Outline

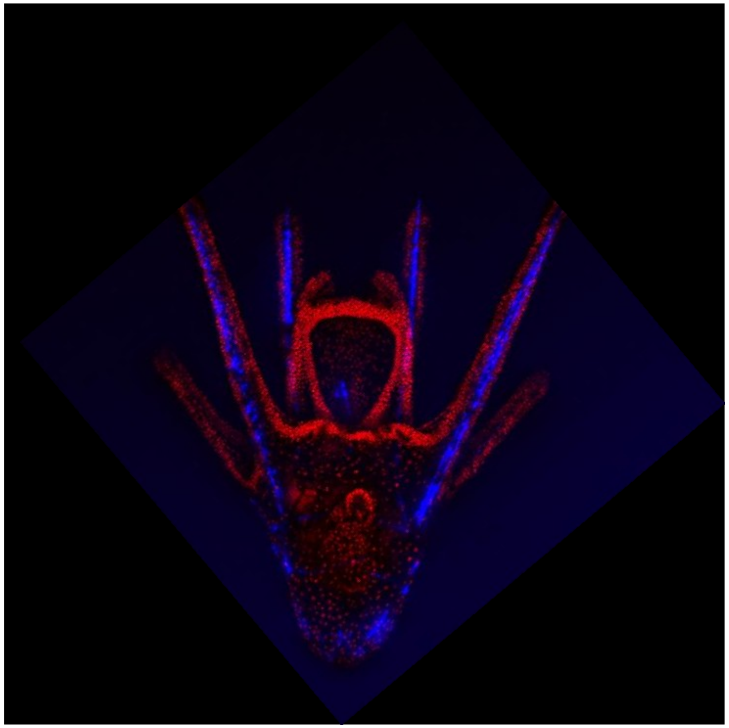
1. Motivations: Large-scale models vs. small-scale heterogeneity

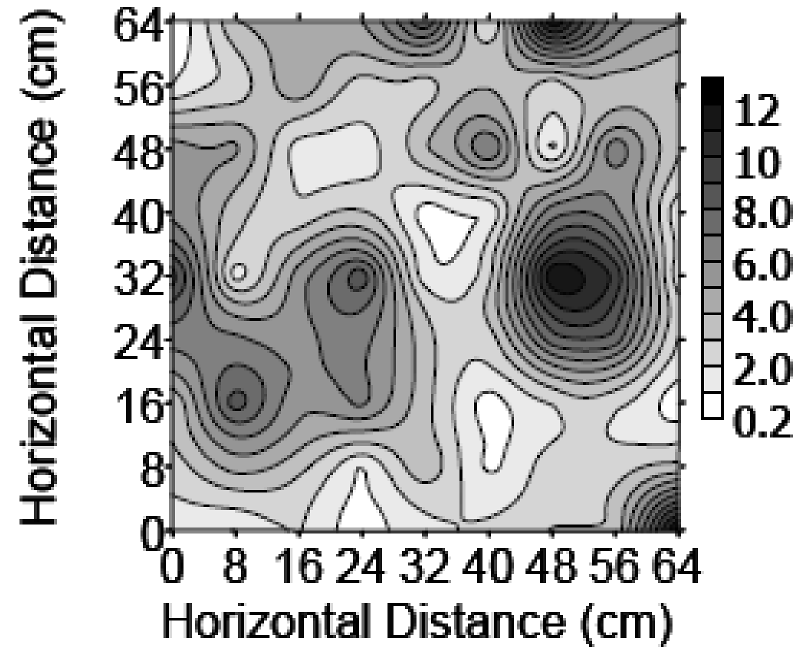
2. Observations: Statistics of movement at the individual level

3. Derivations: Spatially-explicit models of populations

4. $\boxed{\text{4. Approximations: Large-scale effects of unresolved heterogeneity}}$

Interactions between consumers and heterogeneous resources





Key mechanisms: resource patch dynamics, movement, reproduction, consumption

Key parameters:

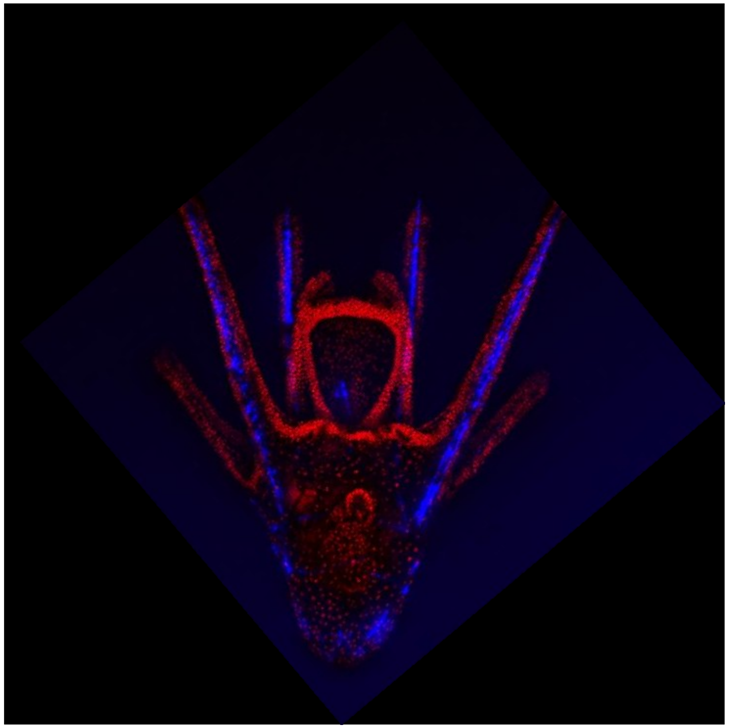
Organism-level

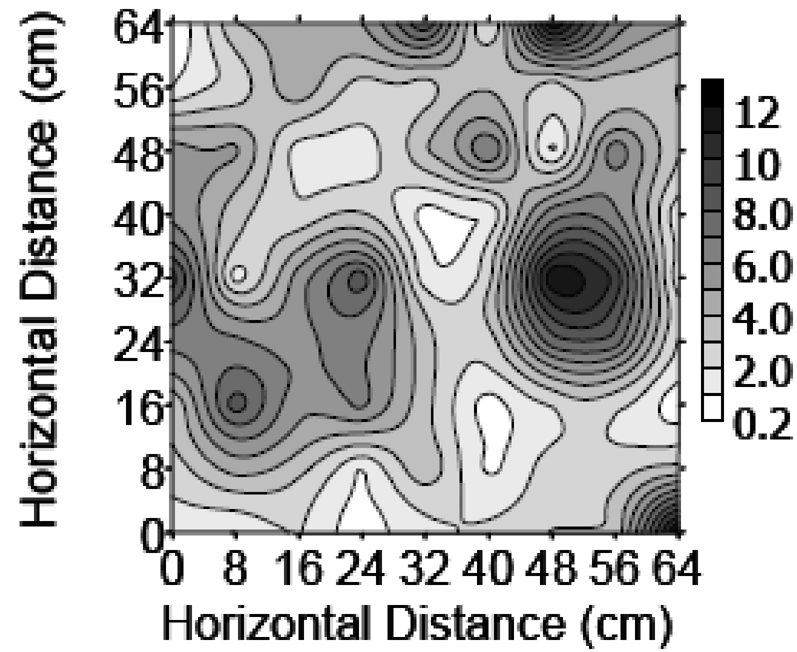
parameter	*interpretation*
c	consumer speed
τ	consumer turning interval
σ_z	max. intrinsic rate of growth of consumer
η	max. specific consumption rate of consumer
μ	mortality rate of consumer

