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The interaction of charged particles with atomic nuclei

Alexander GURBICH
*State Scientific Center; Instit. of Phys. & Power Eng.
Bondarenko Sq. 1
Kaluga Region, 249033
Obninsk
RUSSIA*



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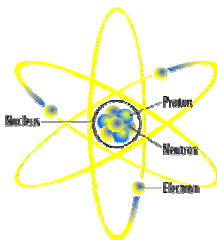


Institute of Physics and Power Engineering
Obninsk, Russia

PLAN OF THE LECTURE

- Nuclear physics fundamentals
- S-matrix formalism
- Projectile-nucleus interaction mechanisms
- Potential scattering
- Optical model
- Resonance interaction
- Compound nucleus
- R-matrix theory
- Proton induced reactions
- Deuteron Induced reactions

ATOMIC NUCLEUS



The atomic nucleus is the centre of an atom. The nucleus radius is much smaller than that of the atom. Thus, the nucleus occupies an extremely small volume inside the atom. Nuclei are composed of protons and neutrons. The number of protons in an atomic nucleus is called the atomic number, and determines which element the atom is. The number of neutrons determines the isotope of the element.

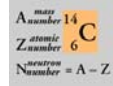
What else do we know about the nucleus?

In addition to its atomic number and mass number, a nucleus is also characterized by its size, shape, binding energy, angular momentum, and (if it is unstable) half-life. The nucleus is now understood to be a quantum system composed of protons and neutrons, particles of nearly equal mass and the same intrinsic angular momentum (spin) of $\frac{1}{2}$. The proton carries one unit of positive electric charge while the neutron has no electric charge. The binding energy of a nucleus is the energy holding a nucleus together.

A nucleus is identified by its atomic number Z (i.e., the number of protons), the neutron number, N , and the mass number, A , where $A = Z + N$.

Mass defect: $\Delta m = (Z m_p + N m_n) - m_X$

Binding energy: $E_B = \Delta m c^2$



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NUCLEAR REACTION

An example: $^{15}\text{N} + ^1\text{H} \rightarrow ^{12}\text{C} + \alpha + \gamma$ (4.965 MeV)

Conservation laws:

- Number of nucleons A
- Electric charge Z
- Energy
- Momentum
- Angular momentum

Application of the conservation of energy gives the Q value of the reaction

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The Q -value of the reaction

$$Q \equiv T_f - T_i$$

where T_i and T_f are the kinetic energies of the system in the initial and the final state, respectively

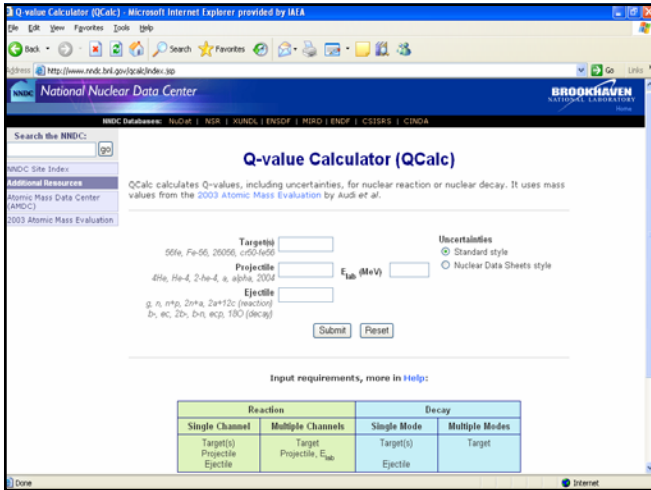
Q is the energy released by the reaction

If $Q > 0$, the reaction proceeds even if $T_i = 0$ (*exoergic reaction*)

If $Q < 0$, the reaction proceeds only if $T_i \geq |Q|$ (*endoergic reaction*)

$|Q|$ is the *threshold energy* of the reaction

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NUCLEAR AND ELECTRICAL FORCES

Nuclear forces

Nuclear forces are:

