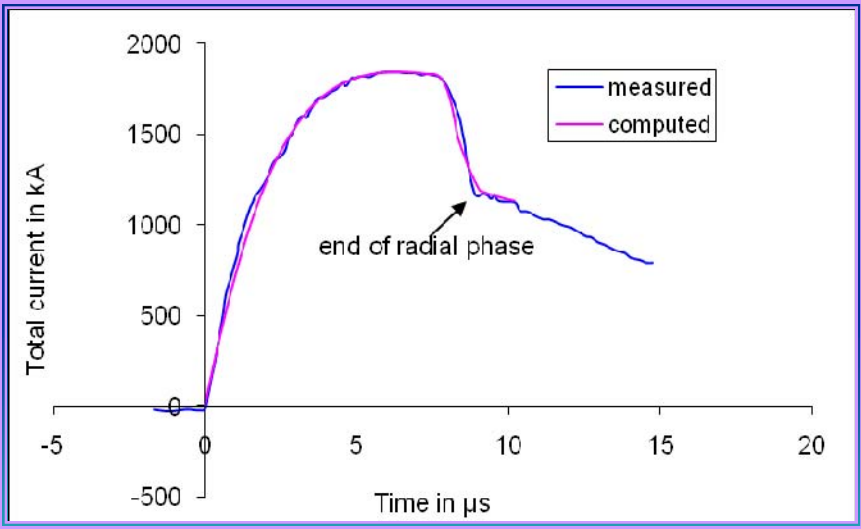
  C_0 = 1332 μF, V_0 = 27 kV, P_0 = 3.5 torr D_2; b = 16 cm, a = 11.55 cm or c=1.39; z_0 = 60 cm; external (or static) inductance L_0= 33.5 nH and; damping factor RESF= 1.22 (or stray resistance r_0=6.1 mΩ).

- 2) Apply the Lee code over a range of C_0

  ranging from 14 μF (8.5 kJ) to 39960 μF (24 MJ):
  - Voltage, V_0 = 35 kV; P_0 = 10 torr deuterium; RESF = 1.22; ratio c=b/a is 1.39.
  - For each C_0, anode length z_0 is varied to find the optimum z_0.
  - For each z_0, anode radius a_0 is varied to get end axial speed of 10 cm/μs.

Scaling laws for neutrons from numerical experiments over a range of energies from 10kJ to 25 MJ (2/4)

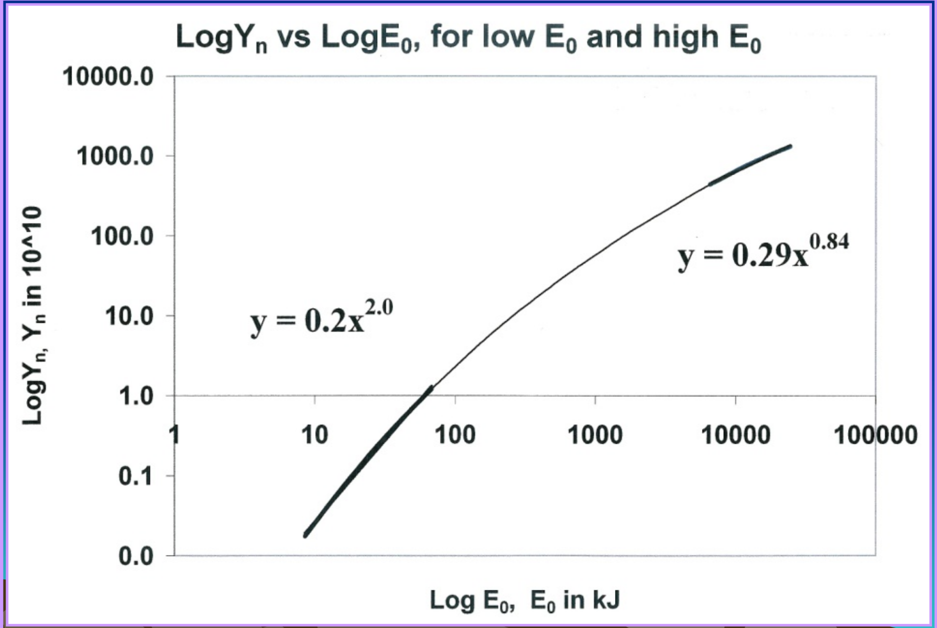
- Fitted model parameters : $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr}=0.65$.
- Computed current trace agrees very well with measured trace through all the phases: axial and radial, right down to the bottom of the current dip indicating the end of the pinch phase as shown below.



PF1000:
C_0 = 1332 μF; V_0 = 27 kV;
P_0 = 3.5 Torr D_2; b = 16 cm;
a = 11.55 cm; z_0 = 60 cm;
L_0= 33.5 nH; r_0 = 6.1 mΩ or
RESF=1.22.

Scaling laws for neutrons from numerical experiments over a range of energies from 10kJ to 25 MJ (3/4)

- Voltage, V_0 = 35 kV; P_0 = 10 torr deuterium; RESF = 1.22; ratio c=b/a is 1.39.
- Numerical experiments: C_0 ranging from 14 μF(8.5 kJ) to 39960 μF (24 MJ)
- For each C_0, anode length z_0 is varied to find the optimum z_0.
- For each z_0, anode radius a_0 is varied to get end axial speed of 10 cm/μs.



Y_n scaling changes:

- $Y_n \sim E_0^{2.0}$ at tens of kJ
- $Y_n \sim E_0^{0.84}$ at the highest energies (up to 25MJ)

Scaling laws for neutrons from numerical experiments over a range of energies from 10kJ to 25 MJ (4/4)

Scaling of Y_n with I_{peak} and I_{pinch}:

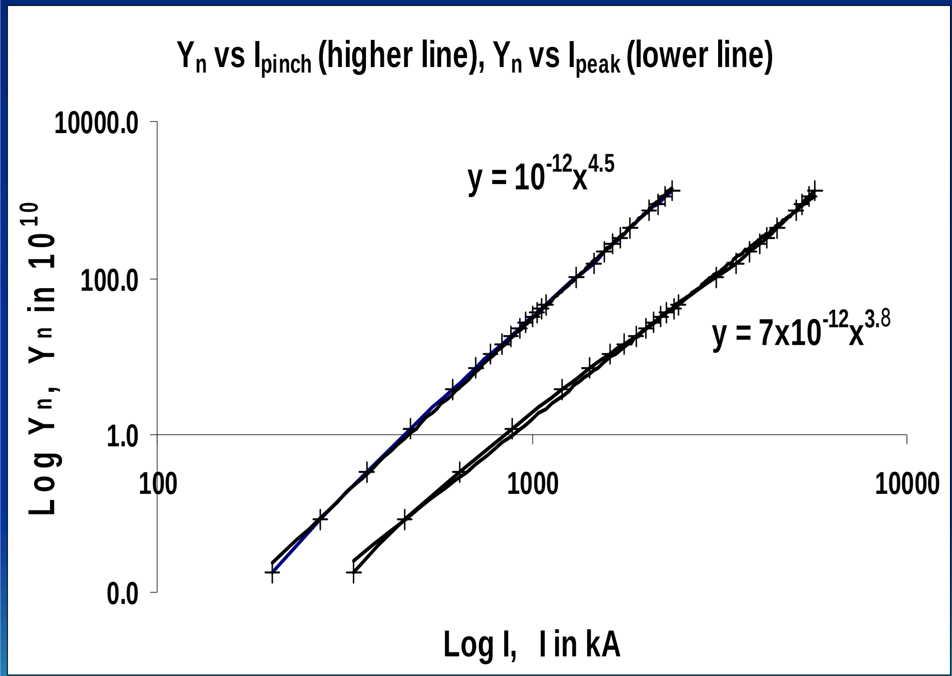
- **$Y_n = 3.2 \times 10^{11} I_{pinch}^{4.5}$**

and

- **$Y_n = 1.8 \times 10^{10} I_{peak}^{3.8}$**

where $I_{peak} = (0.3\text{-}0.7)$MA

and $I_{pinch} = (0.2\text{-}2.4)$MA.

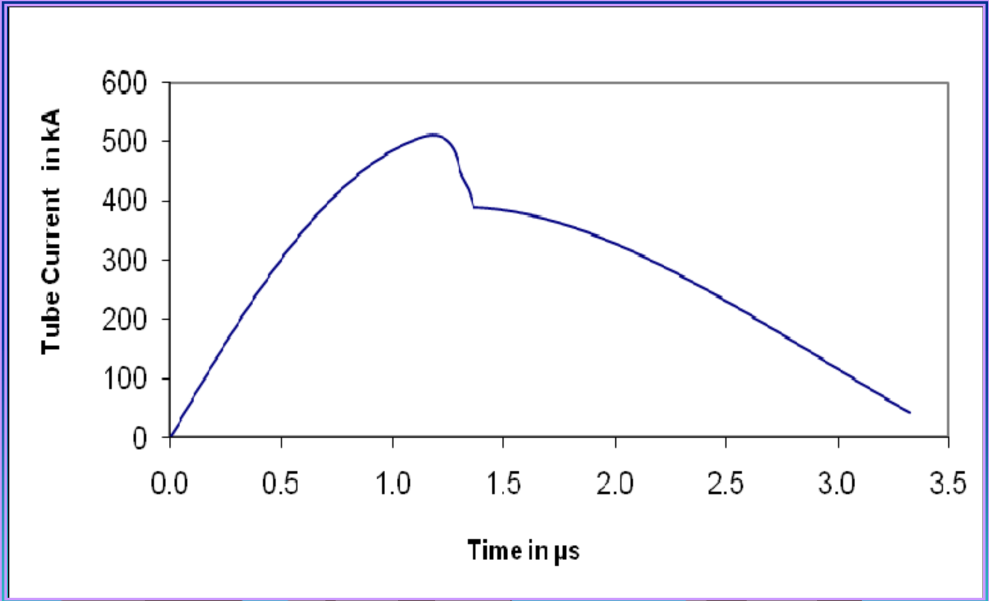


Scaling laws for neon SXR from numerical experiments over a range of energies from 0.2 kJ to 1 MJ (1/4)

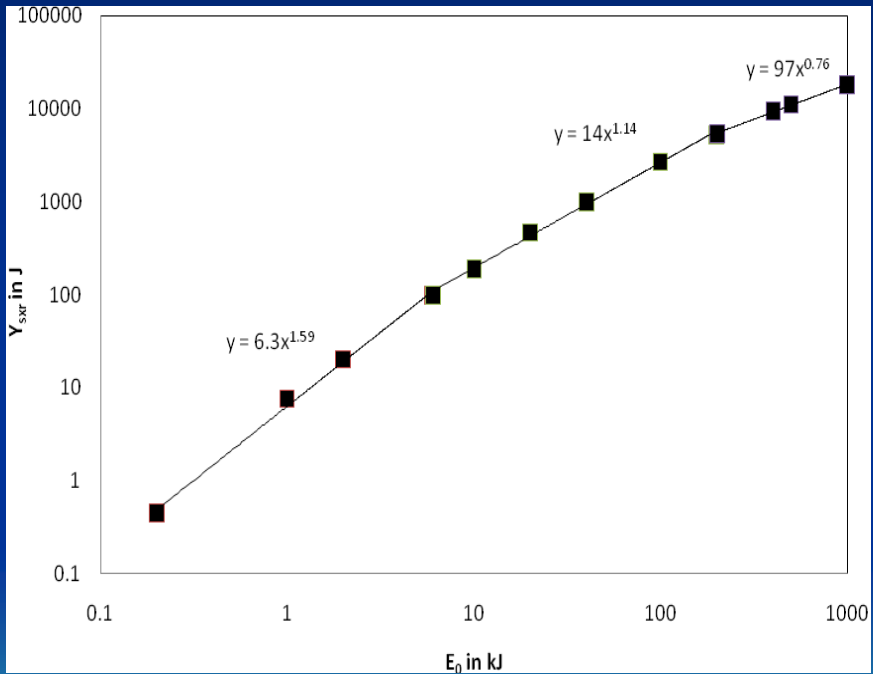
- To study the neon SXR emitted by a modern fast bank energies from 0.2 kJ to 1 MJ.
- Apply the Lee model code to a proposed modern fast plasma focus machine:
  1) With optimised values:
     c=b/a =1.5
     V_0 = 20 kV
     L_0= 30 nH
     RESF = 0.1
     Model parameters : f_m=0.06, f_c=0.7, f_{mr}=0.16, f_{cr}=0.7.
  2) For C_0 varying from 1 μF (0.2 kJ) to 5000 μF (1MJ):
     For each C_0, vary P_0, z_0, and a_0 to find the optimum Y_{sxr}

Scaling laws for neon SXR from numerical experiments over a range of energies from 0.2 kJ to 1 MJ (2/4)

- Computed Total Current versus Time
- For L_0 = 30nH; V_0 = 20 kV; C_0 = 30 uF; RESF = 0.1; c=1.5
- Model parameters : f_m = 0.06, f_c = 0.7, f_{mr} =0.16, f_{cr} = 0.7
- Optimised a=2.29cm; b=3.43 cm and z_0=5.2 cm.



