The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
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IAEA
International Atomic Energy Agency

2177-15

ICTP Latin-American Basic Course on FPGA Design for Scientific Instrumentation

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Digital arithmetic I
(number representations)

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Outline

- Digital CMOS design
  - Boolean algebra
  - Basic digital CMOS gates
  - Combinational and sequential circuits
  - Coding - Representation of numbers

CMOS Circuits

How can I design a digital circuit ?

- A set of gates (cell library)

  combinational gates

  memory elements

CMOS Circuits

How can I design a digital circuit ?

- Method to design combinational circuits

  Karnaugh table (local optimization)

  no method for global optimization

  ⇒ synthesis tools

CMOS Circuits

How can I design a digital circuit ?

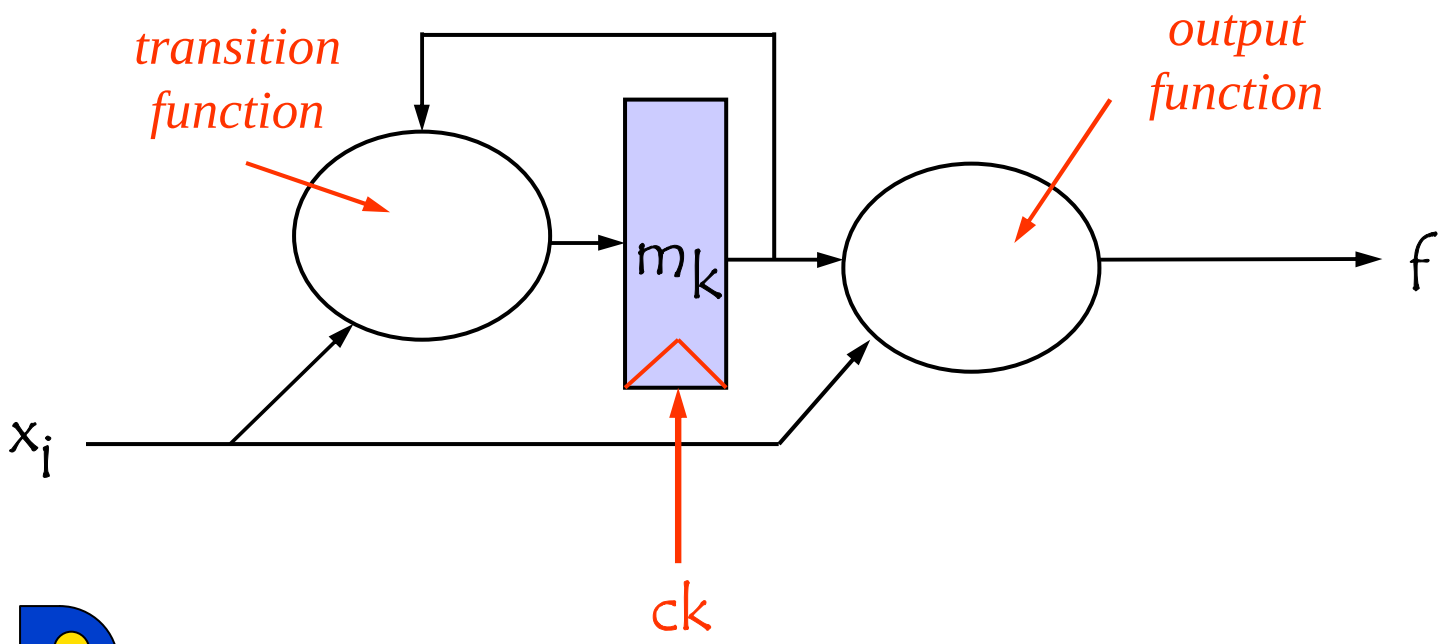
- Specify the circuit

  Combinational circuit : Boolean functions

  Sequential circuit ?