- Strong*
- Attractive*
- Short-range*
- Exchanged*
- Non-central*

Electrical forces

Can be represented as:

$$V_c(r) = \begin{cases} \frac{Zze^2}{r} & \text{for } r \geq R \\ \frac{Zze^2}{2R} \left(3 - \frac{r^2}{R^2} \right) & \text{for } r \leq R \end{cases}$$

$$R \approx (1.1 \div 1.5) \cdot A^{1/3} \text{ fm}$$

$$(1 \text{ fm} = 10^{-13} \text{ cm})$$

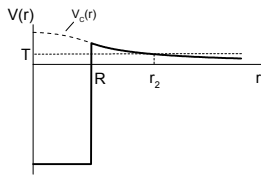
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COMPARISON OF THE FORCE FEATURES

Force properties	<u>Nuclear forces</u>	<u>Electrical forces</u>
Range	~1fm (10^{-13} cm)	Long
Electric charge	Non-sensitive	Sensitive
Saturation	Yes	No
Spin dependence	Yes	No

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NUCLEAR + COULOMB POTENTIAL



The Coulomb repulsion changes to (nuclear) attraction at the distance R .

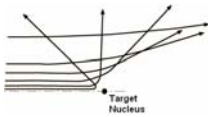
Nuclear potential is pictured in the form of a square potential well which is about 40+50 MeV deep.

Coulomb potential barrier height is $B_c = \frac{Zz e^2}{R}$

Transparency of the Coulomb barrier is $D \approx e^{-\frac{2}{\hbar} \int_{r_1}^{r_2} \sqrt{2\mu(V_c - T)} dr}$

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RUTHERFORD SCATTERING



Dependence of the force on the distance is known \Rightarrow analytical solution is possible

$$\tan \theta = \frac{2Zze^2}{mv^2 b}$$

$$d\sigma = \frac{dN}{N} = 2\pi b db$$

$$d\sigma = n \left(\frac{Zze^2}{mv^2} \right)^2 \frac{d\Omega}{4 \sin^4 \frac{\theta}{2}}$$

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Schrödinger equation

According to quantum mechanics a particle state is described by the wave function ψ , which is obtained as a solution of the wave equation.

For the case of elastic scattering of spinless non-identical particles the wave equation has a form of Schrödinger equation with a spherically symmetric potential $V(r)$

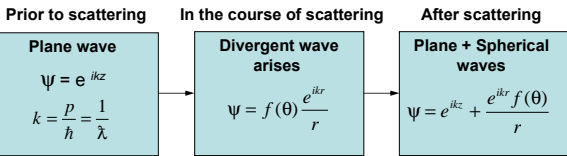
$$\Delta \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

where in spherical coordinates Laplace operator Δ is

$$\Delta = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \varphi^2}$$

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NUCLEAR POTENTIAL SCATTERING

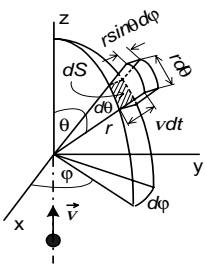


The angular distribution of the scattered particles is defined by the $f(\theta)$ function.

The differential cross-section is $\frac{d\sigma}{d\Omega} = |f(\theta)|^2$

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DEFINITION OF THE DIFFERENTIAL CROSS-SECTION



By definition the differential cross section $d\sigma$ is equal to the fraction dN/N of the projectiles scattered into the given solid angle $d\Omega$.

$$d\Omega = \sin\theta d\theta d\phi$$

Assuming a unity density of particles in the primary beam, the flux density is

$$N = v(\text{s}^{-1} \cdot \text{cm}^{-2}),$$

where v is the particle velocity. The number of particles dN traversing the surface element dS per time unit is determined by the probability of finding particles in the elementary volume

$$dV = v^2 \sin\theta d\theta d\phi,$$

and the probability density is given by the square of the modulus of the scattered wave function:

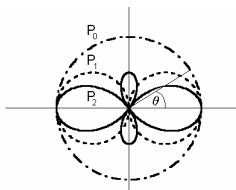
$$dN = \left| f(\theta) \frac{e^{ikr}}{r} \right|^2 v r^2 \sin\theta d\theta d\phi, \quad \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

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SOLUTION OF THE WAVE EQUATION