Landscape-level

parameter	*interpretation*
T	resource time scale
L	resource length scale
$\bar{r}$	char. resource density
$\bar{z}$	char. consumer density

Interactions between consumers and heterogeneous resources





Key mechanisms: resource patch dynamics, movement, reproduction, consumption

Key timescales:

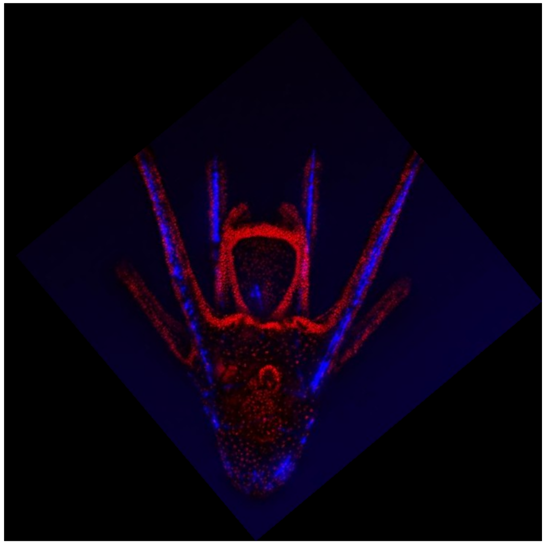
Resource patch duration timescale	T
Consumer motility timescale	$T_{search} = \frac{L^2}{c^2 \tau}$
Consumer reproduction timescale	$T_{reprod} = \frac{1}{\sigma_z}$
Resource consumption timescale	$T_{consumpt} = \frac{\bar{r}}{\eta \bar{z}}$

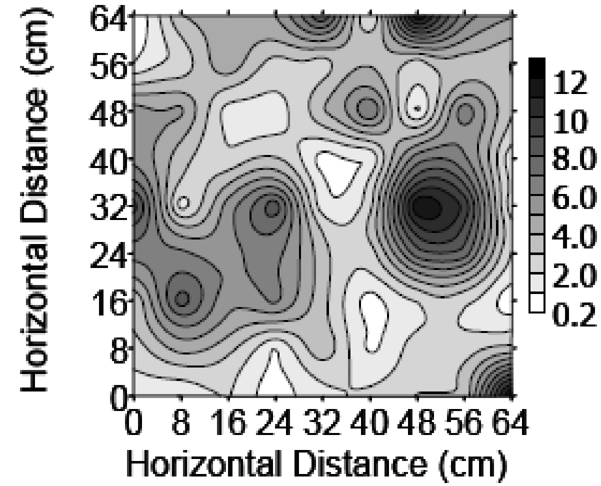
Interactions between consumers and heterogeneous resources

Putative ecological indices:

Definition	*Name*	*interpretation*
$\mathscr{F}r = \frac{T}{T_{search}} = \frac{c^2 T\tau}{L^2}$	Frost number	Relative importance **movement** in z
$\mathscr{S}tr = \frac{T}{T_{reprod}} = T\sigma_z$	Strathmann number	Relative importance **reproduction** in z
$\mathscr{L}e = \frac{T}{T_{consumpt}} = \frac{\eta\bar{z}T}{\bar{r}}$	Lessard number	Relative importance of **consumption** of r by z

