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Hydromagnetic Instabilities in Magnetized Plasmas

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Hydromagnetic Instabilities in Magnetized Plasmas

by

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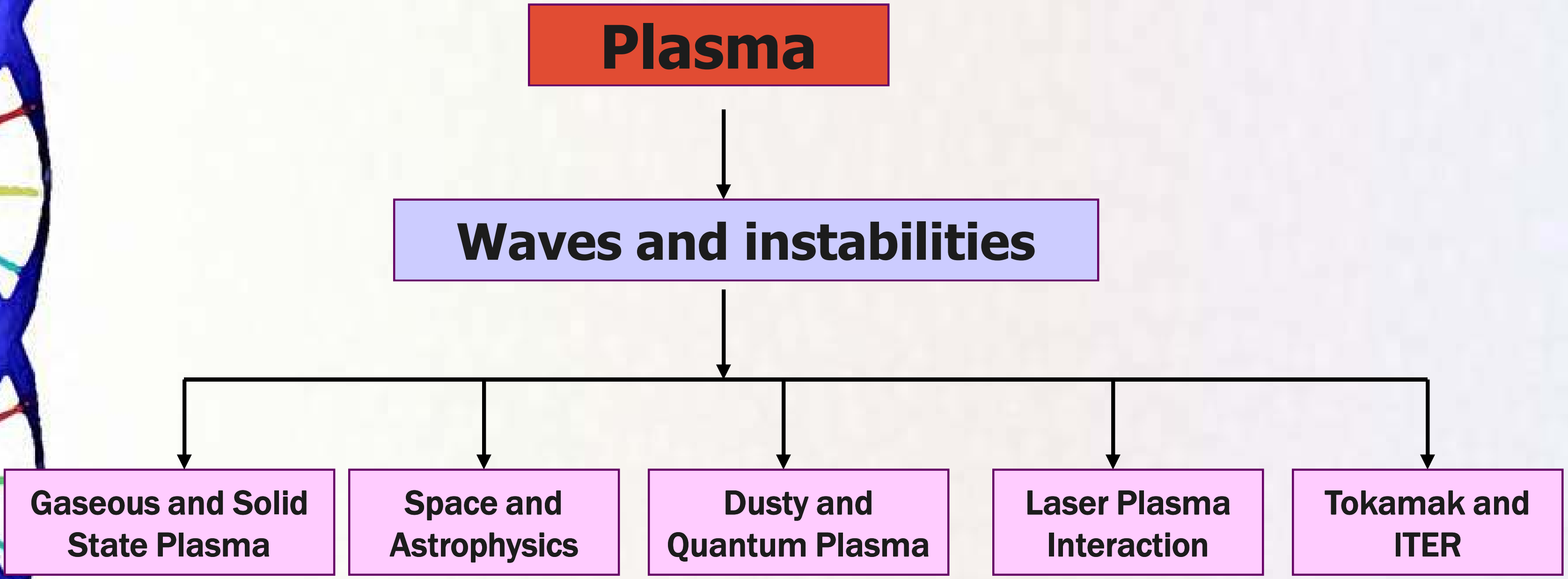
1. Origin of the research work



Plasma

Waves and instabilities

- **Gaseous and Solid State Plasma**
- **Space and Astrophysics**
- **Dusty and Quantum Plasma**
- **Laser Plasma Interaction**
- **Tokamak and ITER**

2. Instabilities in plasma

Because of free-energy sources in plasma, a very large number instabilities can develop.

If the involved scale is:

- Comparable to macroscopic size (bulk scale of plasma,.....)

  Macroinstability (affects plasma globally)

- Comparable to microscopic scale (gyroradius, inertial length,...)

  Microinstability (affects plasma locally)

Theoretical treatment:

 # Macroinstability → **Fluid plasma theory (Single and two fluid)**

 # Microinstability → **Kinetic plasma theory**

Linear instability

The concept of linear instability arises from the consideration of a linear wave function. Assume any variable (density, magnetic field, etc.) here denoted by *A*, the fluctuation of which is δA, that can be Fourier decomposed as

$$\delta A = \sum A \exp(i\mathbf{k}.\mathbf{x} - i\omega t)$$

In general the solution is given as

$$\omega = \omega_r + \gamma$$

For real frequencies disturbances are oscillating waves.

For complex solution
Amplitude A will grow if $\gamma > 0$
and amplitude A will decay if $\gamma < 0$

Nonlinear instability

The nonlinearities come from the harmonic generation involving nonlinear Lorentz force, trapping of particles in the wave potential, ponderomotive force etc.