General solution is $\psi = \sum_{l=0}^{\infty} A_l P_l(\cos\theta) R_l(r)$,

where $R_l(r)$ is a radial wave function and $P_l(\cos\theta)$ are Legendre polynomials $P_0=1, P_1=\cos\theta, P_2=(3\cos^2\theta - 1)/2, \dots$



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RADIAL WAVE FUNCTION ASYMPTOTIC

$e^{-i\left(kr - l\frac{\pi}{2}\right)}$ – convergent partial spherical wave
 $e^{i\left(kr - l\frac{\pi}{2}\right)}$ – divergent partial spherical wave

Initial stage
 $R_{kl}(r) \sim e^{i\left(kr - l\frac{\pi}{2}\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)}$

Final stage
 $R_{kl}(r) \sim e^{i\left(kr - l\frac{\pi}{2} + 2\delta_l\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)} = S_l e^{i\left(kr - l\frac{\pi}{2}\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)}$
 $S_l = e^{2i\delta_l}$

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WAVE FUNCTIONS EXPANSION

Incident Wave:
$$e^{ikz} = \sum_{l=0}^{\infty} \frac{(2l+1) i^l}{2ikr} P_l(\cos\theta) \left[e^{i\left(kr - l\frac{\pi}{2}\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)} \right]$$

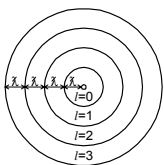
Incident + Divergent:
$$e^{ikz} + f(\theta) \frac{e^{ikr}}{r} = \sum_{l=0}^{\infty} \frac{(2l+1) i^l}{2ikr} P_l(\cos\theta) \left[S_l e^{i\left(kr - l\frac{\pi}{2}\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)} \right]$$

RELATION BETWEEN SCATTERING AMPLITUDE AND PHASES

$$f(\theta) = \frac{1}{2ik} \sum_l (2l+1) (e^{2i\delta_l} - 1) P_l(\cos\theta).$$

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PARTIAL WAVES



Suppose the projectile possesses kinetic momentum p and angular momentum l .

Then from comparison between classical and quantum mechanical relations for the modulus of the angular momentum

$$|\vec{l}| = p\rho = \hbar\sqrt{l(l+1)}$$

follows that

$$\rho = \frac{\hbar}{p} \sqrt{l(l+1)} = \lambda \sqrt{l(l+1)}$$

The initial beam behaves as if it were subdivided in cylindrical zones

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TAKING PROJECTILE CHARGE AND SPIN INTO ACCOUNT

IF PROJECTILE IS CHARGED THEN

$$f(\theta) = f_c(\theta) + \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1)(S_l - 1)e^{2i\sigma_l} P_l(\cos \theta)$$

IF PROJECTILE HAS SPIN $\frac{1}{2}$ THEN

$$d\sigma / d\Omega = |A(\theta)|^2 + |B(\theta)|^2,$$

$$A(\theta) = f_c(\theta) + \frac{1}{2ik} \sum_{l=1}^{\infty} [(l+1)S_l^+ + lS_l^- - (2l+1)] \exp(2i\sigma_l) P_l(\cos \theta);$$

$$B(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (S_l^+ - S_l^-) \exp(2i\sigma_l) P_l(\cos \theta)$$

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OPTICAL MODEL

- In the so-called optical model nucleus is represented by means of a complex potential. The interaction of the projectile with the nucleus is then reduced to de-Broglie's wave refraction and absorption by a opaque sphere. The name of the model originates from the formal analogy with the light plane wave passing through a semitransparent sphere.
- As well as refraction and absorption of the light is described by a complex index
- $n = n_r + ik_a$
- the complex potential of the form
- $U = V + iW$
- is used to take into account scattering and absorption of the projectile by the nucleus. The real part of the potential is responsible for scattering whereas the imaginary part stands for absorption.