CMOS Circuits

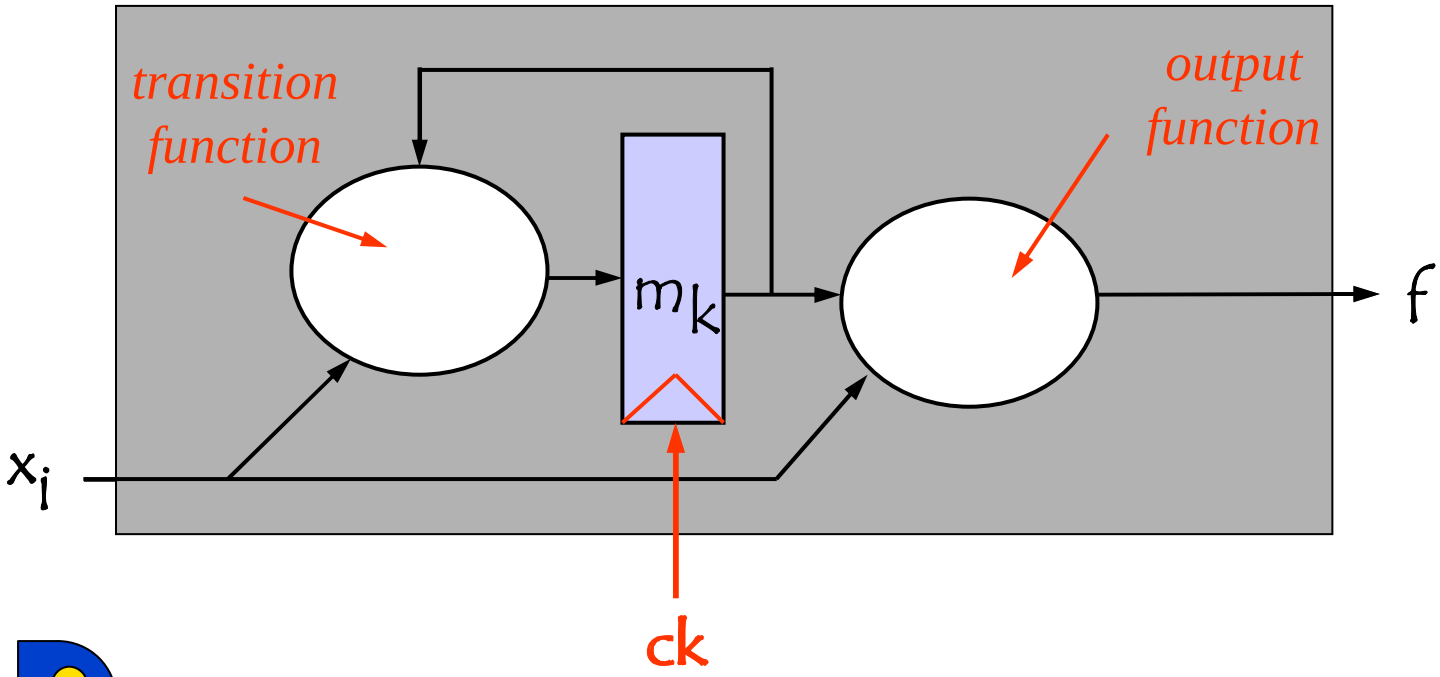
- Sequential Circuit



CMOS Circuits

How can I design a sequential circuit ?

I need a method to go from a black box to a white box



CMOS Circuits

How can I design a sequential circuit ?

The starting point (what I know)

What the circuit is supposed to do

Capture this knowledge into a representation that can be transposed into Boolean functions

The aim

How it will do it

number of required memory elements

CMOS Circuits

Representation of a sequential circuit ?

