The Abdus Salam
International Centre for Theoretical Physics

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IAEA

2168-Presentation

Joint ICTP-IAEA Workshop on Dense Magnetized Plasma and Plasma Diagnostics

15 - 26 November 2010

The effect dust size distribution on quantum dust acoustic waves in ultra-dense dusty plasma

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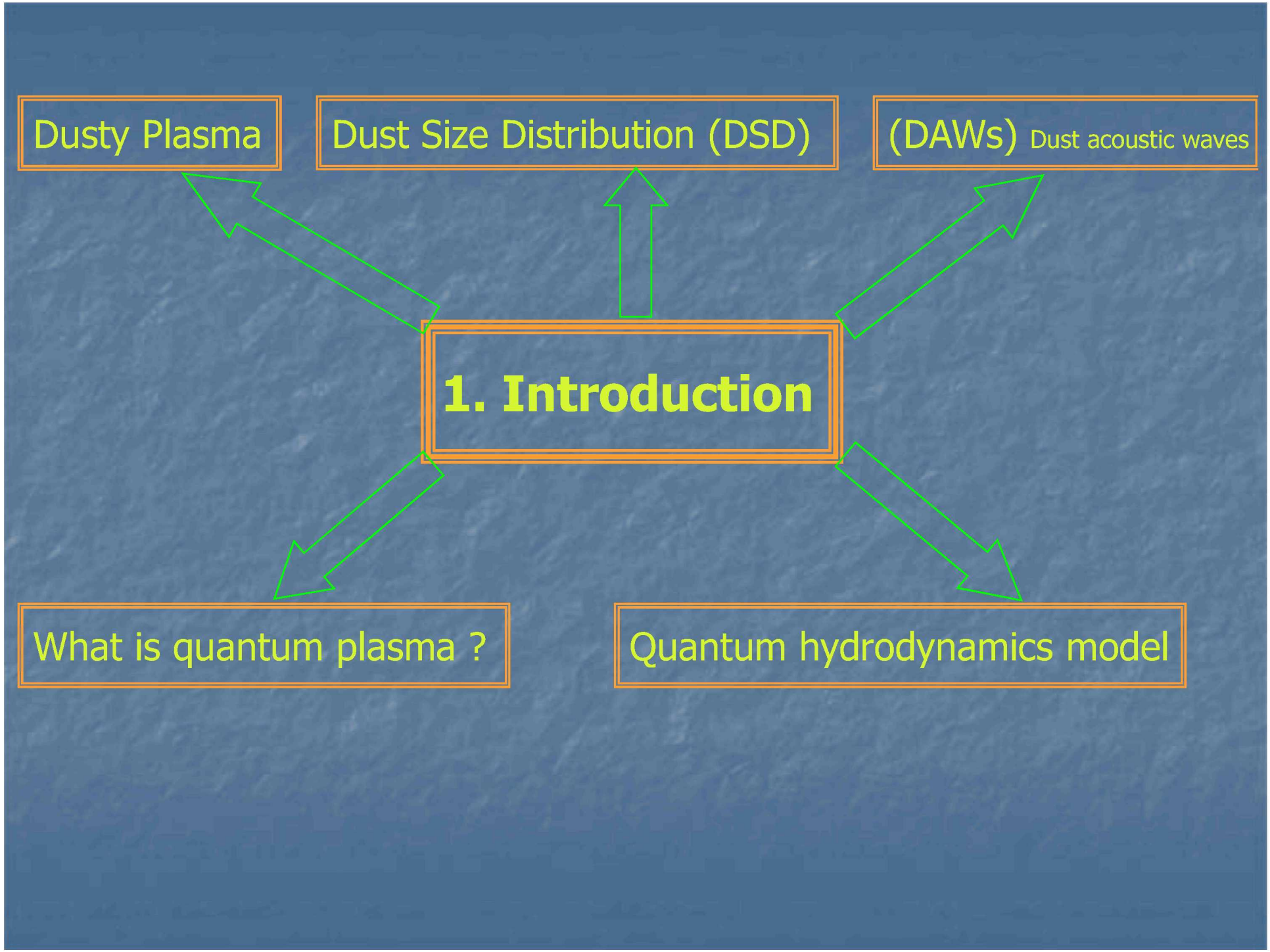
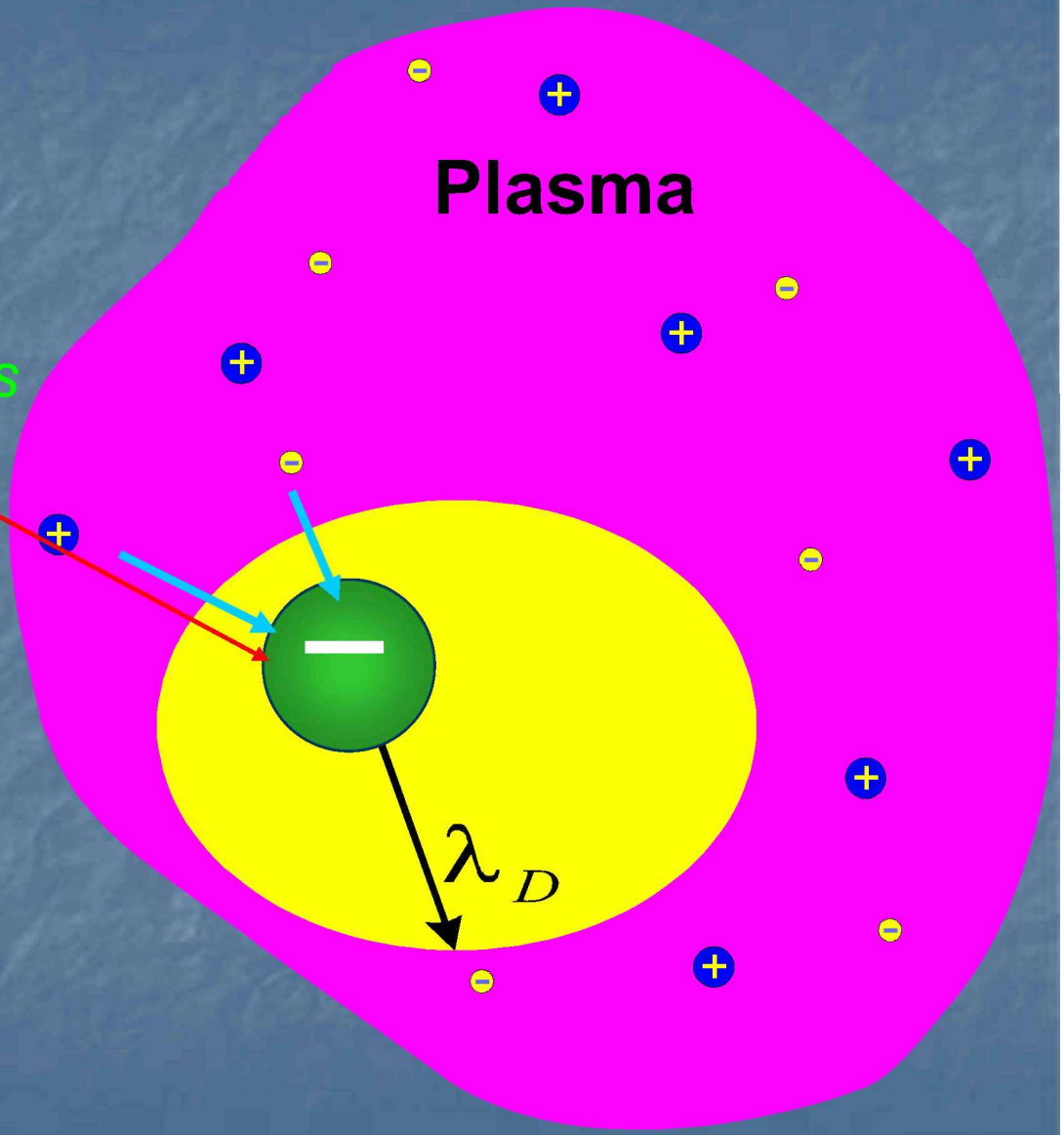

