Stochastic methods in ecosystem modeling

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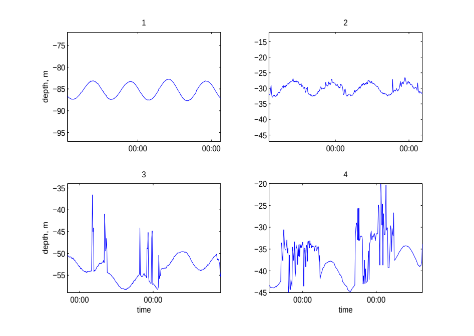
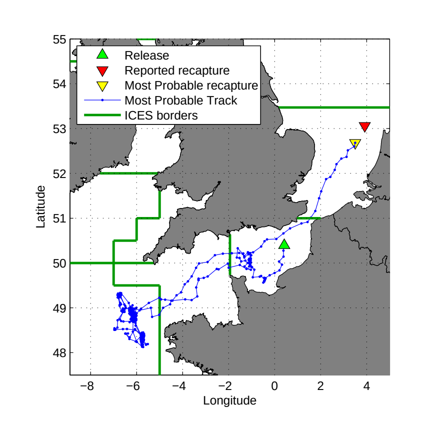
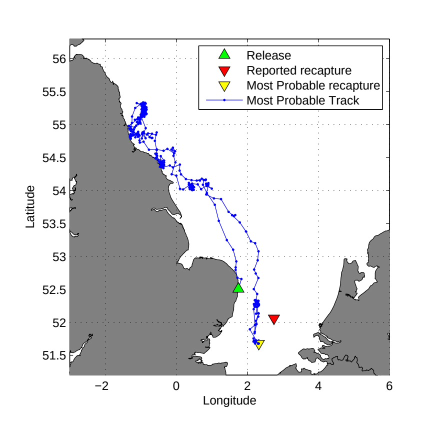
Geolocation:
Where did the fish go?



Hunter et al (2003), Metcalfe & Arnold (1997)

Reconstructed trajectories





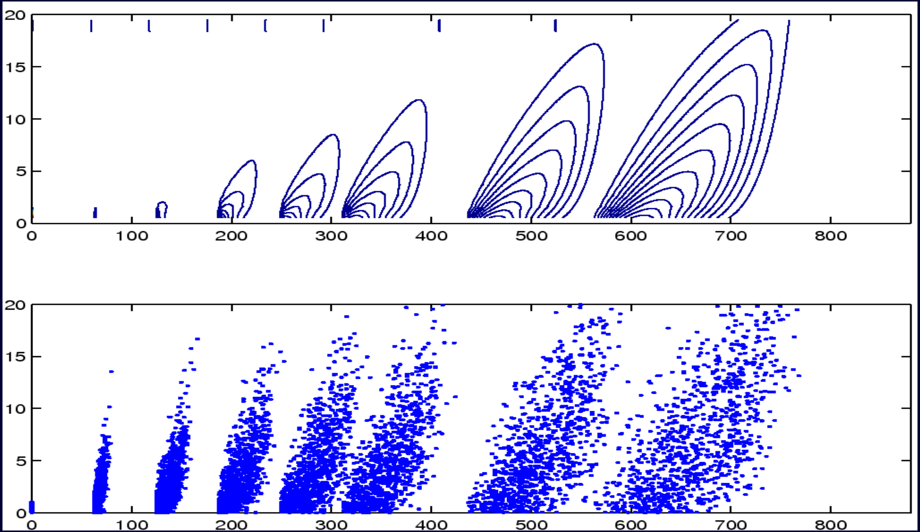
A Hidden Markov Model combing diffusive fish motion and noisy observations.

Pedersen et al (2008), Thygesen et al (2009)

Stochastics supports statistics

Stochastic fluctuations do not always just average out

Turbulence and eddy diffusion: Eulerian vs. Lagrangian models



Thygesen et al, 2007

Birth-death processes

Let the abundance N_t be a Markov process with transitions

$$n \to n + 1 \text{ with rate } \lambda n$$
$$n \to n - 1 \text{ with rate } \mu n$$

Then the *expected* abundance evolves according to

$$\frac{d}{dt}\mathbf{E}N_t = (\lambda - \mu)\mathbf{E}N_t$$

Density dependence

$$\frac{dx_t}{dt} = (\lambda - \mu_1 \cdot x_t)x_t$$

How will the birth-death process behave near the carrying capacity $K = \lambda/\mu_1$?

A diffusion approximation

Continuous, linearized, approximation when N_t is large:

$C(x, t)$ is the probability density of finding the system near state x at time t.

$$\frac{\partial C}{\partial t} = -\frac{\partial}{\partial x}(uC - \lambda \frac{\partial C}{\partial x})$$

with “advection field” $u(x) = \mu_1 \cdot (K - x)$.

Use the theory of noise propagation in linear systems!

Fluctuations and dissipation

The birth/death process will **fluctuate** around K with the **dissipative** time scale

$$\frac{1}{\lambda}$$

The variance of fluctuations is

$$\mathbf{V} N_t = \frac{\lambda}{\mu_1} = K$$

Square root scaling between abundance K and root mean square fluctuations.