↓

Coherent structures as;

Solitary structure

Shock waves

Vortices

3. Methodology and technique

➢ Normal Mode Analysis

Physical quantity = Equilibrium part + Perturbed part

$$\xi = \xi_0 + \xi_1$$

Fourier transform of the perturbed quantity

$$\xi_1 = \xi_1 \exp(\omega t - i\,\boldsymbol{k}.\boldsymbol{r})$$

1. If all the values of ω are purely imaginary: System is said to be stable.
2. If some or all the values of ω are real and positive: System is said to be unstable.
3. If some or all the values of ω are complex i.e. $\omega = \omega_r + i\,\omega$

   Then for stability $\omega_r < 0$

   and for instability $\omega_r > 0$

Routh-Hurwitz criterion for stability analysis:

$$F(\omega) = a_0\,\omega^n + a_1\omega^{n-1} + a_2\omega^{n-2} + \ldots\ldots\ldots\ldots\ldots + a_n = 0.$$

Fluid plasma theory

 # Magnetospheres of planets # stars # Extragalactic jets # Comets tails

The fluid model describes plasma in terms of quantities like density and average velocity around each position.

Single fluid theory ↓ Plasma as single fluid governed by Maxwell's and fluid equations

Two fluid theory ↓ It consists dynamics for each species (electron and ion)

❖ MHD fluid theory (Collisional plasma):

The collisions are so frequent

- **The MHD fluid theory encloses the regime of scalar pressures for both electrons and ions.**
- **The MHD model can be applied only for low frequency phenomena providing the concept of Alfven wave and magneto sonic waves.**

❖ CGL fluid theory (Collisionless plasma):

[Chew et al., Proc. Roy. Soc. A 236, 112 (1956)]

 # Solar wind **# Interstellar Space** **# Nebula**

Multiple Coulomb force dominates over the Charge neutral interaction force

In the absence of collisions the usual scalar pressure be replaced by a pressure tensor due to the presence of strong magnetic field

$$\mathbf{P} = p_{\perp}\mathbf{I} + (p_{\parallel} - p_{\perp})\,\mathbf{n}\,\mathbf{n}$$

where I is unit dyadic and n is the unit vector along the magnetic field.

The pressure equations in terms of pressure component, magnetic field and density are given as

$$\frac{d}{dt}\left(\frac{p_{\parallel} B^2}{\rho^3}\right) = 0 \qquad \text{and} \qquad \frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = 0.$$

❖ Polytrope model:

[B. Abraham-Shrauner, Plasma Phys. 15, 375 (1973)]
[M. Chou and L. N. Hau, Astrophys. J. 611, 1200 (2004)]

MHD approximation → **Collision dominated plasma**

CGL approximation → **Collisionless plasma in presence of strong magnetic field**

Polytrope model → **Transition zone where plasma is neither fully collisional nor collisionless**

$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho^{\varepsilon}B^{\nu}}\right)=0 \qquad \frac{d}{dt}\left(\frac{p_{||}}{\rho^{\beta}B^{-\alpha}}\right)=0$$

Where $\alpha,\beta,\varepsilon \ and \ \nu$ **are the polytropic indices.**

4. Dusty and quantum plasma

A dusty plasma is collection of free electrons and ions with some additional charged dust grains of micron size. A dusty plasma also satisfies the usual quasineutrality condition as

$$n_{i0} = n_{e0} + q_d n_{d0}$$

In classical plasma $\lambda_B << \lambda_D$

$$\lambda_B = \frac{\hbar}{m V_T}$$

i.e. de-Broglie wavelength << Debye length of the system

In case of quantum plasma

$$\lambda_B >> \lambda_D$$

Quantum statistical effects (spin magnetization, particles spin effect etc.)

Quantum Bohm potential (arises due to low temperature)

- Nanoscale technology [H. G. Craighead, Science 290, 1532 (2000)]
- Microelectronic devices [P.A. Markovich et al., S/C Eqn., Springer-Verlag NY 1990]
- Laser Fusion [S. H. Glentzer et al., Phys. Rev. Letters, 98, 065002 (2007)]
- Dense Astrophysical System [C. Kouveliotou et al., Nature, 393, 235 (1998)]