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OPTICAL MODEL POTENTIAL

Complex potential:

$$U = V + iW$$

The standard form of the potential:

$$U(r) = U_c(r) + U_R(r) + iU_I(r) + U_{\omega}(r)$$

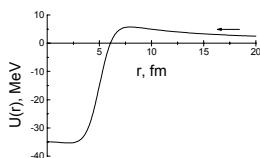
$$U_R(r) = -V_R f_R(r)$$

$$U_I(r) = 4aW_D \frac{df_I(r)}{dr}$$

$$U_{\omega} = \left(\frac{\hbar}{m_p c} \right)^2 V_{\omega} \frac{1}{r} \frac{df_{\omega}}{dr} l \cdot s$$

$$f_R(r) = \left[1 + \exp\left(\frac{r-R_1}{a_1} \right) \right]^{-1}$$

$$R_1 = r_1 A^{1/3}$$



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OPTICAL POTENTIAL PARAMETERS

Low energy peculiarities:

- The strength parameters often have strong energy dependence in the vicinity of the Coulomb barrier.
- The real potential radial dependence is of more complicated than Saxon-Woods form.
- The imaginary part of potential reveals non-systematic dependence on nucleus mass number.
- The imaginary part of potential is close to zero for light nuclei.
- Absorption is peaked at the nucleus surface.
- The radius of the imaginary potential diminishes with decreasing energy while its diffuseness increases.

Conclusion:

Global sets of parameters are inapplicable at low energy!

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Modification of Saxon-Woods form-factor

For deformed nucleus channel coupling is essential. A simple way to take it into account is the modification of the real part of the optical potential by adding a surface term to the Saxon-Woods potential:

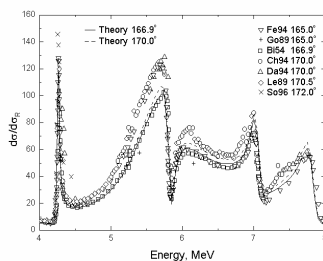
$$U_R(r) = -V_R f(r, r_R, a_R) + 4a_S V_S \frac{d}{dr} f(r, r_S, a_S)$$

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Splitting V_R to take account of exchange mechanism in the case of complex particles scattering

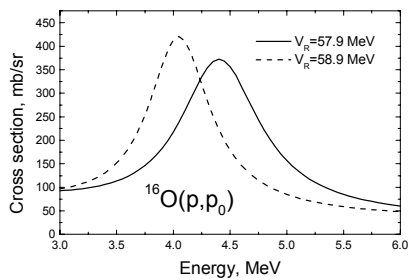
The optimal potential parameters obtained for $^{12}\text{C}(^4\text{He}, ^4\text{He})^{12}\text{C}$ scattering

V_{R0} MeV	V_{R1} MeV	V_{R2} MeV	V_{R3} MeV	V_{R4} MeV	V_{R5} MeV	r_R fm	a_R fm	r_C fm
47.89	50.10	46.30	47.70	42.90	47.00	1.43	0.70	1.45



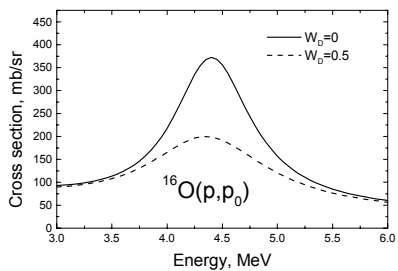
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Shape resonance dependence on V_R



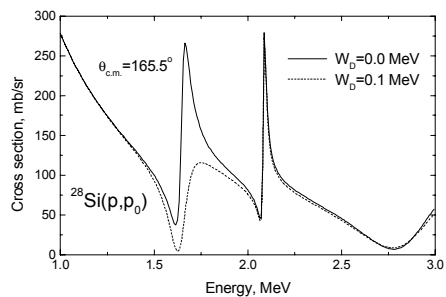
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Shape resonance dependence on W_D



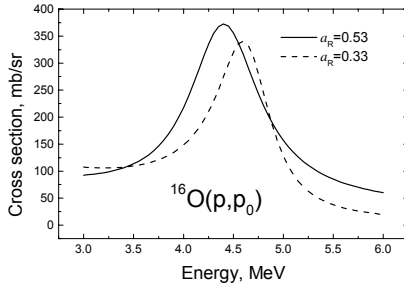
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Shape resonance dependence on W_D



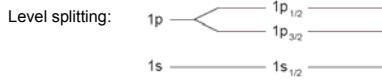
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Shape resonance dependence on a_R