Dusty Plasma
Dust Size Distribution (DSD)
(DAWs) Dust acoustic waves
1. Introduction
What is quantum plasma ?
Quantum hydrodynamics model

Dusty Plasma

plasma = electrons + ions

dusty plasma
= plasma + small particles
of solid matter

- absorbs electrons and ions
- becomes charged
- Debye shielding



DSD

*** The size of the dust grains do not have the same radius, but differs within the range [rmin ,rmax]

*** DSD affects the main characteristics of the plasma systems

*** There two common DSD functions; the power law distribution and the polynomial distribution because of their various applications in space Plasmas and semiconductor technology.

Power law DSD

$$n_d(r)dr = Kr^{-\beta}dr,$$

Polynomial DSD

$$n_d(r)dr = [a_o + a_1 r + a_2 r^2 + a_3 r^3 + \ldots]dr,$$

where $n_d(r)\,dr$ is the number density of the dust grains per unit volume with radii ranges from r to r + dr.

β is the power law index and K is the normalization constant.

The total number density of all grains is given by:

$$N_{tot} = \int_{r_{min}}^{r_{max}} n_d(r) dr$$

The mass and the charge of the j th dust grain can be approximated as:

$$m_{dj} = K_m r_j^3,$$

$$Z_{dj} = K_z r_j.$$

where $K_m \left(\simeq \frac{4}{3}\pi\rho_d\right)$ is the mass constant with the assumption that the mass density of the dust grains ρ_d is the same for all grains, and K_z $(\simeq 4\pi\epsilon_o V_o/e)$ is the charge constant when the electric surface potential of the dust grains V_o is constant. ϵ_o is the permittivity of free space.

For the monosized dust

$$\bar{r} = \frac{\int r n_d(r)\, dr}{N_{tot}}$$

DAWs

- The creation of the DAWs is due to the restoring force comes from the pressure of the inertia less electrons and ions, while the inertia is provided by the dust mass.

What is Quantum Plasma?

When the de-Broglie wavelength of particles composing the plasma can be comparable with the average distance between plasma particles, quantum mechanical effects play an important role

$$\lambda_B \geq d$$
$$d = n^{-1/3}$$
$$n\lambda_B^3 \geq 1$$

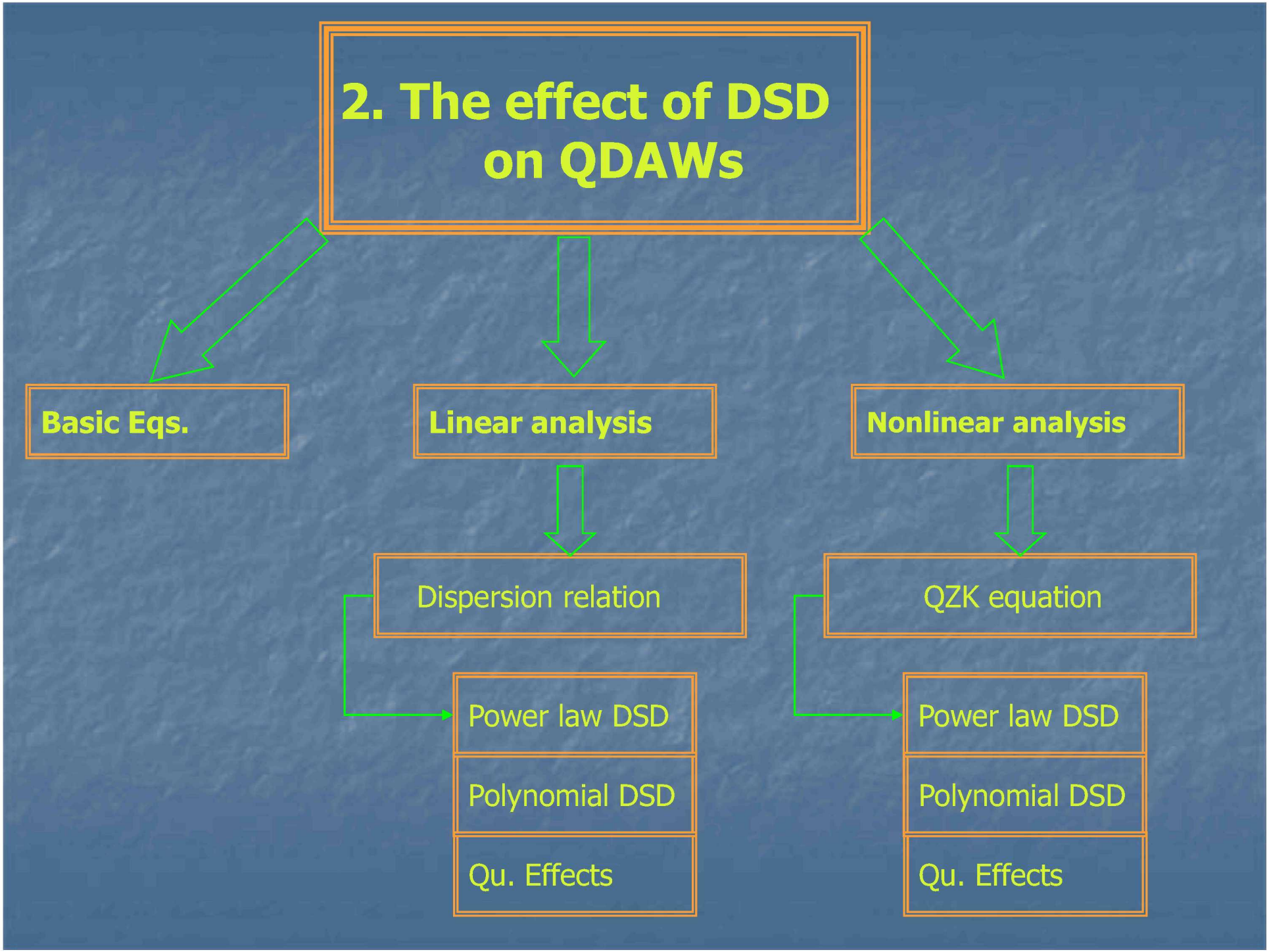
$$\lambda_B = \frac{\hbar}{m\, V_T},$$

Regimes of high n and low T have important quantum effects.