- State graph

G = (X, Y, S, T, O)

- X : input variables
- Y : output variables
- S : states
- T : transitions
- O : output set conditions

CMOS Circuits

- $G = (X, Y, S, T, O)$

  T : set of transitions

  $t \in T, \quad t = (s, s', f)$

  $s \in S$: source state

  $s' \in S$: target state

  $f \in \mathbf{B}_n$: transition condition

CMOS Circuits

- **G** = (X, Y, S, T, O)

  O : output set conditions

  $o \in O, \quad o = (y, s, c)$

  $y \in Y$: output variable

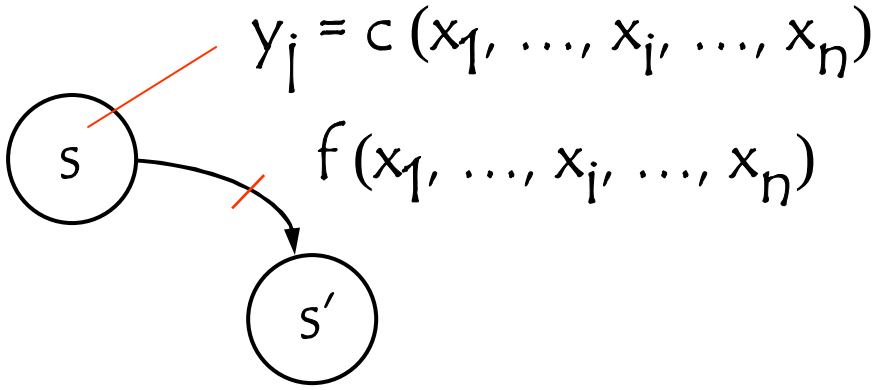
  $s \in S$: state

  $c \in \mathbf{B}_n$: output set condition

CMOS Circuits

- Let consider a graph **G** = (X, Y, S, T, O)

Graphic representation of **G**



CMOS Circuits

Representation of a sequential circuit ?

- State graph

Transpose into a graphic representation the expected behavior of a sequential system