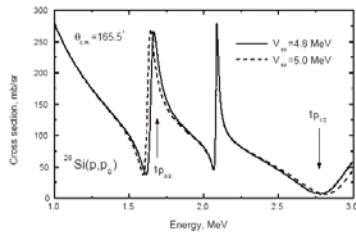


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SPIN-ORBIT INTERACTION



Resonances in proton elastic scattering:

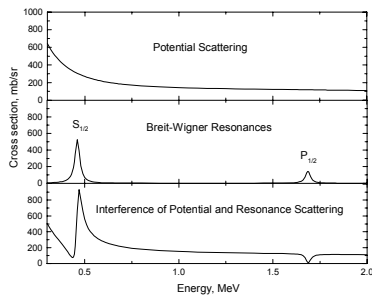


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RESONANCE SCATTERING

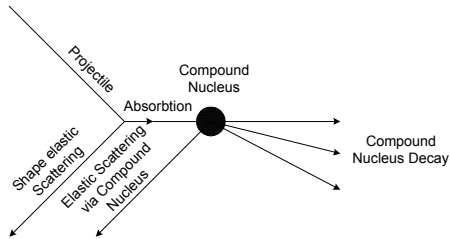
Modification of the scattering matrix element:

$$S_l^\pm = \exp(2i\lambda_l^\pm) \left[\exp(-2i\mu_l^\pm) + \exp(2i\phi_p) \frac{i\Gamma_p}{E_0 - E - \frac{1}{2}i\Gamma} \right]$$



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TYPES OF THE NUCLEAR INTERACTION MECHANISMS AT LOW ENERGIES



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DIFFERENT TYPES OF THE PROJECTILE – NUCLEUS INTERACTION

Elastic scattering: Outgoing particle = Ingoing particle

Outgoing particle energy = Ingoing particle energy

Direct reaction: Energy of the projectile is transferred to one nucleon or to a small group of nucleons.

Outgoing particle – any particle allowed by the conservation laws.

The interaction time is $\sim 10^{-22}$ s

Compound nucleus reaction:

Energy of the projectile is transferred to all the nucleons.

When particle is emitted the residual nucleus may stay both in ground state and in excited one.

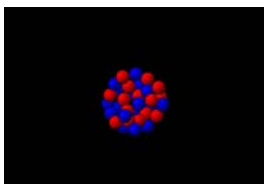
The interaction time is $\sim 10^{-15}$ s.

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SCATTERING VIA COMPOUND NUCLEUS

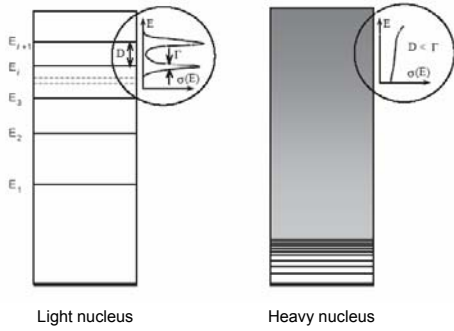
An example: the (d,p)-reaction. The time the compound nucleus stays in an excited state is long as compared with the time needed for the projectile to traverse the domain occupied by the nucleus.

The reaction proceeds in two stages – the compound nucleus is created and in a while it decays.



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NUCLEAR LEVELS DIAGRAM



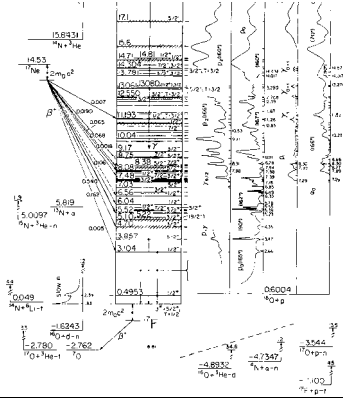
Light nucleus

Heavy nucleus

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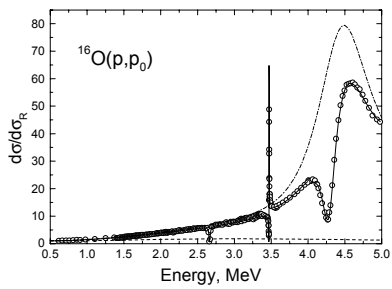
AJZENBERG-SELOVE'S COMPILATION

www.tunl.duke.edu/nucldata/fas/fas_list.shtml



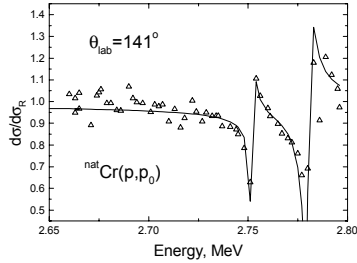
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EFFECT OF SHAPE RESONANCE ON THE ELASTIC SCATTERING OF PROTONS FROM ^{16}O