The quantum hydrodynamics model (QHD)

The QHD model is based on introducing the quantum correction term, the so-called "Bohm Potential", through the usual fluid equations.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{e}{m}\frac{\partial \phi}{\partial x} + \frac{\hbar^2}{2m^2}\frac{\partial}{\partial x}\frac{\frac{\partial^2}{\partial x^2}\sqrt{n}}{\sqrt{n}} - \frac{1}{mn}\frac{\partial P}{\partial x}.$$


2. The effect of DSD on QDAWs
Basic Eqs.
Linear analysis
Nonlinear analysis
Dispersion relation
QZK equation
Power law DSD
Polynomial DSD
Qu. Effects
Power law DSD
Polynomial DSD
Qu. Effects

Basic equation

According to the quantum statistics, the equation of state for plasma particles is given by:

$$P_s = \frac{m_s V_{Fs}^2}{3 n_{so}^2} n_s^3 ,$$

where n_s is the particle number density and n_{so} is its equilibrium value ($s = e$ for electrons, i for ions and dj for the j^{th} dust grain. $V_{Fs} = (2T_{Fs}/m_s)^{1/2}$ is the Fermi thermal speed and m_s is the particle mass. At equilibrium, the charge neutrality condition reads

$$n_{io} = n_{eo} + \sum_{j=1}^{N} Z_{dj} n_{djo} ,$$

Thus, the system of normalized equations will be:

$$\frac{\partial n_{dj}}{\partial t} + \nabla.(n_{dj}u_{dj}) = 0,$$

$$\frac{\partial u_{dj}}{\partial t} + (u_{dj}.\nabla)u_{dj} = \frac{Z_{dj}}{m_{dj}}\nabla\phi - \frac{\sigma_d}{m_{dj}}n_{dj}\nabla n_{dj} - \frac{Z_{dj}}{m_{dj}}\Omega(u_{dj}\times\hat{x}) + \frac{H_d^2}{2m_{dj}^2}\nabla[\frac{\nabla^2\sqrt{n_{dj}}}{\sqrt{n_{dj}}}],$$

$$\nabla^2\phi = \mu_e n_e - \mu_i n_i + \sum_{j=1}^{N} Z_{dj}n_{dj},$$

$$0 = \nabla\phi - \sigma_e n_e\nabla n_e + \frac{H_e^2}{2}\nabla[\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}],$$

$$0 = -\nabla\phi - \sigma_i n_i\nabla n_i + \frac{H_i^2}{2}\nabla[\frac{\nabla^2\sqrt{n_i}}{\sqrt{n_i}}].$$

We considered the following normalization:

$$n_e \to n_e/n_{eo},\ n_i \to n_i/n_{io},\ n_{dj} \to n_{dj}/N_{tot},\ Z_{dj} \to Z_{dj}/\overline{Z_{do}},\ m_{dj} \to m_{dj}/\overline{m_d},\ m_e \to m_e/\overline{m_d},$$
$$m_i \to m_i/\overline{m_d},\ u_{dj} \to u_{dj}/c_d,\ \phi \to e\phi/T_{eff},\ t \to t\omega_{pd},\ \nabla \to \nabla/\lambda_D$$

$$T_{eff} = \frac{T_i T_e}{\mu_i T_i + \mu_e T_e},\quad N_{tot} = \sum_{j=1}^{N} n_{dj},\quad \overline{Z_{do}} = \frac{1}{N_{tot}} \sum_{j=1}^{N} Z_{dj} n_{dj},\quad \overline{m_d} = \frac{1}{N_{tot}} \sum_{j=1}^{N} m_{dj} n_{dj}$$

$$c_d = \sqrt{\frac{2\overline{Z_{do}} T_{eff}}{\overline{m_d}}},\quad \omega_{pd} = \sqrt{\frac{4\pi e^2 \overline{Z_{do}}^2 N_{tot}}{\overline{m_d}}},\quad \lambda_D = \sqrt{\frac{T_{eff}}{2\pi e^2 \overline{Z_{do}} N_{tot}}},\quad \Omega = \frac{e\overline{Z_{do}} B_o}{\overline{m_d}\, c\, \omega_{pd}},$$

$$\sigma_e = \frac{T_e}{T_{eff}},\ \sigma_i = \frac{T_i}{T_{eff}},\ \sigma_d = \frac{T_{Fd}}{\overline{Z_{do}} T_{eff}},\ \mu_i = \frac{n_{io}}{\overline{Z_{do}} N_{tot}},\ \mu_e = \frac{n_{eo}}{\overline{Z_{do}} N_{tot}},$$

$$H_e^2 = \frac{\hbar^2 \overline{Z_{do}} \omega_{pd}^2}{m_e c_d^4},\ H_i^2 = \frac{\hbar^2 \overline{Z_{do}} \omega_{pd}^2}{m_i c_d^4},\ H_d^2 = \frac{\hbar^2 \omega_{pd}^2}{c_d^4 \overline{m_d}^2}.$$

Linear analysis

Linearizing the system of equations and assuming that all perturbed quantities vary as $e^{ikr-\gamma t}$

$$
\begin{aligned}
n_{dj} &= n_{djo} + n_{dj1} e^{ikr-\omega t}, \\
n_i &= 1 + n_{i1} e^{ikr-\omega t}, \\
n_e &= 1 + n_{e1} e^{ikr-\omega t}, \\
u_{dj} &= u_{dj1} e^{ikr-\omega t}, \\
\phi &= \phi_1 e^{ikr-\omega t},
\end{aligned}
$$

The dispersion relation:

$$
\omega^2 = \frac{k^2 \sum_{j=1}^{N} \frac{Z_{dj}^2 n_{djo}}{m_{dj}}}{k^2 + \frac{\mu_e}{\sigma_e + \frac{k^2 H_e^2}{4}} + \frac{\mu_i}{\sigma_i + \frac{k^2 H_i^2}{4}}}.
$$