$$G = (X, Y, S, T, O)$$

CMOS Circuits

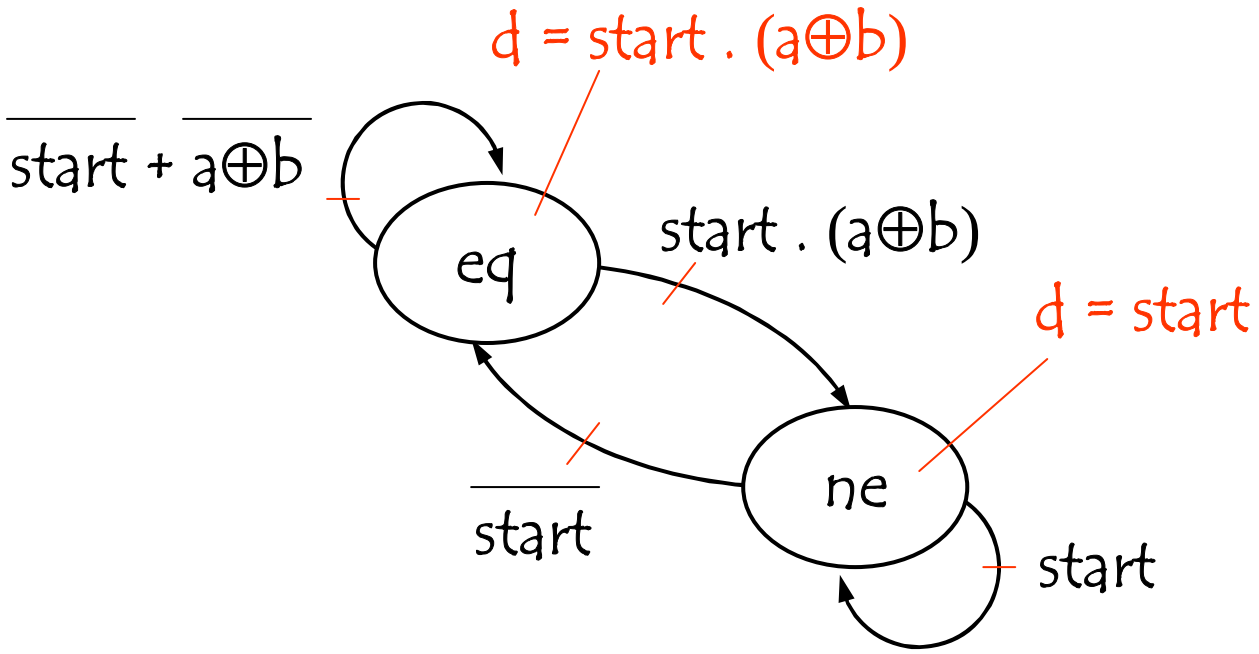
Example

Two signals a, b each transmitting a series of bits (1 bit at a cycle)

Design a system that sets a flag d if the value transmitted by a is different than b

CMOS Circuits

- Example



CMOS Circuits

Representation of a sequential circuit ?

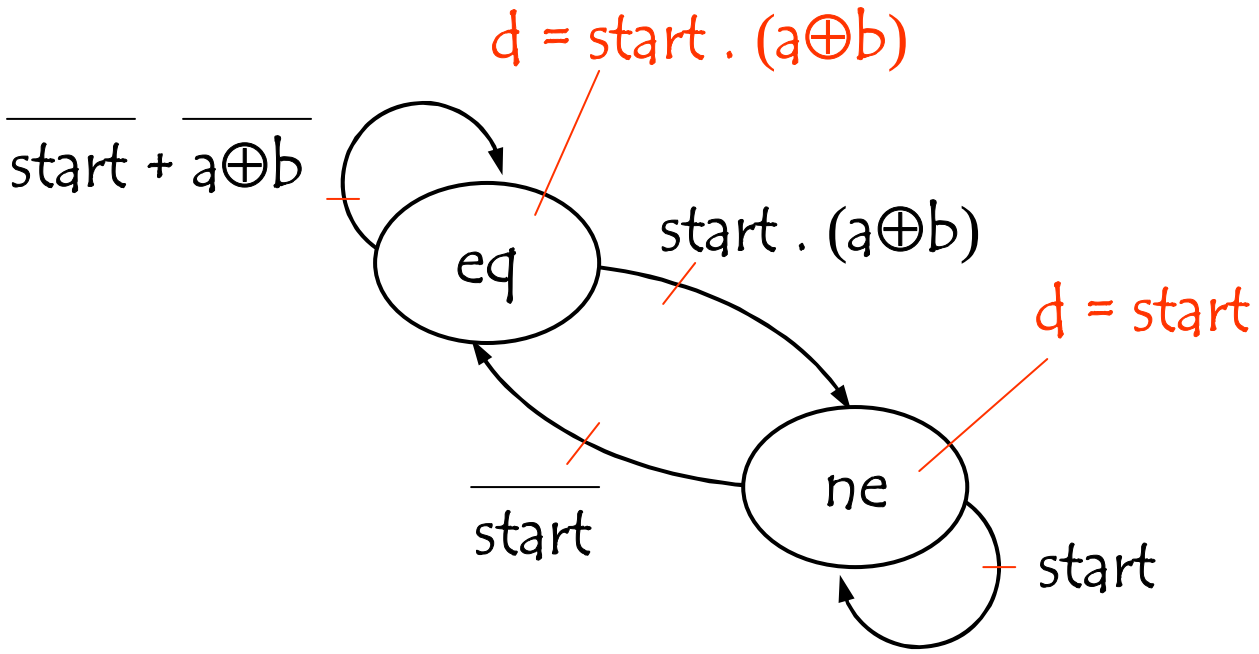
- State graph

$$\mathbf{G} = (X, Y, S, T, O)$$

define the number of memory elements required to represent $S : M$

CMOS Circuits

- Example



2 states ⟶ 1 memory element

CMOS Circuits

Representation of a sequential circuit ?