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Isobaric Analog Resonances in the excitation function for proton elastic scattering



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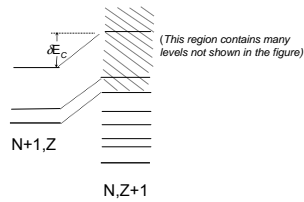
ISOBARIC ANALOG STATES

Isobare: Nuclide with the same mass (A) but different charge (Z)

Except for electrical forces the n-n, n-p, and p-p interactions are similar

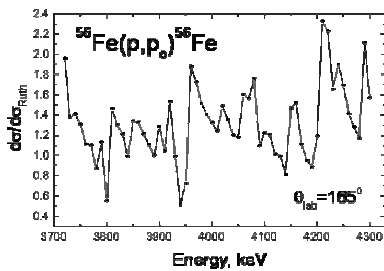
The energy levels of isobaric (equal A) nuclei are relatively insensitive toward the interchange of a proton and a neutron.

The isobaric analog state will have the same properties, but will have a higher energy, δE_C , because of the additional Coulomb energy associated with the extra proton, less the neutron-proton mass difference.



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Ericson's fluctuations in the excitation function for proton elastic scattering from ^{56}Fe



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R-MATRIX THEORY

If a wave-function and its derivative are known at the boundary of the nucleus it can be found everywhere outside the nucleus

The S matrix is expressed through R-matrix which is defined to connect values u_l with its derivative at the nucleus boundary $u_l(a) = R_l a \left(\frac{du_l}{dr} \right)_{r=a}$

R_l can be expressed as $R_l = \sum_{\lambda} \frac{\gamma_{l,\lambda}^2}{E_{\lambda} - E}$

where $\gamma_{l,\lambda} = \left(\frac{\hbar}{2ma} \right)^{1/2} u_{l,\lambda}(a)$

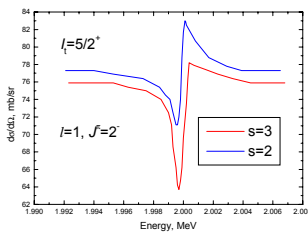
The functions $u_{l,\lambda}$ correspond to actual states E_{λ} of the nucleus

The quantities $\gamma_{l,\lambda}$ are connected with energy width of states

The cross-section can be written in terms of the R-matrix

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THE EFFECT OF THE CHANNEL SPIN ON A RESONANCE SHAPE



For a target of spin I_t and projectile of spin I_p the two spins are coupled to form a channel spin s . This channel spin is then combined with the relative orbital angular momentum l to form the spin of the compound nuclear state J .

Allowed combinations of quantum numbers for elastic proton scattering through the $J = 2^-$ level of the target with $I_t = 5/2^+$

l	s
1	2
1	3
3	2
3	3

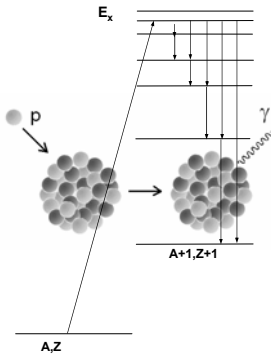
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PROTON INDUCED REACTIONS

- (p,α)-reaction
 - are mainly exoergic (Q~1-3 MeV)
 - the cross section is not large (because of high Coulomb barrier for α-particle)
- (p,n)-reaction
 - are always endoergic (Q < -0.8 MeV)
- (p,γ)-reaction
 - γ-rays of different energy are emitted corresponding to transitions on different levels of the residual nucleus
- (p,p'γ)-reaction
 - is only possible if the projectile energy exceeds the first level energy in the target nucleus