Publications in dusty and quantum plasma

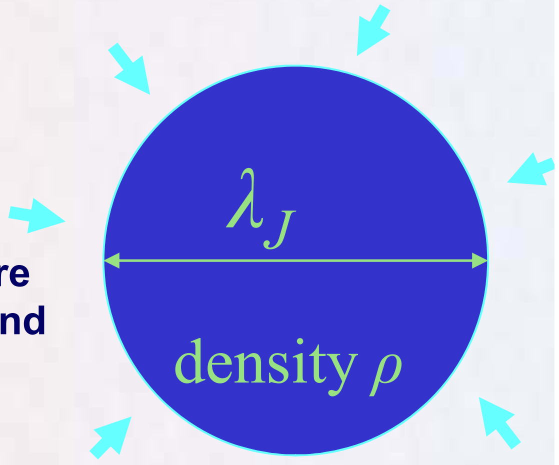
1. **Effect of dust temperature on radiative condensation instability of self-gravitating magnetized dusty plasma**
   R. P. Prajapati and R. K. Chhajlani, **Physica Scripta 81**, 045501 (2010)

2. **Effect of Hall current on Jeans instability of magnetized quantum viscous plasma**
   R. P. Prajapati and R. K. Chhajlani, **Physica Scripta 82**, 055003 (2010)

3. **Effect of magnetic field on Jeans instability of quantum dusty plasma: Application in white dwarf star**
   R. P. Prajapati and R. K. Chhajlani, **Acta Technica** (2010) [In Press]

5. Some hydromagnetic instabilities

5.1 Jeans (gravitational) instability



Mass ≈ 1.4 solar masses

Any self-gravitating object opposes the excess gas pressure due to self-gravitation. This causes collapse of the object and triggers an instability called Self-gravitational or Jeans instability.

Astrophysics, Star formation, ISM formation, Nebula, Molecular cloud formation, Dwarf star and Neutron star formation etc.

Chandrasekhar (1961) has given the Jeans instability criterion

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = -c_s^2 \nabla \, \delta\rho \, + \, \rho \nabla \, \delta\phi$$

(Momentum conservation)

$$\frac{\partial}{\partial t} \delta\rho + \rho \nabla . \boldsymbol{u} = 0$$

(Mass conservation)

$$\nabla^2 \delta\phi = -4\pi G \delta\rho$$

(Poisson’s equation)

The system will be unstable for all the wavenumbers

$$k < k_j \left(= \frac{(4\pi G\rho)^{1/2}}{c_s} \right)$$

S. Chandrasekhar, Hydromagnetic and Hydrodynamic Instability, Oxford (1961).

Publications of Jeans instability

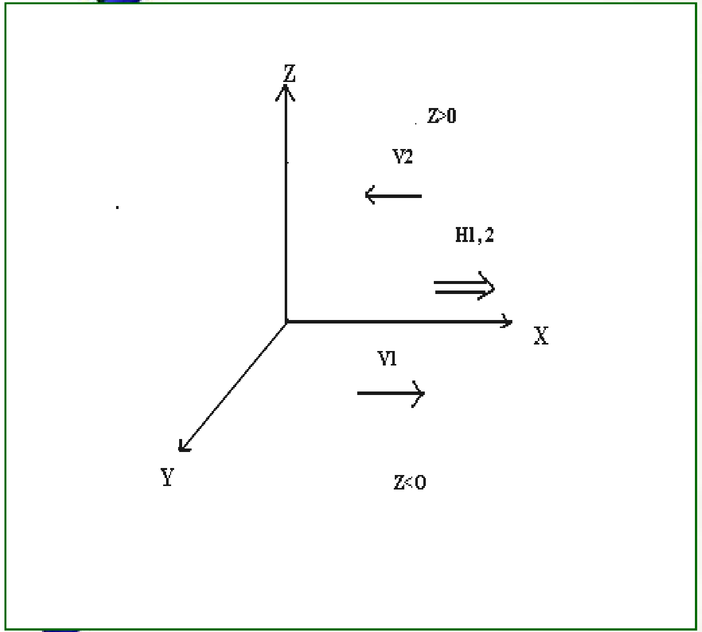
1. **Self-gravitational instability of rotating anisotropic heat-conducting plasma**
   R. P. Prajapati et al., **Physics of Plasmas 15,** 012107 (2008)

2. **Self-gravitating rotating anisotropic pressure plasma in presence of Hall current and electrical resistivity with generalized polytope laws**
   R. P. Prajapati et al., **Physics of Plasmas 15**, 062108 (2008)

3. **Self-gravitational instability of rotating viscous Hall plasma with arbitrary radiative heat-loss functions and electron inertia**
   R. P. Prajapati et al., **Astrophysics & Space Science 327**, 139 (2010)

5.2 Kelvin-Helmholtz (K-H) instability

- The instability arising due to the tangential discontinuity of velocities between two plasma streams is called the K-H instability.
- Two superposed fluids are separated by a horizontal boundary which are in relative motion then the instability arising at the interface is called the K-H instability.
  - Velocity shear is present.
  - There is sufficient velocity difference across the interface between two fluids.