Interpreting marine ecological dynamics with non-dimensional indices

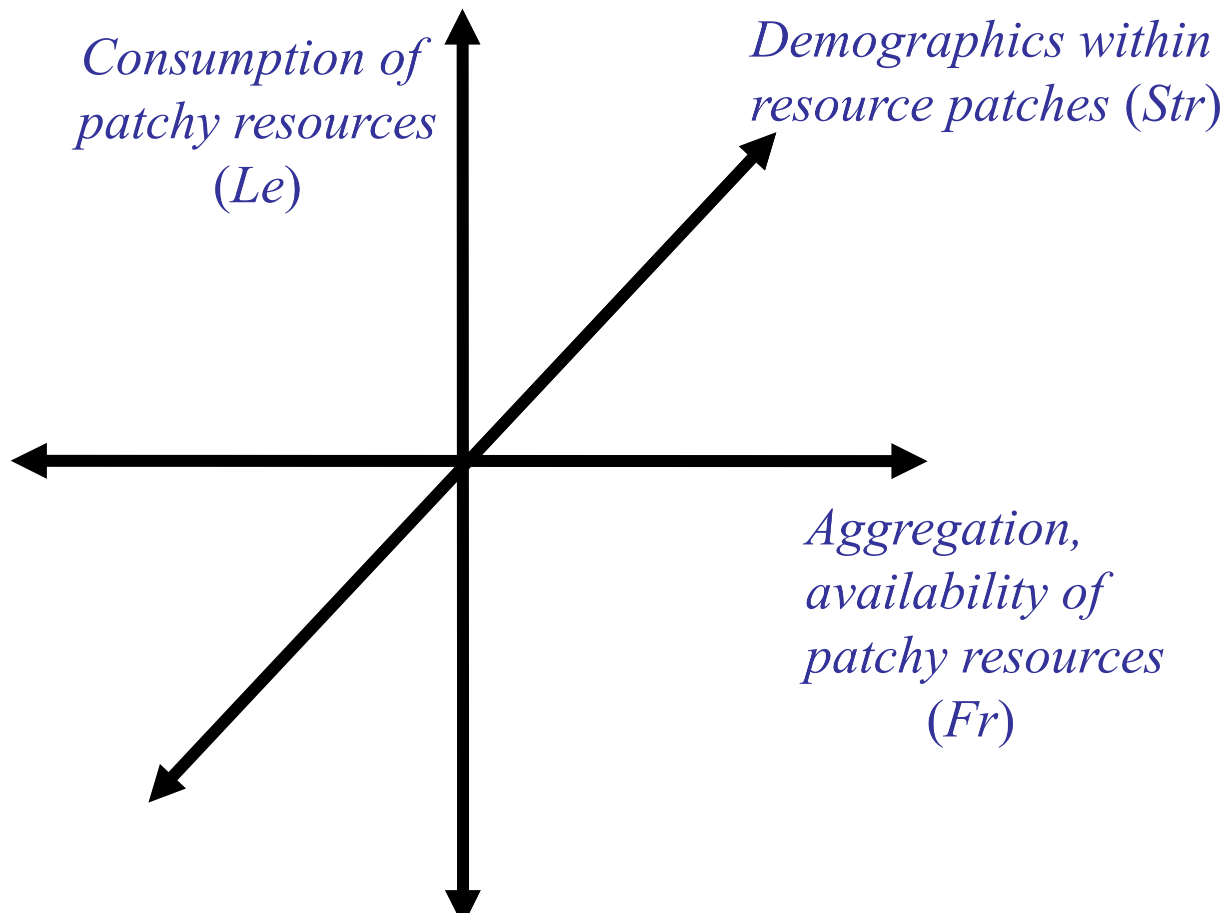




$$\mathcal{F}r = \frac{\text{patch duration}}{\text{search time}}$$

$$\mathcal{S}tr = \frac{\text{patch duration}}{\text{reproduction time}}$$

$$\mathcal{L}e = \frac{\text{patch duration}}{\text{reproduction time}}$$



An ecological modeling framework

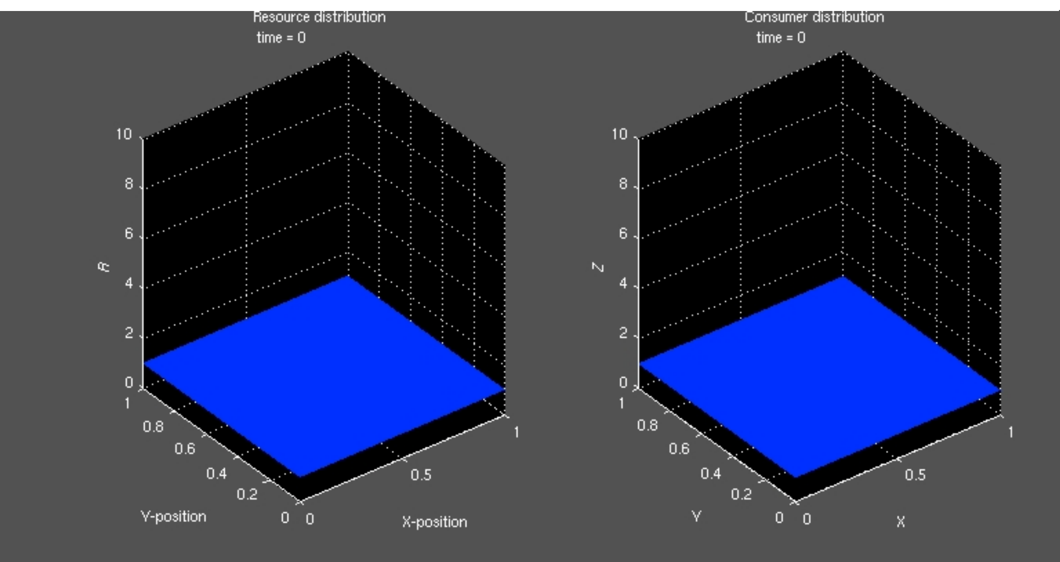
$r(t,x)$ = resource density, $z(t,x)$ = consumer density

$$\frac{\partial r}{\partial t} = \underbrace{\nabla\left(D_e \nabla r - \mathbf{U} r\right)}_{\text{Phys. diffusion, advection of r}} + \underbrace{Q(t,\mathbf{x},r)}_{\text{Production of r}} - \underbrace{\mathcal{L}e\ \hat{H}(t,\mathbf{x},r,z)z}_{\text{Consumption of r}}$$

$$\frac{\partial z}{\partial t} = \underbrace{\mathcal{F}r\ \nabla\left(\hat{D}(r)\nabla z - \hat{\chi}(r)\nabla r z\right)}_{\text{Behav.+Phys. diffusion, migration of z}} - \underbrace{\nabla\left(\mathbf{U} z\right)}_{\text{Advection of z}} + \underbrace{\mathcal{S}tr\ \hat{S}(t,\mathbf{x},r,z)z}_{\text{Demographics of z}}$$

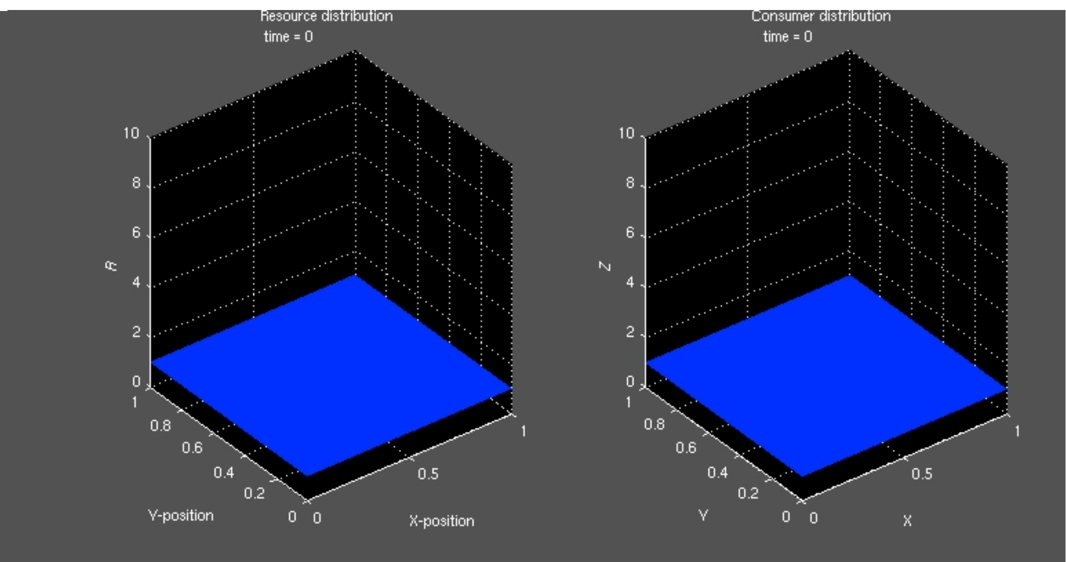
Demography-dominated

Str >> 1 , *Fr* << 1 , *Le* << 1



Motility-dominated

Str << 1, *Fr* >> 1 , *Le* << 1



Interpreting ecological dynamics with non-dimensional indices

Grünbaum 2002

Frost Number: $$\boxed{Fr = \frac{c^2 T \tau}{L^2}}$$

Characteristic values:

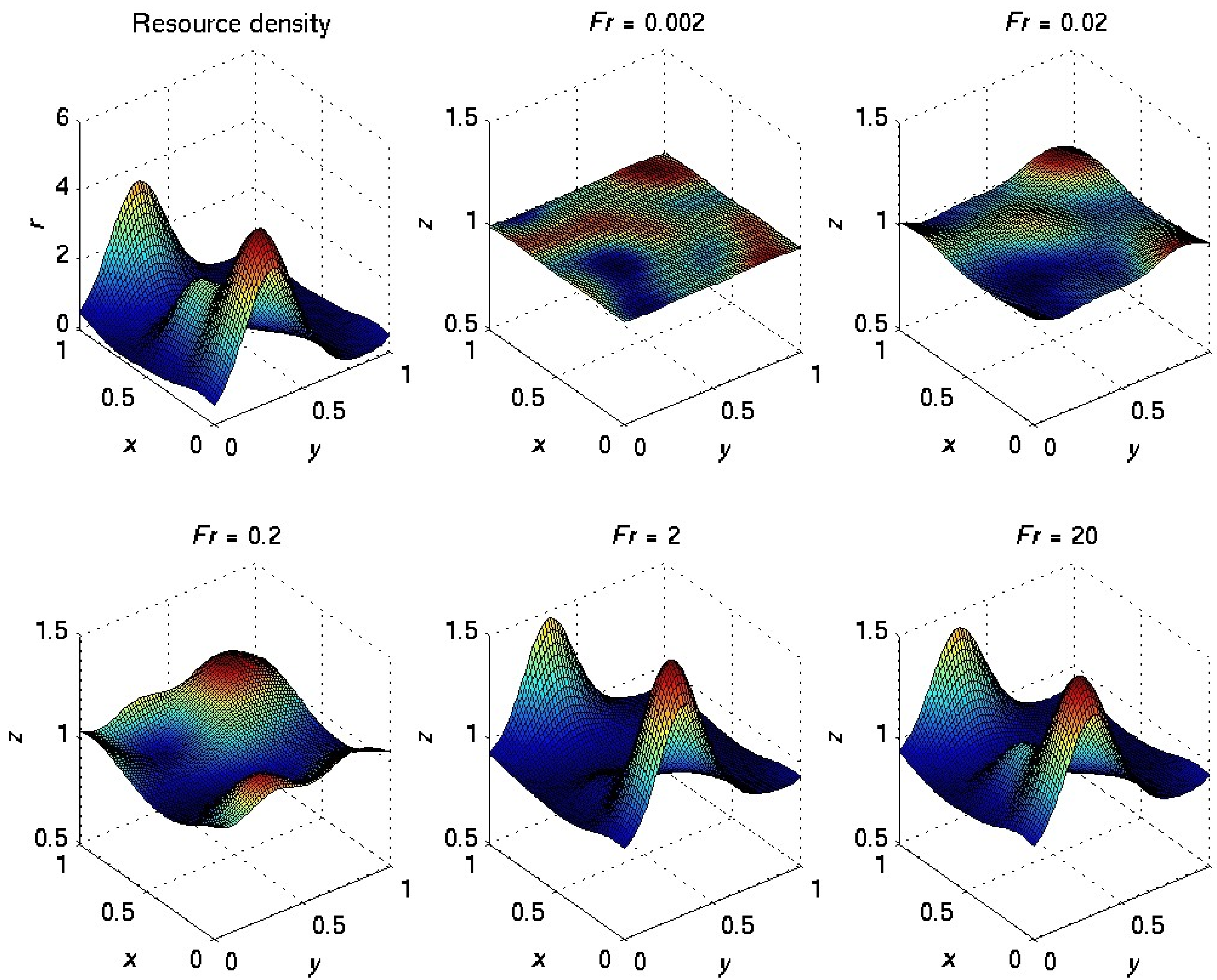
c ~ forager speed

τ ~ forager turning interval

L ~ resource length scale

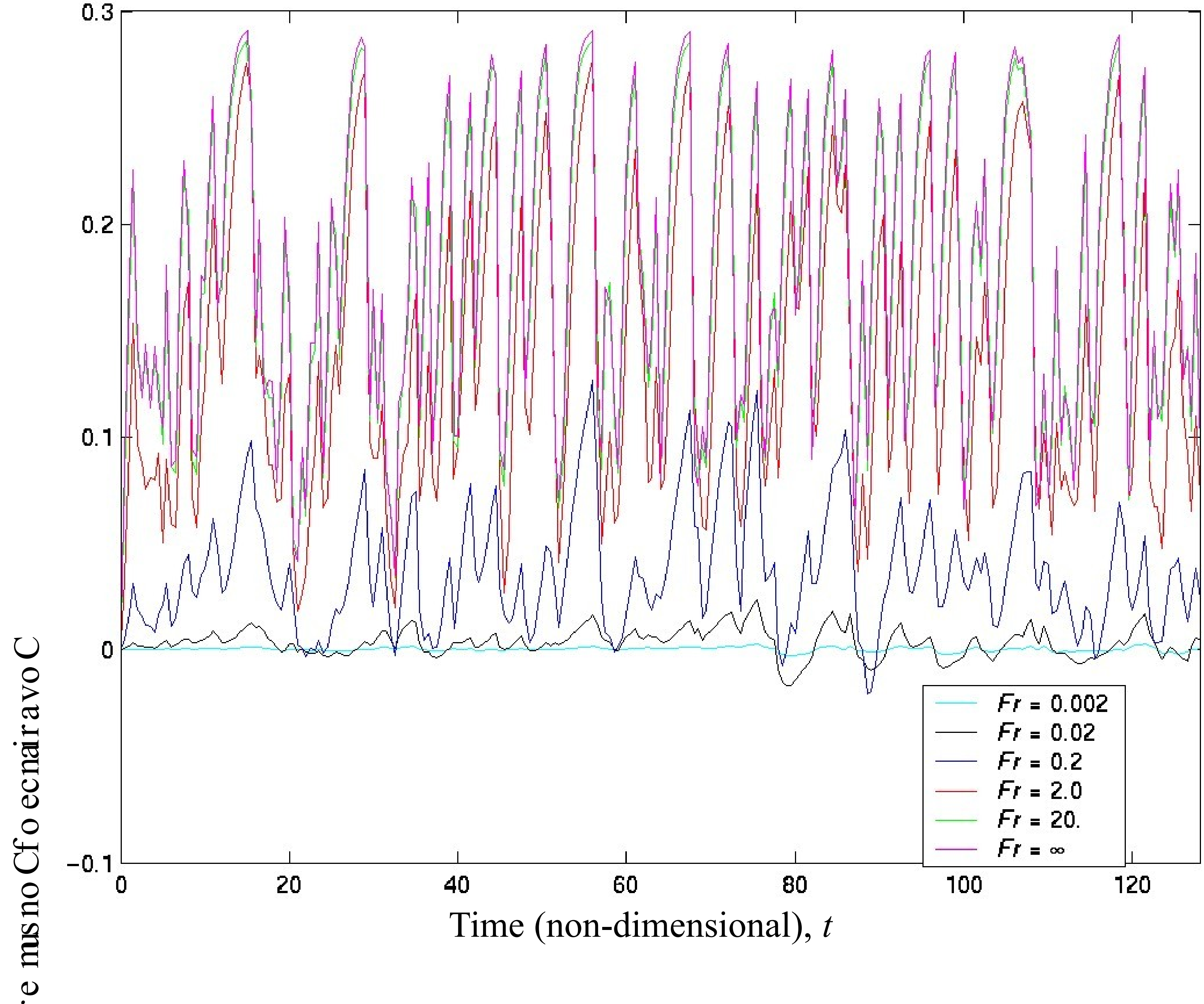
T ~ resource time scale

Aggregation to a spatially and temporally varying resource