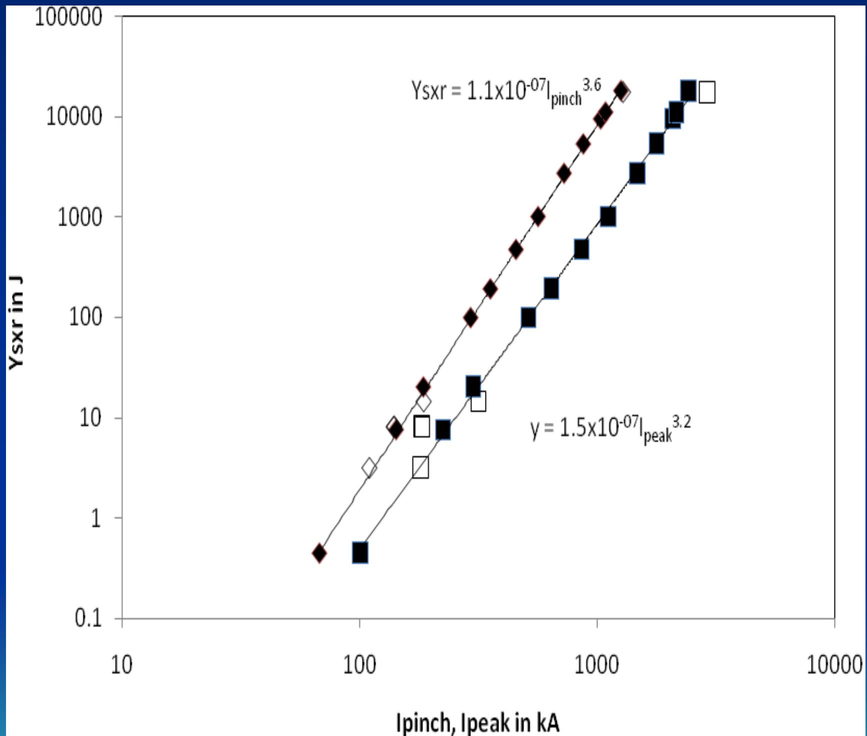
Scaling laws for neon SXR from numerical experiments over a range of energies from 0.2 kJ to 1 MJ (3/4)



Y_{sxr} scales as:

- $E_0^{1.6}$ at low energies in the sub-kJ to several kJ region.
- $E_0^{0.76}$ at high energies towards 1MJ.

Scaling laws for neon SXR from numerical experiments over a range of energies from 0.2 kJ to 1 MJ (4/4)



- Scaling with currents
- **Y_{sxr}~$I_{peak}^{3.2}$ (0.1–2.4 MA) and**
- **Y_{sxr}~$I_{pinch}^{3.6}$ (0.07-1.3 MA)**

- Black data points with fixed parameters RESF=0.1; c=1.5; L_0=30nH; V_0=20 kV and model parameters f_m=0.06, f_c=0.7, f_{mr}=0.16, f_{cr}=0.7.
- White data points are for specific machines with different values for the parameters :c, L_0, V_0 etc.

Summary-Scaling Laws (1/2)

- The scaling laws obtained (at optimized condition) for Neutrons:

- $Y_n \sim E_0^{2.0}$ at tens of kJ to
- $Y_n \sim E_0^{0.84}$ at the highest energies (up to 25MJ)

- $Y_n = 3.2 \times 10^{11} I_{pinch}^{4.5}$ (0.2-2.4 MA)
- $Y_n = 1.8 \times 10^{10} I_{peak}^{3.8}$ (0.3-5.7MA)

Summary-Scaling Laws (2/2)

- The scaling laws obtained (at optimized condition) for neon SXR:

- $\mathbf{Y_{sxr} \sim E_0^{1.6}}$ at low energies
- $\mathbf{Y_{sxr} \sim E_0^{0.8}}$ towards 1 MJ

- $Y_{sxr} \sim I_{\mathbf{peak}}^{3.2}$ (0.1–2.4 MA) and
- $Y_{sxr} \sim I_{pinch}^{3.6}$ (0.07-1.3 MA)

Plasma Focus Numerical Experiments- Trending into the Future

Part I: Scaling Properties & Scaling Laws

Conclusion to Part I

Recent numerical experiments uncovered new **insights** into plasma focus devices including :

(1) **Plasma current limitation effect**, as device static inductance Lo tends towards 0

(2) **Scaling laws** of neutron yield and soft x-ray yield as functions of Eo & I

These effects & scaling laws are a consequence of the scaling properties

(3) **A by-product of the numerical experiments are diagnostic reference points**.