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RADIATIVE CAPTURE OF PROTON

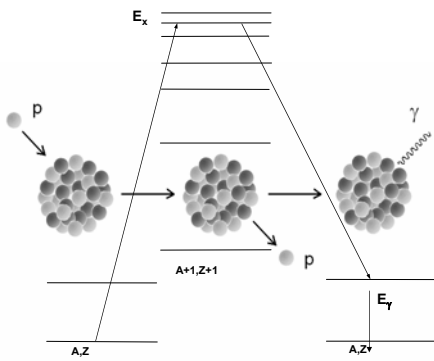


$$E_x = Q + \frac{M_A}{M_A + m_p} E_p$$

$$E_\gamma^{(i)} = E_x - E_{level}^{(i)}$$

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Scheme of the (p,p'γ)-reaction



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DEUTERON INDUCED REACTIONS

Three mechanisms:

- Direct stripping (with amplitude of D)
- Resonant mechanism (with amplitude of R)
- Compound nucleus mechanism (incoherent contribution)

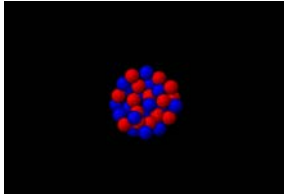
The total amplitude of the process is $D + R$

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DEUTERON INDUCED REACTIONS (STRIPPING)

For deuteron the p-n distance is $\sim 4 \cdot 10^{-13}$ cm (for the rest of nuclei $\sim 2 \cdot 10^{-13}$ cm)

Due to electrical forces deuteron is oriented in such a way that proton is farther from the nucleus than neutron. Because deuteron binding energy is small, neutron may be absorbed by nucleus, while proton keeps moving



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DEUTERON INDUCED REACTIONS (VIA COMPOUND NUCLEUS)

• Binding energy for deuteron is very low:

$E_B \approx 2.2$ MeV \rightarrow 1 MeV/Nucleon (For the rest of nuclei ~ 8 MeV/Nucleon)

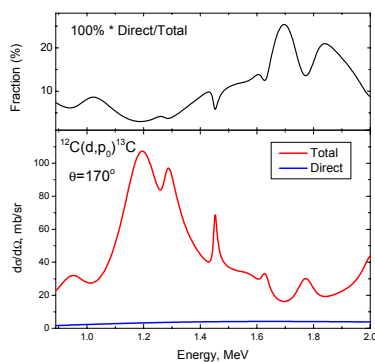
• Since the binding energy of deuteron in a compound nucleus is

$E_B(A,Z) - E_B(A-2,Z-1) - E_B(d) \approx 8A - 8(A-2) - 2,2 \approx 14$ MeV

the excitation energy of the compound nucleus is of order $T+14$ MeV and all the reactions (d, p), (d, n), (d, α) are possible and are highly exoergic.

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Contribution of Direct Reaction Mechanism



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(p,α)-reactions

- The (p,α)-reactions are mainly exoergic (Q~1-3 MeV).
- Because of the high Coulomb barrier for α-particles the cross-section is not large
- For the (p,α)-reactions which lead to the excited states in the residual nucleus the cross-section for low energy protons is as a rule negligible.

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Alphas as projectiles

- The gamma-emission mechanism resembles that for protons.
- The alpha-particle is a strongly bound system the reactions with nucleons in the exit channel are endoergic.
- The compound nucleus produced as a result of the capture of a low energy alpha particle can decay mainly back to the elastic channel or by emitting gammas.
- A threshold for the (α,p)-reaction is usually greater than 1 MeV and due to the Coulomb barrier the cross section for (α,p)-reaction is as a rule small.
- Different mechanisms contribute to the alpha elastic scattering cross section beyond the energy region where it follows the Rutherford law. These are direct (shape elastic) scattering, compound elastic scattering, resonance scattering of a different origin, and exchange processes which consist in exchange of nucleons between alpha-particle and target nucleus in the course of scattering.

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Laboratory and centre of mass frames

The laboratory frame of reference is a frame where detector is placed. In IBA experiments target nucleus is always in rest in this frame.