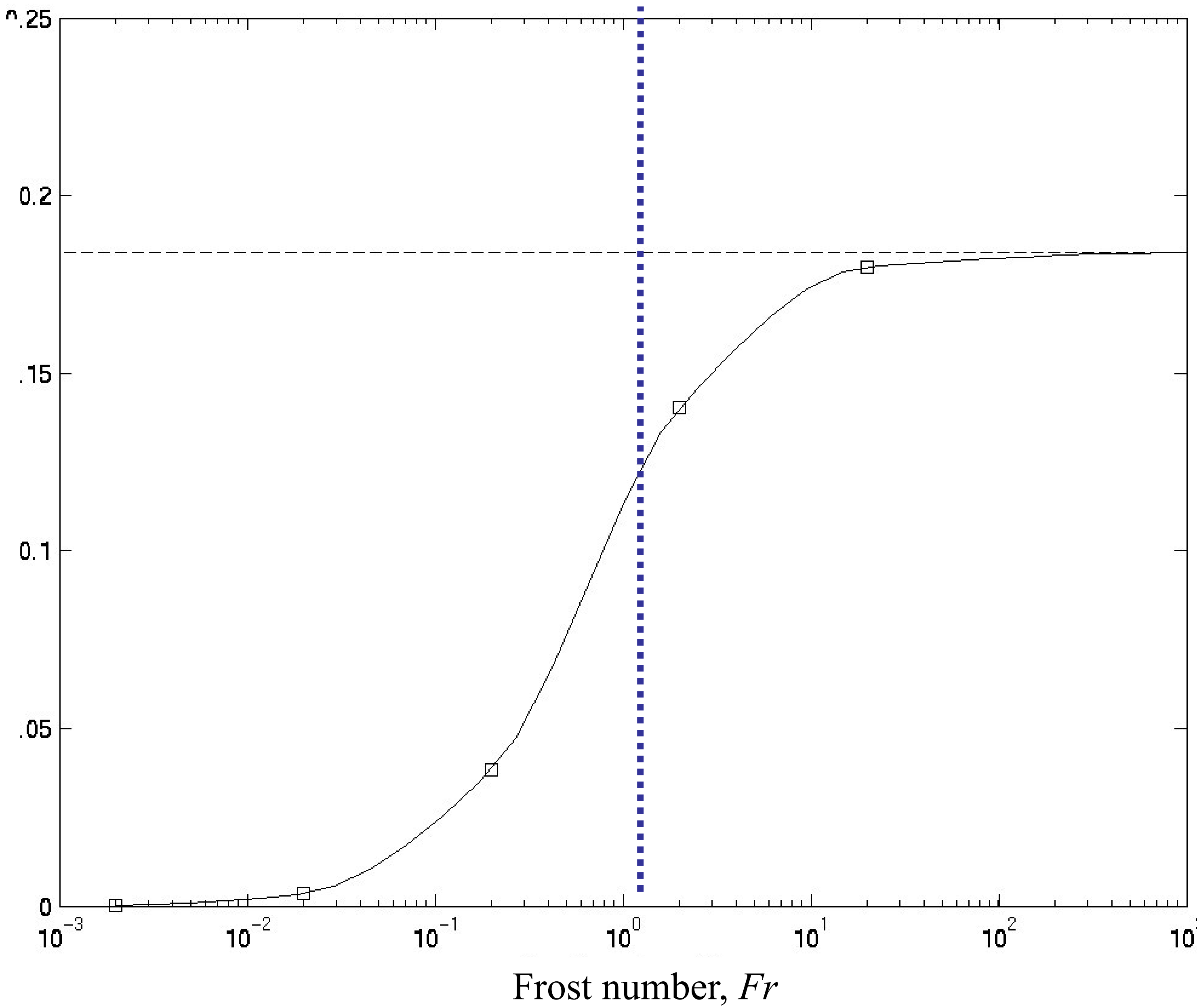
Aggregation to a spatially and temporally varying resource

Aggregation of consumers to resources – time series



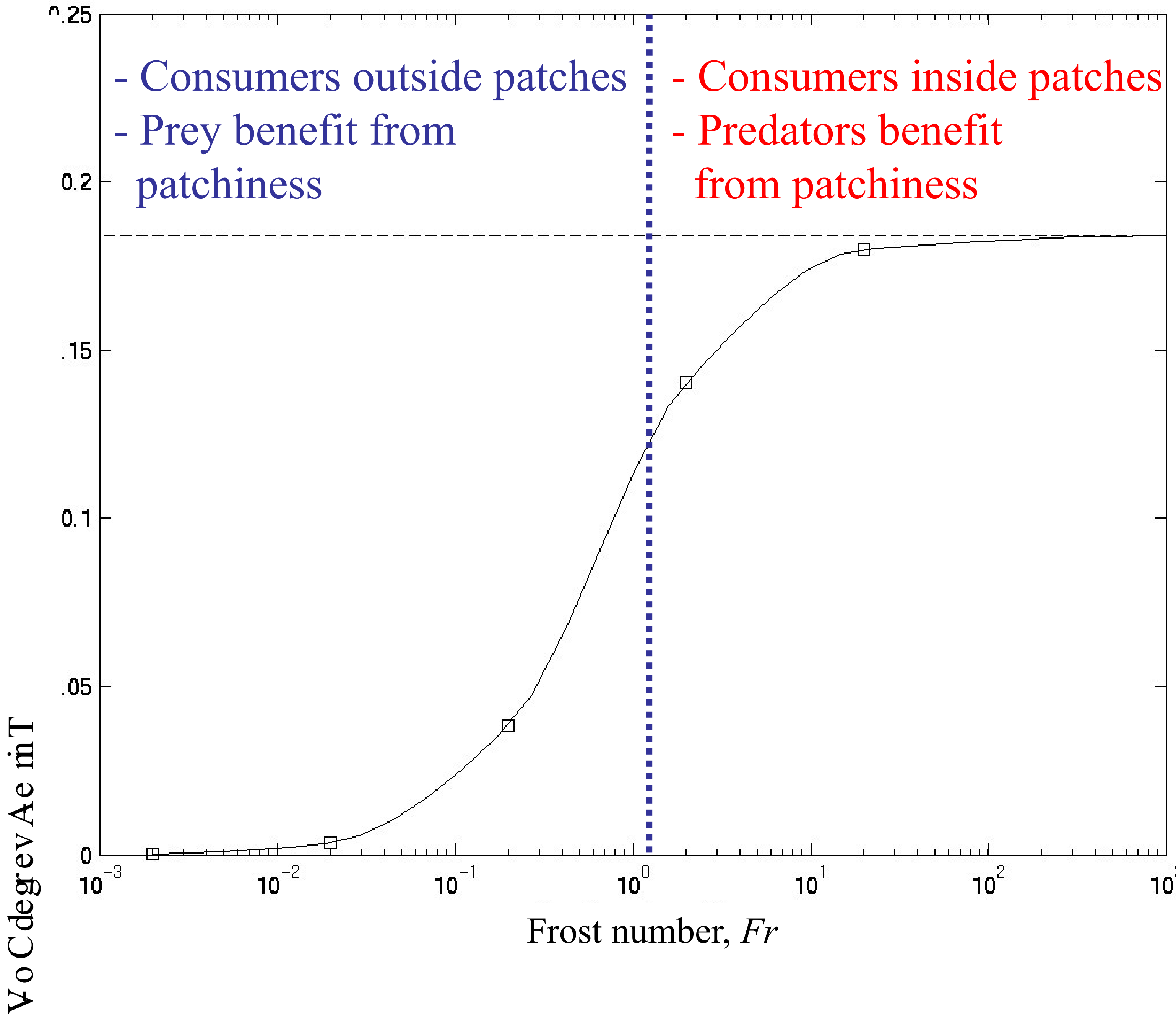
Aggregation to a spatially and temporally varying resource