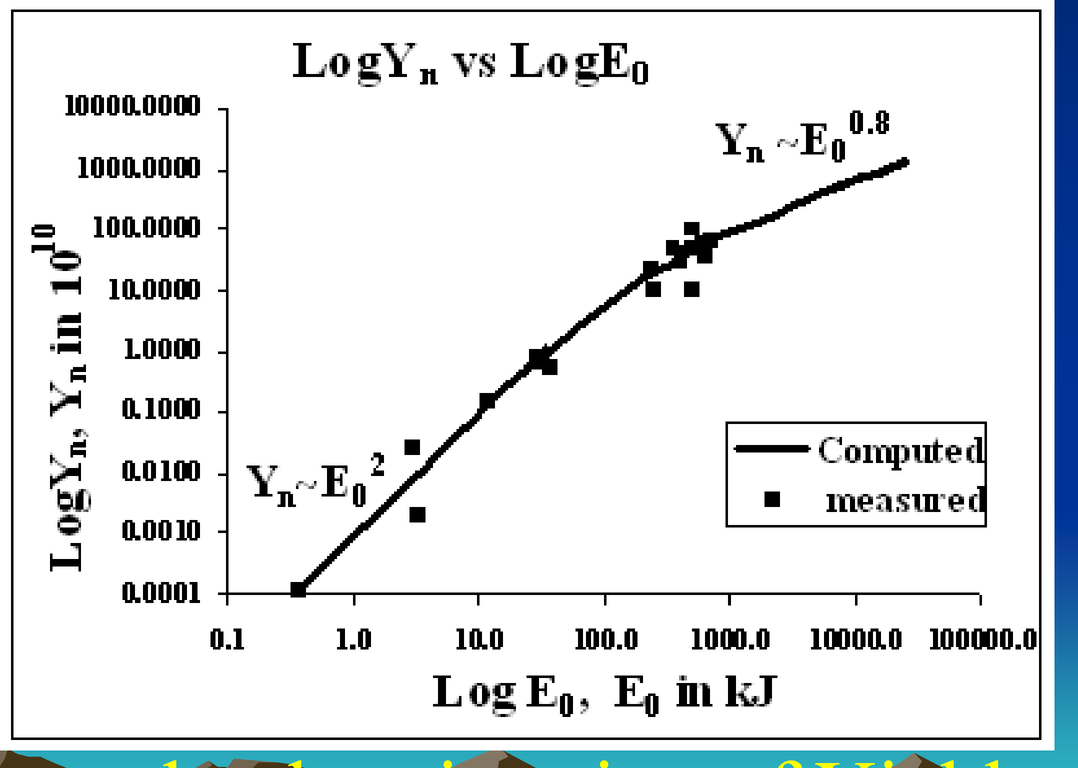
Plasma Focus Numerical Experiments-Trending into the Future

Part II: Concepts into the Future

- Global Neutron scaling law
- Yield deterioration & saturation
- Dynamic Resistance-Cause of "Neutron Saturation"
- Beyond present saturation?

- New classification of plasma focus devices into T1 (Low L_0) & T2 (High L_0)
- T2 requires instability phase modeling
- Simulate by means of anomalous resistance(s)
- Result in new quantitative data of anomalous resistance

Global scaling law, combining experimental and numerical data- Yn scaling , numerical experiments from 0.4 kJ to 25 MJ (solid line), compared to measurements compiled from publications (squares) from 0.4 kJ to 1 MJ.



What causes the deterioration of Yield scaling?

What causes current scaling deterioration and eventual saturation? 1/3

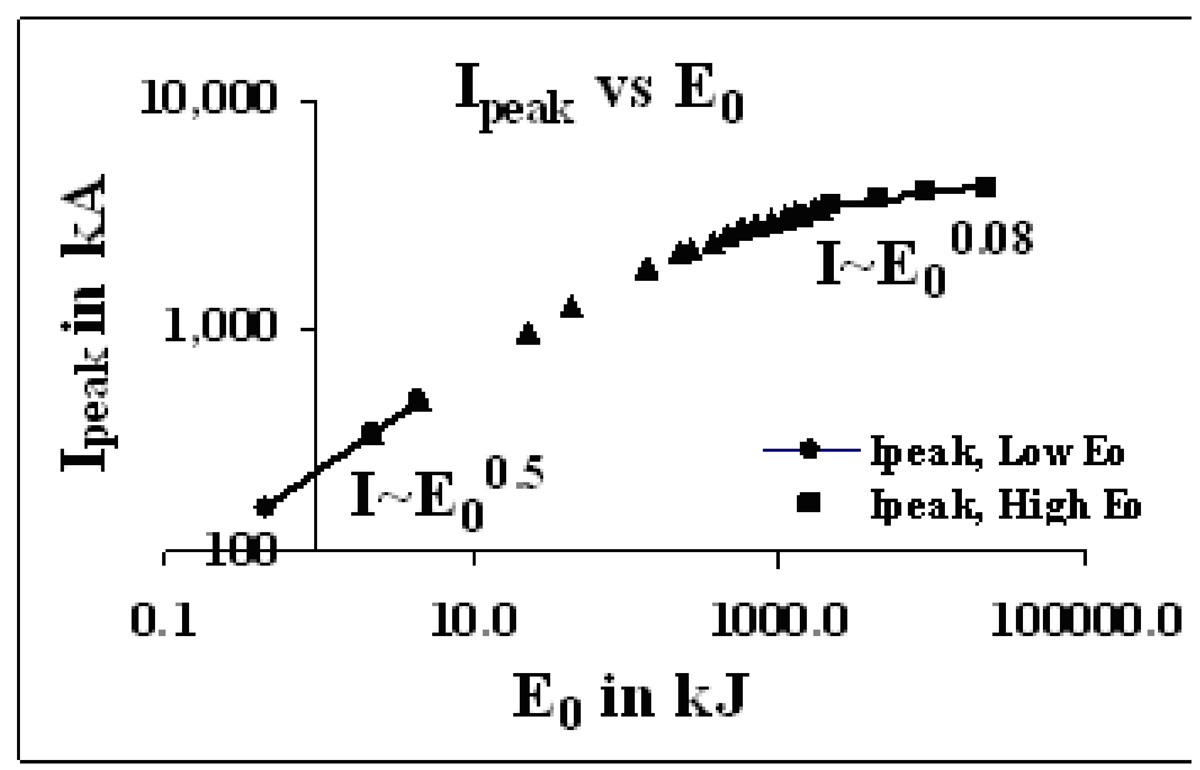
- The axial speed loads the discharge circuit with a dynamic resistance
- The same axial speed over the range of devices means the same dynamic resistance constituting a load impedance DR_0
- Small PF's : have larger generator impedance $Z_0=[L_0/C_0]^{0.5}$ than DR_0
- As energy is increased by increasing C_0, generator impedance Z_0 drops

What causes current scaling deterioration and eventual saturation? 2/3

- At E_0 of kJ and tens of kJ the discharge circuit is dominated by Z_0
- Hence as E_0 increases, $I \sim C_0^{-0.5}$
- At the level typically of 100 kJ, Z_0 has dropped to the level of DR_0; circuit is now no longer dominated by Z_0; and current scaling deviates from $I \sim C_0^{-0.5}$, beginning of current scaling deterioration.
- At MJ levels and above, the circuit becomes dominated by DR_0, current saturates

Deterioration and eventual saturation of I_{peak} as capacitor energy increases

- Axial phase dynamic resistance causes current scaling deterioration as E0 increases



In numerical experiments we showed:

- $Y_n \sim I_{pinch}^{4.5}$
- $Y_n \sim I_{peak}^{3.8}$
- Hence deterioration of scaling of I_{peak} will lead to deteriorationof scaling of Y_n.