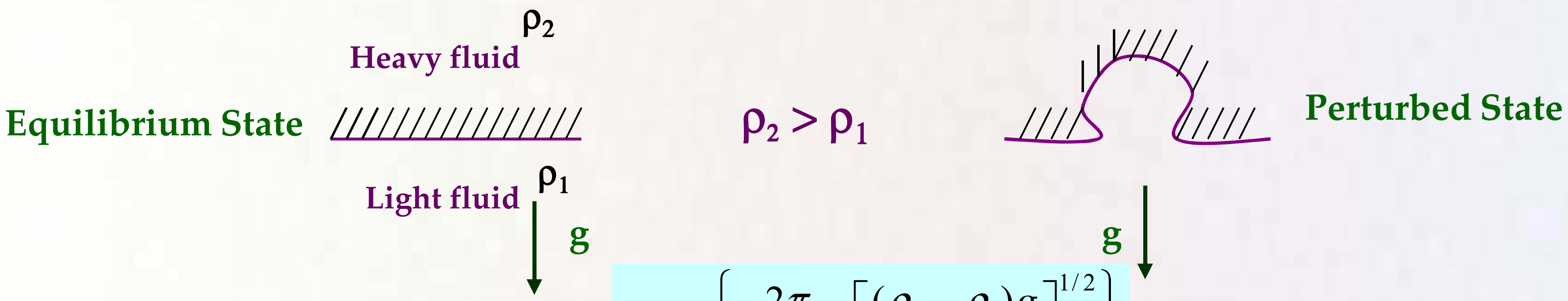


$$k > k_{\min}\left[=\frac{g(\rho_1-\rho_2)}{\rho_1\rho_2(U_1-U_2)^2}\right].$$

- Space plasma
- Comet tails
- Astrophysical plasmas
- Tokamak & ITER
- Industrial plasma

5.3 Rayleigh-Taylor (R-T) instability

The instability at the interface of two superposed fluids of different densities in which the heavy fluid is supported by the light one is generally known as R-T instability.



$$k < k_c \left\{ = \frac{2\pi}{\lambda_c} = \left[\frac{(\rho_2 - \rho_1) g}{T} \right]^{1/2} \right\}.$$

- Inertial Confinement Fusion (ICF)
- Supernova Explosions
- Z-pinches
- Astrophysical plasmas

Publications in K-H and R-T instabilities

1. **Kelvin-Helmholtz and Rayleigh-Taylor instability of two superposed magnetized incompressible fluids with suspended dust particles**
   R. P. Prajapati et al., **Z. Naturforsch** A **64a**, 455 (2009).
2. **Kelvin-Helmholtz instability of magnetized plasmas with surface tension and dust particles**
   R. P. Prajapati and R. K. Chhajlani, **J. Physics Conf. Ser. 208,** 012078 (2010).
3. **Kelvin-Helmholtz instability of anisotropic pressure plasma using generalized polytrope laws**
   R. P. Prajapati et al., J. Phys. Conf. Ser. **208**, 012077 (2010).
4. **Kelvin-Helmholtz and Rayleigh Taylor instability of two superposed fluids with suspended dust particles flowing through porous media**
   R. P. Prajapati and R. K. Chhajlani, **Journal of Porous Media 13**, 765 (2010).
5. **Rayleigh-Taylor instability of two superposed magnetized fluids with suspended dust particles.**
   R. P. Prajapati et al., Thermal Science 14, 11 (2010).
6. **Effect of surface tension and rotation on Rayleigh-Taylor instability of two superposed fluids with suspended dust particles**
   P. K. Sharma, **R. P. Prajapati** and R. K. Chhajlani**, Acta Physica Polonica 118**, 576 (2010).
7. **Effect of pressure anisotropy and flow velocity on Kelvin-Helmholtz instability of anisotropic plasma using generalized polytrope laws**
   R. P. Prajapati and R. K. Chhajlani, **Physics of Plasmas 17**, 1 (2010).

6. Out comes

New extended plasma and astrophysics theory

Formation of stars, molecular cloud, nebula

Interaction between solar wind and Earth's magnetosphere

ISM structure formation

Industrial applications (Multi-fluid flow, oil extraction from rocks, critical flow estimation of fluids)

Working status of tokamak and ITER

Thank You