- State graph

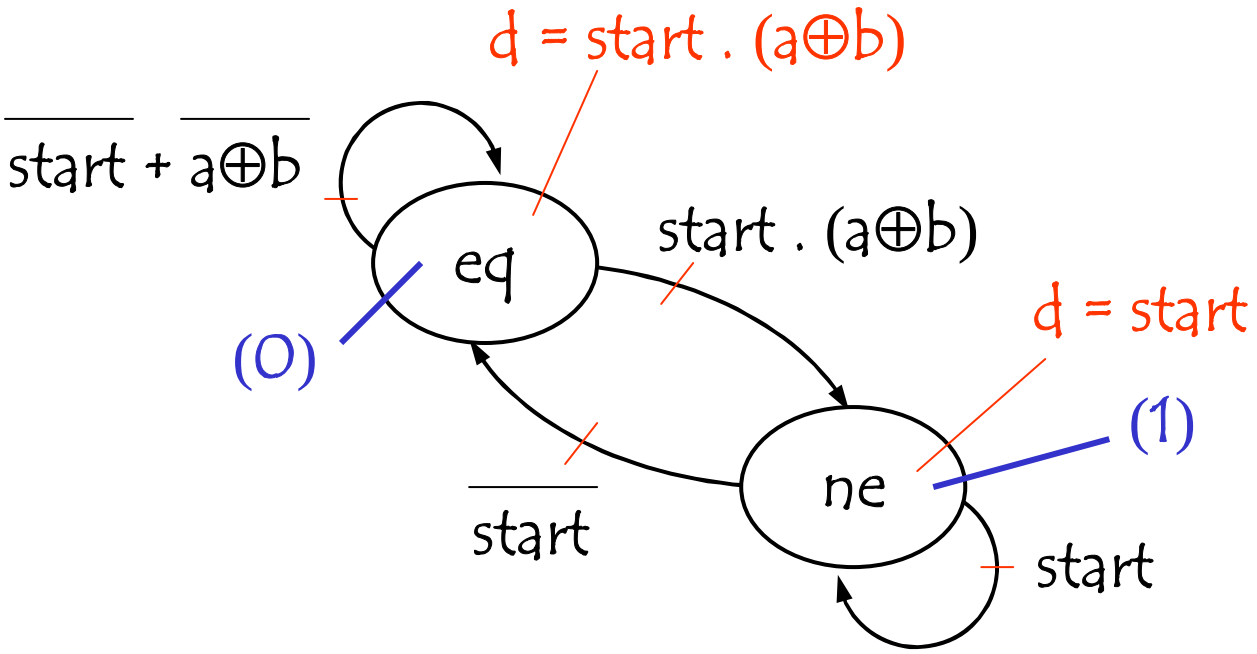
$$\mathbf{G} = (X, Y, S, T, O)$$

define the number of memory elements required to represent $S : M$

represent each state s by a vector of M

CMOS Circuits

- Example



2 states ⟶ 1 memory element

CMOS Circuits

Representation of a sequential circuit ?