What causes current scaling deterioration and eventual saturation? 3/3

- Analysis using the Lee model code has thus shown that the constancy of the dynamic resistance causes the current scaling deterioration resulting in the deterioration of the neutron yield and eventual saturation.
- This puts the global scaling law for neutron yield on a firmer footing

Connecting the scaling properties with the global scaling law (1/3)

- At kJ level; experimentally observed $Y_n \sim E_0^2$
- Ideal scaling at the highest convenient voltage V_0: $I \sim V_0 / Z_0$ at low energy level where Z_0 dominates
- leading to $I \sim E_0^{0.5}$ for optimised low L_0
- and $Y_n \sim I_0^4$

- At higher energy around 100kJ, Z_0 domination ends and current deterioration starts

Connecting the scaling properties with the global scaling law (2/3)

- Lower current increase than the ideal leads to lower increase in anode radius ‘a’
- This leads to lower increase in pinch volume and pinch duration
- Which leads to lower increase in yield

Connecting the scaling properties with the global scaling law (3/3)

- Finally at very high energies, current hardly increases anymore with further increase in energy
- The anode radius should not be increased anymore; only its length should be increased
- Hence pinch volume and duration also will not increase anymore.

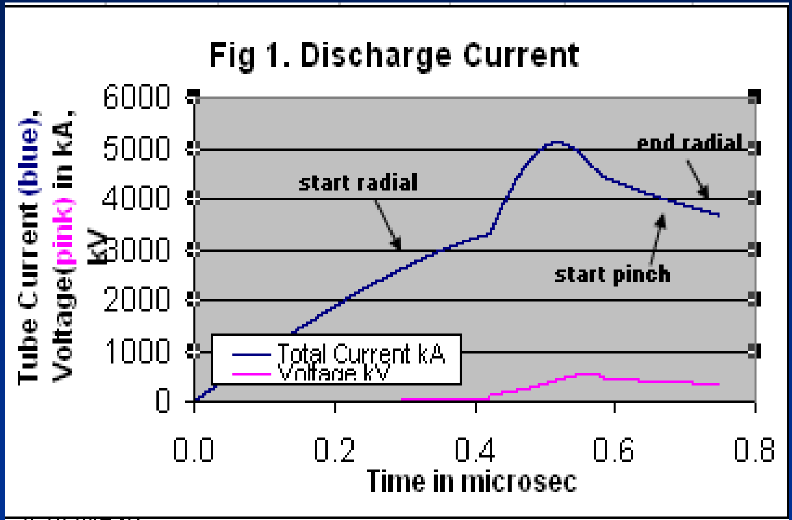
Thus we relate yield scaling deterioration & yield saturation to scaling properties, the fundamental one being the dynamic resistance.

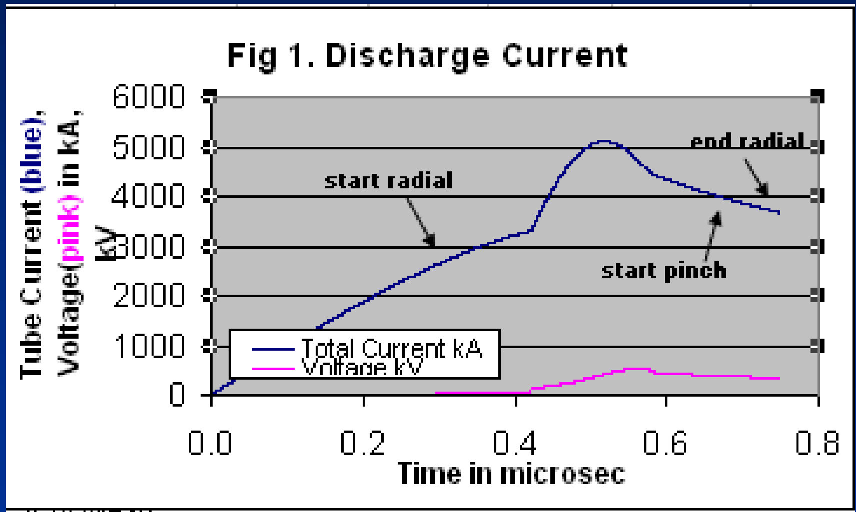
Into the Future-Beyond Saturation Plasma Focus?

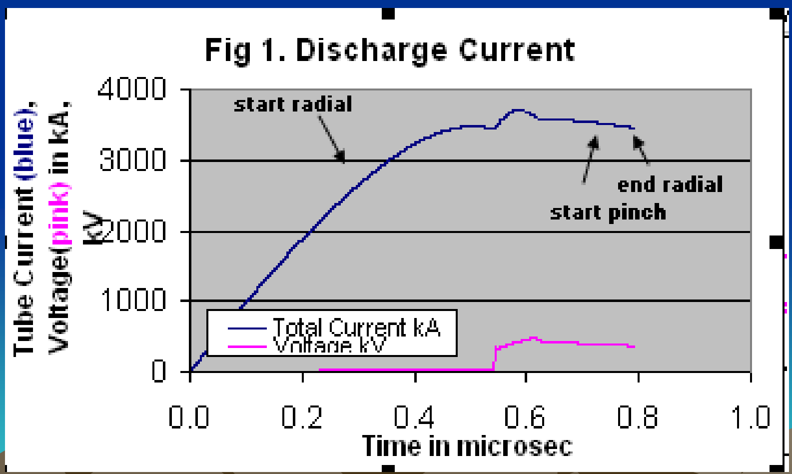
Current Stepped pinch: b= 12cm, a= 8cm, z0= 2cm; 2 capacitor banks: L1= 30nH, C1= 8uF, r0=6mW, V1= 300kV; L2= 15nH, C2= 4 uF, r0=6.3 6mW, V2= 600kV; P0= 12 Torr D