For a projectile of the mass M_1 moving along an x axis towards a target nucleus of the mass M_2 the point with a coordinate x_c defined as

$$x_c = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2}$$

where x_1 and x_2 are the projectile and the target coordinates respectively is a centre of mass (CM) of the system comprised of these two particles.

This point moves in the laboratory reference frame with the velocity

$$\vec{V}_c = \frac{M_1 \vec{v}_1}{M_1 + M_2}$$

where \vec{v}_1 is the projectile velocity. The centre of mass reference frame is ₅₁ defined as a frame with an origin fixed in the point x_c .

König's theorem

The kinetic energy of a system consisted of a projectile and a target is the kinetic energy associated to the movement of the center of mass and the kinetic energy associated to the movement of the particles relative to the center of mass.

For the projectile possessing the kinetic energy E_1 (in the laboratory frame) this means that

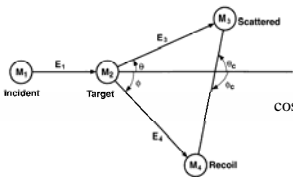
$$E_1 = \frac{(M_1 + M_2)V_C^2}{2} + E_{rel}$$

where E_{rel} is the kinetic energy of colliding particles in their relative motion in the CM system:

$$E_{rel} = \frac{M_2}{M_1 + M_2} E_1$$

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Kinematics of elastic scattering



$$\tan \theta_{lab} = \frac{\sin \theta_{cm}}{\gamma_3 + \cos \theta_{cm}}$$

$$\cos \theta_{cm} = [\cos^2 \theta_{lab} (1 - \gamma_3^2 \sin^2 \theta_{lab})]^{1/2} - \gamma_3 \sin^2 \theta_{lab}$$

$$\gamma_3 = \left(\frac{M_1 M_3}{M_2 M_4} \right)^{1/2} \left(1 + \frac{M_1 + M_2}{M_2} \frac{Q}{E_1} \right)^{-1/2}$$

$$K = \frac{E_3}{E_1} = \frac{\left\{ (M_1 / M_2) \cos(\theta) + [1 - (M_1 / M_2)^2 \sin^2(\theta)]^{1/2} \right\}^2}{(1 + M_1 / M_2)^2}$$

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Cross-section lab-c.m. transformation

The relation between the differential cross-sections expressed in the CM and laboratory frames is derived from the equality of the number of particles emitted in the corresponding solid angles in the two frames:

$$\left. \frac{d\sigma}{d\Omega} \right|_{lab} d\Omega_{lab} = \left. \frac{d\sigma}{d\Omega} \right|_{cm} d\Omega_{cm}$$

$$\left. \frac{d\sigma}{d\Omega} \right|_{lab} \sin \theta_{lab} d\theta_{lab} d\varphi_{lab} = \left. \frac{d\sigma}{d\Omega} \right|_{cm} \sin \theta_{cm} d\theta_{cm} d\varphi_{cm}$$

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Nuclear physics Internet resources relevant to IBA

The resource address	Description
http://www.nndc.bnl.gov/qcalc/	Q-value Calculator
http://nucleardata.nuclear.lu.se/database/masses/	Nuclear structure and decay data NuBase with the Q-value calculator
http://www.nndc.bnl.gov/ensdf/	Evaluated nuclear structure data file (ENSDF)
http://www.nndc.bnl.gov/nudat2/	Nuclear structure and decay data
http://www.nndc.bnl.gov/masses/mass.mas03	Atomic mass adjustment 2003
http://www.tunl.duke.edu/NuclData/	Energy levels of light nuclei, $A=3-20$
http://www-nds.iaea.org/ibandl/	IBA nuclear data library (IBANDL)
http://www-nds.iaea.org/exfor/	Experimental nuclear reaction data (EXFOR)
http://www-nds.iaea.org/sigmacalc/	Evaluated differential cross sections for IBA (SigmaCalc)

Summary

We have discussed:

- What nucleus is.**
- How a projectile interacts with a nucleus.**
- How quantum mechanics depicts the projectile-nucleus interaction.**
- What mechanisms of nuclear reactions exist.**
- How theoretical models are applied to describe the projectile-nucleus interaction.**
- Nuclear reaction kinematics**