Aggregation of consumers to resources – time average



Aggregation to a spatially and temporally varying resource

Aggregation of consumers to resources – time average



Characterizing Ecological Dynamics in the Marine Environment

Frost Number:

$$\boxed{Fr = \frac{c^{2} T \tau}{L^{2}}}$$

Characteristic values:

c ~ forager speed

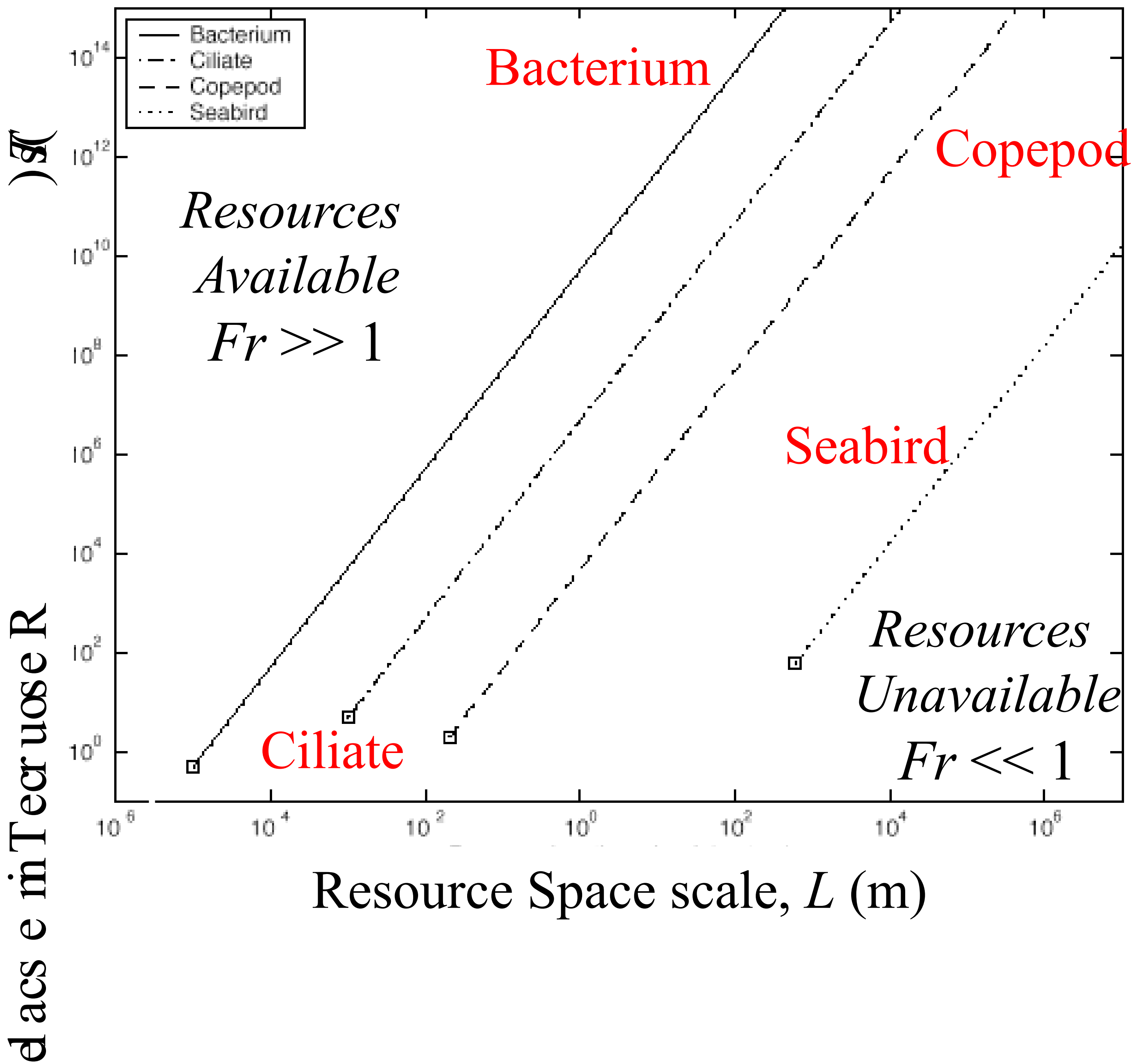
τ ~ forager turning interval

L ~ resource length scale

T ~ resource time scale

$$Fr_{critical} = 1 \longrightarrow \boxed{T_{critical} = \frac{L^{2}}{c^{2}\tau}}$$

Distinguishing available from unavailable resources



An ecological modeling framework

$$\frac{\partial r}{\partial t} = \underbrace{\nabla\left(D_e \nabla r - \mathbf{U} r\right)}_{\text{Phys. diffusion, advection of r}} + \underbrace{Q(t, \mathbf{x}, r)}_{\text{Production of r}} - \underbrace{\mathcal{L}e\ \hat{H}(t, \mathbf{x}, r, z) z}_{\text{Consumption of r}}$$