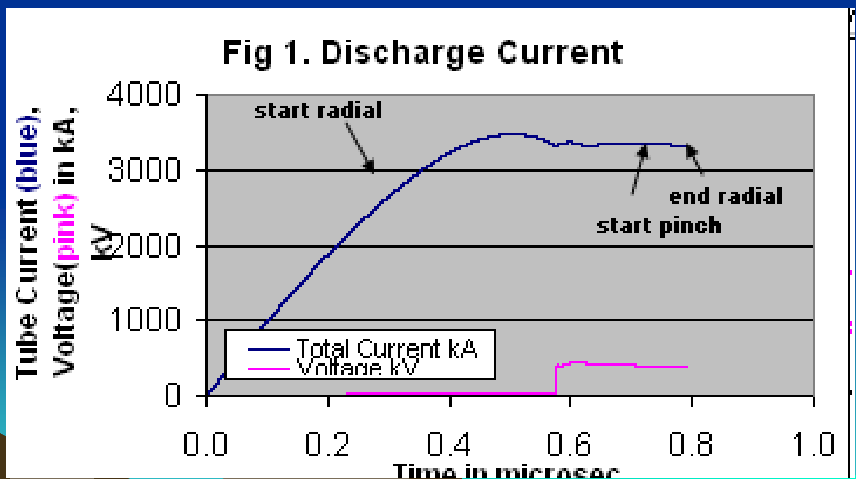
C2 switched after radial start when r=0.8a,Yn= 1..2E12; r=0.6a, Yn= 1.5E12; r=0.5a, Yn= 1.8E12; r=0.4a, Yn= 1.9E12

IPFS-INTI Series 10, 10 October 2010 RADPF15.15d CS







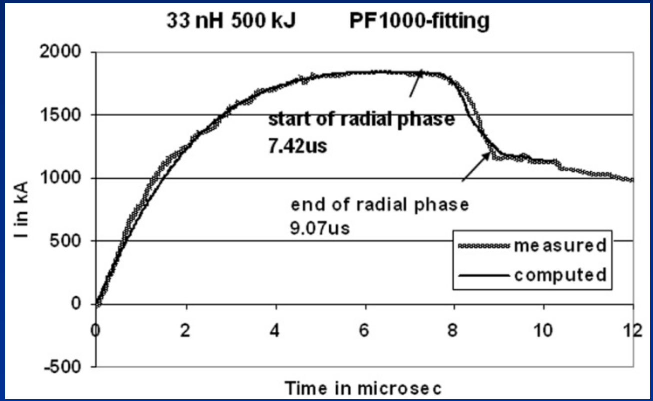


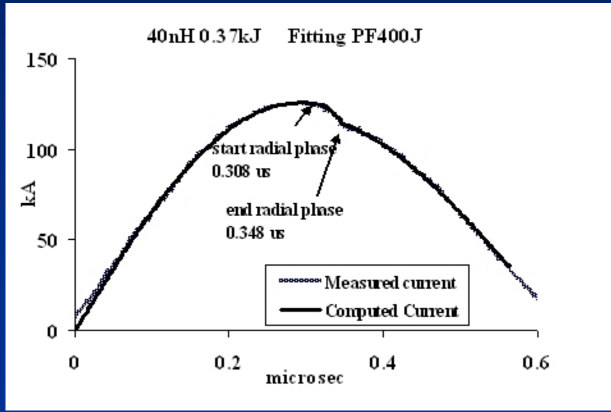
A New Development- 6 Phase Model 1/4

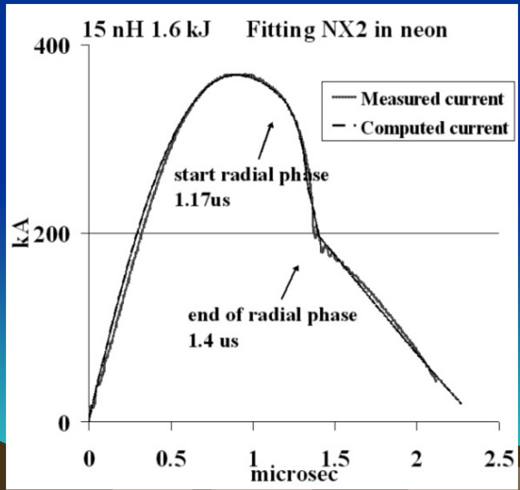
All well-published PF machines are well-fitted:

see following examples and many others;

note: the fit for the axial phase, and for the radial phase



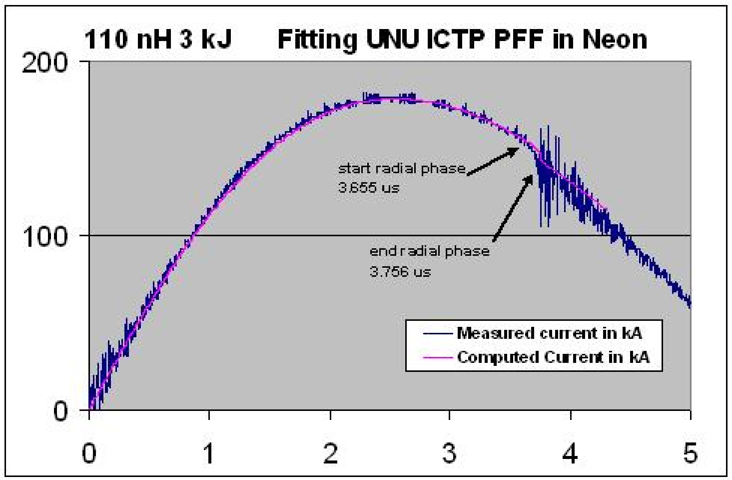




A New Development- 6 Phase Model

Only one well-published machine did not fit

- **UNU ICTP PFF- famed low-cost sharing network; current signal noisy and dip is small; difficult to judge the fitting- suspected ill-fit**
- **Low cost- necessitates single capacitor- hence high inductance L_0**