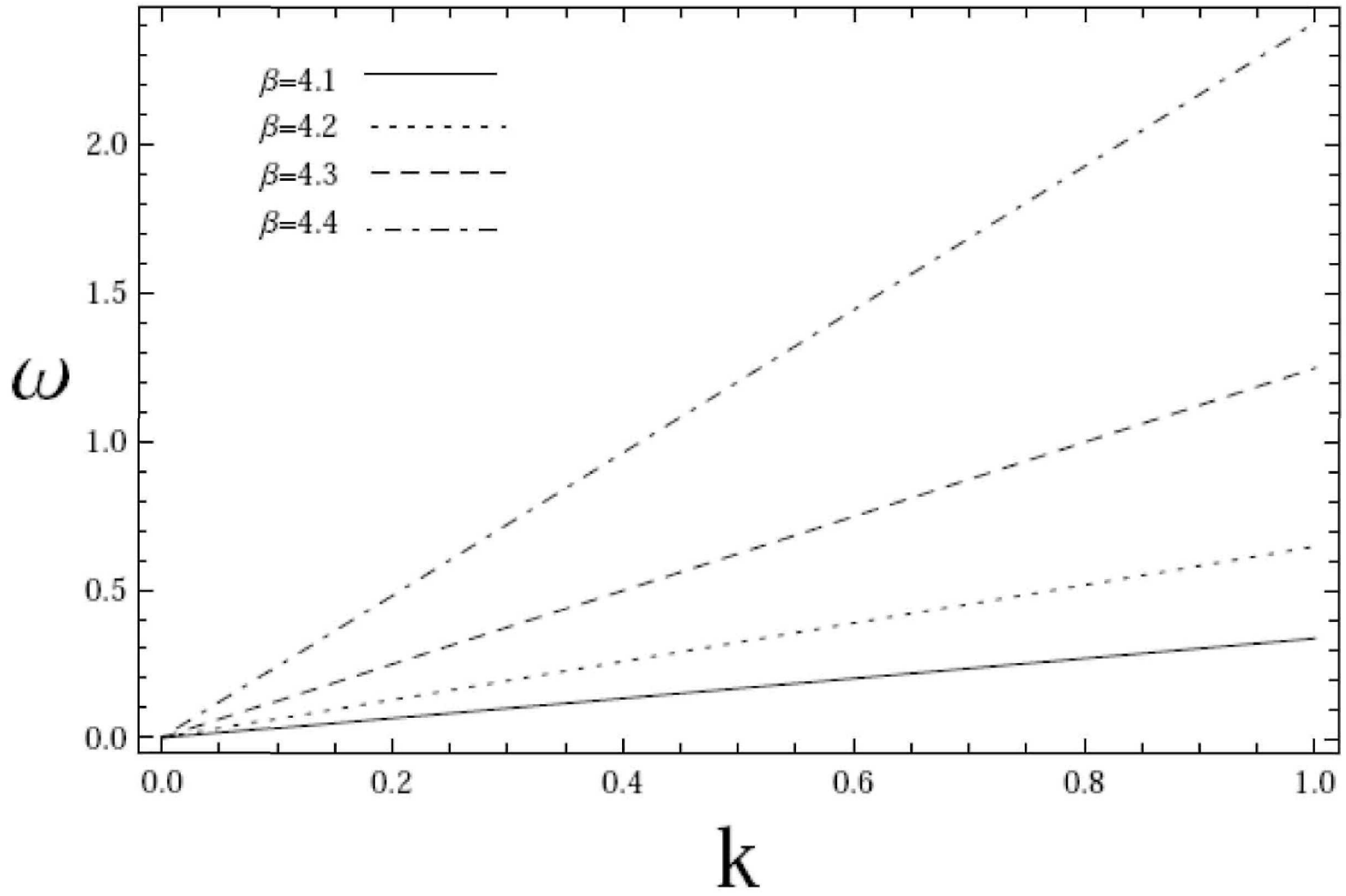


Figure 2.1: The linear dispersion relation of QDAW at different β values for the power law DSD.

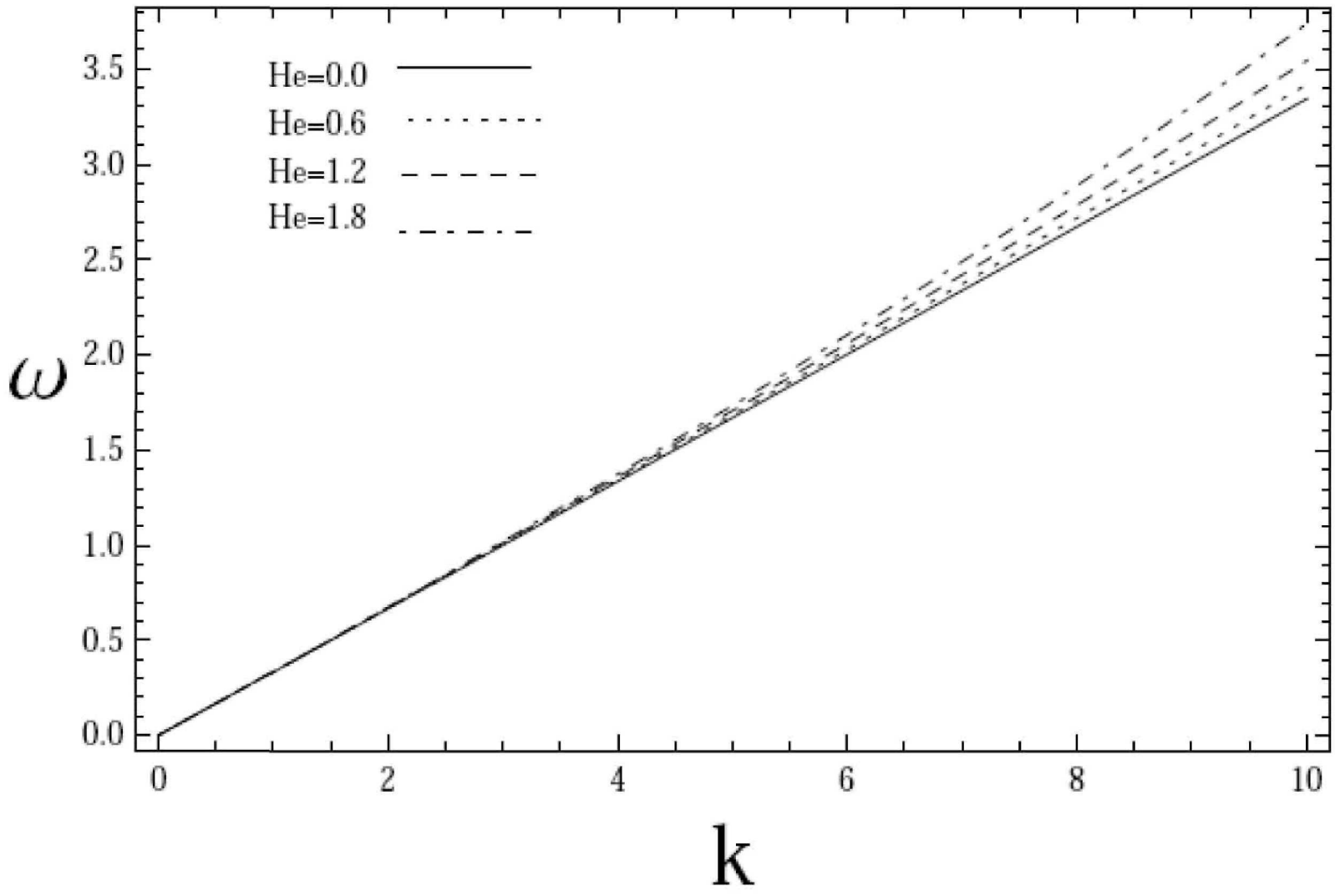


Figure 2.3: The linear dispersion relation of QDAW at different H_e values for the power law DSD at $\beta = 4.1$.