- State graph

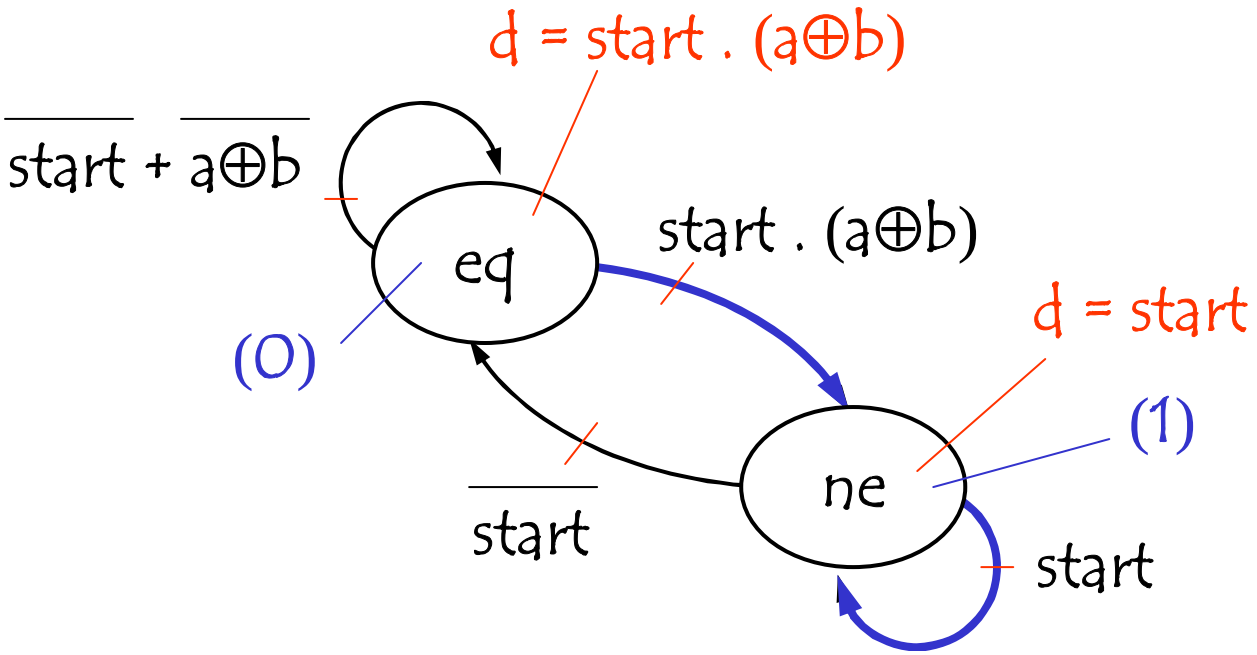
$$G = (X, Y, S, T, O)$$

define the *transition function* : m_k

m_k = sum of the Boolean function of the transitions that have as target a state where $m_k=1$

CMOS Circuits

Example



$$m_1 = \overline{m_1}.start.(a \oplus b) + m_1.start$$

CMOS Circuits

Representation of a sequential circuit ?