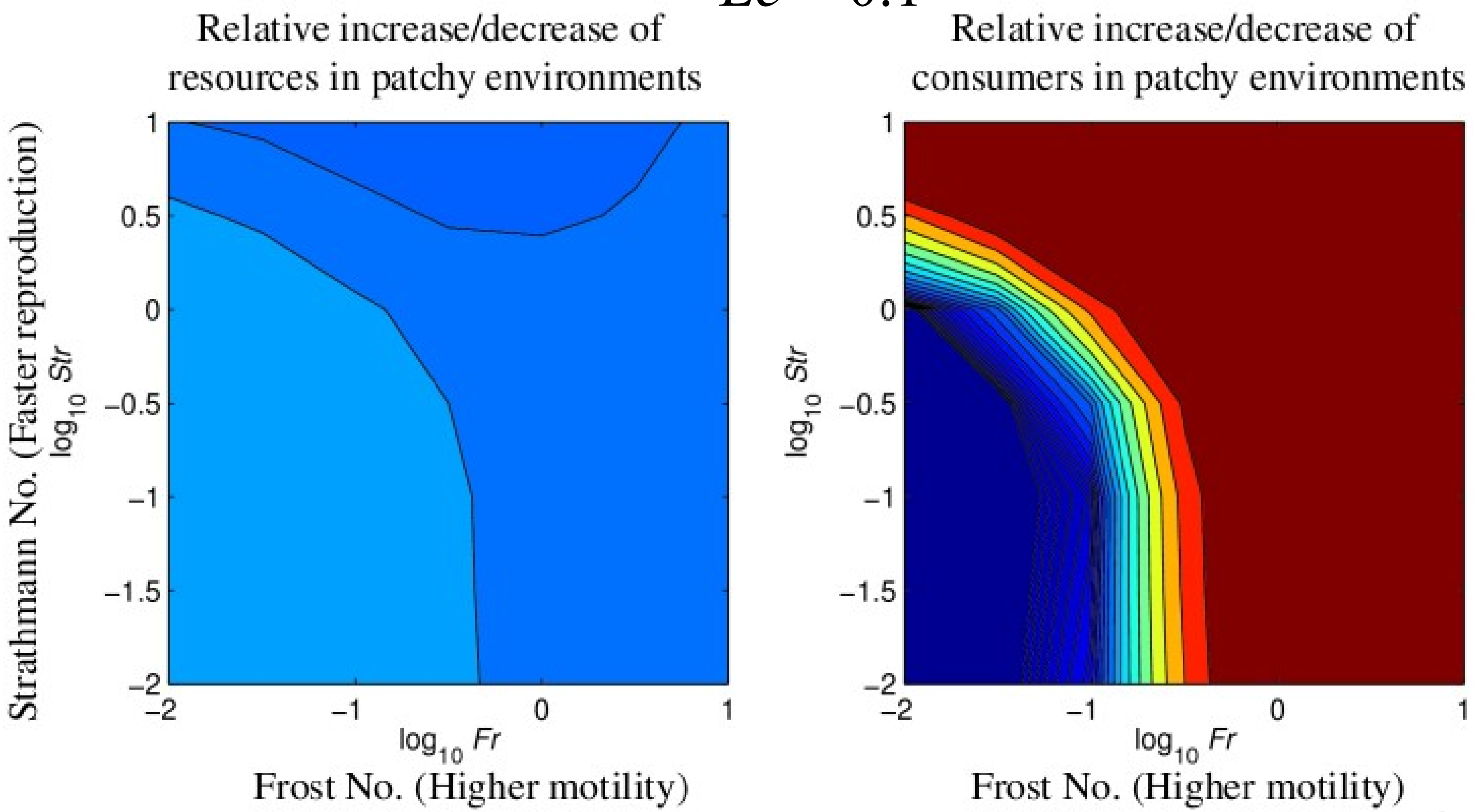
$$\frac{\partial z}{\partial t} = \underbrace{\mathcal{F}r\ \nabla\left(\hat{D}(r)\nabla z - \hat{\chi}(r)\nabla r z\right)}_{\text{Behav.+Phys. diffusion, migration of z}} - \underbrace{\nabla\left(\mathbf{U} z\right)}_{\text{Advection of z}} + \underbrace{\mathcal{S}tr\ \hat{S}(t, \mathbf{x}, r, z) z}_{\text{Demographics of z}}$$

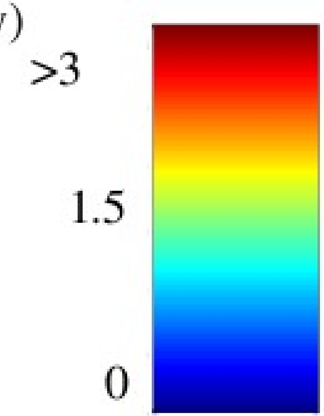
Non-dimensional indices and their interpretations

Definition	*Name*	*interpretation*
$\mathcal{F}r = \frac{c^2 T \tau}{L^2}$	Frost number	$\frac{\textit{patch duration}}{\textit{search time}}$
$\mathcal{S}tr = T\sigma$	Strathmann number	$\frac{\textit{patch duration}}{\textit{consumer generation time}}$
$\mathcal{L}e = \frac{\eta \bar{z} T}{\bar{r}}$	Lessard number	$\frac{\textit{patch duration}}{\textit{consumption time}}$

Spatially and temporally averaged resource and consumer populations

$Le = 0.1$



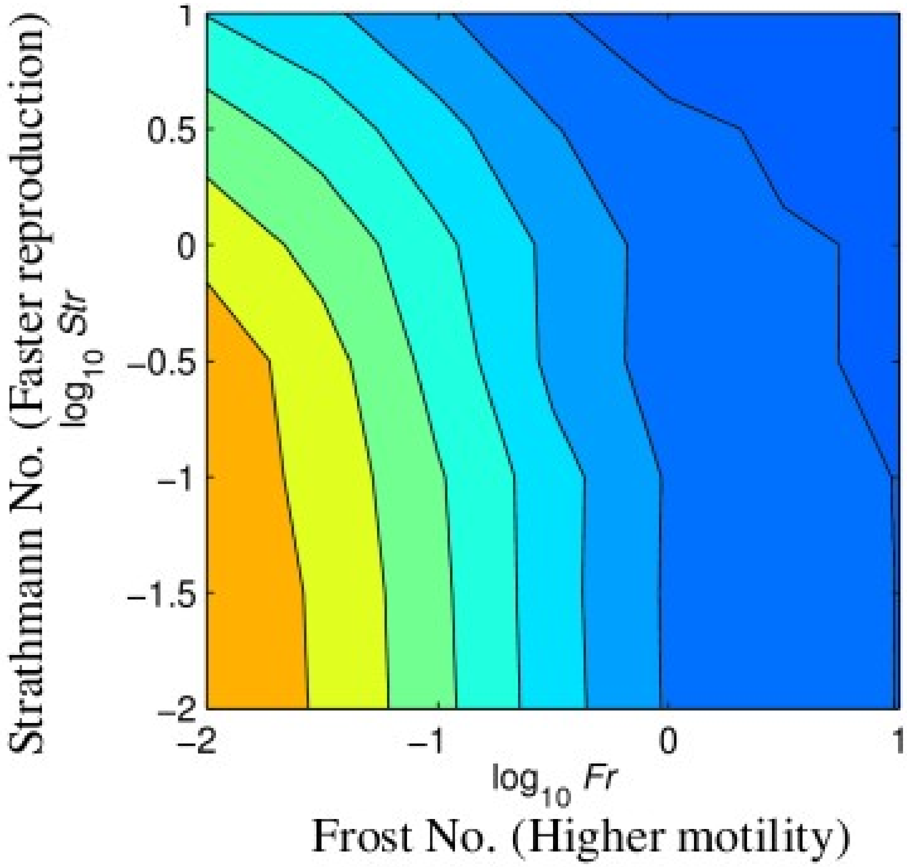


Lessard No. Le << 1 (weak impacts of consumers on resources)