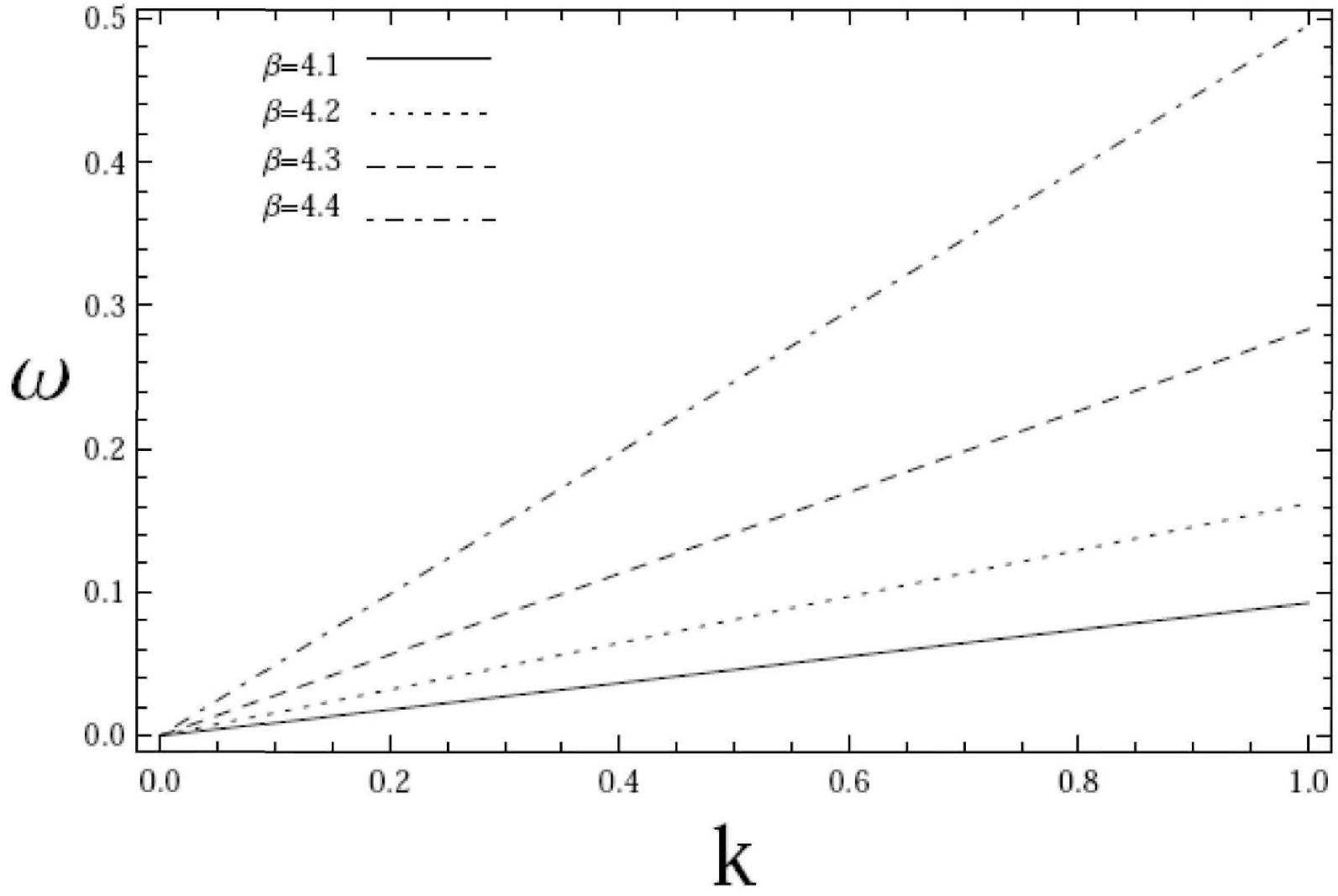


Figure 2.4: The linear dispersion relation of QDAW at different β values for the polynomial DSD.

Nonlinear analysis

Using the RPT method, let us introduce the following stretched coordinates

$$X = \epsilon(x - \lambda t), Y = \epsilon y, \quad Z = \epsilon z, \quad T = \epsilon^3 t.$$

$$\begin{aligned}
n_{dj} &= n_{djo} + \epsilon^2 n_{dj1} + \epsilon^4 n_{dj2} + ... \\
n_e &= 1 + \epsilon^2 n_{e1} + \epsilon^4 n_{e2} + ... \\
n_i &= 1 + \epsilon^2 n_{i1} + \epsilon^4 n_{i2} + ... \\
u_{djx} &= \epsilon^2 u_{djx1} + \epsilon^4 u_{djx2} + ... \\
u_{djy} &= \epsilon^3 u_{djy1} + \epsilon^4 u_{djy2} + ... \\
u_{djz} &= \epsilon^3 u_{djz1} + \epsilon^4 u_{djz2} + ... \\
\phi &= \epsilon^2 \phi_1 + \epsilon^4 \phi_2 + ...
\end{aligned}$$

The soliton velocity is calculated as

$$\lambda^2 = \sum_{j=1}^{N} \frac{n_{djo} Z_{dj}^2}{m_{dj}}.$$

QZK

$$\frac{\partial \phi_1}{\partial T} + A\phi_1 \frac{\partial \phi_1}{\partial X} + B\frac{\partial^3 \phi_1}{\partial X^3} + C\left(\frac{\partial^3 \phi_1}{\partial X \partial Y^2} + \frac{\partial^3 \phi_1}{\partial X \partial Z^2}\right) = 0,$$

$$A = \frac{Z_{dj}^2}{-m_{dj}\lambda^2 + \sigma_d n_{djo}^2}\left[1 + \frac{\lambda}{2} + \frac{\sigma_d n_{djo}^2}{2m_{dj}\lambda} - \frac{\left(-m_{dj}\lambda^2 + \sigma_d n_{djo}^2\right)^2}{2m_{dj}\lambda Z_{dj}}\left(\frac{\mu_e}{\sigma_e^2} - \frac{\mu_i}{\sigma_i^2}\right)\right]$$

$$B = \frac{m_{dj}\lambda^2 - \sigma_d n_{djo}^2}{2m_{dj}\lambda}\left(1 - \frac{\mu_e H_e^2}{4\sigma_e^2} - \frac{\mu_i H_i^2}{4\sigma_i^2}\right) - \frac{H_d^2}{8\lambda m_{dj}^2},$$

$$C = B + \frac{m_{dj}^2 \lambda^3}{2\Omega^2 Z_{dj}^2}.$$

The stationary solution of QZK equation:

$$\phi_1 = \phi_{1m} Sech^2\left(\eta / W\right), \qquad \eta = l_x X + l_y Y + l_z Z - MT,$$

w is the soliton width → $$W = 2\left[\frac{Bl_x^3 + Cl_x(l_y^2 + l_z^2)}{M}\right]^{1/2},$$