For large populations K, fluctuations are relatively small.

Demographic noise: Probability of extinction

$$\mathbf{P}(N_\infty = 0 | N_0 = n) = \min\{1, \left(\frac{\mu}{\lambda}\right)^n\}$$

- $1/\mu$ is the *expected life span* of an individual.
- λ/μ is the *fitness F*: The expected number of offspring.

- If $F \leq 1$, then extinction is inevitable.
- If $F > 1$, then the probability of extinction is $1/F$.

Adaptive dynamics: Mutants arise randomly; birth-death processes determine if they go extinct.

Random Behavior

Animal behaviour is *unpredictable*:

- Unknown cues
- Unknown internal state
- Unknown behavioral strategy

- Sometimes it is *optimal* to be unpredictable.

The Hawk-Dove game

Two conspecifics are competing for a value V.
Each may behave agressively (hawk) or passively (dove).
If two hawks fight, the looser suffers a wound $W > V$.

Payoff matrix:

	H	**D**
H	$V/2 - W/2$	V
D	0	$V/2$

The optimal strategy (Nash, ESS) is to act randomly:

With probability V/W, be a hawk.

Maynard Smith (1982)

The adaptive dynamics

The population is characterized by a trait p: The probability of being a hawk.

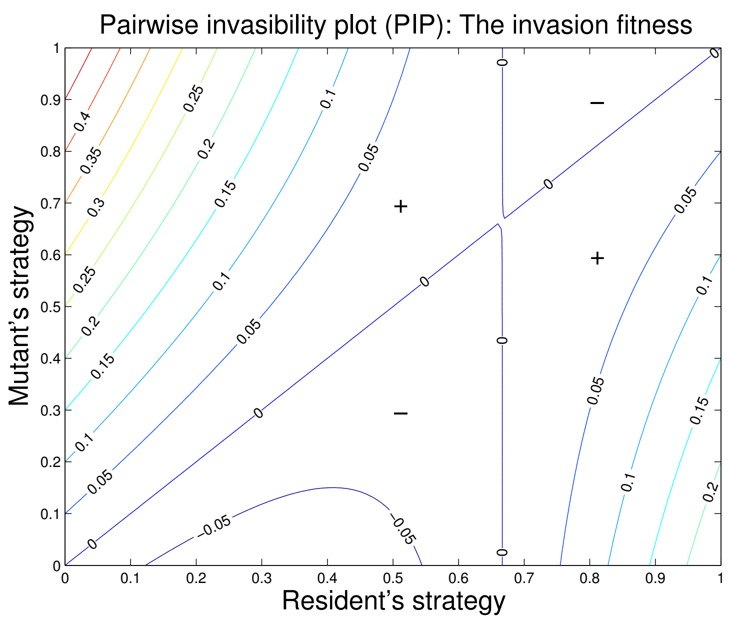
A mutant with trait q arises.

Does it invade?

Its *expected payoff* is

$$q(\frac{v}{2} - \frac{w}{2})p + qv(1-p) + (1-q)0p + (1-q)\frac{v}{2}(1-p)$$

Pairwise invasibility plot



Fluctuating environments

$$N_{t+1} = F_t \cdot N_t$$

where F_t is a stochastic process with mean f and variance σ^2.

$$\mathbf{V}\{N_t + 1 | N_t = n\} = \sigma^2 n^2$$

I.e., linear scaling between abundance and root mean square variance.

Note: Your expected number of descendants is $\langle F_t \rangle$ but the population will persist if $\langle \log F_t \rangle > 0$. If $\langle F_t \rangle = 1$, variance will kill you!

Brownian bugs

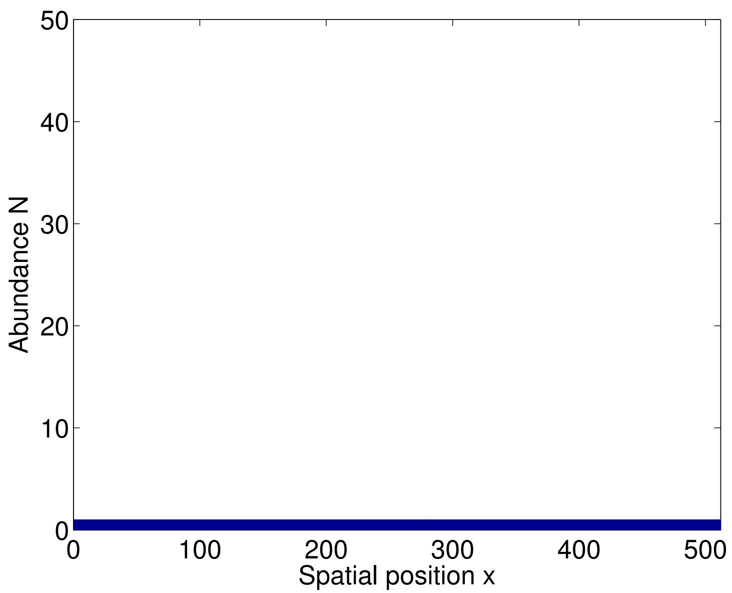
Given a collection of bugs distributed in space.
At each time step, let each bug:

- die
- clone
- move

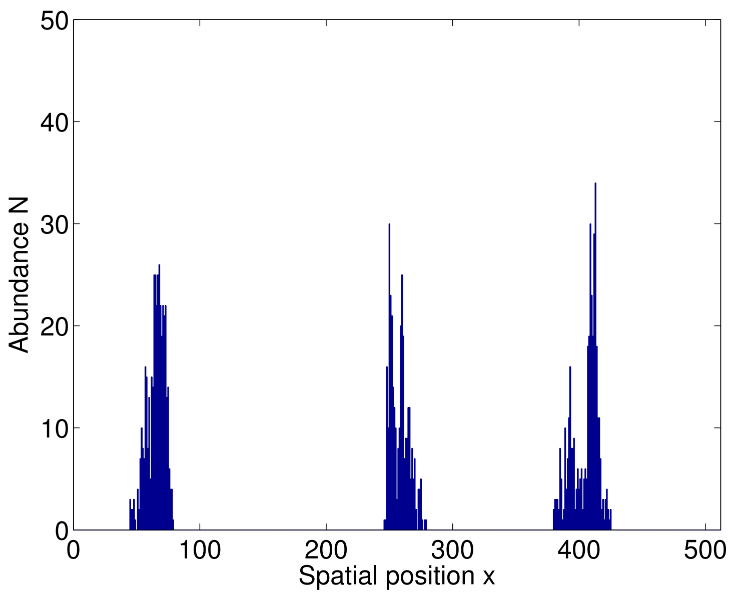
with a specified probability.

Young et al (2001)

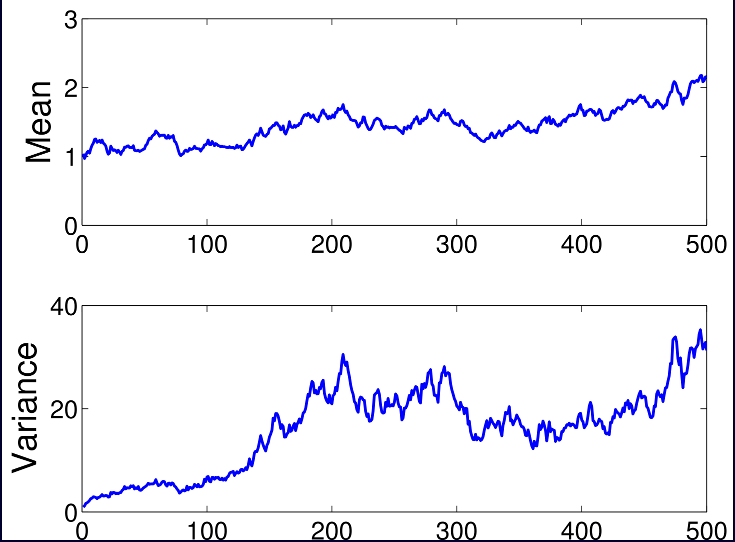
Initial distribution



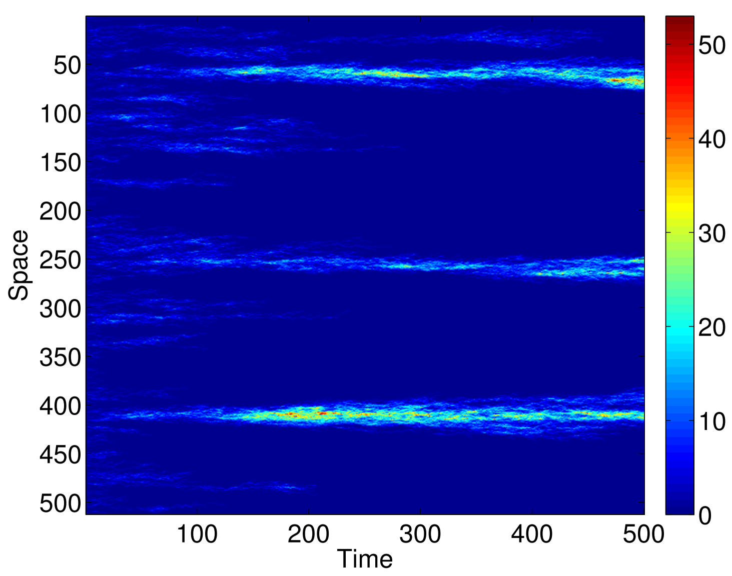
Final distribution



Temporal evolution of spatial statistics



Temporal evolution of spatial density



Conclusions

- Stochastic models supports statistic analysis.
- Individual-level processes are unpredictable
- ... and may be modeled with stochastic processes
- Diffusion approximations are everywhere!
- Unpredictability may be an advantage
- Density dependence dampens fluctuations
- ... but neutral situations are common