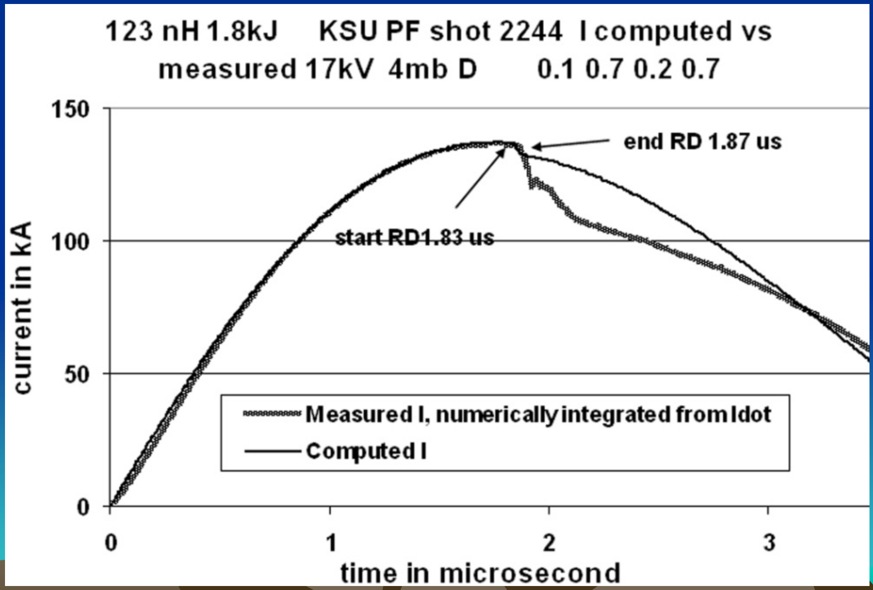


A New Development- 6 Phase Model 3/4

Recently KSU commissioned a machine; a modernised version of the UNU ICTP PFF

- A good Rogowski system was developed to measure dI/dt; which was then numerically integrated resulting in a clean current signal-

Best fit nowhere near the fit of the well-published machines- in fact clearly could only fit a small portion of the radial phase



A New Development- 6 Phase Model 4/4

A study followed; resulting in classifying plasma focus devices into T1 & T2

Table 1. Classification of Plasma Focus Machines (D_2 operation).

PF name	L_0 (nH)	C_0 (μF)	I_{peak} (kA)	R_L	R_{EL}	*RD* dip (%)	Type
Poseidon	17.7	156	3205	0.9	2.5	32	*T1*
PF1000	33.5	1332	1845	1	1.6	34	*T1*
DPF78	55	17.2	869	4.1	12.8	11	*T1*
FN-II	75	7.5	309	4.3	8.5	10	*T1*
FMPF1	31	2.4	81	6.9	8.6	14.5	*T1*
PF-400J	40	1	126	8.8	17.3	8	*T1*
UNUICTP	110	30	163	16.7	29.5	1.9	*T2*
KSU	123	12.5	137	21.4	40	1.5	*T2*

Differentiator: L_0

Better Differentiators: $R_L=(L_0 + L_a)/L_p$

$R_{EL}=(E_{L0}+E_{La})/E_{LPinch}$

Physical explanation 1/2

- RD mechanism for pinch purely compressive
- At end of RD (call this REGULAR DIP), expts show other effects eg instabilities leading to anomalous resistance- these mechanisms not modelled by 5-phase Lee code
- These anomalous resistive effects will absorb further energy from pinch; will result in further current dips- called EXTENDED DIP, ED

Physical explanation 2/2
Our studies further concluded

- T1: Small L_0 lead to big RD and relatively small ED
- T2: Big L_0 lead to small RD and relatively big ED

This explains why the 5-phase model:

For T1: the model parameters can be stretched for the RD to 'absorb' the ED

For T2: the model parameters, stretch how one likes, the RD cannot 'absorb' the ED

Development of the 6th phase 1/2
ie Phase 4a, between 4 and 5

- We have simulated using anomalous resistance of following form:

$$R=R_0[\exp(-t/t_2)-\exp(-t/t_1)]$$

Where R0 is of order of 1 Ohm, t1 controls rise time of the anomalous resistance and t2 controls the fall time (rate)

Use one term to fit one feature; terminate the term

Then use a 2nd term to fit a 2nd feature and so on

Development of the 6th phase 2/2
Simulated Anomalous Resistance Term

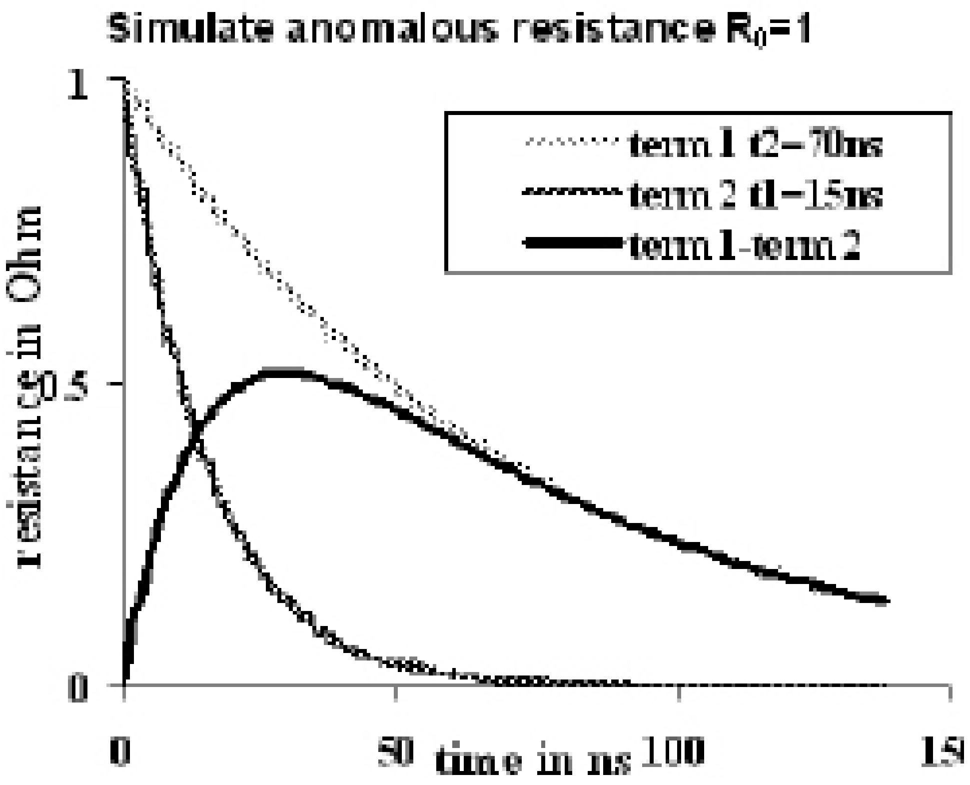


Figure 6. Simulating anomalous resistance.