ϕ_{1m} is the amplitude of the soliton → $$\phi_{1m} = 3M/Al_x,$$

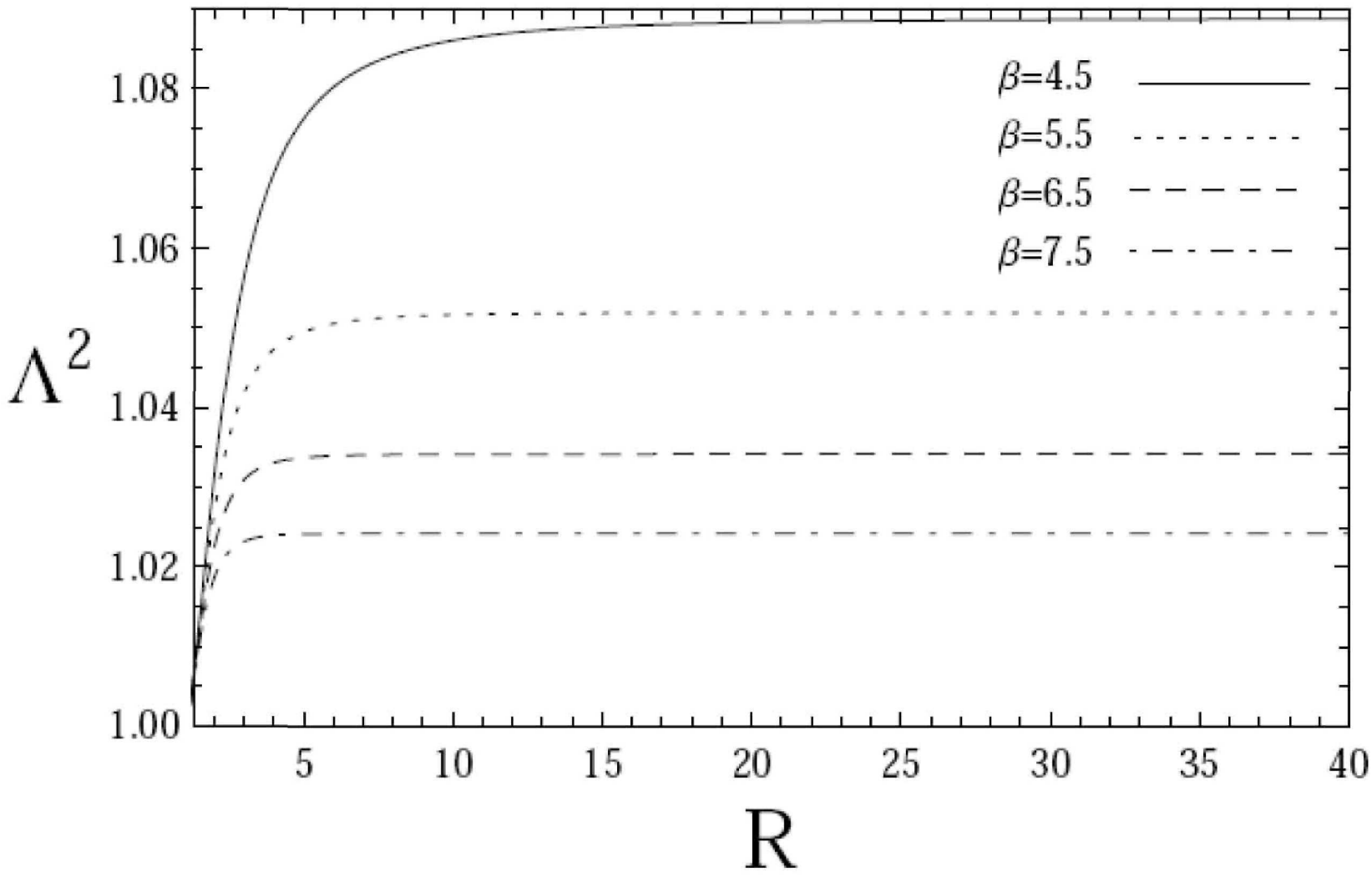


Figure 2.7: The variation of $\Lambda^2 \left(= \lambda^2/\overline{\lambda^2}\right)$ versus R at different β values for power law DSD.