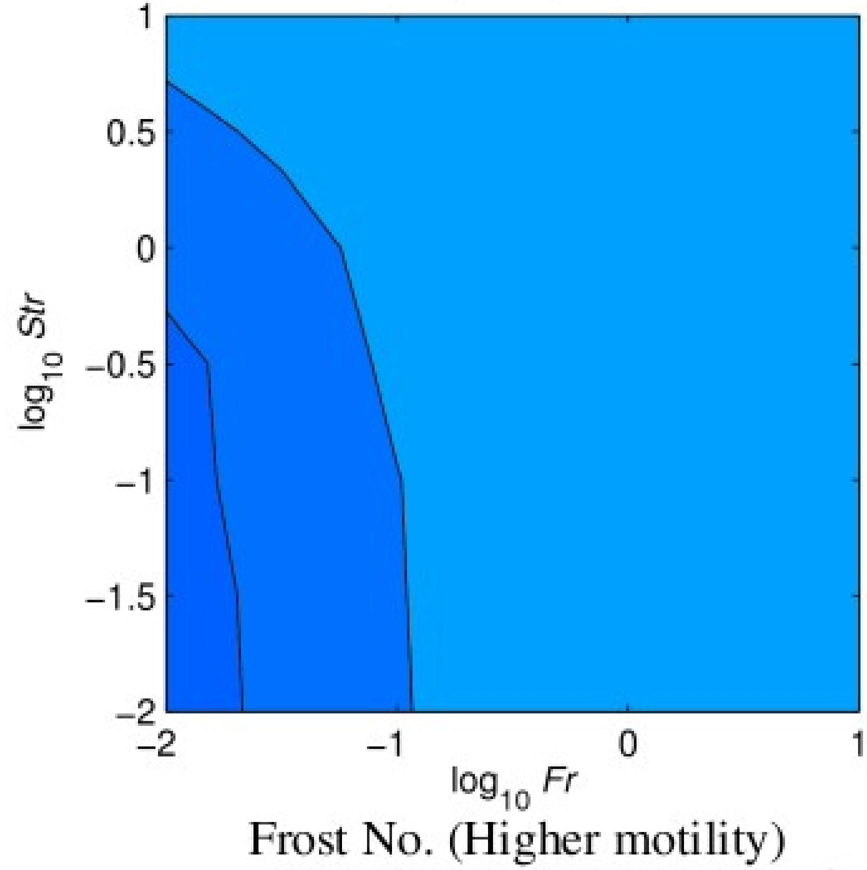
Spatially and temporally averaged resource and consumer populations

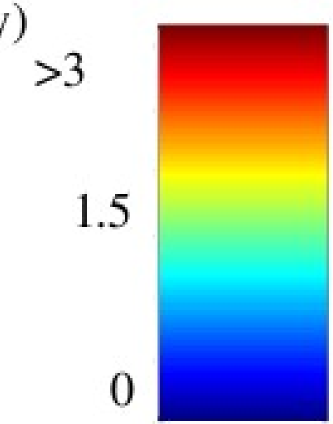
$Le = 10$

Relative increase/decrease of resources in patchy environments



Relative increase/decrease of consumers in patchy environments

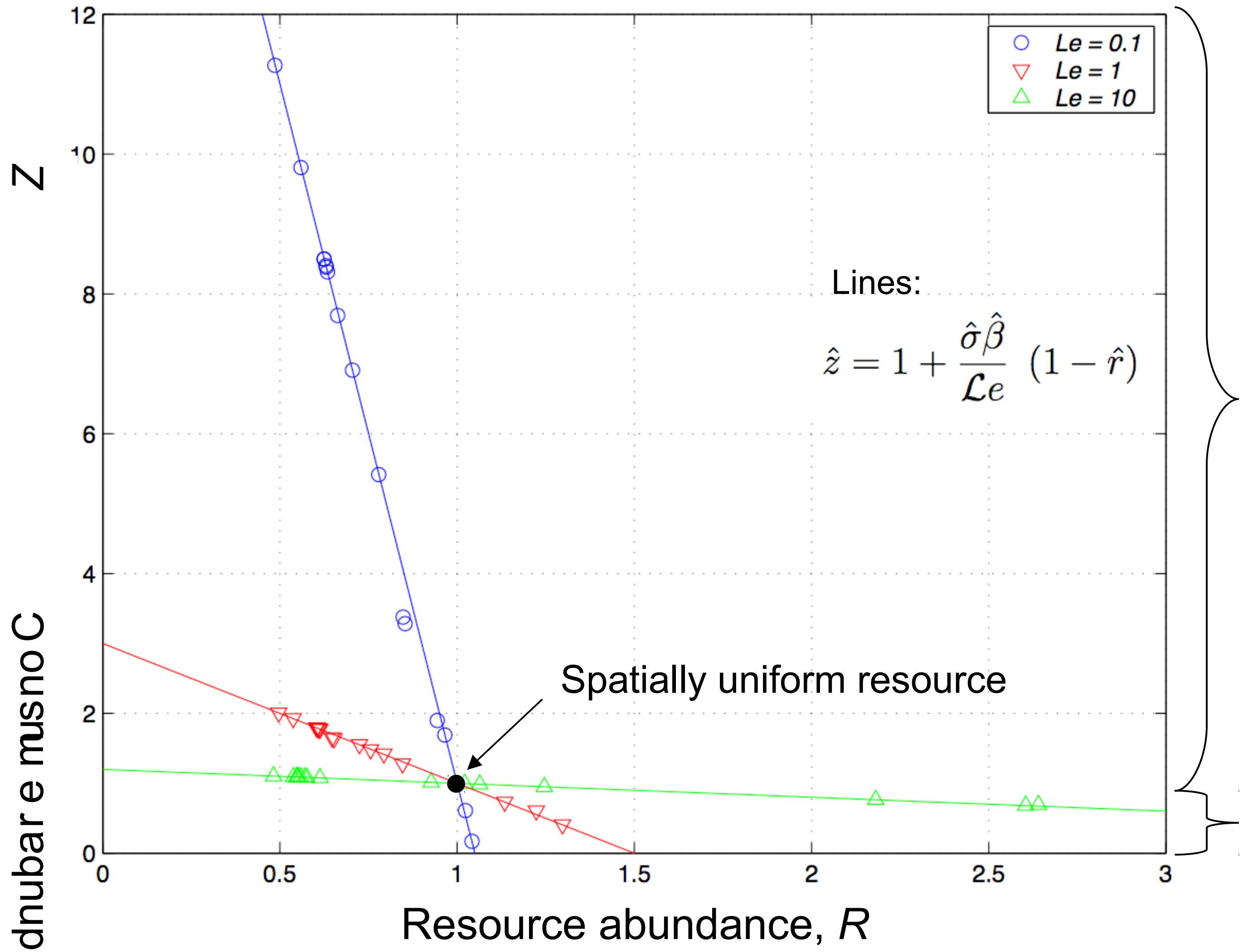




Lessard No. Le >> 1 (strong impacts of consumers on resources)

How valuable is a patchy resource?

Normalized, time-averaged total resource & consumer abundance



Patchiness is good for consumers and bad for resources.

Patchiness is bad for consumers and good for resources.

Statistically stationary results reflect enhanced or reduced <u>effective availability</u> of patchy resource distributions relative to uniform distributions of the same resources.