Result of Phase 4a fitting 1/3
Applied to KSU Current Trace

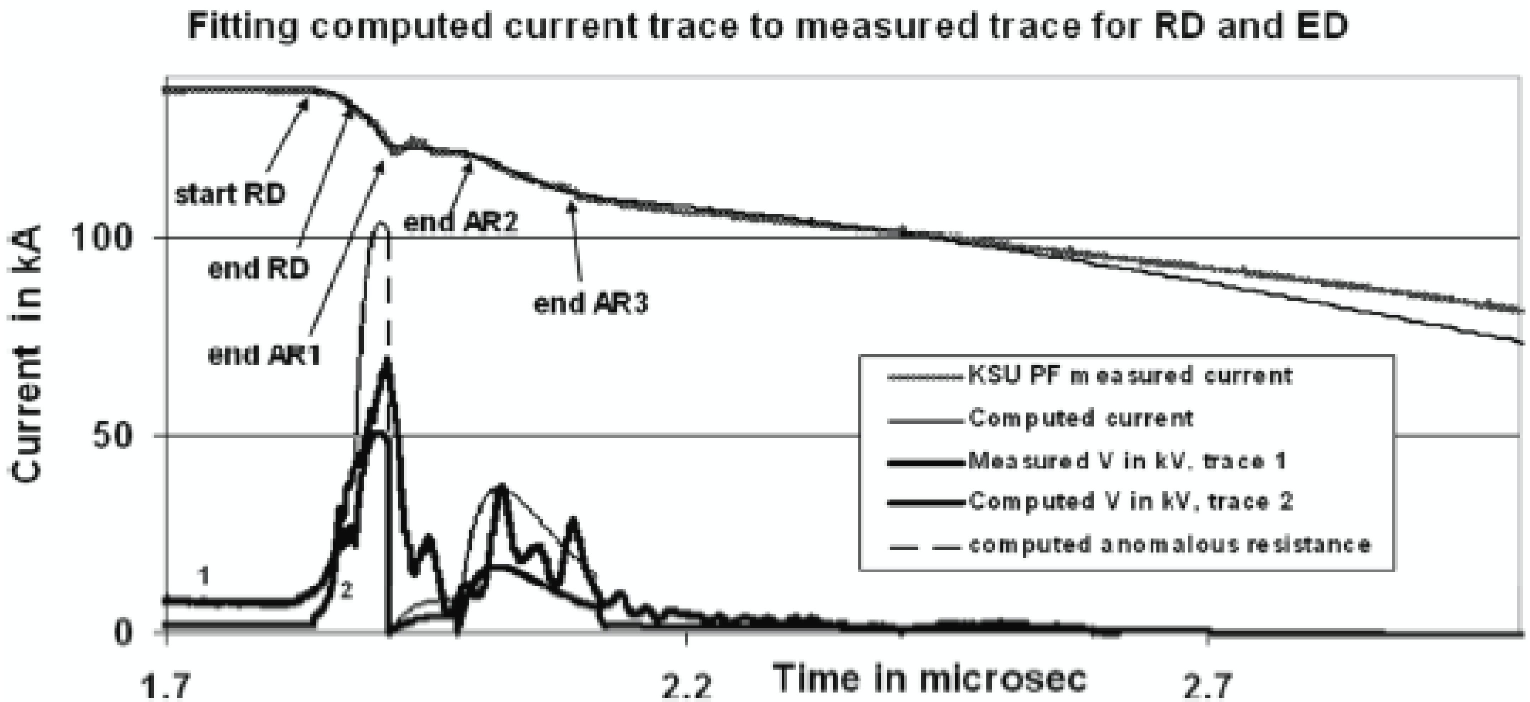


Figure 7. Computed Current (dip region only and expanded to see details) fitted to measured current with inclusion of Phase 4a.

Result of Phase 4a fitting 2/3

Table 3. Anomalous resistances used for the fitting.

	R_0 (Ω)	t_1 (ns)	t_2 (ns)	*endfraction*
Dip 1 *ED*	1.0	70	15	0.53
Dip 2	0.2	70	40	0.4
Dip 3	0.5	70	25	1.0

S Lee, S H Saw, A E Abdou and H Torreblanca- Characterizing plasma focus devices- role of the static inductance- instability phase fitted by anomalous resistances-submitted to Plasma Phys Controlled Fusion for publication

Result of Phase 4a fitting 3/3

- **Current ED now fitted very well**
- **Fig also shows the form of the fitted anomalous resistance (3 terms)**
- **Figure shows that the computed tube voltage waveform also shows features in agreement with the measured tube voltage waveform**
- **The product of this Phase 4a fitting is the magnitude and temporal form of the anomalous resistance. This is an important experimental result. The information is useful to elaborate further on the instability mechanisms.**
- **Moreover even for the T1 current waveforms, we should fit by first just fitting the RD using the 5-phase model; ie the part that fits well with the computed is the RD; the rest of the dip is then fitted using phase 4a.**

Plasma Focus Numerical Experiments-Trending into the Future

Part II: Concepts into the Future

Part II: Conclusion

- Global Neutron scaling law
- Yield deterioration & saturation explained
- Dynamic Resistance-Cause of "Neutron Saturation" thus connecting scaling property to scaling law
- Beyond present saturation?

- New classification of plasma focus devices into T1 & T2 results in the new 6-phase model
- Simulate by means of anomalous resistance(s)
- Result in new quantitative data of anomalous resistance

Papers from Lee model code 1\2

- S Lee and S H Saw, "Pinch current limitation effect in plasma focus," Appl. Phys. Lett. 92, 2008, 021503.
- S Lee and S H Saw, "Neutron scaling laws from numerical experiments," J Fusion Energy 27, 2008, pp. 292-295.
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- S H Saw and S Lee, *Scaling the Plasma Focus for Fusion Energy Considerations*- Int. J. Energy Res. (2010) Int. J. Energy Res. (2010) View this article online at wileyonlinelibrary.com. DOI: 10.1002/er.1758
- S H Saw and S Lee- Scaling laws for plasma focus machines from numerical experiments

  Invited paper *Energy and Power Engineering*, 2010, 65-72 doi:10.4236/epe.2010.21010

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