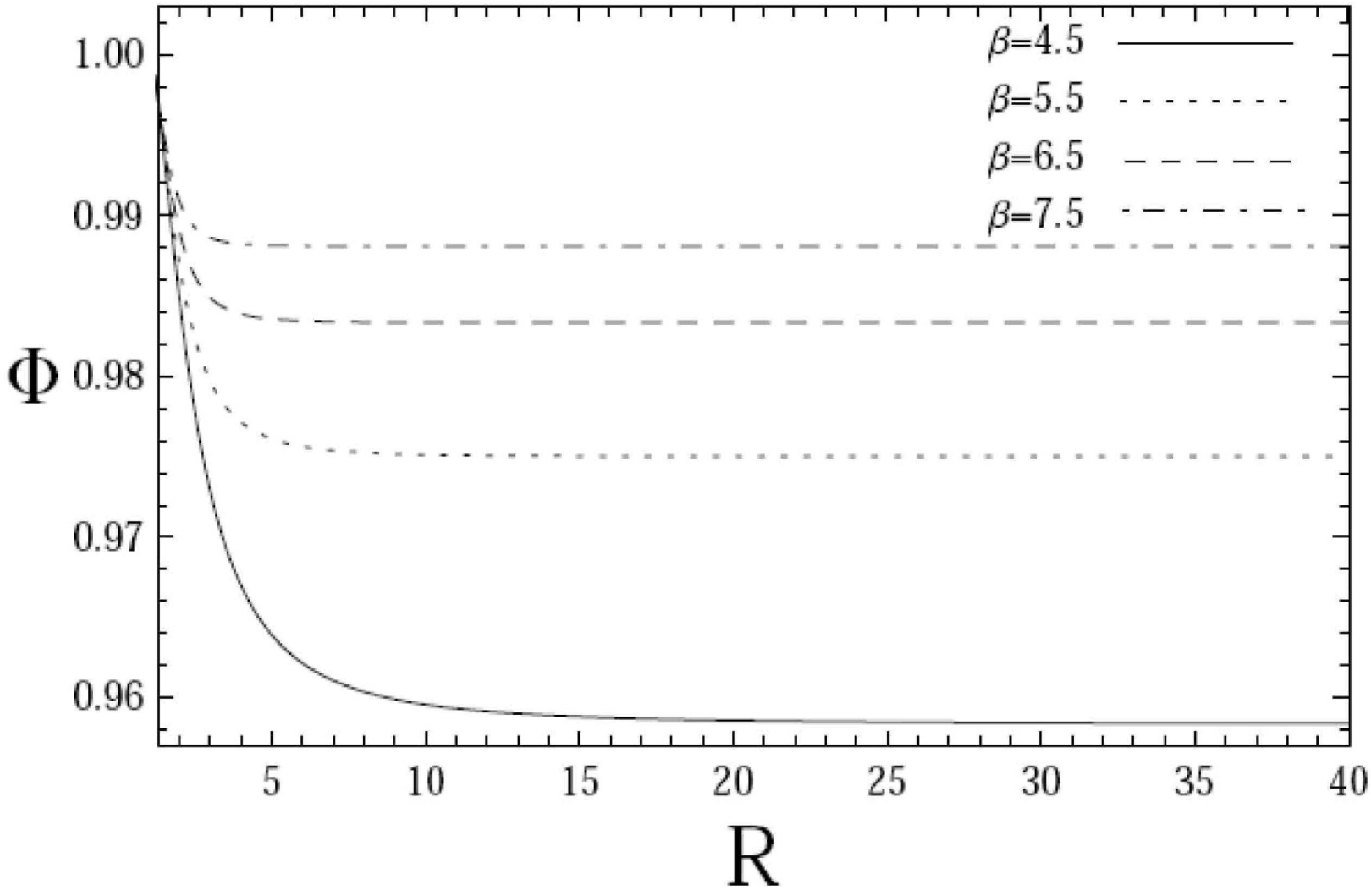


Figure 2.8: The amplitude ratio $\Phi = \phi_{1m} / \overline{\phi_{1m}}$ against R at different β values

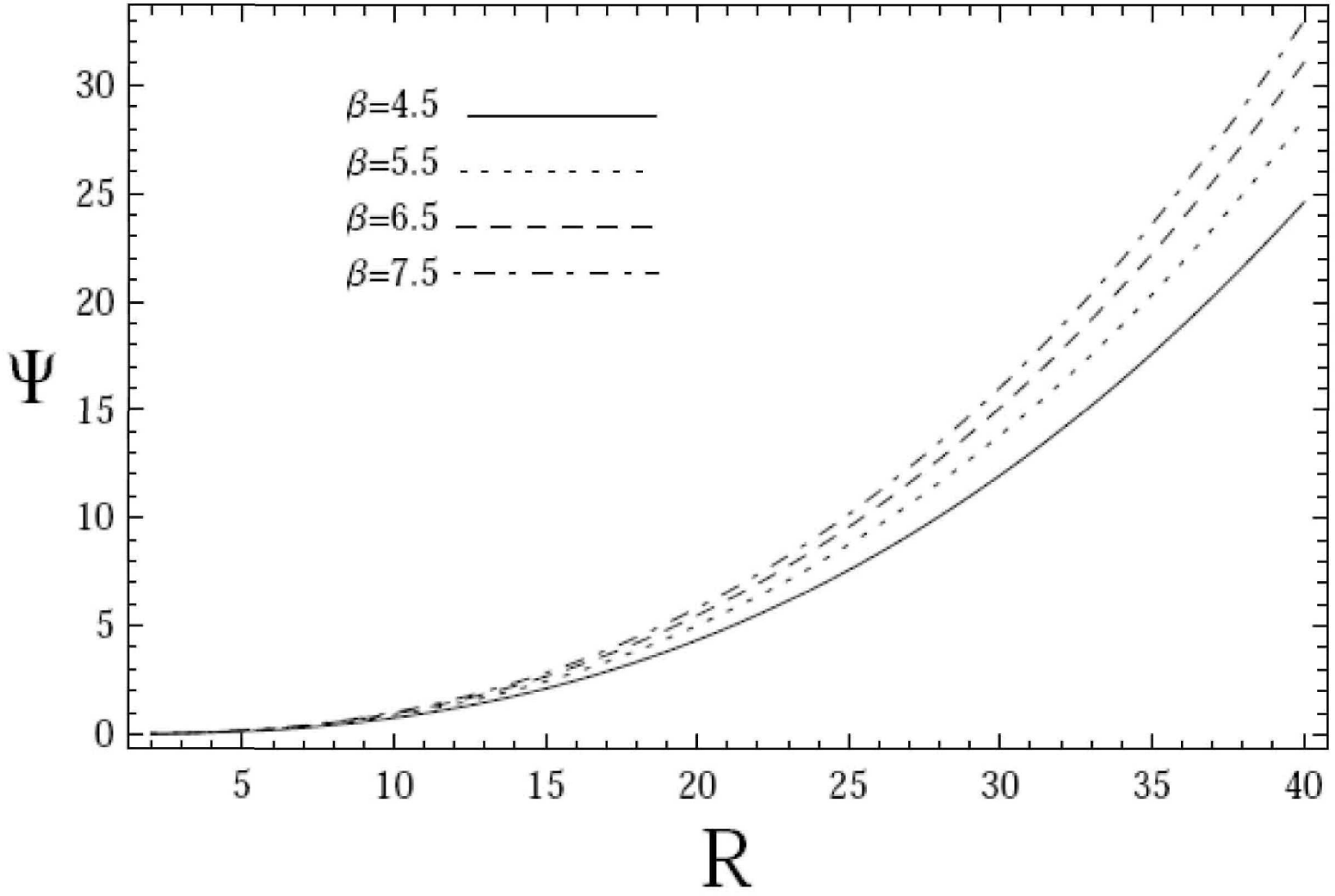


Figure 2.9: The width ratio $\Psi = W / \overline{W}$ versus R at different β values for power law DSD