- State graph

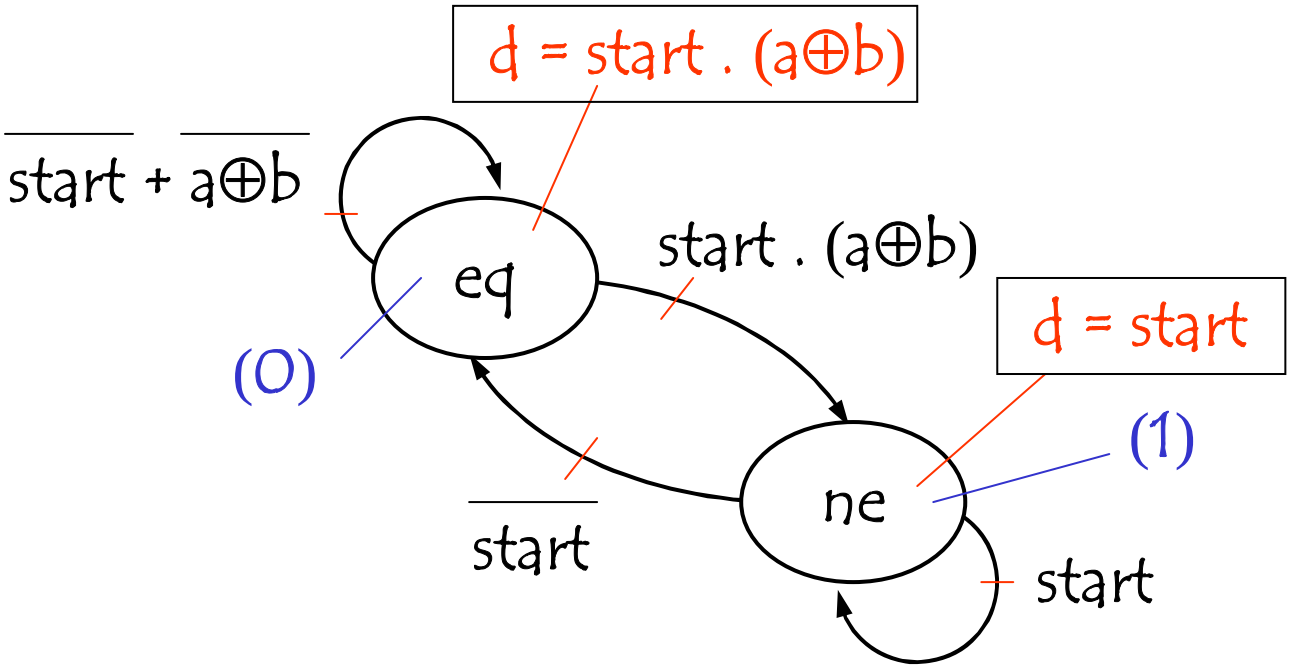
$$\mathbf{G} = (X, Y, S, T, O)$$

define the *output function* : y_i

y_i = sum of the output conditions concerning y_i

CMOS Circuits

Example



$$d = \overline{m_1}.start.(a \oplus b) + m_1.start$$

CMOS Circuits

Representation of a sequential circuit ?

- State graph

$$\mathbf{G} = (X, Y, S, T, O)$$

implement the Boolean functions : m_k , y_j

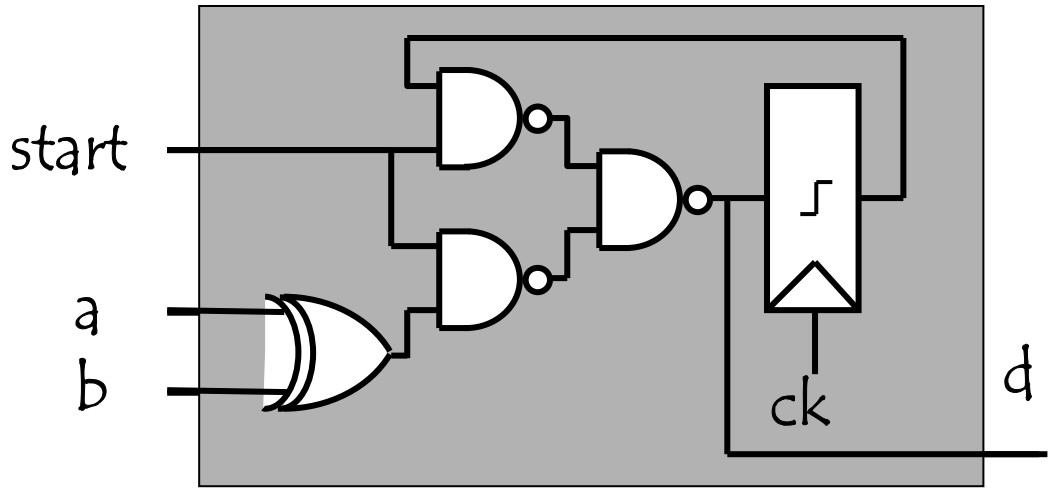
CMOS Circuits

- Example

$$m_1 = \overline{m_1}.start.(a \oplus b) + m_1.start$$

$$d = \overline{m_1}.start.(a \oplus b) + m_1.start$$

$$m_1 = start\ .(a \oplus b) + start.m_1$$



CMOS Circuits

How can I design a sequential circuit (summary) ?

- **G** = (X, Y, S, T,
  - 
    - graphic representation of the behavior
    - define the number of memory elements required to represent S : M
    - represent each state by a vector of M
    - define the *transition function*
    - define the *output function*
    - implement the Boolean functions