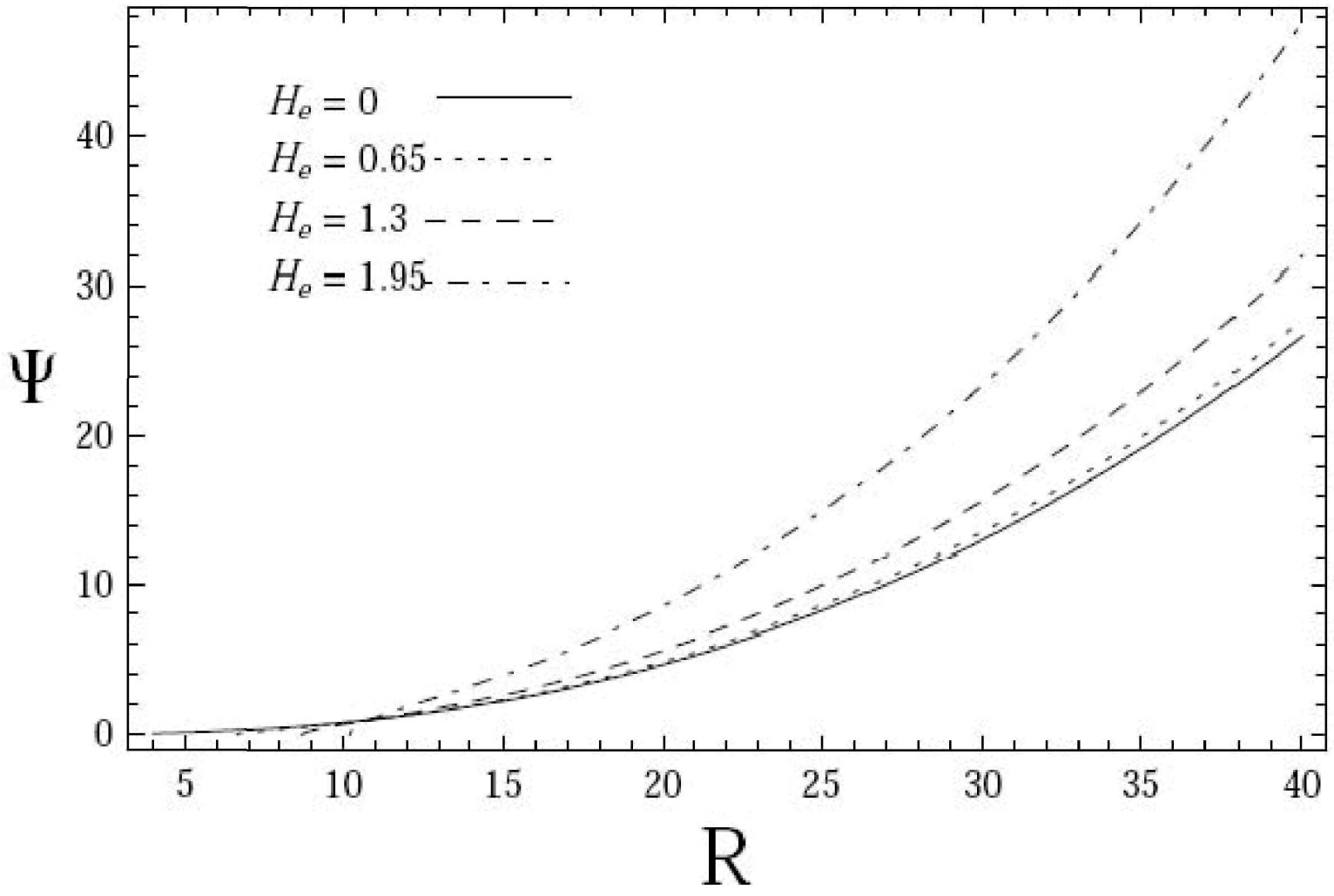


Figure 2.10: The variation of Ψ versus R at defferent H_e values for power law DSD at $\beta = 5$.

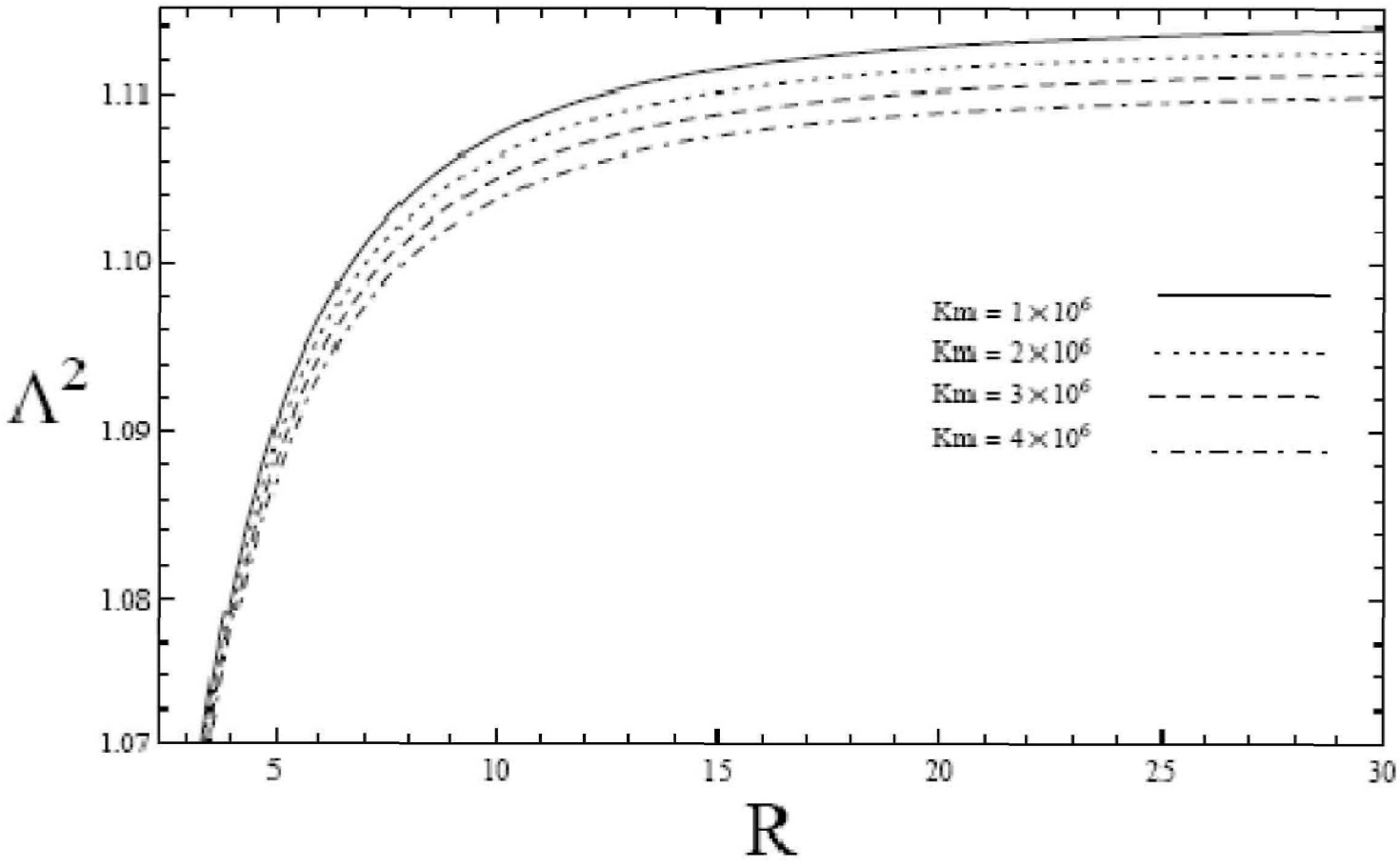


Figure 3.13: The effect of variation of K_m on the relation of Λ^2 and R at $\beta = 4.3$ for the power law DSD.

4. Summary

* Based on QHD theory, proposed model for QDAWs in a magnetized QDP is presented including (DSD) effect.

* Linear analysis is applied to derive the dispersion relation for the wave.

* By the use of RPT, QZK equation are evaluated for the wave description.

* A detailed analytical and numerical investigation of the dependence of the dispersion relation, the soliton velocity, amplitude and width for QDAWs on both of the DSD and quantum effects is illustrated.

*** Quantum effects are presented only in the soliton width and they have no effect on the soliton velocity or its amplitude.

*** The more the mass density (Km) the less the frequency of the wave.

This work

- **S. K. El-Labany, N. M. El-Siragy, W. F. El-Taibany and E. E. Behery, "The effect of dust size distribution on quantum dust acoustic wave", Physics of Plasmas 16, 093701 (2009).**

Thank You